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BACKSTEPPING CONTROL USING NONLINEAR STATE OBSERVER FOR SWITCHED RELUCTANCE MOTOR

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Abstract. Switched reluctance motor (SRM) has many advantages with very strong nonlinearity, hence it is difficult to control. This paper presents a new method to control the speed of switched reluctance motor based on backstepping technique and nonlinear state observation. This controller is first applied for switched reluctance motor with nonlinear drive model. This model is a combination of both the commutator and the motor in the same model. The combined model of switched reluctance motor helps to reduce the influence of nonlinearity due to the switching lock, increasing the accuracy in controlling this motor. The state variables of the controller are approximated by nonlinear state observer, including speed observer, flux observer, and rotor position observer. The observer state variables are compared with directly measured state variables. This nonlinear state observer improves the switched reluctance motor drive system by reducing actual measuring devices, such as incremental encoder and torque transducer. The stability of the closed control loop was analyzed using Lyapunov stability criterion. The simulation is carried out with both traditional backstepping controller and the backstepping controller combined with nonlinear state observation. The quality of the nonlinear state observer and backstepping control system are analyzed theoretically and through numerical simulations.

Keywords: Swiched Redutance Motor; Backstepping Technique; Nonlinear State Observer.

Classification numbers: 4.10.3; 4.10.4.

1. INTRODUCTION

The switched reluctance motor (SRM) is an electric motor with many outstanding advantages such as low manufacturing cost, simple structure, wireless rotor that allows high working temperature, large torque, etc. [1 - 3]. Due to the structure of the SRM and the operating principle of continuous switching between each phase, the SRM has strong nonlinearity. In many works, SRM models have been presented as linear or nonlinear for independent phases [4 - 6]. In the work [7], it was the first combined dynamic model of SRM with logical transitions in one model. Then, the author of this work has linearized the dynamic model to design the linear controller for SRM.

In order to reduce error caused by linear modeling process, in this paper, we propose a new design method based on backstepping technique combined with nonlinear state observer. The research results are verified through numerical simulations.

After general introduction, this paper is organized as follows. Section 2 presents the nonlinear model of SRM (the model including the phase shift switches and dynamics of SRM). In section 3, a controller based on backstepping technique and nonlinear state observer are designed. Section 4 illustrates the simulation results. Finally, section 5 is conclusion.

2. NONLINEAR MODEL OF SRM SYSTEM

The mathematical model of SRM used to design the controller is represented in the form of differential equations based on the basic machine equations. The dynamics of SRM includes voltage equations, torque equations and mechanical equations.

Differential equation of SRM with m phases has the following form:

$$u_j = R.i_j + \frac{d\psi_j}{dt} \tag{1}$$

where: $j = 1, 2, 3, \ldots, m$; u_j is voltage of phase *j; R* is resistor of phase *j;* i_j is current of phase *j;* ψ_j is flux of phase *j*.

From equation (1), flux of any phase *j* is represented:

$$\psi_j = \int_0^T (v_j - R.i_j)dt \tag{2}$$

Flux ψ_j depends on both current i_j and the angle θ, so it is represented in details as follows: $\psi_j\left(i_j, \theta\right)$.

The mechanical equation of SRM:

$$J\frac{d^2\theta}{dt^2} = T_e - T_l \tag{3}$$

where: T_e is torque of one phase; T_l is torque of load; J is moment of inertia.

According to the principle of energy conversion in SRM, the torque generated is equal to the energy variation of magnetic field in stator coil according to rotor angular position.

$$T_j(\theta, i_j) = \frac{\partial \mathrm{W}_j^{'}}{\partial\theta} \tag{4}$$

where:

$$\partial \mathrm{W}_j^{'}(\theta, i_j) = \int_0^{i_j} \psi_j(\theta, i_j)di_j \tag{5}$$

The torque of the SRM is a nonlinear function in terms of current and rotor position. Then, the total torque generated is equal to the total torque of phases:

$$T_e(\theta, i_1, i_2, \ldots, i_m) = \sum_{j=1}^{m} T_j(\theta, i_j) \tag{6}$$

To control the SRM, we need to determine the flux $\psi_j\left(i_j,\theta\right)$ as accurately as possible. For convenience in the research and development of control algorithms, flux property can be approximated as a continuous function [7], as follows:

$$\psi_j(\theta,i_j)=\psi_s(1-e^{-i_j f_j(\theta)}) \tag{7}$$

with j = 1, 2, …, m; ψ_s is magnetic flux saturation.

In general, due to special structure of SRM, the performance of this motor is not the same as that of general electric motor. Rotor of SRM rotates at discrete angles so the function $f_j\left(\theta\right)$ can be represented by a Fourier series as follows:

$$f_j(\theta)=a+\sum_{n=1}^{\infty}\{b_n\sin[nN_r\theta-(j-1)\frac{2\pi}{m}]+c_n\cos[nN_r\theta-(j-1)\frac{2\pi}{m}]\} \tag{8}$$

in which, N_r is the number of rotor poles, and if higher order components in Fourier series is omitted, the simpler function (8) is obtained:

$$f_j(\theta)=a+b\sin[N_r\theta-(j-1)\frac{2\pi}{m}] \tag{9}$$

The torque of phase j is represented as follows:

$$T_j(\theta,i_j)=\frac{\psi_s}{f_j^2(\theta)}\frac{df_j(\theta)}{d\theta}\{1-[1+i_j f_j(\theta)]e^{-i_j f_j(\theta)}\} \tag{10}$$

The state space equations of SRM as follows:

$$\begin{cases}\dfrac{d\theta}{dt}=\omega\\ \dfrac{d\omega}{dt}=\dfrac{1}{J}\left\{\displaystyle\sum_{j=1}^{m}T_j(\theta,i_j)-T_l(\theta,\omega)\right\}\\ \dfrac{di_j}{dt}=-\left(\dfrac{\partial\psi_j}{\partial i_j}\right)\left(Ri_j+\dfrac{\partial\psi_j}{\partial\theta}\omega\right)+\left(\dfrac{\partial\psi_j}{\partial i_j}\right)^{-1}u_j\end{cases} \tag{11}$$

The state model of SRM is illustrated below based on [7]. Considering SRM with m = 4 phases, the state vector is $x=[\theta,\omega,i_1,i_2,i_3,i_4]^T=[x_1,x_2,x_3,x_4,x_5,x_6]^T$. The state equation of motor is:

$$\dot{x}_1=x_2 \tag{12}$$

$$\dot{x}_2 = \frac{1}{J}\left[T_1(\theta, x_3) + T_2(\theta, x_4) + T_3(\theta, x_5) + T_4(\theta, x_6) - T_l(x_1, x_2)\right]$$

$$= \frac{1}{J}\left[\begin{array}{l} \frac{\psi_s}{f_1^2(x_1)}\frac{\partial f_1(x_1)}{\partial x_1} N_r \left\{1-[1+x_3 f_1(x_1)]e^{-x_3 f_1(x_1)}\right\} + \frac{\psi_s}{f_2^2(x_1)}\frac{\partial f_2(x_1)}{\partial x_1} N_r \left\{1-[1+x_4 f_2(x_1)]e^{-x_4 f_2(x_1)}\right\} \\ + \frac{\psi_s}{f_3^2(x_1)}\frac{\partial f_3(x_1)}{\partial x_1} N_r \left\{1-[1+x_5 f_3(x_1)]e^{-x_5 f_3(x_1)}\right\} + \frac{\psi_s}{f_4^2(x_1)}\frac{\partial f_4(x_1)}{\partial x_1} N_r \left\{1-[1+x_6 f_4(x_1)]e^{-x_6 f_4(x_1)}\right\} \\ -Bx_2 - mgl\sin(x_1) \end{array}\right] \tag{13}$$

$$\dot{x}_3 = \left[-\psi_s e^{-x_3 f_1(x_1)} f_1(x_1)\right]^{-1}\left[Rx_3 + \left(\psi_s e^{-x_3 f_1(x_1)}\right)\left(x_3 \tfrac{\partial f_1(x_1)}{\partial x_1}\right)x_2\right] + \left[\psi_s e^{-x_3 f_1(x_1)} f_1(x_1)\right]^{-1} u_1 \tag{14}$$

$$\dot{x}_4 = \left[-\psi_s e^{-x_4 f_2(x_1)} f_2(x_1)\right]^{-1}\left[Rx_4 + \left(\psi_s e^{-x_4 f_2(x_1)}\right)\left(x_4 \tfrac{\partial f_2(x_1)}{\partial x_1}\right)x_2\right] + \left[\psi_s e^{-x_4 f_2(x_1)} f_2(x_1)\right]^{-1} u_2 \tag{15}$$

$$\dot{x}_5 = \left[-\psi_s e^{-x_5 f_3(x_1)} f_3(x_1)\right]^{-1}\left[Rx_5 + \left(\psi_s e^{-x_5 f_3(x_1)}\right)\left(x_5 \tfrac{\partial f_3(x_1)}{\partial x_1}\right)x_2\right] + \left[\psi_s e^{-x_5 f_3(x_1)} f_3(x_1)\right]^{-1} u_3 \tag{16}$$

$$\dot{x}_6 = \left[-\psi_s e^{-x_6 f_4(x_1)} f_4(x_1)\right]^{-1}\left[Rx_6 + \left(\psi_s e^{-x_6 f_4(x_1)}\right)\left(x_6 \tfrac{\partial f_4(x_1)}{\partial x_1}\right)x_2\right] + \left[\psi_s e^{-x_6 f_4(x_1)} f_4(x_1)\right]^{-1} u_4 \tag{17}$$

Where:

$$\frac{\partial f_i}{\partial x_1} = bN_r \cos\left(N_r x_1 - (j-1)\frac{2\pi}{m}\right) \tag{18}$$

Note that, in the upper state spatial model, Bx_2 is an opposite component to the rotation while mgl is the torque of the load.

From (13), we denote:

$$f_a(x) = \frac{1}{J}\left[\frac{\psi_s}{f_1^2(x_1)}\frac{\partial f_1(x_1)}{\partial x_1} N_r \left\{1 - e^{-x_3 f_1(x_1)}\right\}\right]$$

$$g_a(x) = \frac{1}{J}\left[\frac{\psi_s}{f_1^2(x_1)}\frac{\partial f_1(x_1)}{\partial x_1} N_r \left\{-f_1(x_1) e^{-x_3 f_1(x_1)}\right\}\right]$$

$$f_b(x) = \frac{1}{J}\left[\frac{\psi_s}{f_2^2(x_1)}\frac{\partial f_2(x_1)}{\partial x_1} N_r \left\{1 - e^{-x_4 f_2(x_1)}\right\}\right]$$

$$g_b(x) = \frac{1}{J}\left[\frac{\psi_s}{f_2^2(x_1)}\frac{\partial f_2(x_1)}{\partial x_1} N_r \left\{-f_2(x_1) e^{-x_4 f_2(x_1)}\right\}\right]$$

$$f_c(x) = \frac{1}{J}\left[\frac{\psi_s}{f_3^2(x_1)}\frac{\partial f_3(x_1)}{\partial x_1} N_r \left\{1 - e^{-x_5 f_3(x_1)}\right\}\right]$$

$$g_c(x) = \frac{1}{J}\left[\frac{\psi_s}{f_3^2(x_1)}\frac{\partial f_3(x_1)}{\partial x_1} N_r \left\{-f_3(x_1) e^{-x_5 f_3(x_1)}\right\}\right]$$

$$f_d(x) = \frac{1}{J}\left[\frac{\psi_s}{f_4^2(x_1)} \frac{\partial f_4(x_1)}{\partial x_1} N_r \left\{ 1 - e^{-x_6 f_4(x_1)} \right\} \right]$$

$$g_d(x) = \frac{1}{J}\left[\frac{\psi_s}{f_4^2(x_1)} \frac{\partial f_4(x_1)}{\partial x_1} N_r \left\{ -f_4(x_1) e^{-x_6 f_4(x_1)} \right\} \right]$$

Equation (13) can be rewritten as follows:

$$\begin{aligned} \dot{x}_2 = &\left[f_a(x) + g_a(x) x_3 \right] + \left[f_b(x) + g_b(x) x_4 \right] + \left[f_c(x) + g_c(x) x_5 \right] + \\ &\left[f_d(x) + g_d(x) x_6 \right] - \frac{B}{J} x_2 - \frac{mgl}{J} \sin(x_1) \end{aligned} \tag{19}$$

Derivative (19) with respect to time, we have:

$$\begin{aligned} \ddot{x}_2 = &\left[\dot{f}_a(x) + \dot{g}_a(x) x_3 + g_a(x) \dot{x}_3 \right] + \left[\dot{f}_b(x) + \dot{g}_b(x) x_4 + g_b(x) \dot{x}_4 \right] + \\ &\left[\dot{f}_c(x) + \dot{g}_c(x) x_5 + g_c(x) \dot{x}_5 \right] + \left[\dot{f}_d(x) + \dot{g}_d(x) x_6 + g_d(x) \dot{x}_6 \right] - \frac{B}{J} \dot{x}_2 - \frac{mgl}{J} \cos(x_1) \dot{x}_1 \end{aligned} \tag{20}$$

From equation (14) to equation (17), we denote:

$$p_a(x) = \left[-\psi_s e^{-x_3 f_1(x_1)} f_1(x_1) \right]^{-1} \left[R x_3 + \left(\psi_s e^{-x_3 f_1(x_1)} \right) \left(x_3 \tfrac{\partial f_1(x_1)}{\partial x_1} \right) x_2 \right]$$

$$q_a(x) = \left[\psi_s e^{-x_3 f_1(x_1)} f_1(x_1) \right]^{-1}$$

$$p_b(x) = \left[-\psi_s e^{-x_4 f_2(x_1)} f_2(x_1) \right]^{-1} \left[R x_4 + \left(\psi_s e^{-x_4 f_2(x_1)} \right) \left(x_4 \tfrac{\partial f_2(x_1)}{\partial x_1} \right) x_2 \right]$$

$$q_b(x) = \left[\psi_s e^{-x_4 f_2(x_1)} f_2(x_1) \right]^{-1}$$

$$p_c(x) = \left[-\psi_s e^{-x_5 f_3(x_1)} f_3(x_1) \right]^{-1} \left[R x_5 + \left(\psi_s e^{-x_5 f_3(x_1)} \right) \left(x_5 \tfrac{\partial f_3(x_1)}{\partial x_1} \right) x_2 \right]$$

$$q_c(x) = \left[\psi_s e^{-x_5 f_3(x_1)} f_3(x_1) \right]^{-1}$$

$$p_d(x) = \left[-\psi_s e^{-x_6 f_4(x_1)} f_4(x_1) \right]^{-1} \left[R x_6 + \left(\psi_s e^{-x_6 f_4(x_1)} \right) \left(x_6 \tfrac{\partial f_4(x_1)}{\partial x_1} \right) x_2 \right]$$

$$q_d(x) = \left[\psi_s e^{-x_6 f_4(x_1)} f_4(x_1) \right]^{-1}$$

The equations from (14) to (17) are rewritten as follows:

$$\begin{cases} \dot{x}_3 = p_a(x) + q_a(x) u_1 \\ \dot{x}_4 = p_b(x) + q_b(x) u_2 \\ \dot{x}_5 = p_c(x) + q_c(x) u_3 \\ \dot{x}_6 = p_d(x) + q_d(x) u_4 \end{cases} \tag{21}$$

Substituting (21) into (20), we have:

$$\begin{aligned}\ddot{x}_2 &= \left[\dot{f}_a(x)+\dot{g}_a(x)x_3+g_a(x)p_a(x)+g_a(x)q_a(x)u_1\right]+\\ &\left[\dot{f}_b(x)+\dot{g}_b(x)x_4+g_b(x)p_b(x)+g_b(x)q_b(x)u_2\right]+\\ &\left[\dot{f}_c(x)+\dot{g}_c(x)x_5+g_c(x)p_c(x)+g_c(x)q_c(x)u_3\right]+\\ &\left[\dot{f}_d(x)+\dot{g}_d(x)x_6+g_d(x)p_d(x)+g_d(x)q_d(x)u_4\right]-\frac{B}{J}\dot{x}_2-\frac{mgl}{J}\cos(x_1)\dot{x}_1\end{aligned} \tag{22}$$

The SRM operates based on the principle of supplying voltage to each phase. If we consider the number of phases is 4, we have $u_j = k_j u$, with j = 1, 2, 3, 4, k_j is phase shift switch that only allows the values of 0 and 1. Equation (22) can be represented as follows:

$$\begin{aligned}\ddot{x}_2 = &\begin{bmatrix}\dot{f}_a(x)+\dot{g}_a(x)x_3+g_a(x)p_a(x)+\dot{f}_b(x)+\dot{g}_b(x)x_4+g_b(x)p_b(x)+\dot{f}_c(x)+\\ \dot{g}_c(x)x_5+g_c(x)p_c(x)+\dot{f}_d(x)+\dot{g}_d(x)x_6+g_d(x)p_d(x)\end{bmatrix}+\\ &\left[g_a(x)q_a(x)k_1+g_b(x)q_b(x)k_2+g_c(x)q_c(x)k_3+g_d(x)q_d(x)k_4\right]-\frac{B}{J}\dot{x}_2-\frac{mgl}{J}\cos(x_1)\dot{x}_1\end{aligned} \tag{23}$$

Then we denote:

$$\mathrm{F}(x)=\begin{bmatrix}\dot{f}_a(x)+\dot{g}_a(x)x_3+g_a(x)p_a(x)+\dot{f}_b(x)+\dot{g}_b(x)x_4+g_b(x)p_b(x)+\dot{f}_c(x)+\\ \dot{g}_c(x)x_5+g_c(x)p_c(x)+\dot{f}_d(x)+\dot{g}_d(x)x_6+g_d(x)p_d(x)\end{bmatrix}$$

and

$$\mathrm{G}(x)=\left[g_a(x)q_a(x)k_1+g_b(x)q_b(x)k_2+g_c(x)q_c(x)k_3+g_d(x)q_d(x)k_4\right]$$

We have another form of equation (23) as follows:

$$\ddot{x}_2=\mathrm{F}(x)+G(x)-\frac{B}{J}\dot{x}_2-\frac{mgl}{J}\cos(x_1)\dot{x}_1 \tag{24}$$

We denote:

$$\begin{cases} f(x)=\mathrm{F}(x)-\frac{B}{J}\dot{x}_2-\frac{mgl}{J}\cos(x_1)\dot{x}_1 \\ g(x)=G(x)\end{cases} \tag{25}$$

we have:

$$\ddot{x}_2 = f(x)+g(x)u \tag{26}$$

To facilitate the design, we represent equation (26) as a state model. Let $x_2 = z_1$, we have a state model of SRM:

$$\begin{cases}\dot{z}_1 = z_2\\ \dot{z}_2 = f(x)+g(x)u\end{cases} \tag{27}$$

With $f(x), g(x)$ are defined in equation (25).

The model in equation (27) is perfectly suitable to use backstepping technique to design the controller for SRM.

3. BACKSTEPPING CONTROL DESIGN USING NONLINEAR STATE OBSERVER FOR SRM

3.1. Backstepping control

In section 2 of this paper, the dynamic model of SRM is represented as state model (27):

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = f(x) + g(x)u \end{cases} \tag{28}$$

This is a model of a second order tight feedback system. According to the backstepping technique [8 - 11], we need to design in 2 steps.

Step 1: Let speed tracking error $z_d = \omega_d$ is e_1, we have:

$$e_1 = z_1 - z_d \tag{29}$$

Take derivative of the function e_1 over time, we have:

$$\dot{e}_1 = \dot{z}_1 - \dot{z}_d = z_2 - \dot{z}_d \tag{30}$$

Let $e_2 = z_2 - \alpha_1$ in which, α_1 is virtual control signal for the first sub-system.

Substituting to equation (30), we have:

$$\dot{e}_1 = \dot{z}_1 - \dot{z}_d = z_2 - \dot{z}_d = e_2 + \alpha_1 - \dot{z}_d \tag{31}$$

To determine virtual control signal that ensures $e_1 \to 0$, we choose member Lyapunov function:

$$V_1 = \frac{1}{2}e_1^2 \tag{32}$$

Take derivative of V_1 over time we have:

$$\dot{V}_1 = e_1\dot{e}_1 = e_1\left(e_2 + \alpha_1 - \dot{z}_d\right) = -c_1e_1^2 + e_1e_2 \tag{33}$$

To have equation (33), virtual control signal has following form:

$$\alpha_1 = -c_1e_1 + \dot{z}_d \tag{34}$$

in which, c_1 is a positive constant. If $e_1 \to 0$ then $e_2 \to 0$

Step 2:

$$e_2 = z_2 - \alpha_1 \tag{35}$$

Take derivative of the function e_2 over time, we have:

$$\dot{e}_2 = \dot{z}_2 - \dot{\alpha}_1 \tag{36}$$

From (28), we have:

$$\dot{e}_2 = \dot{z}_2 - \dot{\alpha}_1 = f(x) + g(x)u - \dot{\alpha}_1 \tag{37}$$

To determine the signal control *u* that ensures $e_2 \to 0$, we choose Lyapunov function for the closed loop:

$$V_2 = V_1 + \frac{1}{2}e_2^2 \tag{38}$$

Take derivative over time we have:

$$\dot{V}_2 = \dot{V}_1 + e_2 \dot{e}_2 \tag{39}$$

Substituting equation (33) and (37) to equation (39), we have:

$$\dot{V}_2 = -c_1 e_1^2 + e_1 e_2 + e_2 \left[f(x) + g(x) u - \dot{\alpha}_1 \right] \tag{40}$$

Select control signal for system in equation (40):

$$u = \frac{-c_2 e_2 - e_1 - \left[f(x) - \dot{\alpha}_1 \right]}{g(x)} \tag{41}$$

with c_2 is positive constant.

Theorem: SRM has a state model (28) which is controlled by backstepping controller (41) with c_1, c_2 are positive constants to ensure closed loop Lyapunov stable.

Proof. Select Lyapunov function for closed loop as follows:

$$V = \frac{1}{2}\left(e_1^2 + e_2^2\right) = V_1 + \frac{1}{2} e_2^2 = V_2 \tag{42}$$

Take derivative of *V* over time we have:

$$\dot{V} = -c_1 e_1^2 + e_1 e_2 + e_2 \left[f(x) + g(x) u - \dot{\alpha}_1 \right] \tag{43}$$

Substituting *u* from equation (41) to equation (43), we have:

$$\begin{aligned} \dot{V} &= -c_1 e_1^2 + e_1 e_2 + e_2 \left[f(x) - c_2 e_2 - e_1 - \left[f(x) - \dot{\alpha}_1 \right] - \dot{\alpha}_1 \right] \\ \dot{V} &= -c_1 e_1^2 - c_2 e_2^2 \le 0 \end{aligned} \tag{44}$$

➔ What needs to be proven has been proven.

3.2. Nonlinear State Observer

The nonlinear state observer is intended to estimate the states: flux, position and rotor speed from observing directly the value of voltage, current and moment. The structure of nonlinear state observer is illustrated in Figure 1.

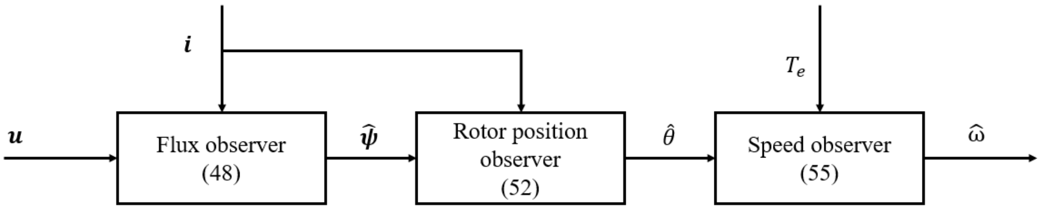

Figure 1. Structure of nonlinear state observer.

3.2.1. Nonlinear flux observer

The model of nonlinear flux observer is shown below. We set a new state:

$$\phi_j = -\ln\left(1 - \frac{\psi_j}{\psi_s}\right) = f_j(\theta) i_j \tag{45}$$

With ψ_j is flux of phase *j*.

The electro dynamic equation of the SRM:

$$\dot{\phi}_j = \left(-ri_j + u_j\right) g_j\left(\phi_j\right) \tag{46}$$

With

$$g_j\left(\phi_j\right) = -\frac{1}{\psi_s} e^{\phi_j} \tag{47}$$

Flux Observer is shown as follows:

$$\dot{\hat{\phi}} = diag\left(-ri_i + u_j\right) g\left(\hat{\phi}\right) + \gamma\Phi(i)\Phi^T(i)\left(ai - \hat{\phi}\right) \tag{48}$$

$$\hat{\psi}_j = \psi_s\left(1 - e^{-\hat{\phi}_j}\right) \tag{49}$$

With $\gamma > 0$ and $g(\phi) = \left[g_1(\phi) \quad g_2(\phi) \quad g_3(\phi) \quad g_4(\phi)\right]^T$

$$\Phi(i) = \begin{bmatrix} i_3 & 0 \\ 0 & i_4 \\ i_1 & 0 \\ 0 & i_2 \end{bmatrix}$$

3.2.2. Rotor's position observer

A model of rotor's position observer is presented in this part. The matrix $X_3(i)$ is calculated from [12]:

$$X_3(i) = \begin{bmatrix} i_1 & 0 \\ 0 & i_2 \\ -i_3 & 0 \\ 0 & -i_4 \end{bmatrix} \tag{50}$$

Matrix $G(i,\phi)$ is defined as:

$$G(i,\phi) := \left[X_3^T(i) X_3(i)\right]^{-1} X_3^T(i) \frac{(\phi - ai)}{b} = \begin{bmatrix} G_1(i,\phi) \\ G_2(i,\phi) \end{bmatrix} \tag{51}$$

The rotor's position observer is shown as follows:

$$\hat{\theta} = \frac{1}{N_r} \arctan\left[\frac{G_2(i,\phi)}{G_1(i,\phi)}\right] \tag{52}$$

3.2.3. Rotor's speed observer

The rotor's speed observer in this part is designed based on equation (11) that illustrates SRM motor and position of rotor observed above. The first step of this design observer is to approximate the torque of load. After that, Luenberger's observer is designed to observe the speed of rotor.

First of all, we determine the filter as follows:

$$G(p) := \frac{1}{(Tp+1)^2} \tag{53}$$

in which: $p := \frac{d}{dt}$ and $T > 0$ is filter time constant.

The torque of load is approximated with the following formula:

$$\hat{\tau}_l = G(p)T_E(i,\theta) - Jp^2G(p)\hat{\theta} \tag{54}$$

Based on equation (11) that illustrates SRM, the motor observer is designed as follows:

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 - \ell_1\left(\hat{x}_1 - \hat{\theta}\right) \\ \dot{\hat{x}}_2 &= -\ell_2\left(\hat{x}_1 - \hat{\theta}\right) + \frac{1}{J}\left[T_E\left(i,\hat{\theta}\right) - \hat{\tau}_L\right] \end{aligned} \tag{55}$$

with $\hat{x}_2 = \hat{\omega}$, $\ell_1, \ell_2 > 0$, $\hat{\tau}_L$ is calculated from equation (54), $\hat{\theta}$ is designed in angular observer of rotor from equation (52).

The accuracy of flux observer, rotor's position observer and rotor's speed observer were verified in [12].

3.3. Backstepping control design using the state observer

The backstepping controller (41) proposed in section 3.1 is only available when the state variables of the SRM are directly observed. In order to control the SRM without measuring the output, we proposed a method of combining the backstepping controller (41) with the observer (Figure 2). Thus, instead of having to measure the flux, position and speed of rotor directly, it is only necessary to measure current, voltage and torque.

The structure of the backstepping controller combined with the state observer is shown in Figure 2.

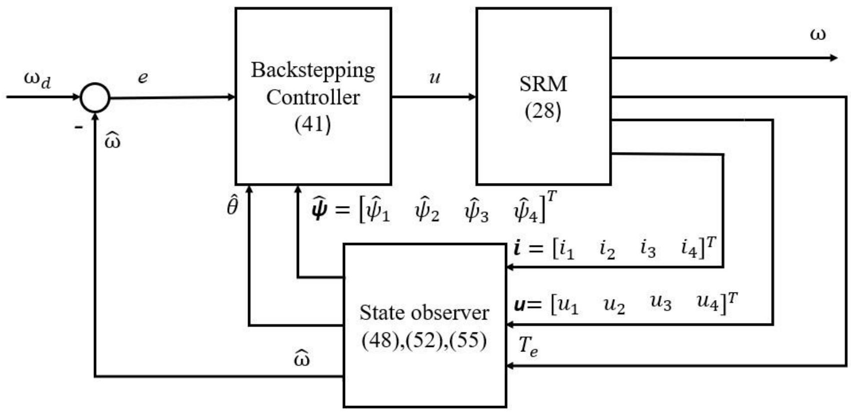

Figure 2. Structure of backstepping control system with nonlinear state observer.

The quality of the nonlinear state observer and system of SRM controller is verified in the next section.

4. NUMERICAL SIMULATION

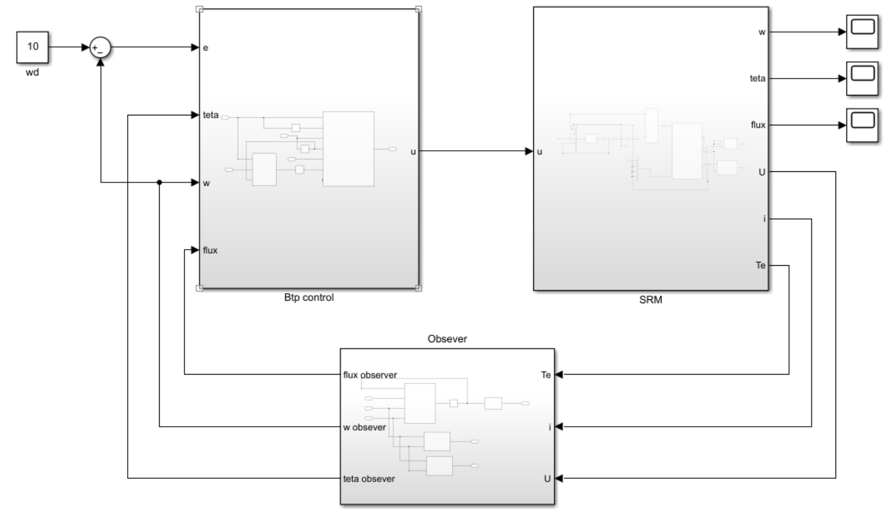

Figure 3. Schematic diagram of SRM backstepping control system.

Table 1. Parameters of SRM model, controller and observer.

$N_r = 6$	$c_1 = 2$	$B = 0.2$
$J = 6.8\times10^3\left(kg/m^2\right)$	$c_2 = 0.1$	$l = 2(m)$
$R = 0.05(\Omega)$	$\gamma = 100$	$l_1 = 100$
$a = 1.5\times10^{-3}(H)$	$T = 0.025$	$l_2 = 2500$
$b = 1.364\times10^{-3}(H)$		

The simulation is performed on MATLAB/Simulink. SRM parameters and selected force parameters of the controller and observer are shown in Table 1. The schematic diagram of SRM backstepping control system is shown in Figure 3.

The results of the nonlinear state observer test are shown in Figures 4, 5, 6; the phase current and torque of the system in Figures 7 and 8, and the control results are shown in Figure 9.

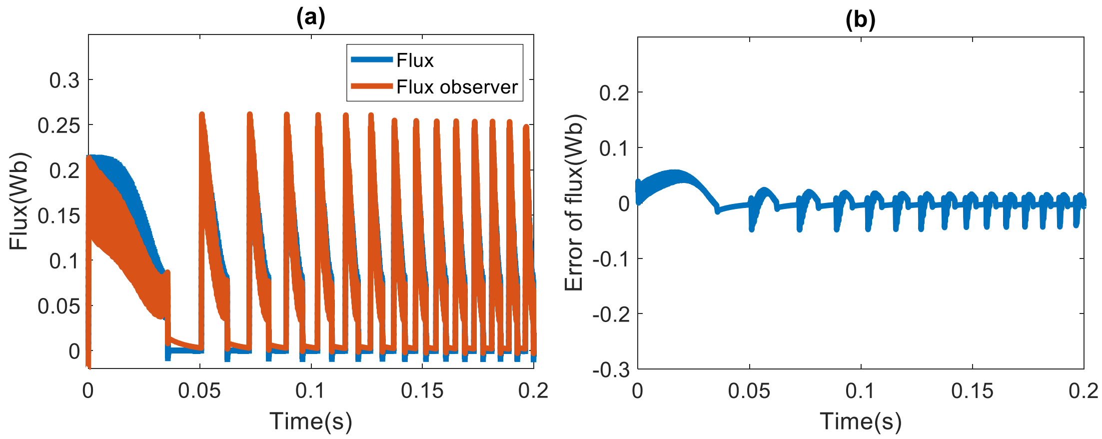

Figure 4. Flux observer and Flux.

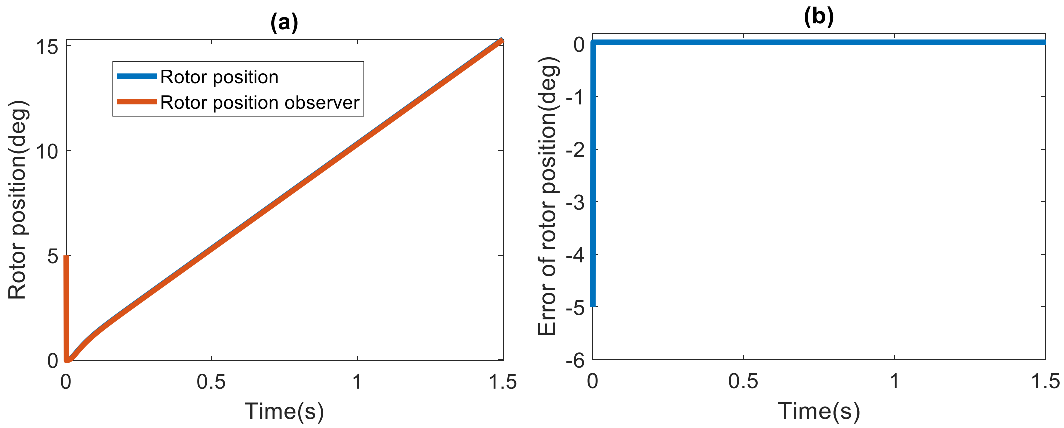

Figure 5. Rotor position observer.

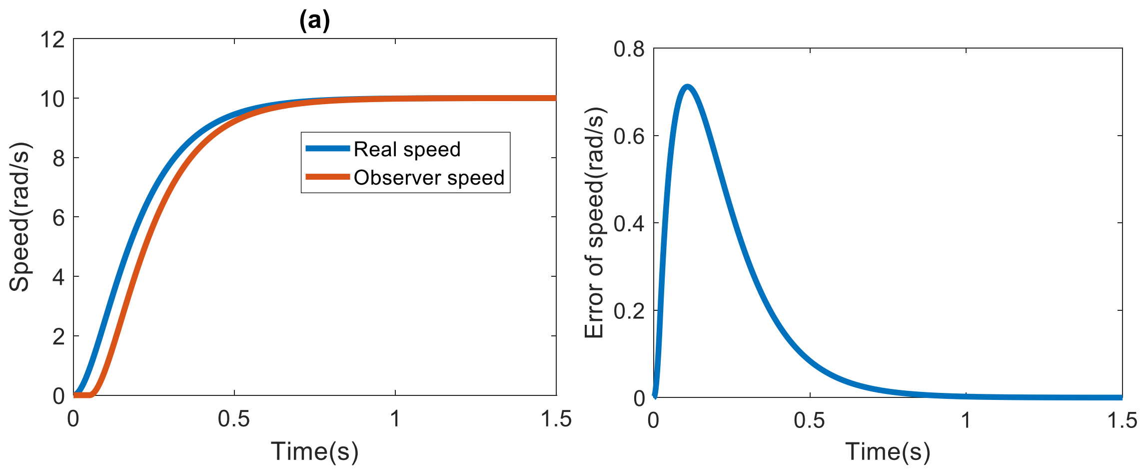

Figure 6. Speed observer.

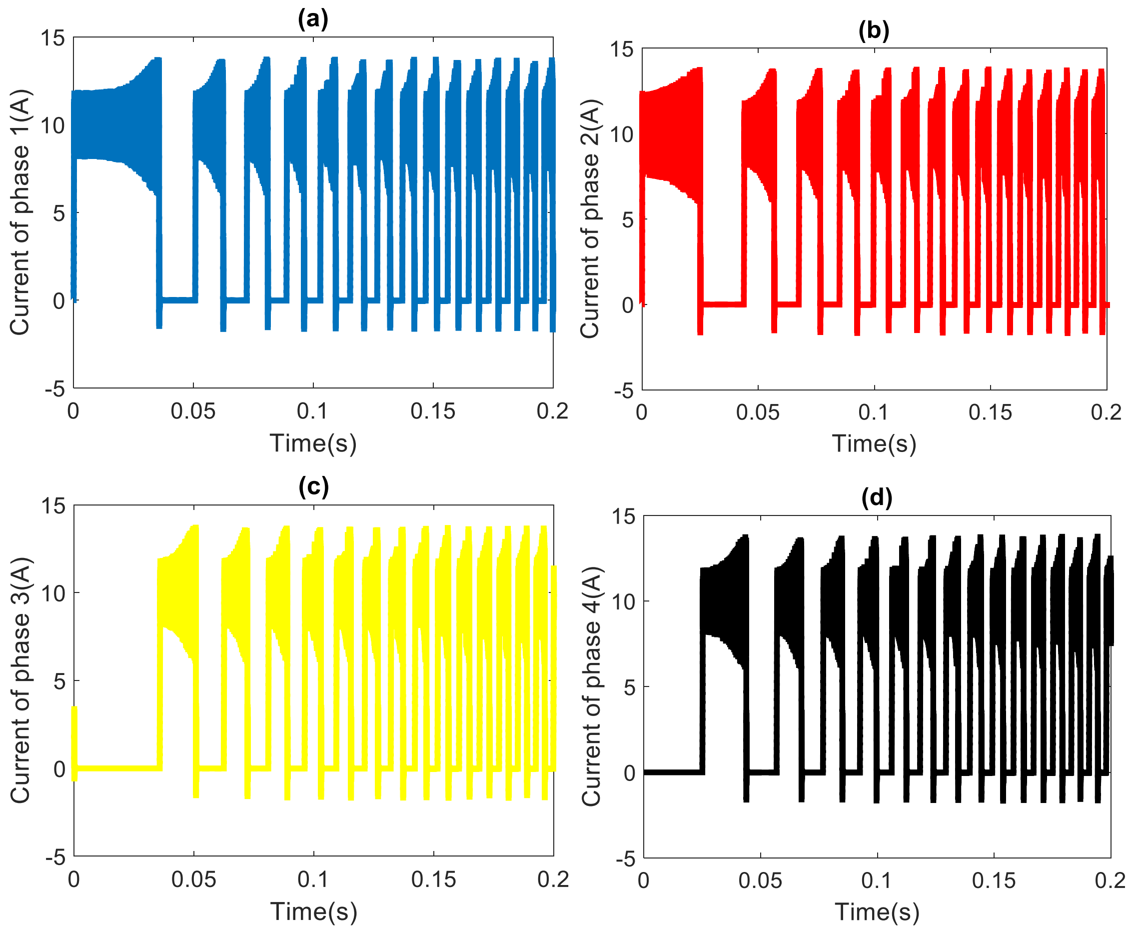

Figure 7. Phase current.

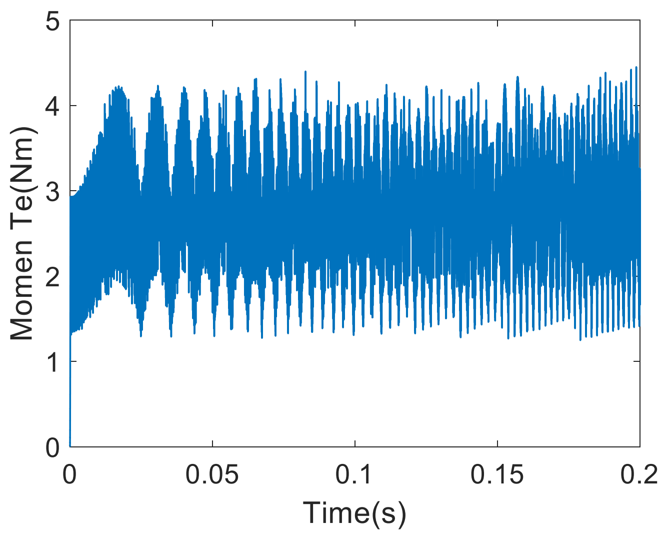

Figure 8. Torque of motor.

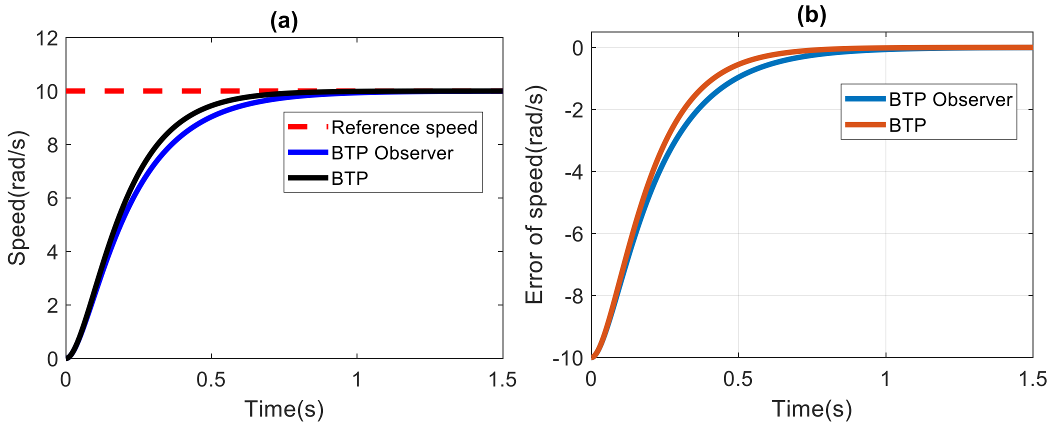

Figure 9. Speed characteristic of SRM.

Table 2. Quality of BTP and BTP based observer.

	BTP	BTP based observer
Steady state error	10^{-4}	10^{-4}
Setting time (seconds)	0.4	0.5
Overshoot (%)	0	0

The simulation results show that the nonlinear state observer achieves the required quality. The observation error for steady state of flux”?) reaches its maximum value at 0.06 Wb (Figure 4), the observation error for steady state of rotor position converges to 0 deg (Figure 5), the observation error for steady state of speed reaches its maximum value at 0.7 rad/s and converges to 0 rad/s (Figure 6). When this observer is combined with the backstepping controller, the control system gives a quality close to that of the backstepping control system through direct observation (Table 2). The torque characteristic is not good because the logic control of the switches is not optimal in terms of time.

5. CONCLUSIONS

The simulation results show that the SRM control system with the backstepping controller combined with the nonlinear state observer completely achieves the desired quality. Thus, in practice we can synthesize the controller without measuring motor output signals such as flux, speed and position of rotor. Besides improving control quality, we need to design logic control of switches.

CRediT authorship contribution statement. Phi Hoang Nha: methodology, formal analysis. Pham Hung Phi, Dao Quang Thuy: supervision. Pham Xuan Dat: formal analysis. Le Xuan Hai: supervision.

Declaration of competing interest. The authorsdeclare that,theyhave no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

REFERENCES

1. L. Zeng and H. Yu. - Research on a novel Rotor Structure Switched Reluctance motor Phys, Procedia. **24** (2012) 320–327. https://doi.org/10.1016/j.phpro.2012.02.048.
2. K. Deguchi, S. Sumita, and Y. Enomoto. - Analytical Method Applying a Mathematical Model for Axial-Gap-Switched Reluctance Motor, Electr. Eng. Japan (English Transl. Denki Gakkai Ronbunshi). **196** (3) (2016) 30–38. https://doi.org/10.1002/eej.22749.
3. L. Shen, J. Wu, S. Yang, and X. Huang. – Fast Flux Measurement for Switched Reluctance Motors Excluding Rotor Clamping Devices and Position Sensors. **62** (1) (2013) 185–191. Doi: 10.1109/TIM.2012.2212598.
4. A. Berdai et al. - Similarity and Comparison of the Electrodynamics Characteristics of Switched Reluctance Motors SRM with Those of Series DC Motors, Engineering. **7** (1) (2015) 36–45. Doi: 10.4236/eng.2015.71004.
5. J. A. Makwana, P. Agarwal, and S. P. Srivastava. - Modeling and Simulation of Switched Reluctance Motor, Lect. Notes Electr. Eng. **442** (2018) 545–558. https://doi.org/10.1007/978-981-10-4762-6_52.
6. A. Nirgude, M. Murali, N. Chaithanya, S. Kulkarni, V. B. Bhole, and S. R. Patel. - Nonlinear mathematical modeling and simulation of switched reluctance motor, IEEE Int. Conf. Power Electron. Drives Energy Syst. PEDES 2016. vol. 2016-Janua, (2017) 1–6. Doi: 10.1109/PEDES.2016.7914462.
7. G. Rigatos, P. Siano, and S. Ademi. - Nonlinear H-infinity control for switched reluctance machines, Nonlinear Eng. **9** (1) (2019) 14–27. Doi:10.1515/nleng-2017-0114.
8. Shankar Sastry - Nonlinear Systems: Analysis, Stability and Control, Spinger, 1999.
9. Alberto Isidori - Nonlinear Control Systems, Spinger, 1995.
10. Hassan Khalli - Nonlinear Systems, 2001.
11. Michael J. Grimble, Pawel Majecki - Nonlinear Industrial Control Sytems: Optimal Polynomial Systems and State) Space Approach, Spinger, 2020.
12. R. Ortega, A. Sarr, A. Bobtsov, I. Bahri, and D. Diallo. - Adaptive state observers for sensorless control of switched reluctance motors, Int. J. Robust Nonlinear Control. **29** (4) (2019) 990–1006. https://doi.org/10.1002/rnc.4420.