ON A ROBUSTNESS OF REDUCED ORDER MODELS BY STATE OPTIMIZATION APPROACH

Nguyen Thuy Anh

Hanoi university of Technology (HUT)

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ABSTRACT

State optimization approach has been proposed to treating various different system problems in optimal projection equations (OPEQ). While OPEQ for problems of open-loop thinking is found consisting of two modified Lyapunov equations, excepting conditions for the rank of measurements matrices whereas required in system identification problems, the one for closed-loop thinking consists of two modified Riccatti or Lyapunov equations excepting conditions for compensating system happened to be in a problem like that of order reduction for controller. Apart from addditonally constrained-conditions and simplicity in the solution form have been obtainable for each problem, it has been found the system problems switching over to computing the solution of OPEQ and the physical nature of medeled states possibly retaining in optimal order reduction problem.

On adopting the state optimization approach to a robustness of reduced order for a nonlinear series-based expressible uncertain model to enjoy the above mentioned advantages is reported in this paper. Necessary conditions for the robustness obtain from those for uncertain model to be the one of stability, joint controllability and observability characteristcs. Sufficient ones for reduced order by state optimization to be applicable for uncertainty of quasilinear model are reported next. Robustness of reduced order interpreting in terms of a concave optimization problem with different initial conditions, bounds and limits are also reported.

1. INTRODUCTION

Reduced order model has been largely accepted to be the first useful sight for System analysis and design and the problem of order reduction for model has been tackled by various different techniques in the last four decades [1]. However, if the discussion is limited to linear models described in the state space equations, the order reduction problem may be regarded to belong to either open- or closed-loop thinking of treatements [2 - 4]. Among the myriad references available in literature, two notable methodology contributions related with this paper are from internally system-theoretic argument and treatment in optimal projection equations (OPEQ).

Internal system philosophy based on the contribution of dynamical elements (state variables) to the system input/output relationship has been originated firstly to so-called singular values by Moore in 1981 [5] for an open-loop thinking system and further developed to characteristic values for a closed-loop thinking one by Jonekheere and Silverman [6], and by

Mustafa and Glover [7]. Contribution of states to the system input/ouput relationship can be measured on the basics of diagonalizing simultaneous both controllability and observability gramians of the system of any loopwise thinking to the very same diagonalized matrix (internally balanced conditions). This methodology is found promising for system problems of both thinking-wises in the analysis part. However, the major drawback lies on the optimality in designing as no where optimal design gives to troublesome in closed-looping like the one for the controller, especially in a problem of projective control. The component cost ranking principle proposed by Skelton [8] on the basics of determining contributions of dynamical elements to a quadratic error criterion, from opinion of the author, may be regarded as a special method of the earlier philosophy since no rigorous guarantee of optimality is possible although the propose has been guided by an optimality consideration. Hence, on combining an optimality consideration and the internally balanced conditions for the design purpose is required in many cases [9].

Last more than three decades, an American scientists group (Bernstein, Haddad and Hyland) have devoted a tremendous effort on establishing OPEQ for different system problems in both loop-wise thinking from the first-order necessary conditions for an optimality consideration of each problem [10 - 14]. Important significance of treatment in OPEQ philosophy lies on the question of multi-extreme since certain constraint conditions, bounds (internally balanced, H_∞ performance, Petersen-Hollt, Guaranteed cost and so on) are able to accommodate suitably in due OPEQs development course for each problem. This methodology is hence found being applicable to both analysis and design purposes. However, with a careful analysis, it is found that the minimization have in all the cases been carried out with respect to parameters, which are inherently non-separable from state-variables for a system. This gives rise to a drawback in regards to some difficulties lying on the complexity of mathematical involvement, also on the optimal projection nature, which in most cases is an oblique one, leading to the requirement of other conditions for computing the solution of OPEQ. Further, although additionally constraint conditions are able to be facilitated in OPEQ, but not a single provision for retaining physical nature of desired states in the result. This disvalues significance of the methodology from the analysis point of view.

Concept of state-optimization has been originated by San [15] from the fact that between two systems of sate-variable equations there exists always a non-similarity transformation on each to other state vectors and then the optimality for back-transform is achieved owing the role of pseudo-inverse of that non-similarity. It has shown that for a given system the non-similarity transformation may be freely chosen; hence the retaining physical nature of modeled states is possible in transformed version [16]. If the non-similarity transformation is factorized in terms of a partial isometry, an orthogonal projection matrix can be formed, facilitating the possibility of obtaining a simpler form for OPEQ. Thus, the state-optimization methodology overcomes the drawbacks and enjoys the merits of both early mentioned approaches.

A robustness of reduced order models on adopting the state-optimization approach in frame work of linear matrix inequality (LMI) is considered in this paper to show above mentioned advantages over the recently proposed methods [17 - 20]. A nonlinear system series expandable around a quasilinear in term of uncertainty with respect to both, parameters and state variables, is considered in this paper. Conditions for the robustness are those for necessary, for sufficient with respect to the perturbations over the quasilinear model.

Arrangement of the paper as follows: Two lemmas proposed for preliminary are retaken in II. The first one is related with defining a criterion for the state-optimization and the other is with factorizing a non-similarity transformation in terms of a partial isometry. In III, states of the problem of model reduction for uncertain systems. In IV, necessary and sufficient conditions for robustness of reduced order models are reported. The necessay conditions are those for reduced order of a quasilinear model by the state-optimization approach and sufficient ones are those for characteristics of perturbated reduced order models to be kept the same as that of the reduced order one. In IV is for concluding remarks, and suggestions for further researches.

2. PRELIMINARY

2.1. Notations

Throughout the paper, following conventions are used

- Bold capital letters are denoted for matrices, while low-case bolt letters are for vectors.

- P stands for real, E(.) for either expectation or average value of (.) when t approaches to infinity.

- ρ(.), $(.)^T$, $(.)^+$ stand for rank, transpose, pseudoinverse of (.).

- Stability matrix is the one having all eigenvalues on the left hand side of the S-plane.

- Non-negative (positive) definite matrix is a symmetric one having only non-negative (positive) eigenvalues.

- All the vectors norms are Euclideans or L^2 norms, $\|\mathbf{x}\|^2 = \left(\sum_j |x_j|^2\right)^{1/2}$.

- Controllability and observability gramians of a system are denoted by

$$\mathbf{W}_c = \int_0^t e^{\mathbf{A}t}\mathbf{B}\mathbf{V}\mathbf{B}^T e^{\mathbf{A}^T t}dt, \ \mathbf{W}_0 = \int_0^t e^{\mathbf{A}^T t}\mathbf{C}^T\mathbf{C}e^{\mathbf{A}t}dt \tag{2.1}$$

Satisfying dual Lyapunov equations

$$\begin{aligned} \mathbf{A}\mathbf{W}_c + \mathbf{W}_c\mathbf{A}^T + \mathbf{B}\mathbf{V}\mathbf{B}^T = 0 \\ \mathbf{W}_o\mathbf{A} + \mathbf{A}^T\mathbf{W}_o + \mathbf{C}^T\mathbf{R}\mathbf{C} = 0 \end{aligned} \tag{2.2}$$

where $\mathbf{V} = E(\mathbf{u}\mathbf{u}^T)$, $\mathbf{R}$ is non-negative weighted matrix of order q.

2.2. Introduction to Pseudo-inverse and Transformation in system problems

Concept of generalized inverse seems to have been first mentioned, called as pseudo-inverse by Fredholm in 1903, originating for integral operator. Generalized inverses have been studied extending to differential operators, Green's functions by numerous authors, in particular by Hilbert in 1904, Myller in 1906, Westfall in 1090, Hurwitz in 1912, etc. Generalized inverse has been antedated to matrices on defining first by Moore in 1920 as general reciprocal. The uniqueness of pseudo-inverse of a finite dimensional matrix has been shown by Penrose in 1955, satisfying four equations [21]

$$\mathbf{TXT} = \mathbf{T} \text{ (i)}, \ \mathbf{XTX} = \mathbf{X} \text{ (ii)}, \ (\mathbf{TX})^* = \mathbf{TX} \text{ (iii)}, \ (\mathbf{XT})^* = \mathbf{XT} \text{ (iv)} \tag{2.3}$$

where, (.)* denotes for conjugate transpose of (.).

The above four equations are commonly known as Moore-Penrose ones and the unique matrix **X** on satisfying these equations is usually referred to as the Moore-Penrose inverse and often denoted by $\mathbf{T}^{+}$.

Assume that an available system (S) and an invited (or assumed) model (AM) are described by

$$\text{(S):}\quad \begin{aligned} \dot{\mathbf{x}}_n &= \mathbf{A}_n\mathbf{x}_n + \mathbf{B}_n\mathbf{u}_n \\ \mathbf{y}_n &= \mathbf{C}_n\mathbf{x}_n \end{aligned} \tag{2.4}$$

$$\text{(AM):}\quad \begin{aligned} \dot{\mathbf{x}}_m &= \mathbf{A}_m\mathbf{x}_m + \mathbf{B}_m\mathbf{u}_m \\ \mathbf{y}_m &= \mathbf{C}_m\mathbf{x}_m \end{aligned} \tag{2.5}$$

where the letters n and m in the subscripts stand for (S) and (AM) also for their order numbers respectively with all of the vectors and matrices are supposed to be appropriately dimensioned.

It was observed that indifferent from orders of the two, there exists always a transformation between two state vectors (referred to as state transformation) and a transformation between two output vectors (named as output transformation). If both (S) and (AM) are subjected to the same input vector, output transformation is seen to be similarity (an invertible matrix) one as dimension of the output vector of (AM) is the same as that of (S), but it is not the case always for state transformation. Even if state transformation is a non-similarity one, the output vectors are match able, however. As non-similarity transformation on state variable vectors is not a bi-directional one, giving rise to the idea of optimization with respect to the state variables.

2.3. Definitions and Lemmas

2.3.1. Definitions

Projection matrix resulted from the first order necessary conditions for an optimality process is termed as an optimal projection. System of equations resulted from the necessary conditions for an optimality expressing in terms of components of optimal projection is called as optimal projection equations (OPEQ).

Conditions for an uncertain model to preserve properties of the respective quasilinear model are the necessay conditions while those for the related model obtained by a theorem applicable for the quasilinear case to be valid in the uncertain case are the sufficient ones.

2.3.2. Lemmas

Lemma 2.1. Let the vector $\mathbf{x}_n$ of n independently specified states of a (S) be given. Assume that an (AM) is chosen having vector $\mathbf{x}_m$ of m independently specified states, $m \leq n$. Then there exists a non-similarity transformation $\mathbf{T} \in P^{mxn}$, $\rho(\mathbf{T}) = m$, on $\mathbf{x}_n$ for obtaining $\mathbf{x}_m$ such that if the number of (S) output is less than or equal to that of (AM) order, $q \leq m$, then $\mathbf{T}^{+}\mathbf{x}_m$ leads to the minimum norm amongst the least-squares of output-errors.

Proof. Details can be found in [11]. It is necessary showing that with the condition mentioned in lemma one can easily obtain the weighted least-squares criterion (L_2) on the output errors

$$J_{\mathrm{Oopt}} = \int_{o}^{\infty} (\mathbf{y}_n - \mathbf{y}_m)^{\mathrm{T}} \mathbf{R} (\mathbf{y}_n - \mathbf{y}_m) \mathrm{d}t\ . \tag{2.6}$$

From the criterion (L_2) for state optimization

$$J_{Sopt} = \int_{o}^{\infty} \| \mathbf{x}_n - \mathbf{T}^{+}\mathbf{x}_m \|_R^2 dt \tag{2.7}$$

where, **R** stands for non-negative weighted matrix of the appropriate dimension.

Usually, order n of (S) is not known, order m of (AM) may be highly chosen. In such a case, the validity of the lemma is kept; see the remark II.1 of [11] for the details of argument.

Lemma 2.2. Let the state vector $\mathbf{x}_n$ of (S) be a transformed state vector $\mathbf{x}_m$ of (AM) as

$$\mathbf{x}_n = \mathbf{T}^{+}\mathbf{x}_m,\ \mathbf{T} \in R^{mxn},\ \rho(\mathbf{T}) = n<m \tag{2.8}$$

Then **T** can be factorized as

$$\mathbf{T = EG = HE} \tag{2.9}$$

where, $\mathbf{E} = E(\mathbf{x}_m\mathbf{x}_n^T) \in P^{mxn}$ is a partial isometry, $\mathbf{G} = E(\mathbf{x}_n\mathbf{x}_n^T) \in P^{nxn}$, $\mathbf{H} = E(\mathbf{x}_m\mathbf{x}_m^T) \in P^{mxm}$, both are non-negative definite matrices.

Proof. See [11] for details.

Remark 2.1.

It is noted that since **T** is constant, hence $\dot{\mathbf{x}}_n = \mathbf{T}^{+}\dot{\mathbf{x}}_m$.

It is known that $\sigma_1 = \mathbf{EE}^T, \sigma_2 = \mathbf{E}^T\mathbf{E}$ are optimal in the sense that one state vector is optimized with respect to the other; moreover both are of orthogonal projection matrix.

Although $\mathbf{x}_n$ and $\mathbf{x}_m$ are definitely specified but **T** is not unique determined due to mismatch between the dimensions of two state vectors. The question arises regarding the construction of **T** so that $\mathbf{x}_n$ is obtainable from the knowledge of $\mathbf{x}_m$.

3. FORMULATION OF THE PROBLEM

3.1. A consideration for study case

Consider a nonlinear dynamic system (S) describable by time-varying parameters and states

$$\dot{\mathbf{x}}_s(t) = \mathbf{A}_s(t)\mathbf{x}_s(t) + \mathbf{B}_s(t)\mathbf{w}(t) \tag{3.1}$$

$$\mathbf{y}_s(t) = \mathbf{C}_s(t)\mathbf{x}_s(t) + \mathbf{D}_s(t)\mathbf{w}(t) \tag{3.2}$$

where, denote $\mathbf{w}(t) \in R^p$, $\mathbf{x}_s(t) \in R^n$, $\mathbf{y}_s(t) \in R^q$ the input, state, output vectors respectively, and $\mathbf{A}_s(t) \in R^{nxn}$, $\mathbf{B}_s(t) \in R^{nxp}$, $\mathbf{C}_s(t) \in R^{qxn}$, $\mathbf{D}_s(t) \in R^{pxq}$ the parameters of system model.

Determine conditions for a reduced model of order $r \le n$, described by

$$\dot{\mathbf{x}}_r(t) = \mathbf{A}_r(t)\mathbf{x}_r(t) + \mathbf{B}_r(t)\mathbf{w}(t) \tag{3.3}$$

$$\mathbf{y}_r(t) = \mathbf{C}_r(t)\mathbf{x}_r(t) + \mathbf{D}_r(t)\mathbf{w}(t) \tag{3.4}$$

where, denote $\mathbf{x}_r(t) \in R^r$, $\mathbf{y}_r(t) \in R^q$ respectively the state, output vector and $\mathbf{A}_r(t) \in R^{rxr}$, $\mathbf{B}_r(t) \in R^{rxp}$, $\mathbf{C}_r(t) \in R^{qxr}$, $\mathbf{D}_r(t) \in R^{pxq}$ the parameters of reduced model,

On satisfying the state-optimization criterion

$$J_{Sopt} = \text{SupE}\left\{\left\|\mathbf{x}_s(t) - \mathbf{T}_s^+(t)\mathbf{x}_r(t)\right\|_R^2\right\},\ \mathbf{T}_s(t) \in P^{mxn} \tag{3.5}$$

and corresponding quadratically weighted output-error criterion

$$J_{Oopt} = \text{SupE}\left\{\left\|\mathbf{y}_r(t) - \mathbf{K}_s(t)\mathbf{y}_s(t)\right\|_R^2\right\},\ \mathbf{K}_s(t) \in P^{pxp},\ \rho(\mathbf{K}_s(t)) = q. \tag{3.6}$$

The above system may represent some system dynamics and parameters that are not precisely known or are difficult to be exactly modelled. However, without losing the generality one may assume $\mathbf{D}_s(t) = \mathbf{0}_{pxq}$ and a quasilinear model having constant, nominal values $(\mathbf{A}_0, \mathbf{B}_0, \mathbf{C}_0)$ so that the system described in (3.1) and (3.2) can be expanded in series as

$$\dot{\mathbf{x}}_s(t) = \left[\mathbf{A}_0 + \sum_{i=1}^{k} \mathbf{A}_i \mathbf{r}_i(t)\right]\left[\mathbf{x}_0(t) + \sum_{i=1}^{n} \boldsymbol{\varepsilon}_i \boldsymbol{\eta}_i(t)\right] + \left[\mathbf{B}_0 + \sum_{i=1}^{l} \mathbf{B}_i \mathbf{s}_i(t)\right]\mathbf{w}(t) \tag{3.7}$$

$$\mathbf{y}_s(t) = \left[\mathbf{C}_0 + \sum_{i=1}^{p} \mathbf{C}_i \mathbf{v}_i(t)\right]\left[\mathbf{x}_0(t) + \sum_{i=1}^{n} \boldsymbol{\varepsilon}_i \boldsymbol{\eta}_i(t)\right] \tag{3.8}$$

where, denotes $\boldsymbol{\varepsilon}_i = \alpha_i \lambda_i^{'}$ for uncertain state variables and $\mathbf{A}_i = d_i e_i^{'}$, $\mathbf{B}_i = f_i g_i^{'}$, $\mathbf{C}_i = h_i e_i^{'}$ of the rank 1 for uncertain parameters and the respective vector of uncertain state $\boldsymbol{\eta}(t) \in \chi$, and the matrices of uncertain parameters $\mathbf{r}(t) \in \mathsf{R}$, $\mathbf{s}(t) \in \mathsf{S}$, $\mathbf{v}(t) \in \mathsf{V}$ are bounded within sets

$$\chi = \{\eta \in \mathbb{R}^n : |\eta_i| \le \bar{\eta},\ i = 1, 2, \ldots, n\};\ \bar{\eta} \ge 0 \tag{3.9.a}$$

$$\mathsf{R} = \{r \in \mathbb{R}^k : |r_i| \le \bar{r},\ i = 1, 2, \ldots, k\};\ \bar{r} \ge 0 \tag{3.9.b}$$

$$\mathsf{S} = \{s \in \mathbb{R}^l : |s_i| \le \bar{s},\ i = 1, 2, \ldots, l\};\ \bar{s} \ge 0 \tag{3.9.c}$$

$$\mathsf{V} = \{v \in \mathbb{R}^p : |v_i| \le \bar{v},\ i = 1, 2, \ldots, p\};\ \bar{v} \ge 0. \tag{3.9.d}$$

Assume that the above mentioned uncertain $\boldsymbol{\eta}(t)$, $\mathbf{r}(t)$, $\mathbf{s}(t)$ and $\mathbf{v}(t)$ are of measurable vector functions Lebesgue for all $t \ge 0$ that $\boldsymbol{\eta}(t) \in \chi$, $r(t) \in \mathsf{R}$, $s(t) \in \mathsf{S}$ and $v(t) \in \mathsf{V}$ [24]. Then, for short, one can denote

$$\Delta\mathbf{x} = \sum_{i=1}^{n} \boldsymbol{\varepsilon}_i \boldsymbol{\eta}_i(t),\ \ \Delta\mathbf{A} = \sum_{i=1}^{k} \mathbf{A}_i \mathbf{r}_i(t),\ \Delta\mathbf{B} = \sum_{i=1}^{l} \mathbf{B}_i s_i(t),\ \Delta\mathbf{C} = \sum_{i=1}^{p} \mathbf{C}_i \mathbf{v}_i(t) \tag{3.10}$$

Moreover, one can also represent the uncertain state and output vectors as respective variation around the constant, nomional values of the quasilinear model as follows

$$\dot{\mathbf{x}}_s(t) = \dot{\mathbf{x}}_0 + \sum_{i=1}^{n} \boldsymbol{\varepsilon}_i \dot{\boldsymbol{\eta}}_i(t) = (\dot{\mathbf{x}}_0 + \Delta\dot{\mathbf{x}}) \text{ and } \mathbf{y}_s(t) = (\mathbf{y}_0 + \Delta\mathbf{y}) \tag{3.11}$$

It is clearly seen that the conditions in (3.9.a-d), (3.10) and (3.11) are those for the validity of expanding in series of respective functions in the convergent sense of each expansion, also for preserving the stabilty, controllability and observability properties of the quasilinear model.

It is also seen that the nonlinearity problem can be treated to be a robustness one on adopting the perturbation method. However, in the present case whereas variations of the state variable can directly come in to the scene to be dealt with, which is the major different aspect from the earlier contributions on the parameter-based optimization procedure.

3.2. Statement of the problem

Given a nonlinear system described by uncertain quasilinear model of order n

$$\dot{\mathbf{x}}_{\mathbf{s}}(t) = [\mathbf{A}_0 + \Delta\mathbf{A}][\mathbf{x}_0 + \Delta\mathbf{x}] + [\mathbf{B}_0 + \Delta\mathbf{B}]\mathbf{w}(t) \tag{3.12}$$

$$\mathbf{y}_{\mathbf{s}}(t) = [\mathbf{C}_0 + \Delta\mathbf{C}][\mathbf{x}_0 + \Delta\mathbf{x}] \tag{3.13}$$

Determine a reduced order uncertain quasilinear model of order r, $r \le n$

$$\dot{\mathbf{x}}_{\mathbf{r}}(t) = [\mathbf{A}_r + \Delta\mathbf{A}_r][\mathbf{x}_r + \Delta\mathbf{x}_r] + [\mathbf{B}_r + \Delta\mathbf{B}_r]\mathbf{w}(t) \tag{3.14}$$

$$\mathbf{y}_r(t) = [\mathbf{C}_r + \Delta\mathbf{C}_r][\mathbf{x}_r + \Delta\mathbf{x}_r] \tag{3.15}$$

and related robust conditions on satisfying state-optimization (3.5) and quadratically weighted output-error criterions (3.6).

The above stated problem can be solved in two steps reporting in the next paragraph. The first one is related to order reduction for the quasilinear model of the nonlinear uncertain system. The next is related to the necessery and sufficient conditions for the robustness.

4. SOLUTION OF THE PROBLEM

4.1. Reduced order of quasilinear model

Theorem 4.1. For an n order quasilinear model with appropriately dimensioned matrices and vectors

$$\dot{\mathbf{x}}_0 = \mathbf{A}_0\mathbf{x}_0 + \mathbf{B}_0 u \tag{4.1}$$

$$\mathbf{y}_0 = \mathbf{C}_0\mathbf{x}_0 \tag{4.2}$$

there exists in the set of r-th order, $q \le r \le n$, jointly controllable and observable models, the one called optimal model on satisfying L_2 model-reduction criterion with optimal parameters expressed in term of an rxn partial isometry **E** and nxn non-negative definite matrix **H**

$$\mathbf{A}_r = \mathbf{E}\mathbf{H}\mathbf{A}_0\mathbf{H}^+\mathbf{E}^T,\ \mathbf{B}_r = \mathbf{E}\mathbf{H}\mathbf{B}_0,\ \mathbf{C}_r = \mathbf{C}_0\mathbf{H}^+\mathbf{E}^T \tag{4.3}$$

Further, there exists an nxn optimal projector $\boldsymbol{\sigma}$ and two nxn non-negative definite matrices **Q** and **P** such that the coupled Lyapunov equations are to be satisfied

$$\boldsymbol{\sigma}\left[\mathbf{H}\mathbf{A}_0\mathbf{Q} + \mathbf{Q}\mathbf{A}_0^T\mathbf{H} + \mathbf{H}\mathbf{B}_0\mathbf{V}_1\mathbf{B}_0^T\mathbf{H}\right] = \mathbf{0} \tag{4.4}$$

$$\left[\mathbf{H}^+\mathbf{A}_0^T\mathbf{P} + \mathbf{P}\mathbf{A}_0\mathbf{H}^+ + \mathbf{H}^+\mathbf{C}_0^T\mathbf{R}_2\mathbf{C}_0\mathbf{H}^+\right]\boldsymbol{\sigma} = \mathbf{0} \tag{4.5}$$

where, $\mathbf{V}_1 = \mathrm{E}\left(\mathbf{uu}^{\mathrm{T}}\right)$, $\mathbf{R}_2$ is weighted matrix in the criterion for order reduction.

Proof. By the use of Lemma 2.2, first (2.8) then (2.9), relations in (4.3) are derived. Adopting dual Lyapunov equations to the reduced order model with defined optimal projection and two mentioned optimal non-negative definite matrices, (4.4) and (4.5) are obtained.

Converse of Theorem 4.1. Let the r-th order model jointly controllable and observable with $q \le r \le n$, with parameters determined by (4.3) on satisfying (3.4) and (3.5). Then, $\boldsymbol{\sigma}$, **Q** and **P** are optimal.

Proof. It is evident to show that the optimal is achieved in the sense of satisfying the criterion for state-optimization and the quadratically weighted output-errors.

Remark 4.1. Physical significances of various particularly quasilinear modeled states can be retained in the reduced one. Considerable effort is reduced for finding the global amongst multi-extreme due to the effect of partial isometrics **E**. Actually, **H**, **Q** and **P** consists of the states measurements, controlability and observability gramians of quasilinear model.

If the quasilinear model of order n is yet to be known, the reduced order model in such a case has to be considered as a mis-order modelling case. The theorem deals with the measurements of quasilinear models controllability and observability gramians. However, if (4.4) and (4.5) are solvable, **Q** and **P** are obtainable and **E** follows. Parameters of reduced order model are determinable irrespective of the measurability of controllability and observability gramians. A difficulty in solving these equations stands on the fact that no standard algorithm is available yet regarding the guarantee for convergence of solutions.

It shows next that conditions for robustness of reduced-order model can be found in the same manner as that for robustness of modeling. However, great effort would be reduced in tackling the mentioned robustness by adopting state-optimization approach with respect to parameter-based optimization technique.

2.2. Robustness of reduced order model

Under perturbation, quasilinear model (4..1), (4.2) becomes the one with uncertain parameters and states described by (3.12), (3.13) in the present consideration, which requires the reduced order model obtained by theorem 4.1 to have also uncertain parameters and states. Conditions for the model described by (3.12), (3.13) to preserve the properties of the quasilinear model described by (4.1), (4.2) are known as the necessay ones while those for the reduced model obtained by the theorem 4.1 to be valid in the uncertain model case are sufficient ones.

There may establish several methods to obtain necessay and sufficient conditions on adopting equivalent vector and matrix norms in either time, frequency domains or combined both (L^2 limit, H_2, H_∞ bounds), or in other space also.

Theorem 4.2 (Necessay conditions)

For a nonlinear dynamic system described by uncertain model with time-varying parameters and uncertain states (3.12) and (3.13) to preserve the stability, controllabitity and observability properties of the quasilinear model desribed by (4.1) and (4.2), following conditions are to be satisfied by the uncertain parameters

$$\|\mathbf{D\,A}\| £ \; 2(a_{\mathrm{n}})^{1/2}\big/n\,,\;\; \|\mathbf{D\,B}\| £ \; (b_1)^{1/2}\big/(1+n)\,,\;\; \|\mathbf{D\,C}\| £ \; (g_1)^{1/2}\big/(1+n) \tag{4.6}$$

and by the controllability, observability gramians of uncertain states

$$\mathbf{AD\,Q} + \mathbf{D\,QA}^{\mathrm{T}} + \mathbf{W}(\mathbf{Q})\, £ \; 0 \tag{4.7}$$

$$\mathbf{\Delta PA} + \mathbf{A}^{\mathrm{T}}\mathbf{\Delta P} + \mathbf{\Omega(P)}\, £ \; \mathbf{0} \tag{4.8}$$

where, $\mathbf{W}(\mathbf{Q}) = \mathbf{D\,AQ} + \mathbf{QD\,A}^{\mathrm{T}} + 2\mathbf{BVD\,B}^{\mathrm{T}};\mathbf{W}(\mathbf{P}) = \mathbf{PD\,A} + \mathbf{D\,A}^{\mathrm{T}}\mathbf{P} + 2\mathbf{D\,C}^{\mathrm{T}}\mathbf{RC}$.

Proof. For a linear model of canonical (minimal) realization to be stability, controllability and observability properties, all poles positions predicted by $\mathbf{A}_0\mathbf{A}_0^{\mathbf{T}} = diag(\,a_1 \;...\; a_n)$ are on the left hand side of complex-plane having number higher than the number of nulles related with $\mathbf{B}_0\mathbf{B}_0^{\mathbf{T}} = diag(\,b_1 \;...\; b_p)$ and $\mathbf{C}_0^{\mathbf{T}}\mathbf{C}_0 = diag(\,g_1 \;...\; g_q)$.

To preserve stability property of the quasilinear model under the uncertainty perturbation with Petersen-Hollot bound and presume (i). Model (3.12), (3.13) with transform matrix norm be consistent with the state vector norm $\|\mathbf{x}_{\mathrm{s}}\| £ \; \|\mathbf{x}_0\| + \|\mathbf{D\,x}\| = 1 + \mathrm{n}$; (ii). Positions of poles related to the smallest eigenvalue of $\mathbf{A}_0$ be not shifted to the righ hand side of complex-plane due to variations of parameters in $\mathbf{A_s}$, of states in $\mathbf{x}_{\mathrm{s}}$; (iii). All non-zero eigenvalues of $\mathbf{B}_{\mathrm{s}}\mathbf{B}_{\mathrm{s}}^{\mathrm{T}}, \mathbf{C}_{\mathrm{s}}^{\mathrm{T}}\mathbf{C}_{\mathrm{s}}$ are unchanged by number (no eigenvalue of $\mathbf{B}_{\mathrm{s}}\mathbf{B}_{\mathrm{s}}^{\mathrm{T}}, \mathbf{C}_{\mathrm{s}}^{\mathrm{T}}\mathbf{C}_{\mathrm{s}}$ are annulled by eigenvalues of $\mathbf{A}_{\mathrm{s}}$ due to $\mathbf{D\,A}$, $\mathbf{\Delta B}$, $\mathbf{D\,C}$). By some arithmetic manipulations the relations (4.6) are then obtained.

Substituting the values of controllability and observability gramians of uncertain model (3.12), (3.13) then (4.7), (4.8) are obtained. These relations imply that variations of states with respect to both, input and output sides, are to be bounded.

It is clearly seen that the relations in (4.6)-(4.8) give a strictly bounded range for parameters, state variables than the limited range in (3.9). The fact lies on the characteristics of stability, jointly controllability and observability to be satisfied by the model of uncertainty.

Theorem 4.3 (Sufficient conditions). Let a reduced order model be obtained by theorem 4.1 for a given quasilinear model. Assume that the quasilinear model satisfies the necessay conditions for uncertainty perturbation stated in theorem 4.2. Then, the obtained reduced order model has to satisfy following conditions

a) For optimal transformation:

$$\|\mathbf{H}\| = (l_1)^{1/2}.\left\{1 + (l_{\mathrm{n}})^{1/2}\big/((l_{\mathrm{n}})^{1/2} + n)\right\}$$
$$\|\mathbf{\Delta H}\|\big/\|\mathbf{H}\| = (l_{\mathrm{n}})^{1/2}\big/((l_{\mathrm{n}})^{1/2} + n),\;\; \|\mathrm{D}\mathbf{H}^{+}\|\big/\|\mathbf{H}^{+}\| = (l_{\mathrm{n}})^{1/2}\big/n \tag{4.9.a}$$

$$\mathbf{J}_{\mathrm{Oopt}} \; £ \; \|\mathrm{D}\mathbf{x}\| + \|\mathrm{D}\mathbf{H}^{+}\|.\|\mathbf{x}_{\mathrm{n}}\| = 2$$
$$\|\mathbf{K}_{\mathrm{s}}\| = 1\big/\|\mathbf{H}_{\mathrm{s}}\|,\;\; \|\mathbf{\Delta K}\| = \left(3(l_{\mathrm{n}})^{1/2} + 2n\right)\Big/(l_1)^{1/2}.\left(2(l_{\mathrm{n}})^{1/2} + n\right) \tag{4.9.b}$$

b) For variation of parameters:

$$\|\mathbf{A}_{\mathrm{s}}\| £ \; (a_1 l_1)^{1/2}\left(2(l_{\mathrm{n}})^{1/2} + n\right)\Big/\left(n + (l_{\mathrm{n}})^{1/2}\right),\;\; \|\mathrm{D}\mathbf{A}_{\mathrm{n}}\| £ \; 2(a_1 l_1)^{1/2}\big/n \tag{4.10.a}$$

$$\|\mathbf{B}_s\| \le (\beta_1)^{1/2}\left((\lambda_n)^{1/2}+\nu\right)\Big/\left(\nu+(\lambda_n)^{1/2}\right),\ \|\Delta\mathbf{B}_n\| \le (\beta_1)^{1/2}\big/\nu \qquad (4.10.b)$$

$$\|\mathbf{C}_s\| \le (\alpha_1\lambda_1)^{1/2}\left(2(\lambda_n)^{1/2}+\nu\right)\Big/\left(\nu+(\lambda_n)^{1/2}\right),\ \|\Delta\mathbf{C}_n\| \le (\gamma_1\lambda_1\lambda_n)^{1/2}\Big/\left((\lambda_n)^{1/2}+\nu\right) \qquad (4.10.c)$$

$$1\big/(\lambda_n)^{1/2} \le \|\mathbf{x}_s\| \le 1\big/(\lambda_m)^{1/2} \qquad (4.10.d)$$

c) For controllability and observability:

- $2(\alpha_1\lambda_1)^{1/2}/\nu \le (\alpha_{sn})^{1/2}$, $(\beta_1)^{1/2}/\nu \le (\beta_{sn})^{1/2}$, $(\gamma_1\lambda_1\lambda_n)^{1/2}\big/\left((\lambda_n)^{1/2}+\nu\right) \le (\gamma_{sn})^{1/2}$,
- $\mathbf{H}^{+}\mathbf{A}_m\Delta\mathbf{Q}+\Delta\mathbf{Q}\mathbf{A}_m^T\mathbf{H}^{+}=\Omega(\mathbf{Q}),\ \mathbf{H}\mathbf{A}_m^T\Delta\mathbf{P}+\Delta\mathbf{P}\mathbf{A}_m\mathbf{H}=\Omega(\mathbf{P})$,
- $\mathbf{H}^{+}\mathbf{A}_m\mathbf{Q}+\mathbf{Q}\mathbf{A}_m^T\mathbf{H}^{+}+\Omega(\mathbf{Q})+\mathbf{H}^T\mathbf{B}_m\mathbf{B}_m^T\mathbf{H}^{+}=\mathbf{0}$,
  $\mathbf{H}\mathbf{A}_m^T\mathbf{P}+\mathbf{P}\mathbf{A}_m\mathbf{H}+\Omega(\mathbf{P})+\mathbf{H}\mathbf{C}_m^T\mathbf{C}_m\mathbf{H}=\mathbf{0}$
- $\|\Delta\mathbf{Q}\|$, $\|\Delta\mathbf{P}\|$ are bounded.

Proof. From (2.8), (2.9) one gets $\|\mathbf{H}\| = (\lambda_1)^{1/2}, \|\mathbf{H}^{+}\| = 1/(\lambda_n)^{1/2}$ where λ_1, λ_n are the maximum and the least non-zero eigenvalues of $\mathbf{HH}^T$. State-optimization criterion with $\mathbf{T} + \Delta\mathbf{T} = \mathbf{T}_s$ and related quadratically weighted output-error criterion with $\mathbf{K} + \Delta\mathbf{K} = \mathbf{K}_s$ are satisfied for robust performance with bounds of $\mathbf{A}_s$, $\mathbf{B}_s$ and $\mathbf{C}_s$ so that the reduced model (3.14), (3.15) is to be controllable and observable. So, conditions for optimal transformations (4.9s) are obtained.

3.2.2. Solution of problem

1. Sufficient conditions for robust performance
2. Uncertainty structure:

a) Assumption:

- $\mathbf{V} = \mathbf{I}_p$, $\mathbf{K} = \mathbf{R} = \mathbf{I}_q$,
- $\mathbf{A}_m = diag(-\alpha_1 ... -\alpha_m), \mathbf{B}_m\mathbf{B}_m^T = diag(\beta_1...\beta_m), \mathbf{C}_m^T\mathbf{C}_m = diag(\gamma_1...\gamma_m)$,
- Maximum variations of parameters are computed by theorem 3.1.

b) Variation of parameters:

- $\|\Delta\mathbf{A}_n\| \le 2(\alpha_1\lambda_1)^{1/2}/\nu$, $\|\Delta\mathbf{B}_n\| \le (\beta_1)^{1/2}/\nu$, $\|\Delta\mathbf{C}_n\| \le (\gamma_1\lambda_1\lambda_n)^{1/2}\big/\left((\lambda_n)^{1/2}+\nu\right)$,
- $\|\mathbf{A}_s\| \le (\alpha_1\lambda_1)^{1/2}\left(2(\lambda_n)^{1/2}+\nu\right)\big/\left(\nu+(\lambda_n)^{1/2}\right)$,
- $\|\mathbf{B}_s\| \le (\beta_1)^{1/2}\left((\lambda_n)^{1/2}+\nu\right)\big/\left(\nu+(\lambda_n)^{1/2}\right)$,
- $\|\mathbf{C}_s\| \le (\alpha_1\lambda_1)^{1/2}\left(2(\lambda_n)^{1/2}+\nu\right)\big/\left(\nu+(\lambda_n)^{1/2}\right)$.

3. Stability, Controllabity and Observability

   a). Assumption:

- Positions of poles corresponding to $-\alpha_{sn}$ be not shifted to R.H.S of complex-plane,
- Number of non-zero eigenvalues of $\mathbf{B}_s\mathbf{B}_s^T$ and of $\mathbf{C}_s^T\mathbf{C}_s$ be kept unchanging (none of eigenvalues of $\mathbf{B}_s\mathbf{B}_s^T$ and $\mathbf{C}_s^T\mathbf{C}_s$ be annulled due to $\Delta\mathbf{B}_n$ and $\Delta\mathbf{C}_n$),
- n eigenvalues of $\mathbf{B}_s\mathbf{B}_s^T$ be differed from those of $\mathbf{C}_s^T\mathbf{C}_s$,
- $\left\{\Delta\boldsymbol{\sigma}\mathbf{H}^+\mathbf{A}_m, \Delta\boldsymbol{\sigma}\mathbf{H}^+\left(\mathbf{B}_m\mathbf{B}_m^T\right)^{1/2}\right\}, \left\{\left(\mathbf{C}_m^T\mathbf{C}_m\right)^{1/2}\mathbf{H}\Delta\boldsymbol{\sigma}, \mathbf{A}_m\mathbf{H}\Delta\boldsymbol{\sigma}\right\}$ be stabilizable, detectable.

b) Conditions:

- $2(\alpha_1\lambda_1)^{1/2}/n \le (\alpha_{sn})^{1/2}$, $(\beta_1)^{1/2}/n \le (\beta_{sn})^{1/2}$, $(\gamma_1\lambda_1\lambda_n)^{1/2}\Big/\left((\lambda_n)^{1/2}+n\right) \le (\gamma_{sn})^{1/2}$,
- $\mathbf{H}^+\mathbf{A}_m\Delta\mathbf{Q}+\Delta\mathbf{Q}\mathbf{A}_m^T\mathbf{H}^+ = \Omega(\mathbf{Q})$, $\mathbf{H}\mathbf{A}_m^T\Delta\mathbf{P}+\Delta\mathbf{P}\mathbf{A}_m\mathbf{H} = \Omega(\mathbf{P})$,
- $\mathbf{H}^+\mathbf{A}_m\mathbf{Q}+\mathbf{Q}\mathbf{A}_m^T\mathbf{H}^+ + \Omega(\mathbf{Q}) + \mathbf{H}^T\mathbf{B}_m\mathbf{B}_m^T\mathbf{H}^+ = \mathbf{0}$,
  $\mathbf{H}\mathbf{A}_m^T\mathbf{P}+\mathbf{P}\mathbf{A}_m\mathbf{H} + \Omega(\mathbf{P}) + \mathbf{H}\mathbf{C}_m^T\mathbf{C}_m\mathbf{H} = \mathbf{0}$
- $\|\Delta\mathbf{Q}\|$, $\|\Delta\mathbf{P}\|$ are bounded.

In term of convex optimization

Denote transformation between the input and output of model (3.12), (3.13) by Π_s and that of model (3.14), (3.15) by Π_r as

$$\Pi_s \triangleq \left\{\begin{bmatrix}\mathbf{A}_0+\Delta\mathbf{A} & \mathbf{B}_0+\Delta\mathbf{B}\\ \mathbf{C}_0+\Delta\mathbf{C} & \end{bmatrix}\begin{bmatrix}\mathbf{x}_0+\Delta\mathbf{x}\\ \mathbf{u}+\mathbf{w}\end{bmatrix}\right\},\ \Pi_r \triangleq \left\{\begin{bmatrix}\mathbf{A}_r+\Delta\mathbf{A}_r & \mathbf{B}_r+\Delta\mathbf{B}_r\\ \mathbf{C}_r+\Delta\mathbf{C}_r & \end{bmatrix}\begin{bmatrix}\mathbf{x}_r+\Delta\mathbf{x}_r\\ \mathbf{u}+\mathbf{w}\end{bmatrix}\right\} \quad (4.15)$$

Then, there exists an error transformation representing the mismatch between Π_s and Π_r

$$\Pi(\Pi_s,\Pi_r) \triangleq \begin{cases}\dot{\mathbf{x}}_e = \mathbf{A}_e\mathbf{x}_e + \mathbf{B}_e\mathbf{u}_e\\ \mathbf{y}_e = \mathbf{C}_e\mathbf{x}_e\end{cases}. \quad (4.16)$$

On satisfying criterions (3.5), (3.6) with the initial conditions of Π_r (parameters determinated by (4.3) $\mathbf{A}_r = \mathbf{E}\mathbf{H}\mathbf{A}_0\mathbf{H}^+\mathbf{E}^T$, $\mathbf{B}_r = \mathbf{E}\mathbf{H}\mathbf{B}_0$, $\mathbf{C}_r = \mathbf{C}_0\mathbf{H}^+\mathbf{E}^T$), and

$$\mathbf{A}_e = \begin{bmatrix}\mathbf{A}_0+\Delta\mathbf{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{A}_r+\Delta\mathbf{A}_r\end{bmatrix},\ \mathbf{B}_e = \begin{bmatrix}\mathbf{B}_0+\Delta\mathbf{B} & \mathbf{0}\\ \mathbf{0} & \mathbf{B}_r+\Delta\mathbf{B}_r\end{bmatrix},$$

$$\mathbf{C}_e^T = \begin{bmatrix}(\mathbf{C}_0+\Delta\mathbf{C})^T\\ (\mathbf{C}_r+\Delta\mathbf{C}_r)^T\end{bmatrix}^T,\ \mathbf{x}_e = \begin{bmatrix}\mathbf{x}_0+\Delta\mathbf{x}\\ \mathbf{x}_r+\Delta\mathbf{x}_r\end{bmatrix}.$$

The robustness of reduced order model becomes a convex optimization that minimizes errors arising in (4.16). That is, one has to find the minimum amongst the values obtainable by (3.5), (3.6). However, it has been shown that (3.6) is deducible from (3.5) in time domain […]. Here, different measurements in frequency domain are considered.

Defining H_2-error (H_2 norm): The H_2 norm of model (4.16) is defined as

$$\left\|\mathbf{P}(\mathbf{P}_s,\mathbf{P}_r)\right\|_2 \cong Sup\left\|y_e\right\|_2$$

$$Subj.\ to:\begin{cases}\mathbf{P}_s: \textit{Stability, Controllability, observability}\\ \mathbf{P}_r: \textit{Stability, Controllability, observability}\end{cases} \qquad (4.17)$$

That is, a guaranteed H_2 error bound for $\|\Delta\mathbf{Q}\|$, $\|\Delta\mathbf{P}\|$ on inferimizing quadractically stable of (3.6) from optimization (4.17) with respect to both controllability and observability gramians.

Defining H_∞-error (H_∞ norm): The H_∞ of the model (4.16) is given by

$$\left\|\mathbf{P}(\mathbf{P}_s,\mathbf{P}_r)\right\|_\infty \cong Sup\frac{\left\|y_e\right\|_\infty}{\left\|u_e\right\|_\infty}$$

$$Subj.\ to:\begin{cases}\mathbf{P}_s: \textit{Stability, Controllability, observability}\\ \mathbf{P}_r: \textit{Stability, Controllability, observability}\end{cases} \qquad (4.18)$$

An H_∞ characterization for (4.16) is bounded by a real value for $\|\Delta\mathbf{Q}\|$, $\|\Delta\mathbf{P}\|$ on optimization of (4.18) so that for an uncertain parameter model of stability, controllability and observability jointly, an uncertain reduced model can be obtained of the same characteristics.

4. CONCLUDING REMARKS

Optimal projection equation (OPEQ) has been recognized to play an important contribution to finding the uniqueness amongst multi-extreme in the effect sense of an aditionally constrained condition. However, a complexity happened to be in mathematical involvement of that OPEQ on adopting parameter-optimization process from both aspects; in the establishment and in the solution to the mentioned OPEQ. State-optimization has been found removing that complexity due to the role of factorization in term of a partial isometry and mentioned factorization has an effect of that of an additionally constrained condition to the optimization process.

State-optimization approach can be employed to treating different various problems where an optimization is asked for. In the case of an infinite-dimensional (S) like distributed parameter, non-linear modeled by a series, ect., where partial or functional equations are required, then the concept of generaliazed Green function and its inverse are to be adopted, however. This may gives rise to the concept of a poly-optimization in stead of state-optimization and various researches can be carried out in this direction apart from treating the above mentioned infinite-dimensional (S) also for treating many different optimization problems happened to be in non-finite dimensional space.

It will show in the coming report, through consideration of a typical uncertain closed-loop thinking problems (robustness of reduced state estimator), great efforts would be reduced with respect to parameter-optimization approach on adopting the results obtained for opened-loop thinking ones.

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REFERENCES

1. Chen C. T. - Linear system theory and design, Holt-Saunders International Edition, 1984.
2. Moore B. C. - Principal component analysis in linear system Controllability, observbility and model reduction, IEEE Trans. Auto. Contr. **AC-26** (1) (1981) 17-32.
3. Jonckheere E. D. and Silverman L. D. - A new set of invariant for linear systems - Application to reduced order compensator design, IEEE trans. Auto. Contr. **AC-28** (10) (1983) 953-964.
4. Mustafa D. and Glover K. - Controller reduction by H_∞-Balanced truncation, IEEE Trans. Auto. Contr. **AC-36** (6) (1991) 668-682.
5. Skelton A. R. - Cost decomposition of linear system: Application to model reduction, Int. J. Contr. **32** (1980) 1031-1055.
6. Hyland D. C. and Bernstein D. S. - Optimal projection equations for fixed-order dynamic compensation, IEEE Trans. Auto. Contr. **AC-29** (11) (1984) 1034-1037.
7. Hyland D. C. and Bernstein D. S. - Optimal projection equations for model reduction and the relationship among the methods of Wilson, Skelton and Moore, IEEE Trans. Auto. Contr. **AC-30** (12) (1985) 1201-1211.
8. Haddad W. M. and Bernstein D. S. - Combine L_2/H_∞ model reduction, Int. J. Contr. **49** (5) (1989) 1523-1535.
9. Haddad W. M. and Bernstein D. S. - Robust reduced-order modeling via the optimal projection equations with Petersen-Hollot bounds, IEEE Trans. Auto. Contr. **AC-33** (7) (1988) 692-695.
10. Berntein D. S. and Haddad W. M. - LQG controller with an H_∞ performance bound: A Riccati equation approach, IEEE Trans. Auto Contr., Vol. AC-34, No. 4, pp. 293-305, 1989.
11. San N. N. - On an approach to the estimation state variable descriptive parameters for linear, co models, Optimization **33** (1994) 201-217.
12. Isreal A. R. and Greville N. E. - Generalized inverse of matrices: Theory and application, John Wiley & Sons, NewYork,
13. San N. N. - State-optimization menthod for order reduction of linear models and of state estimators, Optimization **34** (1995) 341-357.
14. Anh N. T. and San N. N. - Controller reduction by state-optimization approach, Optimization **35** (1996) 621-640.
15. San N. N. and Nath N. G. - Input-error approach to parameter estimation of linear, time-invariant, continuous-time models in state-variable description, Optimization **30** (1) (1994) 69-87.
16. San N. N. - The system-assumed model-reduced model relationship: Optimal projection equation with input-error, Contr. and Cybernetics **22** (3) (1993) 47-68.
17. Francis B. A. - A course in $H\infty$ control theory, Springer-Verlag, NewYork, 1987.
18. L. H. Khanh, P. V. Van and N. N. San - On robustness of ATM electronic switches, J. Science and Technology (Vietnam Academy of Science) **41** (3) (2003) 1-14.

19. P. T. Phong, P. V. Van and N. N. San - On a method of modeling for telecommunication network: Formulations of typical problems, J. Science and Technology (Vietnam Academy of Science) **42** (1) (2004) 15-24.
20. P. T. Phong, P. V. Van and N. N. San - On a method of modeling for telecommunication network: Solutions of typical problems, J. Science and Technology (Vietnam Academy of Science) **42** (2) (2004) 1-17.
21. P. T. Phong and N. N. San - On an approach to construction of TMN network, J. Science and Technology (Vietnam Academy of Science) **42** (3) (2004) 9-20.
22. N. N. San - Identification of llinear, continuous-time systems, Science and technique Publishing house, Hanoi, 2006.

Corresponding author:

Nguyen Thuy Anh,

Hanoi university of Technology (HUT)

Email: *ngth.anh74@gmail.com*