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ON THE STATE-OPTIMIZATION APPROACH TO SYSTEM PROBLEMS: OPENED LOOP THINKING SOLUTIONS

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ABSTRACT

State-optimization approach has been proposed to treating various different system problems in optimal projection equations (OPEQ). While the OPEQ for problems of open-loop thinking is found consisting of two modified Lyapunov equations, excepting the rank conditions whereas required in system identification and its related robust problems, the one for closed-loop thinking consists of two modified either Reccatti or Lyapunov equations, excepting conditions for compensating system happened to be in a problem like that of order reduction for controller.

Apart from addditonally constrained-conditions and simplicity in the solution form have been obtainable for each problem, it has been found the system identification problem switching over to computing the solution of OPEQ and the physical nature of medeled states possibly retaining in optimal order reduction problem.

1. INTRODUCTION

System problems may be divided into four major parts which are modeling, setting up the mathematical equations, analysis and design [1]. However, if the discussion is limited to linear systems described in the state space equations, the system problems may be then regarded to belong to either open- or closed-loop thinking ones. There have many research workers been devoted to tackling various different aspects of open- and closed-loop thinking problems from both theoretical and practical angles. Among the myriad references available in literature, two notable methodology contributions related with present paper are from the internally system-theoretic argument and from the treatment in optimal projection equations (OPEQ).

Internal system philosophy based on the contribution of dynamical elements (state variables) to the system input/output relationship has been originated firstly to so-called singular values by Moore in 1981 [2] for an open-loop thinking system and further developed to characteristic values for a closed-loop thinking one by Jonekheere and Silverman [3], and by Mustafa and Glover [4]. The contribution of states to the system input/ouput relationship can be measured on the basics of diagonalizing simultaneous both controllability and observability gramians of the system of any loopwise thinking to the very same diagonalized matrix (internally balanced conditions). This methodology is found promising for system problems of both thinking-wises in the analysis part. However, the major drawback lies on the optimality in designing as no where optimal design gives to troublesome in closed-looping like the one for the controller, especially in a problem of projective control. The component cost ranking principle proposed by Skelton [5] based on determining contributions of dynamical elements to a quadratic errors criterion, from the opinion of the authors, may be regarded as a special method of the earlier philosophy since no rigorous guarantee of optimality is possible although the propose has been guided by an optimality consideration. However, it suggests researches to be carried out on combining an optimality consideration and the internally balanced conditions for the design purpose.

Last more than three decades, an American scientists group (Bernstein, Haddad and Hyland) have devoted a tremendous effort in publishing a series of research papers on different system problems in both loop-wise thinking [6 - 10]. From the first-order necessary conditions for an optimality consideration of each problem, an optimal projection matrix has been realized and used for developing suitable OPEQ. Important significance of treatment in OPEQ philosophy lies on the question of multi-extreme as certain constraint conditions, bounds like internally balanced condition, H_∞ performance bounds, Petersen-Hollt, Guaranteed cost bounds and so on, are able to be accommodated suitably in due OPEQ development course for each problem. This methodology is hence found being applicable to both analysis and design purposes. With a careful analysis, it is found that the minimization has in all the cases been carried out with respect to parameters, which are inherently non-separable from state-variables for an output function. This gives rise to a drawback in regards to some difficulties lying on the complexity of mathematical involvement also on the optimal projection nature, which in most of the cases is an oblique one, leading to the requirement of other conditions for computing the solution of OPEQ. Further, although additionally constraint conditions are able to be facilitated in OPEQ, but not a single provision for retaining the physical nature of desired states in the result. This disvalues significance of the methodology from the analysis point of view.

Concept of state-optimization has been originated by San [11] from the fact that between two systems of sate-variable equations there exists always a non-similarity transformation on each to other state vectors and then the optimality for back-transform is achieved owing the role of pseudo-inverse of that non-similarity. San has shown that for a given system the non-similarity transformation may be freely chosen; hence the retaining physical nature of modeled states is possible in transformed version [17]. If the non-similarity transformation is factorized in terms of a partial isometry, an orthogonal projection matrix can be formed, facilitating the possibility of obtaining a simpler form for OPEQ. Thus, the state-optimization methodology overcomes the drawbacks and enjoys the merits of both early mentioned approaches.

Arrangement of the paper as follows: Two lemmas proposed for preliminary are retaken in 2. The first one is related with defining a criterion for the state optimization and the other is with factorizing a non-similarity transformation in terms of a partial isometry. In III, the results of three problems in open-loop thinking and related issues are reported. The first result is for a problem of system identification, more exactly the one of parameter estimation, the second is related with robust modeling [11] and reduced-order model is the last reported one [13]. In 4 is for concluding remarks, and suggestions for further researches.

2. PRELIMINARY

2.1. Notations

Throughout the paper, following conventions are used

- All systems are taken to be linear, time-invariant, causal and multi-variable.

- Bold capital letters are denoted for matrices, while low-case bolt letters are for vectors.

- P stands for real, E(.) for either expectation or average value of (.) when t approaches to infinity.

- ρ(.), $(.)^T$, $(.)^+$ stand for rank, transpose, pseudoinverse of (.).

- Stability matrix is the one having all eigenvalues on the left hand side of the S-plane.

- Non-negative (positive) definite matrix is a symmetric one having only non-negative (positive) eigenvalues.

- All the vectors norms are Euclideans or l^2 norms, $\|\mathbf{x}\|^2 = \left(\sum_j |x_j|^2\right)^{1/2}$.

- Controllability and observability gramians of a system are denoted by

$$\mathbf{W}_c = \int_0^t e^{\mathbf{A}t}\mathbf{B}\mathbf{V}\mathbf{B}^T e^{\mathbf{A}^T t} dt\,,\ \mathbf{W}_0 = \int_0^t e^{\mathbf{A}^T t}\mathbf{C}^T\mathbf{C}e^{\mathbf{A}t} dt \tag{2.1}$$

satisfying dual Lyapunov equations

$$\begin{aligned} \mathbf{A}\mathbf{W}_c + \mathbf{W}_c\mathbf{A}^T + \mathbf{B}\mathbf{V}\mathbf{B}^T = 0 \\ \mathbf{W}_0\mathbf{A} + \mathbf{A}^T\mathbf{W}_0 + \mathbf{C}^T\mathbf{R}\mathbf{C} = 0 \end{aligned} \tag{2.2}$$

where $\mathbf{V} = \mathrm{E}(\mathbf{u}\mathbf{u}^T)$, $\mathbf{R}$ is non-negative weighted matrix of order q.

2.2. Introduction to Pseudo-inverse and Transformation in system problems

Concept of generalized inverse seems to have been first mentioned, called as pseudo-inverse by Fredholm in 1903, originating for integral operator. Generalized inverses have been studied extending to differential operators, Green's functions by numerous authors, in particular by Hilbert in 1904, Myller in 1906, Westfall in 1090, Hurwitz in 1912, etc. Generalized inverse has been antedated to matrices on defining first by Moore in 1920 as general reciprocal. The uniqueness of pseudo-inverse of a finite dimensional matrix has been shown by Penrose in 1955, satisfying four equations [12]

$$\mathbf{TXT = T}\ \text{(i)},\ \mathbf{XTX = X}\ \text{(ii)},\ \mathbf{(TX)^* = TX}\ \text{(iii)},\ \mathbf{(XT)^* = XT}\ \text{(iv)} \tag{2.3}$$

where (.)* denotes for conjugate transpose of (.).

The above four equations are commonly known as Moore-Penrose ones and the unique matrix **X** on satisfying these equations is usually referred to as the Moore-Penrose inverse and often denoted by $\mathbf{T}^+$.

Assume that an available system (S) and an invited (or assumed) model (AM) are described in the state-space equations as

$$\text{(S):}\quad \begin{aligned} \mathbf{x}'_n &= \mathbf{A}_n\mathbf{x}_n + \mathbf{B}_n\mathbf{u}_n \\ \mathbf{y}_n &= \mathbf{C}_n\mathbf{x}_n \end{aligned} \tag{2.4}$$

$$\text{(AM):}\quad \begin{aligned} \mathbf{x}'_m &= \mathbf{A}_m\mathbf{x}_m + \mathbf{B}_m\mathbf{u}_m \\ \mathbf{y}_m &= \mathbf{C}_m\mathbf{x}_m \end{aligned} \tag{2.5}$$

where the letters n and m in the subscripts stand for (S) and (AM) also for their order numbers respectively with all of the vectors and matrices are supposed to be appropriately dimensioned.

It was observed that indifferent from orders of the two, there exists always a transformation between two state vectors (referred to as state transformation) and a transformation between two output vectors (named as output transformation). If both (S) and (AM) are subjected to the same input vector, output transformation is seen to be similarity (an invertible matrix) one as dimension of the output vector of (AM) is the same as that of (S), but it is not the case always for state transformation. Even if state transformation is a non-similarity one, the output vectors are match able, however. As non-similarity transformation on state variable vectors is not a bi-directional one, giving rise to the idea of optimization with respect to the state variables.

2.3. Definitions and Lemmas

2.3.1. Definitions

Problem that deals with system be tackled inherently in closed-loop configuration is referred to as closed-loop thinking one [1].

Projection matrix resulted from the first order necessary conditions for an optimality process is termed as an optimal projection. System of equations resulted from the necessary conditions for an optimality expressing in terms of components of optimal projection is called as optimal projection equations (OPEQ) [7, 11].

2.3.2. Lemmas

Lemma 2.1. Let the vector $\mathbf{x}_n$ of n independently specified states of a (S) be given. Assume that an (AM) is chosen having vector $\mathbf{x}_m$ of m independently specified states, $m \leq n$. Then there exists a non-similarity transformation $\mathbf{T} \in P^{mxn}$, $\rho(\mathbf{T}) = m$, on $\mathbf{x}_n$ for obtaining $\mathbf{x}_m$ such that if the number of (S) output is less than or equal to that of (AM) order, $q \leq m$, then $\mathbf{T}^{+}\mathbf{x}_m$ leads to the minimum norm amongst the least-squares of output-errors.

Proof. Details can be found in [11]. It is necessary showing that with the condition mentioned in lemma one can easily obtain the weighted least-squares criterion on the output errors

$$J_{\mathrm{Oopt}} = \int_{o}^{\infty} (\mathbf{y}_{\mathrm{n}} - \mathbf{y}_{\mathrm{m}})^{\mathrm{T}} \mathbf{R} (\mathbf{y}_{\mathrm{n}} - \mathbf{y}_{\mathrm{m}}) dt \tag{2.6}$$

from the criterion for state optimization

$$J_{\mathrm{Sopt}} = \int_{o}^{\infty} \| \mathbf{x}_{\mathrm{n}} - \mathbf{T}^{+}\mathbf{x}_{\mathrm{m}} \|_{\mathbf{R}}^{2} dt \tag{2.7}$$

with **R** stands for non-negative weighted matrix of the appropriate dimension.

Usually, order n of (S) is not known, order m of (AM) may be highly chosen. In such a case, the validity of the lemma is kept; see the remark 1.1 of [11] for the details of argument.

Lemma 2.2. Let the state vector $\mathbf{x}_n$ of (S) be a transformed state vector of (AM) as

$$\mathbf{x}_{\mathrm{n}} = \mathbf{T}^{+}\mathbf{x}_{\mathrm{m}}, \ \mathbf{T} \in R^{\mathrm{mxn}}, \ \rho(\mathbf{T}) = \mathrm{n} < \mathrm{m} \tag{2.8}$$

then **T** can be factorized as

$$\mathbf{T} = \mathbf{EG} = \mathbf{HE} \tag{2.9}$$

where, $\mathbf{E} = \mathrm{E}(\mathbf{x}_m \mathbf{x}_n^T) \in P^{mxn}$ is a partial isometry, $\mathbf{G} = \mathrm{E}(\mathbf{x}_n \mathbf{x}_n^T) \in P^{nxn}$, $\mathbf{H} = \mathrm{E}(\mathbf{x}_m \mathbf{x}_m^T) \in P^{mxm}$, both are non-negative definite matrices.

Proof. See [11] for details.

Remark 2.1. It is noted that since **T** is constant $\mathbf{x}_n^{\cdot} = \mathbf{T}^{+}\mathbf{x}_m^{\cdot}$ is also valid.

It is known that $\sigma_1 = \mathbf{EE}^T, \sigma_2 = \mathbf{E}^T\mathbf{E}$ are optimal in the sense that one state vector is optimized with respect to the other; moreover both are of orthogonal projection matrix.

Although $\mathbf{x}_n$ and $\mathbf{x}_m$ are definitely specified but **T** is not unique determined due to mismatch between the dimensions of two state vectors. The question arises regarding the construction of **T** so that $\mathbf{x}_n$ is obtainable from the knowledge of $\mathbf{x}_m$.

3. TYPICAL PROBLEMS IN OPEN-LOOP THINKING

3.1. Problem of parameter estimation

State-descriptive models have been shown avoiding the usage of linear dynamical operators in supplying derivative measurements of (S) input and output signals for identification purpose [15, 16]. It has also been shown that by parameter-optimization methodology, the complexity of mathematical involvement is un-avoided however and optimization with respect to the state variables is obtained as secondary effects.

3.1.1. Statement of the problem

Let an n-th order (S) be in the state-space equations described by (2.4) and let an m-th order, known parameters (AM) in the same space be available by (2.5) subjecting to the (S) input

$$\text{(S):}\quad \begin{aligned} \mathbf{x}_n^{\cdot} &= \mathbf{A}_n \mathbf{x}_n + \mathbf{B}_n \mathbf{u}_n \\ \mathbf{y}_n &= \mathbf{C}_n \mathbf{x}_n \end{aligned} \tag{3.1}$$

$$\text{(AM):}\quad \begin{aligned} \mathbf{x}_m^{\cdot} &= \mathbf{A}_m \mathbf{x}_m + \mathbf{B}_m \mathbf{u}_n \\ \mathbf{y}_m &= \mathbf{C}_m \mathbf{x}_m \end{aligned} \tag{3.2}$$

where $\mathbf{u}_n$, $\mathbf{y}_n$ and $\mathbf{y}_m$ are p-, q- and q-dimensional vectors, matrices $\mathbf{A}_n$, $\mathbf{B}_n$, $\mathbf{C}_n$, $\mathbf{A}_m$, $\mathbf{B}_n$ and $\mathbf{C}_m$ are appropriately dimensioned.

Assume all the requirements happened to be in the parameter estimation process are satisfied. The parameters of (S) are estimated on adopting the state-optimization criterion.

3.1.2. Solution of the problem

Theorem 3.1. Let the measurements of a system (S) of order n be available for the parameter estimation. Let a controllable and observable (AM) of order m, m > n, be chosen with known parameters. Then there exists an optimal orthogonal projection matrix $\boldsymbol{\sigma} = \mathbf{EE}^T \in P^{mxm}$, $\rho(\boldsymbol{\sigma}) = n$, and two non-negative definite matrices $\mathbf{Q} = \mathbf{HEW_cE}^T$, $\mathbf{P} = \mathbf{H^+EW_oE}^T \in P^{mxm}$, both of rank n, such that the parameters of the controllable and observable part of (S) are computable from

$$\mathbf{A}_n = \mathbf{E}^T\mathbf{H}^+\mathbf{A}_m\mathbf{HE},\ \mathbf{B}_n = \mathbf{E}^T\mathbf{H}^+\mathbf{B}_m,\ \mathbf{C}_n = \mathbf{KC}_m\mathbf{HE} \tag{3.3}$$

which satisfy the following conditions

$$\boldsymbol{\sigma}(\mathbf{H}^+\mathbf{A}_m\mathbf{Q} + \mathbf{QA}_m^T\mathbf{H}^+ + \mathbf{H}^+\mathbf{B}_m\mathbf{VB}_m^T\mathbf{H}^+)\boldsymbol{\sigma}^T = \mathbf{0} \tag{3.4}$$

$$\boldsymbol{\sigma}^T(\mathbf{HA}_m^T\mathbf{P} + \mathbf{PA}_m\mathbf{H} + \mathbf{HC}_m^T\mathbf{K}^T\mathbf{RKC}_m\mathbf{H})\boldsymbol{\sigma} = \mathbf{0} \tag{3.5}$$

where $\mathbf{E} = E(\mathbf{x_m}\mathbf{x}_n^T) \in P^{mxn}$ is a partial isometry, $\mathbf{H} = E(\mathbf{x}_m\mathbf{x}_m^T) \in P^{mxm}$ is a positive definite matrix, $\mathbf{W}_c$ and $\mathbf{W}_0$ are the controllability and observability gramians of the system and $\mathbf{K}$ is a similarity transformation for matching the output of (AM) with that of the system (S).

Proof. See [11]. Eqns (3.3) - (3.5) are termed as optimal projection equations (OPEQ).

Converse of Theorem 3.1: Let a controllable and observable (AM) of order m, m > n, be chosen. Assume that the parameters of (S) are determinable with (3.3) satisfying (3.4) and (3.5). Then, $\boldsymbol{\sigma}$, $\mathbf{Q}$ and $\mathbf{P}$ are optimal.

Proof. It requires to show optimal in the sense of satisfying the criterion for state-optimization and the quadratically weighted output-errors. The detail is available in [11].

Remark 3.1. Theorem deals with the measurements $\mathbf{W}_c$ and $\mathbf{W}_o$ of (S). If (3.4) and (3.5) are solvable, $\mathbf{Q}$ and $\mathbf{P}$ are obtainable and $\mathbf{E}$ follows. Then, parameters of (S) are determinable irrespective of the measurability of $\mathbf{W}_c$ and $\mathbf{W}_o$. A difficulty in solving these equations stands on the fact that no standard algorithm is available regarding the guarantee for convergence of solutions.

Eqns (3.4) and (3.5) are seen to be readily decoupled owing the role of partial isometry ($\boldsymbol{\sigma}$ is always an orthogonal projection matrix). Thus, factorizing $\mathbf{T}$ in terms of an isometry has an effect equivalent to that of an additional constrained-condition.

System identification problem has been shown to switch over to the development of suitable algorithm for solving the OPEQ, which permits one to avoid using linear dynamical operators and to get ride off the persistently exciting property (to be imposed on (S) input), meeting the demand of real-time estimation of (S) parameters.

3.2. Robustness of modeling

A linear uncertain (S) was interpreted to have real-valued, structured parameter uncertainty [17]. A more reasonable argument, the mentioned (S) has been considered to have uncertain perturbations on the nominal values of its states. That is, $\mathbf{x}_n(t) + \Delta\mathbf{x}_n(t) = \mathbf{x}_s(t)$ [11].

3.2.1. Statement of the problem

For a linear uncertainty (S) order n described by

$$\mathbf{x}_s^{\cdot}(t) = \mathbf{A}_s\mathbf{x}_s(t) + \mathbf{B}_s\mathbf{w}(t) \tag{3.6}$$

$$\mathbf{y}_{\mathrm{s}}(t) = \mathbf{C}_{\mathrm{s}}\mathbf{x}_{\mathrm{s}}(t) \tag{3.7}$$

and an invited (AM) of order m > n, described by

$$\dot{\mathbf{x}}_{\mathrm{m}}(t) = \mathbf{A}_{\mathrm{m}}\mathbf{x}_{\mathrm{m}}(t) + \mathbf{B}_{\mathrm{m}}\mathbf{w}(t) \tag{3.8}$$

$$\mathbf{y}_{\mathrm{m}}(t) = \mathbf{C}_{\mathrm{m}}\mathbf{x}_{\mathrm{m}}(t) \tag{3.9}$$

with q-dimensional vectors $\mathbf{y}_s(t) = \mathbf{y}_n(t) + \Delta\mathbf{y}_n(t)$ and $\mathbf{y}_m(t)$, appropriately dimensioned matrices $\mathbf{A}_s = \mathbf{A}_n(t) + \Delta\mathbf{A}_n(t)$, $\mathbf{B}_s = \mathbf{B}_n(t) + \Delta\mathbf{B}_n(t)$, $\mathbf{C}_s = \mathbf{C}_n(t) + \Delta\mathbf{C}_n(t)$, $\mathbf{A}_m$, $\mathbf{B}_m$ and $\mathbf{C}_m$, there exists state-optimization criterion with $\mathbf{T} + \Delta\mathbf{T} = \mathbf{T}_s$

$$\mathrm{J}_{\mathrm{Sopt}} = \mathrm{SupE}\left\{\left\|\mathbf{x}_{\mathrm{s}} - \mathbf{T}_{\mathrm{s}}^{+}\mathbf{x}_{\mathrm{m}}\right\|_{\mathbf{R}}^{2}\right\},\ \mathbf{T}_{\mathrm{s}} \in \mathrm{P}^{\mathrm{mxn}} \tag{3.10}$$

and corresponding quadratically weighted output-error criterion with $\mathbf{K} + \Delta\mathbf{K} = \mathbf{K}_s$

$$\mathrm{J}_{\mathrm{Oopt}} = \mathrm{SupE}\left\{\left\|\mathbf{y}_{\mathrm{m}} - \mathbf{K}_{\mathrm{s}}\mathbf{y}_{\mathrm{s}}\right\|_{\mathbf{R}}^{2}\right\},\ \mathbf{K}_{\mathrm{s}} \in \mathrm{P}^{\mathrm{pxp}},\ \rho(\mathbf{K}_{\mathrm{s}}) = \mathrm{q}. \tag{3.11}$$

Determine conditions for robust performance, bounds of $\mathbf{A}_s$, $\mathbf{B}_s$ and $\mathbf{C}_s$ so that (S) described by (3.6) and (3.7) is to be controllable and observable.

3.2.2. Solution of problem

1. Sufficient conditions for robust performance:

a) Assumption

- Vector norm and matrix norm be consistent,
- For a chosen (AM), $\|\mathbf{x}_{\mathrm{m}}\| = n$, which is constant,
- $\|\mathbf{T}\| = (\lambda_1)^{1/2}, \|\mathbf{T}^{+}\| = 1/(\lambda_n)^{1/2}$ where λ_1 and λ_n are the maximum and the least non-zero eigenvalues of $\mathbf{TT}^{\mathrm{T}}$.

b) Conditions

- $\|\Delta\mathbf{T}\|/\|\mathbf{T}\| = (\lambda_n)^{1/2}/((\lambda_n)^{1/2} + n),\ \|\mathbf{T}_{\mathrm{s}}\| = (\lambda_1)^{1/2}.\left\{1 + (\lambda_n)^{1/2}/((\lambda_n)^{1/2} + n)\right\},$
- $\|\Delta\mathbf{T}^{+}\|/\|\mathbf{T}^{+}\| = (\lambda_n)^{1/2}/n,$
- $\mathrm{J}_{\mathrm{Oopt}} \le \|\Delta\mathbf{x}_{\mathrm{n}}\| + \|\Delta\mathbf{T}^{+}\|.\|\mathbf{x}_{\mathrm{n}}\| = 2,$
- $\|\mathbf{K}_{\mathrm{s}}\| = 1/\|\mathbf{T}_{\mathrm{m}}\|,\ \|\Delta\mathbf{K}\| = \left(3(\lambda_n)^{1/2} + 2n\right)/(\lambda_1)^{1/2}.\left(2(\lambda_n)^{1/2} + n\right),$
- $1/(\lambda_n)^{1/2} \le \|\mathbf{x}_{\mathrm{s}}\| \le 1/(\lambda_m)^{1/2}.$

2. Uncertainty structure

a) Assumption

- $\mathbf{V} = \mathbf{I}_p$, $\mathbf{K} = \mathbf{R} = \mathbf{I}_q$,
- $\mathbf{A}_{\mathrm{m}} = diag(-\alpha_1 \ldots -\alpha_{\mathrm{m}}), \mathbf{B}_{\mathrm{m}}\mathbf{B}_{\mathrm{m}}^{\mathrm{T}} = diag(b_1 \ldots b_{\mathrm{m}}), \mathbf{C}_{\mathrm{m}}^{\mathrm{T}}\mathbf{C}_{\mathrm{m}} = diag(g_1 \ldots g_{\mathrm{m}}),$
- Maximum variations of parameters are computed by theorem 3.1.

b) Variation of parameters

- $\|\Delta\mathbf{A}_{\mathrm{n}}\| \le 2(\alpha_1\lambda_1)^{1/2}/\nu$, $\|\Delta\mathbf{B}_{\mathrm{n}}\| \le (\beta_1)^{1/2}/\nu$, $\|\Delta\mathbf{C}_{\mathrm{n}}\| \le (\gamma_1\lambda_1\lambda_{\mathrm{n}})^{1/2}\Big/\big((\lambda_{\mathrm{n}})^{1/2}+\nu\big)$,
- $\|\mathbf{A}_{\mathrm{s}}\| \le (\alpha_1\lambda_1)^{1/2}\big(2(\lambda_{\mathrm{n}})^{1/2}+\nu\big)\Big/\big(\nu+(\lambda_{\mathrm{n}})^{1/2}\big)$,
- $\|\mathbf{B}_{\mathrm{s}}\| \le (\beta_1)^{1/2}\big((\lambda_{\mathrm{n}})^{1/2}+\nu\big)\Big/\big(\nu+(\lambda_{\mathrm{n}})^{1/2}\big)$,
- $\|\mathbf{C}_{\mathrm{s}}\| \le (\alpha_1\lambda_1)^{1/2}\big(2(\lambda_{\mathrm{n}})^{1/2}+\nu\big)\Big/\big(\nu+(\lambda_{\mathrm{n}})^{1/2}\big)$.

3. Stability, Controllabity and Observability

a) Assumption

- Positions of poles corresponding to $-\alpha_{\mathrm{sn}}$ be not shifted to R.H.S of complex-plane,
- Number of non-zero eigenvalues of $\mathbf{B}_{\mathrm{s}}\mathbf{B}_{\mathrm{s}}^{\mathrm{T}}$ and of $\mathbf{C}_{\mathrm{s}}^{\mathrm{T}}\mathbf{C}_{\mathrm{s}}$ be kept unchanging (none of eigenvalues of $\mathbf{B}_{\mathrm{s}}\mathbf{B}_{\mathrm{s}}^{\mathrm{T}}$ and $\mathbf{C}_{\mathrm{s}}^{\mathrm{T}}\mathbf{C}_{\mathrm{s}}$ be annulled due to $\Delta\mathbf{B}_{\mathrm{n}}$ and $\Delta\mathbf{C}_{\mathrm{n}}$),
- n eigenvalues of $\mathbf{B}_s\mathbf{B}_s^{\mathrm{T}}$ be differed from those of $\mathbf{C}_{\mathrm{s}}^{\mathrm{T}}\mathbf{C}_s$,
- $\left\{\Delta\boldsymbol{\sigma}\mathbf{H}^{+}\mathbf{A}_{\mathrm{m}}, \Delta\boldsymbol{\sigma}\mathbf{H}^{+}\left(\mathbf{B}_{\mathrm{m}}\mathbf{B}_{\mathrm{m}}^{\mathrm{T}}\right)^{1/2}\right\}$, $\left\{\left(\mathbf{C}_{\mathrm{m}}^{\mathrm{T}}\mathbf{C}_{\mathrm{m}}\right)^{1/2}\mathbf{H}\Delta\boldsymbol{\sigma}, \mathbf{A}_{\mathrm{m}}\mathbf{H}\Delta\boldsymbol{\sigma}\right\}$ be stabilizable, detectable.

b) Conditions

- $2(\alpha_1\lambda_1)^{1/2}/\nu \le (\alpha_{\mathrm{sn}})^{1/2}$, $(\beta_1)^{1/2}/\nu \le (\beta_{\mathrm{sn}})^{1/2}$, $(\gamma_1\lambda_1\lambda_{\mathrm{n}})^{1/2}\Big/\big((\lambda_{\mathrm{n}})^{1/2}+\nu\big) \le (\gamma_{\mathrm{sn}})^{1/2}$,
- $\mathbf{H}^{+}\mathbf{A}_{\mathrm{m}}\Delta\mathbf{Q}+\Delta\mathbf{Q}\mathbf{A}_{\mathrm{m}}^{\mathrm{T}}\mathbf{H}^{+}=\Omega(\mathbf{Q})$, $\mathbf{H}\mathbf{A}_{\mathrm{m}}^{\mathrm{T}}\Delta\mathbf{P}+\Delta\mathbf{P}\mathbf{A}_{\mathrm{m}}\mathbf{H}=\Omega(\mathbf{P})$,
- $\mathbf{H}^{+}\mathbf{A}_{\mathrm{m}}\mathbf{Q}+\mathbf{Q}\mathbf{A}_{\mathrm{m}}^{\mathrm{T}}\mathbf{H}^{+}+\Omega(\mathbf{Q})+\mathbf{H}^{\mathrm{T}}\mathbf{B}_{\mathrm{m}}\mathbf{B}_{\mathrm{m}}^{\mathrm{T}}\mathbf{H}^{+}=\mathbf{0}$,

$\mathbf{H}\mathbf{A}_{\mathrm{m}}^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{A}_{\mathrm{m}}\mathbf{H}+\Omega(\mathbf{P})+\mathbf{H}\mathbf{C}_{\mathrm{m}}^{\mathrm{T}}\mathbf{C}_{\mathrm{m}}\mathbf{H}=\mathbf{0}$

- $\|\Delta\mathbf{Q}\|$, $\|\Delta\mathbf{P}\|$ are bounded.

Proof. See [11] for the details.

Remark 3.2. If (AM) is not properly chosen, estimated (S) may turn out to be uncontrollable, unobservable. (AM) plays the role as that of initial linear model in a recursive process.

State-optimization approach permits the norms of vectors and of matrices to be employed in tackling different robust problems while optimality equations serve as sufficient conditions for characterizing the robustness.

3.3. Problem of order reduction for model

3.3.1. Statement of the problem

Given an n-th order (S) described in state-variable equations with appropriately dimensioned matrices and vectors as follows

$$\mathbf{x}_n^{\cdot} = \mathbf{A}_n\mathbf{x}_n + \mathbf{B}_n\mathbf{u} \tag{3.12}$$

$$\mathbf{y}_n = \mathbf{C}_n\mathbf{x}_n \tag{3.13}$$

Determine a model of order $r, q \le r \le n$

$$\mathbf{x}_r^{\cdot} = \mathbf{A}_r\mathbf{x}_r + \mathbf{B}_r\mathbf{u} \tag{3.14}$$

$$\mathbf{y}_r = \mathbf{C}_r\mathbf{x}_r \tag{3.15}$$

Satisfying following coditions

- L_2 model-reduction criterion,
- $(\mathbf{A}_r, \mathbf{B}_r, \mathbf{C}_r)$: Controllable and observable; $(\mathbf{A}_r, \mathbf{B}_r)$: Stabilizable, $(\mathbf{A}_r, \mathbf{C}_r)$: Detectable.

3.3.2. Solution of the problem

Theorem 3.2. For a given linear, time-invariant (S) of the order n, there exists always an rxn partial isometry **E** and an nxn non-negative definite matrix such that the optimal parameters of the reduced-order model are given by

$$\mathbf{A}_r = \mathbf{EHA}_n\mathbf{H}^+\mathbf{E}^T,\ \mathbf{B}_r = \mathbf{EHB}_n,\ \mathbf{C}_r = \mathbf{C}_n\mathbf{H}^+\mathbf{E}^T \tag{3.16}$$

Further, there exists an nxn optimal projector $\boldsymbol{\sigma}$ and two nxn non-negative definite matrices **Q** and **P** such that if the optimal model is to be controllable and observable, then the following conditions are to be satisfied

$$\boldsymbol{\sigma}\left[\mathbf{HA}_n\mathbf{Q} + \mathbf{QA}_n^T\mathbf{H} + \mathbf{HB}_n\mathbf{V}_1\mathbf{B}_n^T\mathbf{H}\right] = \mathbf{0} \tag{3.17}$$

$$\left[\mathbf{H}^+\mathbf{A}_n^T\mathbf{P} + \mathbf{PA}_n\mathbf{H}^+ + \mathbf{H}^+\mathbf{C}_n^T\mathbf{R}_2\mathbf{C}_n\mathbf{H}^+\right]\boldsymbol{\sigma} = \mathbf{0} \tag{3.18}$$

where $\mathbf{V}_1 = E(\mathbf{uu}^T)$, $\mathbf{R}_2$ is weighted matrix in the criterion for order reduction.

Proof. See [13, 22] for the details.

Remark 3.3. Non-similarity transformation **T** is chosen rather freely, which permits physical significances of various different particularly modeled states to be retained in the reduced model.

A considerable effort is reduced for finding the global amongst multi-local extreme due to the effect of factorizing **T** in terms of a partial isometrics **E**.

Robustness of reduced-order model can be carried out by adopting the same manner as that of robustness modeling. A great effort would be reduced in tackling the mentioned robustness by adopting the state-optimization approach with respect to the parameter-optimization technique.

It is found also that robust problems play an important role in estimating technology standard, which is on the direction for further researches.

5. CONCLUDING REMARKS

Optimal projection equation (OPEQ) has been recognized to play an important contribution to finding the uniqueness amongst multi-extreme in the effect sense of an aditionally constrained condition. However, a complexity happened to be in mathematical involvement of that OPEQ on adopting parameter-optimization process from both aspects; in the establishment and in the solution to the mentioned OPEQ. State-optimization has been found removing that complexity due to the role of factorization in term of a partial isometry and mentioned factorization has an effect of that of an additionally constrained condition to the optimization process.

State-optimization approach can be employed to treating different various problems where an optimization is asked for. In the case of an infinite-dimensional (S) like distributed parameter, non-linear modeled by a series, ect., where partial or functional equations are required, then the concept of generaliazed Green function and its inverse are to be adopted, however. This may gives rise to the concept of a poly-optimization in stead of state-optimization and various researches can be carried out in this direction apart from treating the above mentioned infinite-dimensional (S) also for treating many different optimization problems happened to be in non-finite dimensional space.

It will show in the coming report, through the consideration some typical closed-loop thinking problems, great efforts would be reduced with respect to parameter-optimization approach on adopting the results obtained for opened-loop thinking ones.

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TÓM TẮT

VỀ PHƯƠNG PHÁP TỐI ƯU THEO TRẠNG THÁI VỚI CÁC BÀI TOÁN HỆ THỐNG: XỬ LÍ THEO TƯ DUY HỆ HỞ

Có thể phân các bài toán thuộc lĩnh vực lí thuyết hệ thống thành 4 nhóm chính: mô phỏng, xác lập phương trình toán học, phân tích hệ và thiết kế hệ thống. Khi giới hạn những bàn luận đối với một hệ thống được mô tả bởi hệ phương trình trong không gian trạng thái thì có thể phân các bài toán thành nhóm phụ thuộc vào kiểu xử lí: cách của tư duy hệ hở và cách của tư duy hệ kín. Gần đây nhất, có hai phương pháp tiếp cận đáng chú ý đối với cả hai kiểu xử lí là sử dụng điều kiện cân bằng nội và hệ phương trình quy chiếu tối ưu (OPEQ). Phương pháp đề xuất trên cơ sở điều kiện cân bằng nội có ưu điểm nổi trội là sử dụng được tính bất biến về đóng góp của động học vào quá trình tạo ra quan hệ vào ra của hệ, nhưng lại bị hạn chế lớn nhìn trên quan điểm tối ưu do không biết được nghiệm tối ưu mặc dù đôi chỗ vẫn có dùng một tiêu chí nào đấy. Phương pháp

xây dựng OPEQ loại bỏ được hạn chế về tính tối ưu nhưng lại đối mặt với tính phức tạp về mặt sử dụng toán học trong quá trình phát triển, tìm nghiệm của OPEQ, tuy rằng phương pháp OPEQ được xác định là tìm ra điều kiện ràng buộc thêm vào các điều kiện ban đầu của bài toán tối ưu. Phương pháp tối ưu theo trạng thái do San đề xuất được minh chứng đã thụ hưởng các ưu điểm, bỏ lại hạn chế của cả hai phương pháp đã nêu mà còn tạo ra hiệu ứng như của một điều kiện ràng buộc mới thêm vào nhờ vào việc thừa số hoá phép biến đổi không đồng nhất (non-similarity transformation) giữa các vector trạng thái của hai hệ động học theo đẳng cự thành phần (partial isometry).

Phần đầu của bài báo này giành để giới thiệu tổng quát về nội dung của bài báo. Phần thứ hai giành để tóm tắt hai đề xuất cơ bản liên quan đến tiêu chí tối ưu trạng thái và thừa số hoá biến đổi không đồng nhất làm sở cứ để giải quyết các bài toán điển hình của lí thuyết hệ thống cần được xử lí bằng cách của tư duy hệ hở được trình bày trong phần thứ ba. Tuy các phép chiếu tối ưu tìm thấy bởi phương pháp tối ưu trạng thái đều vuông làm đơn giản đáng kể, nhưng tính phức tạp về mặt toán học vẫn còn hiện diện khá rõ nét ở quá trình xử lí, xây dựng hệ phương trình OPEQ đối với hầu hết các quá trình xác định nghiệm của các bài toán về tính bền vững.

Phần cuối cùng giành để bình phẩm, nêu định hướng nghiên cứu áp dụng kết quả đã thu được đối với những chủ đề kế tiếp của các bài toán thuộc lĩnh vực lí thuyết hệ thống, kể cả những nội dung sẽ công bố trong công trình tiếp theo.