INVESTIGATING EFFECT OF SURFACE ROUGHNESS PATTERN ON DYNAMIC PERFORMANCE OF MEMS RESONATORS IN VARIOUS TYPES OF GASES AND GAS RAREFACTION

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Abstract. In ambient gas environment, the squeeze film damping (SFD) is a dominant damping source to reduce the quality factor (Q-factor) of micro-beam resonators. At a thin gap spacing, the surface roughness pattern effect becomes more strongly because a gas flow is more restricted by the surface roughness. The average modified molecular gas lubrication (MMGL) equation, which is modified with the pressure flow factors and the effective viscosity, is utilized to analyze the squeeze film damping (SFD) on micro-beam resonators considering the effect of surface roughness pattern in various types of the gas and gas rarefaction. Three types of roughness pattern (longitudinal, isotropic, and transverse roughness) are considered. The thermoeelastic damping (TED) and support loss are involved to calculate the total Q-factor of micro-beam resonator. The effect of surface roughness pattern (film thickness ratio and Peklenik number) on Q-factors of micro-beam resonators is then discussed. It is found that the effect of roughness pattern becomes more considerably on Q-factor in lower gas rarefaction (higher pressure) and higher effective viscosity of the gas. While, the effect of roughness pattern is significantly reduced as the effective viscosity of the gas decreases in higher mode of resonator and higher gas rarefaction.

Keywords: Quality factor of MEMS resonator, squeeze film damping, surface roughness pattern, gas rarefaction, types of gases.

Classification numbers: 5.2.4, 5.4.3, 5.4.4.

1. INTRODUCTION

Micro vibrational structures (such as micro-beam, bridge, plate, membrane, etc.), which are the most important structures of micro-electro-mechanical system (MEMS) resonators, can be
used in many sensing applications (such as gas, temperature, relative humidity, pressure, etc.),
and high precision actuations [1]. In MEMS resonators, the resonant frequency and the quality
factor (Q-factor) are important dynamic characteristics of the mechanical resonator. High Q-
factor is a key requirement for high resolution, frequency stability, and high sensitivity of
MEMS resonators.

In MEMS resonators, there are many dominant damping sources (such as external gas
damping and internal structural damping) affecting their dynamic performance. In ambient gas
environment, the squeeze film damping (SFD) is a dominant external damping source to reduce
Q-factor of MEMS resonators as a gas flow is resisted in a small gap spacing during their
transverse motion. To lower the external SFD and improve the Q-factor, a lower ambient
pressure (p) is introduced within the thin gap spacing (h0), thus the effect of gas rarefaction
becomes important [2]. Also, the effect of surface roughness [3] becomes an important factor to
be discussed because of the large surface area and volume ratio under gas ambient conditions.
To model the SFD, the conventional Reynolds equation [4] was derived using the conventional
lubrication theory. Fukui and Kaneko [5] derived the modified Reynolds equation with
Poiseuille flow rate (Qp) to model the effect of gas rarefaction. Also, the surface roughness
effect can be solved by (1) mixed average film thickness functions [6], (2) average flow factors
[3,7-10] for all surface roughness pattern directions, and (3) using the fractal model [11] to
generate functions for random surface roughness. To consider the surface roughness patterns,
Patir and Cheng [12] first proposed the modified Reynolds equation using flow factors. Bhushan
and Tonder [13,14] extended the flow factors to consider the slip flow. Flow factors could be
conveniently used for the modified molecular gas lubrication (MMGL) equation to model the
SFD considering various surface roughness patterns. Li et al. [3] and Li and Weng [7] proposed
a flow factor analysis to modify the molecular gas lubrication (MMGL) equation for the effect of
surface roughness pattern. Li [8] used pressure flow factors to modify the linearized MMGL
equation including the coupled effects of roughness and gas rarefaction in MEMS devices.

Generally, the surface roughness pattern effect is characterized by the film thickness ratio
(H3) and the Peklenik number (γ). Also, the effect of gas rarefaction is represented by the
inverse Knudsen number (D) and the accommodation coefficients, ACs (α). Flow factors [15, 16]
are used to modify the MMGL equation to discuss the effects of gas rarefaction and surface
roughness in MEMS devices. However, the effect of ACs has not been considered. Li [17]
proposed a complete database of Poiseuille flow rates (Qp(D,α1,α2)) in a wide range of gas
rarefaction (D (0.01 ≤ D ≤ 100)) and ACs (0.1 ≤ α1, α2 ≤ 1.0) conditions. The effect of surface
roughness on the dynamic coefficients of SFD in MEMS devices with symmetric ACs (α1 = α2)
and non-symmetric ACs (α1 ≠ α2) is discussed by Li in [9] and [10], respectively. In the
previous works, Nguyen and Li examined the effects of gas rarefaction (D and ACs) [18]. Also,
the coupled effects of surface roughness and gas rarefaction on the quality factors of MEMS
resonators [19] are discussed in a wide range of resonator modes. In addition, the influences of
temperature (Nguyen and Li [20]) and relative humidity (Nguyen et al. [21]) on the Q-factor of
the MEMS resonator under a variety of gas rarefaction (D and ACs (α1,α2)) conditions were
investigated. However, the effect of surface roughness has not been examined for various types
of the gas in gas rarefaction. In this study, the average MMGL equation for the SFD, which is
modified with the pressure flow factors (ϕ^p_, ϕ^p_γ), dynamic viscosity (µ) given by Sutherland
[22] for various types of the gas, and the database of Qp(D,α1,α2) [17], is used to examine the
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effects of surface roughness pattern under various types of the gas and gas rarefaction conditions. The internal structural damping sources (thermoelastic damping (TED) and support loss) are also involved to calculate the total Q-factor of micro-beam resonator. Finally, the roughness pattern effect \( (H_s, \gamma') \) on the Q-factor of MEMS resonators in various types of the gas, gas rarefaction \( (D, \text{ and } ACs) \), and mode of resonator is considered.

2. MATERIALS AND METHODS

In this section, the main energy dissipation sources of the MEMS resonators such as the SFD, TED, and support loss are taken into account under different operation conditions. The main governing equations are: (1) the average MMGL equation (which represents the effect of the surface roughness pattern on the SFD in various types of the gas and gas rarefaction), (2) the transverse motion equation with their boundary conditions of the micro-beam.

2.1. The squeeze film damping (SFD) in MEMS resonators

In a gaseous ambient, the transverse vibration of micro-beam resonators is restricted by an applied load of gas film as structure of micro-beam is squeezed in small gap spacing as depicted in Figure 1. The Poiseuille flow rate occurs as a gas flow is squeezed periodically in a small gap spacing in the normal direction with a substrate. Also, when the transverse movement of micro-beam is influenced by a gas applied load in a thin gap spacing, the resonant frequency and the quality factor of micro-beam resonators can be changed in various type of the gas and gas rarefaction.

![Figure 1. Transverse motion behaviors of micro-beam resonators under the squeeze film damping conditions in various types of the gas.](image)

In an ultra-thin gap spacing, gas film is resisted between two surfaces, \( z_1 \) (vibration surface) with the transverse motion in \( z \)-direction and \( z_2 \) (stationary one) as shown in Figure 2, where the random variables \( (\delta_1' \text{ and } \delta_2') \) represent a stationary stochastic process distributed with standard deviations of the composite surfaces \( (\sigma_1 \text{ and } \sigma_2) \), respectively (Li et al. [3]). Under the assumption that the two variables \( (\delta_1' \text{ and } \delta_2') \) are uncorrected, the standard deviation \( (\sigma) \) of the roughness combination \( (\delta_1 - \delta_2) \) is given simply by \( (\sigma_1^2 + \sigma_2^2)^{1/2} \). Thus, at a small gas...
pressure and thin gap spacing, the effects of surface roughness and gas rarefaction become important and must be considered in the SFD analysis.

![Figure 2. Schematic representation of two roughness surfaces [3].](image)

To consider the effects of roughness pattern in various types of the gas, the pressure flow factors were derived [3] for the average MMGL equation. The complete database of $Q_p(D, \alpha_1, \alpha_2)$ reported by Li [17] is used to modify the average MMGL equation to consider the effect of gas rarefaction. The pressure distribution of the gas flow over the small gap spacing is modeled using the average MMGL equation to consider the influence of the roughness pattern in a wide range of gas rarefaction and various types of the gas as follows:

$$\frac{\partial}{\partial x} \left( \frac{\phi_{xx}^p \rho h^3}{12 \mu_{eff}} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\phi_{yy}^p \rho h^3}{12 \mu_{eff}} \frac{\partial p}{\partial y} \right) = \frac{\partial}{\partial t} (\rho h)$$

(1)

where $p$, $\rho$, and $h$ are the pressure, the density and the gas film spacing, respectively; $\phi_{xx}^p$ and $\phi_{yy}^p$ ($\gamma, \sigma, H_s, D_0, \alpha$) are the pressure flow factors in $x$ and $y$ directions, respectively; $H_s$ ($= h_i / \sigma$) is the film thickness ratio (the roughness height); $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$ is the roughness height standard deviation of the two surfaces.

The pressure flow factors ($\phi_{xx}^p$ and $\phi_{yy}^p$ ($\gamma, \sigma, H_s, D_0, \alpha$)) are derived as a function of the Peklenik number ($\gamma_i$) and the standard deviation ($\sigma_i$) of the $i$-surface, where $i = 1, 2$. They are also functions of film thickness ratio ($H_s$), and gas rarefaction (inverse Knudsen numbers ($D_0$) and accommodation coefficients, ACs ($\alpha_1 = \alpha_2$)). Thus, the pressure flow factors [3] are represented as below:

$$\phi_{xx}^p = 1 + g \times \left( \frac{\sigma}{h} \right)^2 \left\{ 1 - \frac{f^2}{g} \frac{1}{\gamma + 1} \right\}$$

(2)
\[
\phi_{\gamma_3}^\rho = 1 + g \times \left( \frac{\sigma}{h} \right)^2 \left\{ 1 - \frac{f^2}{g} \frac{\gamma}{\gamma + 1} \right\} 
\]

(3)

\[
\phi_{\gamma_2}^\rho (\gamma_1, \gamma_2) = \phi_{\gamma_3}^\rho \left( \frac{1}{\gamma_1}, \frac{1}{\gamma_2} \right) 
\]

(4)

Though the diagonal flow factors are the same, the off-diagonal flow factors are different.

\[
\frac{1}{\gamma + 1} = \left( \frac{\sigma_2}{\sigma} \right)^2 \frac{1}{\gamma_2 + 1} + \left( \frac{\sigma_1}{\sigma} \right)^2 \frac{1}{\gamma_1 + 1} 
\]

(5)

\[
f(D, \alpha_1, \alpha_2) = 3 \frac{D \cdot \partial Q_p / \partial D}{Q_p} 
\]

(6)

\[
g(D, \alpha_1, \alpha_2) = 3 + \frac{3D \cdot \partial Q_p / \partial D}{Q_p} + \frac{D^2 \cdot \partial^2 Q_p / \partial D^2}{2 Q_p} 
\]

(7)

where \( \gamma \) is the Peklenik number. As shown in Figure 3, three types of roughness patterns are represented.

Figure 3. (a) longitudinal type roughness pattern (\( \gamma > 1 \)), (b) isotropic type roughness pattern (\( \gamma = 1 \)), and (c) transverse type roughness pattern (\( \gamma < 1 \)) [8].
The effective viscosity \( \mu_{\text{eff}} \) described by Veijola et al. [2] is used to consider the gas rarefaction effect as follows:

\[
\mu_{\text{eff}} = \frac{\mu}{Q_p(D, \alpha_1, \alpha_2)}
\]  

(8)

where \( \mu \) is the dynamic viscosity of gas, and \( Q_p \) is the Poiseuille flow rate corrector, which is the measure of gas flow restriction on the motion of the micro-beams in gas rarefaction.

Sutherland [22] deduced an expression for dynamic viscosity for various types of ideal gas over a wide range of temperatures \((0 < T < 555 \text{ K})\) as follows:

\[
\mu = \mu_0' \left( \frac{T'}{T} \right)^{3/2} \left( 1 + \frac{C}{T - T_0'} \right)
\]

(9)

where \( \mu_0' \) is the reference viscosity at the reference temperature \((T_0')\) and \( C \) is a Sutherland’s constant. Sutherland’s constants including \( C, \mu_0', T_0' \) for various types of ideal gas are provided in Table 1 for analytical calculations of \( \mu \).

The database of \( \tilde{Q}_p(D, \alpha_1, \alpha_2) \) obtained by solving the linearized Boltzmann equation was used by Li [17] to consider the gas rarefaction effect in a wide range of \( D \) \((0.01 \leq D_0 \leq 100)\) and ACs \((0.1 \leq \alpha_1, \alpha_2 \leq 1.0)\) as follows:

\[
\tilde{Q}_p(D, \alpha_1, \alpha_2) = \exp \left[ \sum_{n=1}^{13} C_n (\ln D)^{13-n} \right]
\]

(10)

where ACs \((\alpha_1 = \alpha_2)\) are the surface accommodation coefficients representing the gas molecules collisions with \( i \)-solid surface (specular reflection \((\alpha = 1.0)\) and diffusive reflection \((\alpha = 0.1)\)).

The Poiseuille flow rate corrector of \( Q_p(D, \alpha_1, \alpha_2) \) is calculated as the ratio of \( \tilde{Q}_p(D, \alpha_1, \alpha_2) \) for rarefied flow to that for continuum flow \((\tilde{Q}_{\text{con}}(D) = D/6)\), i.e.

\[
Q_p(D, \alpha_1, \alpha_2) = \frac{\tilde{Q}_p(D, \alpha_1, \alpha_2)}{\tilde{Q}_{\text{con}}(D)} = \frac{6}{D} \tilde{Q}_p(D, \alpha_1, \alpha_2)
\]

(11)

The first and second derivations of gas rarefaction coefficients \((\tilde{Q}_p)\) with respect to \((D, \alpha_1, \alpha_2)\) are given by the following expressions:

\[
\frac{\partial \tilde{Q}_p}{\partial D} = -\frac{6}{D^2} \tilde{Q}_p + \frac{6}{D} \frac{\partial \tilde{Q}_p}{\partial D}
\]

(12a)

\[
\frac{\partial^2 \tilde{Q}_p}{\partial D^2} = \frac{1}{D} \tilde{Q}_p \left[ \sum_{n=1}^{12} (13-n)C_n (\ln D)^{12-n-2} \right]
\]

(12b)
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\[
\frac{\partial^2 \bar{Q}_p}{\partial D^2} = \frac{12}{D^2} \frac{\partial \bar{Q}_p}{\partial D} - \frac{12}{D^2} \frac{\partial \bar{Q}_p}{\partial D} + 6 \frac{\partial^2 \bar{Q}_p}{\partial D^2} \tag{13a}
\]

\[
\frac{\partial^2 \bar{Q}_p}{\partial D^2} = \left[ -\frac{1}{D} \frac{\partial \bar{Q}_p}{\partial D} \right] + \left[ \frac{1}{\bar{Q}_p} \left( \frac{\partial \bar{Q}_p}{\partial D} \right)^2 \right] + \frac{1}{D^2} \frac{\partial \bar{Q}_p}{\partial D} \left[ \sum_{n=1}^{12} (13 - n)(12 - n)C_n \ln(D)^{1-n} \right] \tag{13b}
\]

The inverse Knudsen number \((D)\), which is an important gas rarefaction indicator, is calculated by

\[
D = \frac{\sqrt{\pi}}{2K_p} = \frac{\sqrt{\pi} h}{2\lambda} \tag{14}
\]

where \(K_p = \lambda_p / h_0\) is the Knudsen number representing the gas rarefaction.

The mean free path of gas \((\lambda)\) can be evaluated in the physical model [23] as below:

\[
\lambda = \frac{k_B T}{\sqrt{2\pi \cdot d^2 \rho}} \tag{15}
\]

where \(k_B = 1.380658 \times 10^{-23} \text{ (J/K)}\) is the Boltzmann constant, and \(d\) is the diameter of the cross section of particles given by Kennard [24].

An alternative method to calculate the mean free path of the gas \((\lambda)\) as a function of pressure \((p)\) at a constant temperature [16] is given by

\[
\lambda = \lambda_0 \frac{P_0}{p} \tag{16}
\]

where \(\lambda_0\) is the reference mean free path of the gas at reference pressure \((p_0 = 101325 \text{ Pa})\) and 300 K for different types of the gas as shown in Table 1.

Table 1. The reference parameters for dynamic viscosities \((\mu)\) and mean free paths \((\lambda)\) for various types of ideal gas.

<table>
<thead>
<tr>
<th>Gas</th>
<th>(C (K)) Sutherland [22]</th>
<th>(T_0' (K)) Sutherland [22]</th>
<th>(\mu_0' (\mu\text{Pa.s})) Sutherland [22]</th>
<th>(d (\text{Å})) Kennard [24]</th>
<th>(\lambda_0, \text{(nm)}) (Eq.(15)) at 101325 Pa &amp; 300 K</th>
<th>(\mu, \text{(\mu\text{Pa.s})}) (Eq.(9)) at 300 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>120</td>
<td>291.15</td>
<td>18.27</td>
<td>3.72</td>
<td>66.487</td>
<td>18.71</td>
</tr>
<tr>
<td>Helium (He)</td>
<td>79.4</td>
<td>273</td>
<td>19</td>
<td>2.18</td>
<td>193.6</td>
<td>20.33</td>
</tr>
<tr>
<td>Oxygen (O₂)</td>
<td>127</td>
<td>292.25</td>
<td>20.18</td>
<td>3.61</td>
<td>70.601</td>
<td>20.61</td>
</tr>
<tr>
<td>Carbon dioxide (CO₂)</td>
<td>240</td>
<td>293.15</td>
<td>14.8</td>
<td>4.59</td>
<td>43.672</td>
<td>15.13</td>
</tr>
<tr>
<td>Hydrogen (H₂)</td>
<td>72</td>
<td>293.85</td>
<td>8.76</td>
<td>2.74</td>
<td>122.55</td>
<td>8.887</td>
</tr>
<tr>
<td>Nitrogen (N₂)</td>
<td>111</td>
<td>300.55</td>
<td>17.81</td>
<td>3.75</td>
<td>65.428</td>
<td>17.79</td>
</tr>
<tr>
<td>Argon (Ar)</td>
<td>133</td>
<td>298</td>
<td>22.6</td>
<td>3.64</td>
<td>69.442</td>
<td>22.72</td>
</tr>
</tbody>
</table>

2.2. Transverse vibration of micro-beam resonators

In a small gap spacing, the vibration of the micro-beam is resisted by a gas fluid force. We consider the transverse vibrations of micro-beam affected by a net external force, \(f_{ext} = \Delta p(x, y, t)\) per unit area on the boundary of the micro-beams (Figure 1). Under the small
variation of beam deflection, the linear equation for the transverse vibration of the microplate [25] is given by

\[
D_b \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + \rho_s t_b \frac{\partial^2 w}{\partial t^2} = -\Delta p(x, y, t) \tag{17}
\]

where \( w(x, y, t) \) is the small transverse deflection of the micro-beams as a function of positions \( x, y \), and time \( t \); \( D_b (= E t_b^3 / 12(1 - \nu^2)) \) is the rigidity of the beam structure; \( E \) is the Young’s modulus of the beam; \( \rho_s \) is the density of the beam; \( t_b \) is the beam thickness, and \( \nu \) is the Poisson’s ratio.

The boundary conditions of the micro-beams are given by clamped edge at \( x = 0 \)

\[
w(x, y, t) = 0; \quad \frac{\partial w(x, y, t)}{\partial x} = 0 \tag{18}
\]

and free edges at \( y = 0, \ y = w_b \), and \( x = l_b \)

\[
\frac{\partial^2 w(x, y, t)}{\partial x^2} = \frac{\partial^3 w(x, y, t)}{\partial x^3} = 0 \tag{19}
\]

\[
\frac{\partial^2 w(x, y, t)}{\partial y^2} = \frac{\partial^3 w(x, y, t)}{\partial y^3} = 0 \tag{20}
\]

For the SFD, the eigenvalues of the average MMGL equation (Eq.(1)), the transverse motion equation (Eq.(17)), and boundary conditions of micro-beam resonators (Eqs. (18 - 20)) are simultaneously solved by the Finite Element Method (FEM) [26]. Thus, the eigenvalues \( (\bar{\lambda} = \delta + i\omega) \) including the damping factor \( (\delta = \text{Re}(\bar{\lambda})) \) and the natural frequencies \( (\omega = \text{Im}(\bar{\lambda})) \) are obtained from solving these equations [18].

2.3. Quality factor

For MEMS resonators, the Q-factor is the ratio between the resonant frequency \( (\omega_0) \) and its bandwidth \( (\Delta\omega) \) of the resonant spectrum as given by

\[
Q = \frac{2\pi W_0}{\Delta \omega} = \frac{\omega_0}{\Delta \omega} \tag{21}
\]

In the eigenvalue problem, an equivalent definition (Nguyen and Li [18]) becomes more accurate to calculate the Q-factor of MEMS resonators as below:

\[
Q = \frac{\omega_0}{2\delta} = \frac{\text{Im}(\bar{\lambda})}{2\text{Re}(\bar{\lambda})} \tag{22}
\]

from the eigenvalues for \( n \)-transverse modes of MEMS resonator \( (\bar{\lambda}_n = \delta_n + i\omega_n) \), the Q-factor for the SFD problem \( (Q_{SFD}) \) can be evaluated for \( n \)-modes of MEMS resonators.

In micro-beam resonators, the total Q-factor \( (Q_T) \) can be calculated by summing the contributions of the Q-factors from dominant damping sources such as SFD \( (Q_{SFD}) \), TED \( (Q_{TED}) \) and support loss \( (Q_{sup}) \) [18, 19] as follows:
\[
\frac{1}{Q_T} = \frac{1}{Q_{SFD}} + \frac{1}{Q_{TED}} + \frac{1}{Q_{sup}} = \frac{1}{Q_{SFD}} + \frac{1}{Q_{TA}}
\]  

(23)

where \(Q_{SFD}\) can be obtained by solving the complex eigenvalue (\(\vec{\lambda}\)) in the eigenvalue problem of the linear equations (Eq.(1) and Eq. (17)) with their appropriate boundary conditions (Eqs. (18 - 20)). \(\frac{1}{Q_{TA}} = \frac{1}{Q_{TED}} + \frac{1}{Q_{sup}}\) is the internal structural damping of the micro-beam resonators. \(Q_{TED}\) and \(Q_{sup}\) can be obtained from Table 3 and Table 4, respectively, as described by Nguyen and Li [18, 19].

3. RESULTS AND DISCUSSION

In thin clearance, the effect of surface roughness must be considered under gas ambient conditions. For the SFD problem, pressure flow factors ((\(\phi^P_{xx}\) and (\(\phi^P_{yy}\))) (Eqs.(2)-(7)) are correctors for surface roughness. The effective viscosity (\(\mu_{\text{eff}}\)) is defined as the ratio between dynamic viscosity (\(\mu\)) by Sutherland [22] (considering various types of ambient gas) and Poiseuille flow rate (\(Q_p(D,\alpha_1,\alpha_2)\)) by Li [17] (Eqs.(10-13)) (the correctors for the gas rarefaction). In addition, the database of \(Q_p(D,\alpha_1,\alpha_2)\) reported by Li [17] is applicable for use in arbitrary \(D_0\) and ACs. Basic operating conditions are listed in Table 2. The average MMGL equation (Eq. (1)) for the SFD problem is modified by effective viscosity (\(\mu_{\text{eff}}\)). Finally, the effect of surface roughness pattern (\(H_S, \gamma\)) on the Q-factors of micro-beam resonator in various types of the gas, gas rarefaction (\(D_0\) and ACs), and resonator mode is considered.

Table 2. Basic operating conditions of micro-beam resonators.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ell_b)</td>
<td>length of micro-beam</td>
<td>300 (\mu m)</td>
</tr>
<tr>
<td>(w_b)</td>
<td>width of micro-beam</td>
<td>22 (\mu m)</td>
</tr>
<tr>
<td>(t_b)</td>
<td>thickness of micro-beam</td>
<td>4 (\mu m)</td>
</tr>
<tr>
<td>(E)</td>
<td>Young modulus of poly-silicon</td>
<td>(160 \times 10^9) Pa</td>
</tr>
<tr>
<td>(\rho_S)</td>
<td>density of poly-silicon</td>
<td>(2330) Kg/m(^3)</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Poisson’s ratio of poly-silicon</td>
<td>0.22</td>
</tr>
<tr>
<td>(\alpha_S)</td>
<td>thermal expansion coefficient of poly-silicon</td>
<td>(2.6 \times 10^{-4}) 1/K</td>
</tr>
<tr>
<td>(\kappa)</td>
<td>thermal conductivity of poly-silicon</td>
<td>(90) W/(m K)</td>
</tr>
<tr>
<td>(C_p)</td>
<td>specific heat capacity of poly-silicon</td>
<td>(700) J/(Kg K)</td>
</tr>
<tr>
<td>(T_0)</td>
<td>temperature</td>
<td>300 K</td>
</tr>
<tr>
<td>(\mu)</td>
<td>dynamic viscosity</td>
<td>(1.871 \times 10^{-3}) Pa s</td>
</tr>
<tr>
<td>(h_0)</td>
<td>gas film thickness</td>
<td>4 (\mu m)</td>
</tr>
<tr>
<td>(p_0)</td>
<td>reference pressure of air</td>
<td>101325 Pa</td>
</tr>
<tr>
<td>(\lambda_{p0})</td>
<td>reference molecular mean free path of air at (p_0)</td>
<td>66.5 nm</td>
</tr>
<tr>
<td>(p_{\text{basic}})</td>
<td>basic ambient pressure for air</td>
<td>190.0786 Pa</td>
</tr>
<tr>
<td>(D)</td>
<td>inverse Knudsen number</td>
<td>0.1</td>
</tr>
<tr>
<td>ACs</td>
<td>Surface accommodation coefficients ((\alpha_i))</td>
<td>1.0</td>
</tr>
<tr>
<td>(H_S)</td>
<td>Film thickness ratio of surface roughness height</td>
<td>3</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Longitudinal Peklenik number of roughness pattern</td>
<td>9</td>
</tr>
</tbody>
</table>
3.1. Pressure flow factors, ($\phi_{xx}^P$)

**Figure 4.** Pressure flow factor ($\phi_{xx}^P$) plotted with inverse Knudsen number ($D_0$) for different Peklenik numbers ($\gamma$) in air.

**Figure 5.** Pressure flow factor ($\phi_{xx}^P$) plotted with film thickness ratio ($H_S$) for different Peklenik numbers ($\gamma$).
In Figure 4, pressure flow factor ($\phi_{xx}^P$) increases as $D_0$ increases (gas rarefaction decreases and gas flow becomes more restricted) for the longitudinal type roughness pattern ($\gamma = 9$). Meanwhile, $\phi_{xx}^P$ decreases and moves further away for the case of smooth surface ($\phi_{xx}^P = 1$) as $D_0$ increases and $\gamma$ decreases for the isotropic type roughness ($\gamma = 1$) and the transverse type roughness ($\gamma = 1/9$). Therefore, it is possible for the present model to consider the surface roughness pattern effects ($H_S$, $\gamma$) on the Q-factors of MEMS resonators in a wide range of gas rarefaction ($D$ and $ACs(\alpha_1, \alpha_2)$).

In Figure 5, the pressure flow factor ($\phi_{xx}^P$) is plotted as a function of the film thickness ratio ($H_S$) for different Peklenik numbers ($\gamma$). The results show that $\phi_{xx}^P$ gradually decreases to nearly 1 (smooth case) when the film thickness ratio ($H_S$) increases for the longitudinal surface roughness pattern ($\gamma = 9$). The surface roughness pattern effect is reduced as $H_S$ increases or surface roughness height decreases. In the meantime, $\phi_{xx}^P$ increases to nearly 1 as $H_S$ increases from the isotropic type surface roughness ($\gamma = 1$) to the transverse type surface roughness ($\gamma = 1/9$) ($\phi_{xx}^P (\gamma = 1) > \phi_{xx}^P (\gamma = 1/9)$). At lower $H_S$, $\phi_{xx}^P$ changes with $\gamma$ more considerably because the gas flow in thin gap spacing becomes more restricted and the surface roughness pattern effect becomes more strongly.

### 3.2. Effective viscosity, $\mu_{eff} (= \mu / Q_p)$

The reference parameters used to calculate the dynamic viscosity ($\mu$) and the mean free path ($\lambda$) for various types of the gas are listed in Table 1. In Figure 6, the results show that $\lambda$ decreases as $p$ increases (Figure 6(a)), and then $Q_p$ decreases as $p$ increases for various types of the gas (Figure 6(b)). In Figure 6(c), $\mu_{eff} (= \mu / Q_p)$ increases as pressure ($p$) increases for various types of the gas. Different types of the gas exhibit different behaviors of $\mu_{eff}$ over a wide pressure range because they have different values of $\mu$ (Table 1) and $Q_p$ (Figure 6(b)) under various pressure conditions. Furthermore, $\mu_{eff}$ for He and H$_2$ is less than those for the other gases (Ar, O$_2$, Air, N$_2$, and CO$_2$) in a wide range of pressure conditions. The obtained results can be applied to discuss the effects of surface roughness pattern ($H_S$, $\gamma$) to improve the Q-factors of MEMS resonators under various types of the gas and gas rarefaction conditions.

### 3.3. Resonant frequency ($\omega$), damping factor (δ), and quality factor ($Q_{SFD}$)

It can be seen from Figure 7(a) that $\omega$ increases and then decreases with increasing pressure because the spring force of the gas film increases more than the damping force as the pressure increases [8]. In Figure 7(b), the damping factor ($\delta$) of the SFD increases and reaches its maximum value as the ambient pressure increases. Thus, $Q_{SFD}$ reduces and reaches its minimum value as the pressure increases (Figure 7(c)) because the gas flow becomes more restricted when the pressure is increased or higher SFD is produced. To consider the surface
roughness effect, three types of surface roughness pattern (γ = 9, 1, 1/9) are used. The results show that \( \omega_n \) varies significantly as \( \gamma \) increases (Figure 7(a)). Moreover, \( \omega_n \) varies with \( \gamma \) more considerably in the case of air than in the case of \( \text{H}_2 \) (because the effective viscosity (\( \mu_{eff} \)) of air is greater than that of \( \text{H}_2 \) as seen in Figure 6(c)).

Figure 6. (a) Mean free path of gas (\( \lambda \)), (b) Poiseuille flow rate (\( Q_p \)), and (c) effective viscosity (\( \mu_{eff} \)) plotted with ambient pressure (\( p \)) for different types of the gas.
Figure 7. (a) Resonant frequency ($\omega_n = 2\pi f_n$), (b) Damping factor ($\delta$), and (c) Q-factor for the SFD ($Q_{SFD} = \omega_n/(2 \cdot \delta)$) plotted with pressure for different Peklenik numbers ($\gamma$) and different types of the gas (Air and H$_2$).

Also, $\delta$ increases as $\gamma$ increases (Figure 7(b)) and thus, $Q_{SFD}$ decreases with increasing $\gamma$, i.e. $Q_{SFD} (\gamma=1/9) > Q_{SFD} (\gamma=1) > Q_{SFD} (\gamma=1/1)$ (Figure 7(c)). Apart from that, the effect of surface roughness pattern ($\gamma$) on $Q_{SFD}$ becomes more considerably as the
ambient pressure \((p)\) and the effective viscosity \((\mu_{\text{eff}})\) of gases increase. Thus, the surface roughness pattern effect \((\gamma')\) on Q-factor of SFD \((Q_{\text{SFD}})\) is enhanced and becomes more significantly in lower gas rarefaction (higher pressure) and higher effective viscosity \((\mu_{\text{eff}})\) of gases.

3.3. Quality factors, \(Q_{\text{SFD}}\) and \(Q_T\)

In Figure 8, \(Q_{\text{SFD}}\) and total Q-factor \((Q_T)\) are plotted for different types of the gas in a wide range of ACs \((\alpha_1 = \alpha_2)\) under the smooth and roughness \((H_S = 3, \gamma = 9)\) conditions. \(Q_{\text{SFD}}\) and \(Q_T\) increase simultaneously as ACs \((\alpha_1 = \alpha_2)\) decrease because the gas film is less restricted and lower SFD is produced as gas rarefaction increases (ACs decrease). Also, the values of \(Q_{\text{SFD}}\) almost approach those of \(Q_T\) in the smooth case for various types of the gas in a wide range of ACs \((\alpha_1 = \alpha_2)\) because the SFD is dominant. In addition, the gas flow is squeezed and then enhanced for the longitudinal type of roughness pattern \((H_S = 3, \gamma = 9)\). Then, the \(Q_T(H_S = 3, \gamma = 9)\) is less than \(Q_{\text{SFD}}\) and \(Q_T\) (smooth case) for different types of the gas. Thus, the surface roughness patterns \((H_S \text{ and } \gamma)\) clearly affect \(Q_{\text{SFD}}, Q_T\) for different types of the gas in a wide range of ACs \((\alpha_1 = \alpha_2)\).
In Figure 9, $Q_{SFD}$ and $Q_T$ are plotted as the functions of film thickness ratio ($H_S$) for different Peklenik numbers ($\gamma$) in the 1st mode of resonator. The results show that $Q_{SFD}$ increases as surface roughness pattern varies from the longitudinal type ($\gamma=9$) to isotropic type ($\gamma=1$) and transverse type ($\gamma=1/9$) because of the change in the pressure flow factor ($\phi^P_{st}$) under various $\gamma$ and $H_S$ conditions (Figure 5). In addition, the values of $Q_{SFD}$ almost approach those of $Q_T$ in a wide range of surface roughness patterns ($H_S$ and $\gamma$) in the 1st mode of resonator because the SFD is dominant. Also, $Q_{SFD}$ and $Q_T$ tend to approach the smooth surface as $H_S$ increases for various surface roughness patterns ($\gamma$). The obtained results can be used to discuss changes in the ratio of $Q_T / (Q_T)_{smooth}$ for different surface roughness patterns ($H_S$, $\gamma$) in various types of the gas, gas rarefaction and mode of resonator.

In Figure 10, the ratio of $Q_T / (Q_T)_{smooth}$ is plotted as a function of the film thickness ratio ($H_S$) at different Peklenik numbers ($\gamma$) with various types of the gas for different resonator modes in high gas rarefaction ($p = 1000$ Pa). It can be seen from Figure 10(a) that the ratio of $Q_T / (Q_T)_{smooth}$ varies with the effect of surface roughness pattern ($H_S$ and $\gamma$) in a way similar to that of $Q_{SFD}$ and $Q_T$ in Figure 9. In addition, the variation of the $Q_T / (Q_T)_{smooth}$ ratio with respect to $H_S$ and $\gamma$ is almost unchanged for various types of the gas in the 1st resonator mode (SFD is dominant), while the $Q_T / (Q_T)_{smooth}$ ratio tends to approach the smooth case more quickly for various types of the gas as the mode of resonator increases (see Figures 10(b-d)). That is because the internal structural damping (TA) increases (an increase of $(Q_{TA})^{-1}$ in Eq.

![Figure 9. $Q_{SFD}$ and $Q_T$ versus film thickness ratios ($H_S$) for different Peklenik numbers ($\gamma$).](image-url)
(23)) and the external SFD decreases (a decrease of $(Q_{SFD})^{-1}$ in Eq. (23)) as the resonator mode increases.

![Graphs showing the ratio of $Q_T / (Q_{T\text{smooth}})$ plotted with film thickness ratio ($H_s$) at different Peklenik numbers ($\gamma'$) with various types of the gas in gas rarefaction ($p = 1000$ Pa) for (a) 1$^{st}$ mode, (b) 2$^{nd}$ mode, (c) 3$^{rd}$ mode, and (d) 4$^{th}$ mode of resonator.]

Figure 10. Ratio of $Q_T / (Q_{T\text{smooth}})$ plotted with film thickness ratio ($H_s$) at different Peklenik numbers ($\gamma'$) with various types of the gas in gas rarefaction ($p = 1000$ Pa) for (a) 1$^{st}$ mode, (b) 2$^{nd}$ mode, (c) 3$^{rd}$ mode, and (d) 4$^{th}$ mode of resonator.

Also, the $Q_T / (Q_{T\text{smooth}})$ ratio varies with surface roughness patterns ($H_s$ and $\gamma'$) more considerably and tends to approach the smooth case more quickly with gases of higher effective
viscosity (such as Ar, O\textsubscript{2}, Air, N\textsubscript{2}, CO\textsubscript{2}) compared to lower effective viscosity gases (He and H\textsubscript{2}) in a higher mode of resonator and gas rarefaction ($p = 1000$ Pa) (see Figures 10(b-d)). Thus, the effect of surface roughness pattern ($H_s$ and $\gamma$) on $Q_T/(Q_T)_{smooth}$ decreases more significantly as the effective viscosity of gases decreases in higher mode of resonator and higher gas rarefaction.

4. CONCLUSIONS

The average MMGL equation is modified with the pressure flow factors, $(\phi^p_{xx}, \phi^p_{yy})$, the database of $Q_p(D,\alpha_1,\alpha_2)$ provided by Li [17], and the dynamic viscosity determined by Sutherland [22] to consider the effect of surface roughness pattern for various types of the gas and gas rarefaction. The Q-factors of the external SFD and the internal structural damping (TA) are taken into account to calculate the total Q-factor. Thus, the Q-factor due to the SFD ($Q_{SFD}$) and the total Q-factor ($Q_T$) are studied under the effect of surface roughness pattern ($H_s$ and $\gamma$) for various types of the gas, gas rarefaction (pressure and ACs), and mode of the resonator. Some important results are summarized below:

a) The effect of surface roughness ($H_s$, $\gamma$) on $\omega_n$, $\delta_{SFD}$, and $Q_{SFD}$ become more significant at higher effective viscosity ($\mu_{eff}$) of the gas and lower gas rarefaction (higher ambient pressure and ACs) due to the increased SFD in the 1\textsuperscript{st} mode of the resonator.

b) The effect of surface roughness pattern ($H_s$, $\gamma$) on the $Q_T/(Q_T)_{smooth}$ ratio is almost insensitive to various types of the gas in the 1\textsuperscript{st} mode of the resonator and higher gas rarefaction. Meanwhile, this effect is significantly reduced as the effective viscosity of the gas decreases in the higher mode of the resonator and higher gas rarefaction.

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