

# EFFECTS OF VARIABLE VISCOSITY ON MAGNETIC FIELD INTENSITY WITH HEAT, MASS TRANSFER OVER A VERTICAL PLATE

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**Abstract.** Heat and mass transfer flow along a vertical plate under the combined buoyancy force of thermal and species diffusion in the presence of a magnetic field is investigated. This paper examines the effect of variable viscosity with heat and mass transfer over a vertical layer on the magnetic field intensity. The governing equation of continuity, energy, momentum, and concentration is transformed into ordinary differential equations and then solved the governing transformed ordinary differential equations through the technique of perturbation. Expressions were generated for the distribution of velocity, temperature, and concentration. Tests for parameter values of importance are described graphically and addressed quantitatively from a practical point of view. The findings demonstrate that as the amount of *Grashof* ( $Gc$ ) increases, the effects of the concentration bounce increase, and hence the velocity of the fluid decreases. Velocity and temperature decrease as the *Prandtl* number ( $Pr$ ) increases. As parameter ( $K$ ) of the magnetic field increases, the intensity decreases. The velocity and concentration decrease even as the Schmidt number increases.

**Keywords:** Vertical plate, heat, and mass transfer, magnetic field, variable viscosity.

**Classification numbers:** 2.2.1, 2.2.2, 2.2.3, 5.4.4.

## 1. INTRODUCTION

The viscosity of a fluid can be termed as friction involving a fluid. It is a measure of internal resistance to the flow of a fluid when subjected to shear stress or tensile stress. Fluids that show stress resistance are termed viscous fluids while those that do not exhibit resistance to shear stress are referred to as ideal fluids or inviscid fluids (Hassan, Lawal, & Amurawaye, 2020) [1]. The viscosity of a fluid depends on various factors such as temperature, pressure, and shear rate. The temperature affects the viscosity of both liquids and gases. In the case of liquids,

increasing the temperature of the liquid leads to a reduction in its viscosity as this can be explained using the particle theory. Heat transfer through thermal radiation is of significance to most engineering and industrial processes occurring at higher temperatures and its knowledge is important in the design of most engineering equipment [2 - 5]. Such equipment is used as devices for propelling air crafts, missiles, satellites, and hypersonic flights, in rocket combustion chambers, gas turbines, and gas-cooled nuclear reactors. Variable viscosity property of a fluid and energy transfer have been of vast interest and issues of discussion among researchers in petroleum and chemical engineering due to its technological and industrial application. Some of the applications of flow can be established in well-arranged chemical reactors, geothermal containers, physical transformation companies, convertible exhaust schemes to mention a few. Investigations on fluid behavior cannot be adequately described based on variable viscosity properties, convective cooling, thermal radiation, heat source/sink, porosity, among the few [1 - 2, 6].

Many researchers have conducted thorough studies in this field, which include studies in [7 - 11]. However, in the preceding studies the radiation, chemical reaction, variable suction, and variable viscosity effects were neglected. However, exceptions are observed in [12] by analyzing the effect of radiation flows over an elastic stretching surface on heat and fluid. Mukhopadyyay [13] examined the influence of thermal radiation on elastic boundary layers of combined convection heat transfer issue from a vertical stretching surface incorporated in porous media. Based on the above research the fluid's viscosity was believed to be stable. Several researchers studied and investigated the influence of vector properties for fluid viscosity and thermal conductivity over a continuous flowing surface at fluid heat transfer. In the case of turbulent flow, [14] measured the effect of radiation and variable viscosity on a free MHD convection flow through a semi-infinite flat plate with an associated magnetic field. The influence of chemical reaction and variable viscosity on hydromagnetic combined convection heat and mass transfer for Hiemenz flow across porous media in the presence of radiation and magnetic field has been studied in [15]. Dandapat *et al.* [16] conducted a study on the effects of variable thermal conductivity, variable viscosity, and thermocapillary on flow and heat transfer on a horizontal stretch sheet in a laminar liquid film. Hayat *et al.* [17] experimented to examine the effect of variable viscosity and thermal conductivity in the three-dimensional mixed convection flow of viscous fluid across an exponentially stretching surface with heat transfer. Siddiqa *et al.* [18] studied the numerical solutions for the normal convection flow along a vertical wavy cone centered in the thermally radiating stream. Recently, Nadeem *et al.* ) [19] proposed exponential surface temperature and recommended exponential heat flux because of the movement of micropolar fluids on a Riga board. Krishna *et al.* [20] proposed a theoretical analysis of the influence of magnetohydrodynamic systems on classical heat transfer. However, there has been no study conducted on the effect of variable viscosity with heat and mass transfer over a vertical layer with particular reference to the magnetic field intensity.

The objective of this paper is to study the effects of variable viscosity on magnetic field intensity with heat, mass transfer over a vertical fixed plate, and other parameters on the flow such as modified *Grashof* number ( $G_c$ ), Schmidt number ( $Sc$ ), thermal *Grashof* number ( $Gr$ ), Prandtl ( $Pr$ ), permeability parameter ( $K_1$ ), and chemical reaction parameter ( $K$ ).

## 2. MATERIALS AND METHODS

### 2.1. Mathematical Formulation

Consider a steady, free convective flow of an incompressible viscous electrically conductive fluid through a semi-infinite vertical layer at a constant temperature  $T_w'$  and a uniform transverse magnetic field. The fluid temperature far from the plate is  $T_\infty'$ . The axis is taken upwards around the wall, and y-axis is taken to it as normal. The gravitational force descends almost vertically. As the fluid's density is small the viscous dissipative heat is believed to be zero. It is often assumed that the magnetic field of constant strength is naturally applied to the vertical layer, so that the electrical conductivity of the fluid is supposed to be minimal so that the magnetic field generated can be ignored relative to the magnetic field applied. The temperature and concentration of the fluid at infinity are considered in this paper to be  $T_\infty'$  and  $C_\infty'$  everywhere and the temperature and concentration at the plate are  $T_w'$  and  $C_w'$  respectively. The plate is taken along  $x'$ -axis in vertically upward direction and  $y'$ -axis is taken normal to the plate as shown in Fig.1

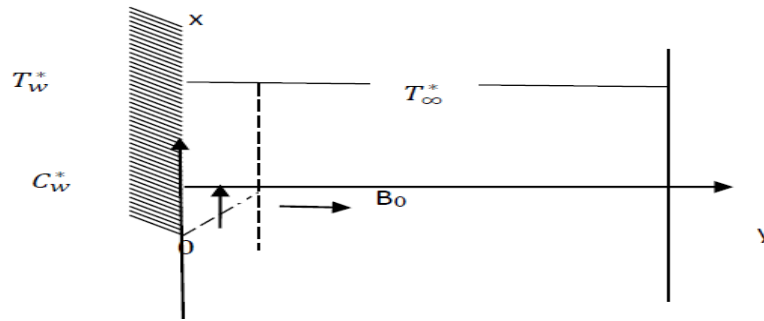


Fig 1. The physical configuration of the model.

The governing equations of the fluid flow can be written as follows

$$\frac{1}{4} \frac{\partial u'}{\partial t'} - \nu_0 (1 + \varepsilon e^{i\omega t'}) \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 u'}{\partial y'^2} - k_0 \left( \frac{\partial^3 u'}{\partial t' \partial y'^2} - \nu_0 (1 + \varepsilon e^{i\omega t'}) \frac{\partial^3 u'}{\partial y'^3} \right) - \frac{a_1 u'}{1 + d \varepsilon e^{i\omega t'}} + \frac{\sigma \mu e^2}{\rho} \frac{\partial B'}{\partial y'} + g \beta (T' - T'_\infty) + g \beta_2 (C' - C'_\infty) \quad (1)$$

$$\frac{1}{4} \frac{\partial B'}{\partial t'} - \nu_0 (1 + \varepsilon e^{i\omega t'}) \frac{\partial B'}{\partial y'} = \beta_1 \frac{\partial^2 B'}{\partial y'^2} + \frac{\partial u'}{\partial y'} \quad (2)$$

$$\frac{1}{4} \frac{\partial T'}{\partial t'} - \nu_0 (1 + \varepsilon e^{i\omega t'}) \frac{\partial T'}{\partial y'} = \left( \frac{K_T}{\rho C_p} \right) \frac{\partial^2 T'}{\partial y'^2} \quad (3)$$

$$\frac{1}{4} \frac{\partial C'}{\partial t'} - \nu_0 (1 + \varepsilon e^{i\omega t'}) \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - K (C' - C'_\infty) \quad (4)$$

where  $u'$  and  $v'$  refer to the velocity components in the directions of  $x$  and  $y$ , respectively,  $g$  is defined as the acceleration due to gravity,  $\beta$  and  $\beta_2$  denote the volume expansion coefficient,  $k_0$  and  $\nu$  represent the kinematic viscoelasticity and the kinematic viscosity, respectively,  $\rho$  is

defined as the density,  $\mu$  represents the viscosity,  $K_T$  is described as the thermal conductivity,  $C_p$  is defined as a fluid heat at constant pressure,  $\sigma$  refers to the electrical conductivity of the fluid,  $\mu_e$  is defined as the magnetic permeability,  $D$  is defined as the molecular diffusivity,  $T'_\omega$  and  $T'_\infty$  represent the temperature of the plate and the temperature of the fluid far away from plate, respectively.  $C'_\omega$  and  $C'_\infty$  refer to the concentration of the plate and the concentration of the fluid far away from the plate, respectively. For  $v_0 > 0$ , it is indicated that the negative sign suction is towards the plate.

The boundary conditions of the problem are

$$\left. \begin{aligned} y = 0 : U = 0, B = 0, T = T'_\omega + \varepsilon(T'_\omega - T'_\infty)e^{i\omega t}, \\ C = C'_\omega + \varepsilon(C'_\omega - C'_\infty)e^{i\omega t} \\ y \rightarrow 1 : U = 0, B = \theta = 0, C \rightarrow 0 \end{aligned} \right\} \quad (5)$$

Introducing the following non-dimensional quantities

$$\left. \begin{aligned} y' = \frac{yv_0}{\nu}; t' = \frac{tv_0^2}{4\nu}; u' = \frac{u}{v_0}; \theta = \frac{T' - T'_\infty}{T'_\omega - T'_\infty}; C = \frac{C' - C'_\infty}{C'_\omega - C'_\infty} \\ Gr = \frac{\nu g \beta (T'_\omega - T'_\infty)}{v_0^3}; Pr = \frac{\mu C_p}{K}; K = \frac{k_0 v_0^2}{\nu^2}; S^2 = \frac{\sigma \mu e^2 \nu}{\rho v_0^2} \\ Rm = \frac{\beta_1 v_0^2}{\nu^2}; Gc = \frac{\nu g \beta^* (C'_\omega - C'_\infty)}{v_0^3}; Sc = \frac{\nu}{D}; B = \frac{B' - B'_\infty}{B'_\omega - B'_\infty} \end{aligned} \right\} \quad (6)$$

The dimensionless variables are substituted in (6) into (1) to (5), we get (dropping the bars)

$$\frac{1}{4} \frac{\partial U}{\partial t} - (1 + \varepsilon e^{i\omega t}) \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\partial^2 U}{\partial y^2} - K_1 \left\{ \frac{1}{4} \frac{\partial^3 U}{\partial t \partial y^2} - (1 + \varepsilon e^{i\omega t}) \frac{\partial^3 U}{\partial y^3} \right\} - \frac{a_1 U}{1 + d \varepsilon e^{i\omega t}} + S^2 \frac{\partial B}{\partial y} + Gr \theta + Gc \quad (7)$$

$$\frac{1}{4} \frac{\partial B}{\partial t} - (1 + \varepsilon e^{i\omega t}) \frac{\partial B}{\partial y} = \frac{1}{Rm} \frac{\partial^2 B}{\partial y^2} + \frac{\partial U}{\partial y} \quad (8)$$

$$\frac{1}{4} \frac{\partial \theta}{\partial t} - (1 + \varepsilon e^{i\omega t}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \quad (9)$$

$$\frac{1}{4} \frac{\partial C}{\partial t} - (1 + \varepsilon e^{i\omega t}) \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - KC \quad (10)$$

with the following boundary conditions

$$\left. \begin{aligned} U = 0, B = 0, \theta = 1 + \varepsilon e^{i\omega t}, C = 1 + \varepsilon e^{i\omega t} \text{ at } y = 0 \\ U = 0, B = 0, \theta = 0, C = 0 \text{ at } y = 1 \end{aligned} \right\} \quad (11)$$

where  $Gr$  is defined as the *thermal Grashof number*,  $Gc$  the *mass Grashof number*,  $Sc$  the Schmidt number,  $Pr$  the Prandtl number,  $Rm$  the Reynold number,  $K$  the viscoelastic Parameter, and  $S$  the permeability.

## 2.2. The Boussinesq approximation Model

The reason of this assumption is that there are flows in which all the temperature varies slightly and thus the density varies slightly, but in which the buoyancy controls the motion. For certain natural convection flows, you can achieve easier convergence with the *Boussinesq approximation* model by creating a fluid density problem as a function of temperature (Cherkasov *et al.* 2018). This model represents density as a constant value in all the equations that have been resolved, except for the buoyancy term in the momentum as follows:

$$(\rho - \rho_0)g \approx \rho_0\beta(T - T_0)g \tag{12}$$

where  $\rho_0$  describes the (constant) density of the flow,  $T_0$  represents operating temperature and  $\beta$  describes the thermal expansion coefficient. Equation (12) is generated by obeying the Boussinesq approximation  $\rho = \rho_0(1 - \beta\Delta T)$  to eliminate  $\rho$  the buoyancy term. This approximation is valid as long as the original density changes are low; specifically, the Boussinesq approximation is correct when equation (13) is satisfied.

$$\beta(T - T_0) \leq 1 \tag{13}$$

### 3. METHODS OF SOLUTION

Magnetic field strength, also called magnetic intensity or magnetic field intensity, is the part of the magnetic field in a material that arises from an external current and is not intrinsic to the material itself. It is expressed as a vector and is measured in units of amperes per meter. In order to solve (7) to (10) according to the boundary conditions (11), we assume the following equations:

$$U(y, t) = U_0(y) + U_1(y) \varepsilon e^{i\omega t} \tag{14}$$

$$B(y, t) = B_0(y) + B_1(y) \varepsilon e^{i\omega t} \tag{15}$$

$$\theta(y, t) = \theta_0(y) + \theta_1(y) \varepsilon e^{i\omega t} \tag{16}$$

$$C(y, t) = C_0(y) + C_1(y) \varepsilon e^{i\omega t} \tag{17}$$

where  $U_0(y)$ ,  $B_0(y)$ ,  $\theta_0(y)$ ,  $C_0(y)$ ,  $U_1(y)$ ,  $B_1(y)$ ,  $\theta_1(y)$  and  $C_1(y)$  can be determined.

Substituting (14) to (17) into (7) to (11), respectively, and comparing harmonic terms with non-harmonic terms, yield the following equations:

$$K_1 U_0''' + U_0'' + U_0' - a_1 U_0 = -1 - S^2 B_0' - Gr \theta_0 - Gc C_0 \tag{18}$$

$$B_0'' + Rm B_0' = -Rm U_0' \tag{19}$$

$$\theta_0'' + Pr \theta_0' = 0 \tag{20}$$

$$C_0'' + Sc C_0' - Sc K C_0 = 0 \tag{21}$$

$$K_1 U_1''' + \left(1 - \frac{Ki\omega}{4}\right) U_1'' + U_1' - \left(1 + \frac{i\omega}{4}\right) U_1 = -K_1 U_0''' - U_0'' - a_1 U_0' - S^2 B_1' - Gr \theta_1 - Gc C_1 \tag{22}$$

$$B_1 + Rm B_1' - Rm \frac{i\omega}{4} B_1 = -Rm B_0' - Rm U_1' \tag{23}$$

$$\theta_1'' + Pr \theta_1' - \frac{i\omega Pr}{4} \theta_1 = -Pr \theta_0' \tag{24}$$

$$C_1'' + ScC_1' - Sc\left(K + \frac{i\omega}{4}\right)C_1 = -ScC_0' \quad (25)$$

with the following boundary conditions

$$\left. \begin{aligned} U_0 = 0, U_1 = 0, B_0 = B_1 = 0, \theta_0 = \theta_1 = 1, C_0 = C_1 = 1 \quad \text{at } y = 0 \\ U_0 = 0, U_1 = 0, B_0 = B_1 = 0, \theta_0 = \theta_1 = 0, C_0 = C_1 = 0 \quad \text{at } y = 1 \end{aligned} \right\} \quad (26)$$

where the primes denote differentiation concerning  $y$ .

Solving equations (18) to (25) according to boundary conditions (26), and substituting the solutions obtained with (14) and (16), respectively, the field of velocity can be represented as follows:

$$U(y,t) = \left( \begin{aligned} &-\frac{B_2 m_7}{Rm} e^{m_7 y} + \frac{B_3 m_8}{Rm} e^{-m_8 y} - \frac{B_4 m_9}{Rm} \cos m_9 y - \frac{B_5 m_{10}}{Rm} \sin m_{10} y + \frac{B_7 Pr}{Rm} e^{-Pr y} - \frac{B_8 m_3}{Rm} e^{m_3 y} \\ &+ \frac{B_9 m_4}{Rm} e^{-m_4 y} - B_2 e^{m_7 y} - B_3 e^{m_8 y} - B_4 \cos m_9 y - B_5 \sin m_{10} y - B_6 - B_7 e^{-Pr y} - B_8 e^{m_3 y} - B_9 e^{-m_4 y} \end{aligned} \right) \\ + \varepsilon e^{i\omega t} \left\{ \begin{aligned} &D_1 e^{n_7 y} + D_2 e^{-n_2 y} + D_3 e^{n_3 y} + D_4 \cos n_4 y + D_5 \sin n_5 y + D_6 N_1 e^{m_1 y} + D_7 N_2 e^{-m_2 y} \\ &+ D_8 N_3 e^{m_3 y} + D_9 N_4 e^{-m_4 y} + D_{10} N_5 e^{m_5 y} + D_{11} N_6 e^{m_6 y} + D_{12} N_7 e^{m_7 y} + D_{13} N_8 e^{-m_8 y} \\ &+ D_{14} N_9 e^{-Pr y} + (N_{10} D_{15} + N_{11} D_{18}) \cos m_9 y + (N_{10} D_{16} + N_{11} D_{17}) \sin m_9 y \\ &- B_2 e^{m_7 y} - B_3 e^{m_8 y} - B_4 \cos m_9 y - B_5 \sin m_5 y - B_6 - B_7 e^{-Pr y} - B_8 e^{m_3 y} - B_9 e^{-m_4 y} \end{aligned} \right\} \quad (27)$$

the induced magnetic field becomes

$$B(y,t) = B_2 e^{m_7 y} + B_3 e^{-m_8 y} + B_4 \cos m_9 y + B_5 \sin m_{10} y + B_6 + B_7 e^{-Pr y} + B_8 e^{m_3 y} + B_9 e^{-m_4 y} \\ + \varepsilon e^{i\omega t} \left\{ \begin{aligned} &B_1(y) = D_1 e^{n_7 y} + D_2 e^{-n_2 y} + D_3 e^{n_3 y} + D_4 \cos n_4 y + D_5 \sin n_5 y + D_6 N_1 e^{m_1 y} + D_7 N_2 e^{-m_2 y} \\ &+ D_8 N_3 e^{m_3 y} + D_9 N_4 e^{-m_4 y} + D_{10} N_5 e^{m_5 y} + D_{11} N_6 e^{m_6 y} + D_{12} N_7 e^{m_7 y} + D_{13} N_8 e^{-m_8 y} \\ &+ D_{14} N_9 e^{-Pr y} + (N_{10} D_{15} + N_{11} D_{18}) \cos m_9 y + (N_{10} D_{16} + N_{11} D_{17}) \sin m_9 y \end{aligned} \right\} \quad (28)$$

and, by the temperature field, we have:

$$\theta(y,t) = A + B e^{-Pr y} + \varepsilon e^{i\omega t} (A_1 e^{m_1 y} + A_2 e^{-m_2 y} + A_3 e^{-Pr y}) \quad (29)$$

Similarly, the concentration distribution yields:

$$C(y,t) = A_4 e^{m_3 y} + A_5 e^{-m_4 y} + \varepsilon e^{i\omega t} (A_6 e^{m_5 y} + A_7 e^{-m_6 y} + A_8 e^{m_3 y} + A_9 e^{-m_4 y}) \quad (30)$$

#### 4. RESULTS AND DISCUSSION

The results on velocity, temperature, and concentration distributions have been discussed for the following parameters: modified Grashof number ( $Gc$ ), Thermal Grash of number ( $Gr$ ), Prandtl number ( $Pr$ ), Schmidt number ( $Sc$ ), Permeability parameter ( $K_1$ ), and chemical reaction parameter ( $K$ ).

##### 4.1. Velocity profiles

Figures 2 to 7 reflect the respective velocity profiles with changing parameters.

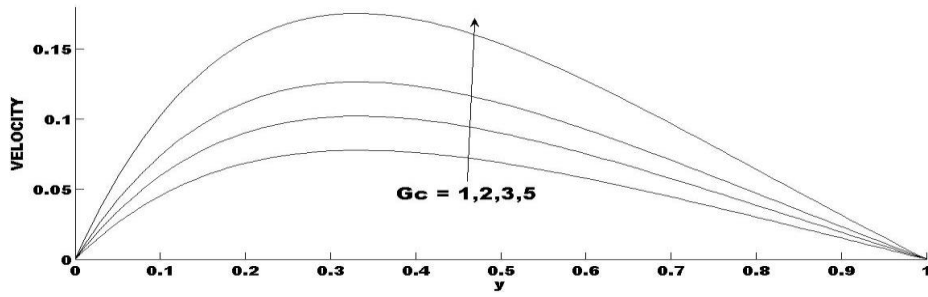


Figure 2. Profile velocity of various values of  $G_c$ .

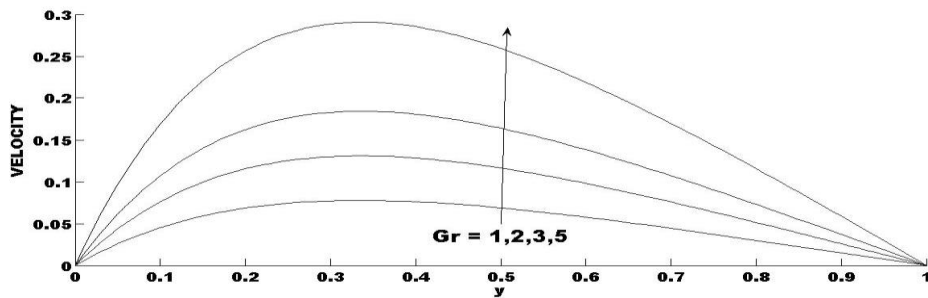


Figure 3. Profile velocity of various values of  $Gr$ .

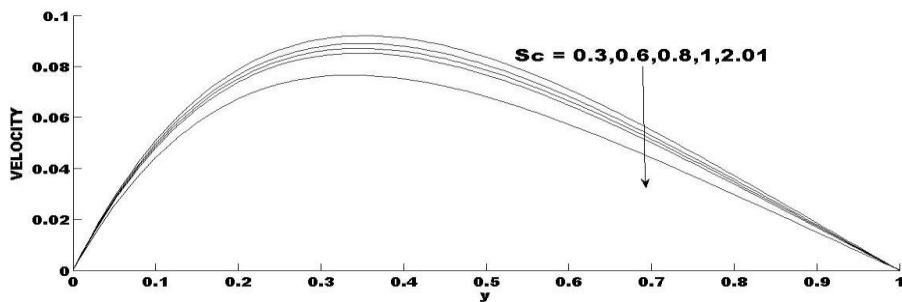


Figure 4. Profile velocity of various values of  $Sc$ .

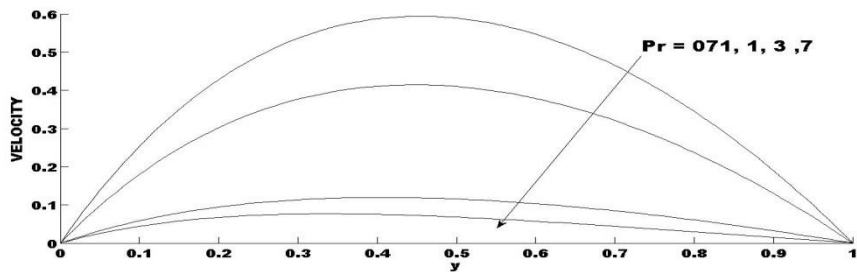


Figure 5. Profile velocity of various values of  $Pr$ .

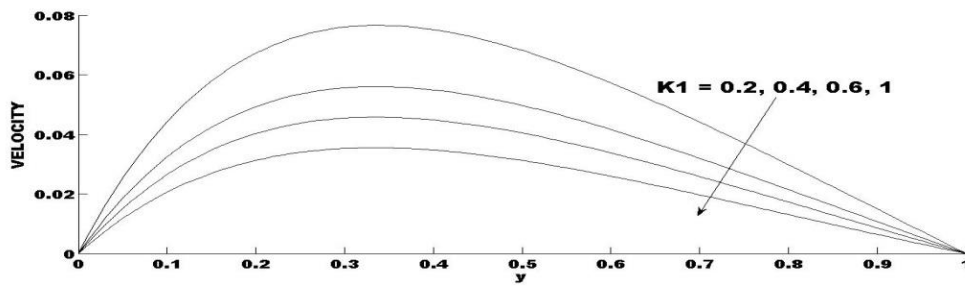


Figure 6. Profile velocity of various values of  $K_1$ .

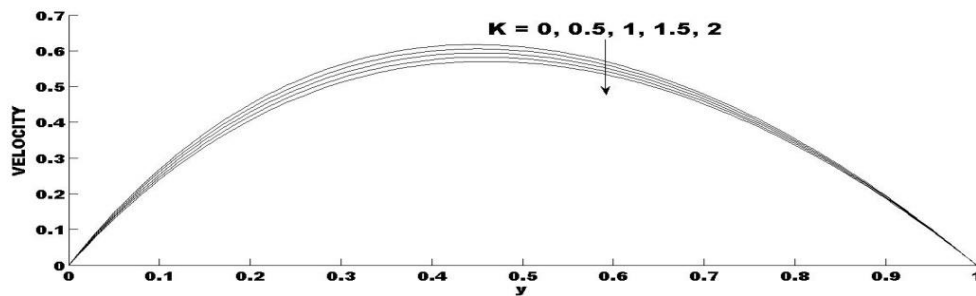


Figure 7. Profile velocity of various values of  $K$ .

The influence of the modified  $G_c$  on the velocity is displayed in Figure 2. It is revealed that when all other parameters are kept constant, the velocity increases with increasing the modified  $G_c$ . The influence of the thermal  $Gr$  on the velocity is displayed in Figure 3, showing that increasing  $Gr$  leads to an increase in efficiency as all other parameters occurring in the area of velocity are kept constant. Figure 4 presents the influence of  $Sc$  on the velocity. It is found that when all the competing parameters are kept constant, and an increase in  $Sc$  reduces the velocity. The effect of  $Pr$  on the velocity field is illustrated in Figure 5, where the velocity field is observed to increase with  $Pr$  when all other parameters occurring in the velocity field are kept constant. Figure 6 illustrates the speed profiles for various parameters of  $K_1$ . It is clear that the peak value of the velocity continues to decrease as  $K_1$  increases when all other contributing parameters are kept constant. It can be seen from Figure 7, describing the effect of  $K$  on the velocity, that when all other participating parameters are kept constant, the chemical reaction parameter  $K$  is decreased, the velocity, in general, decreases with reducing the chemical reaction parameter  $K$ . It's also found that as we're heading away from the ground, the velocity of the fluid is going down.

#### 4.2. Temperature and Concentration profiles

The temperature and concentration profiles are presented in Figure 8 and Figure 9, respectively. The effect of  $Pr$  on the temperature field is illustrated in Figure 8, from which it is observed that the temperature field decreases with increasing  $Pr$  when all other parameters that appear in the temperature field are held constant. Figure 9 displays the influence of  $Sc$  on the concentration, where one can see that as all other participating parameters are held constant, increasing  $Sc$  leads to a decrease in the concentration.



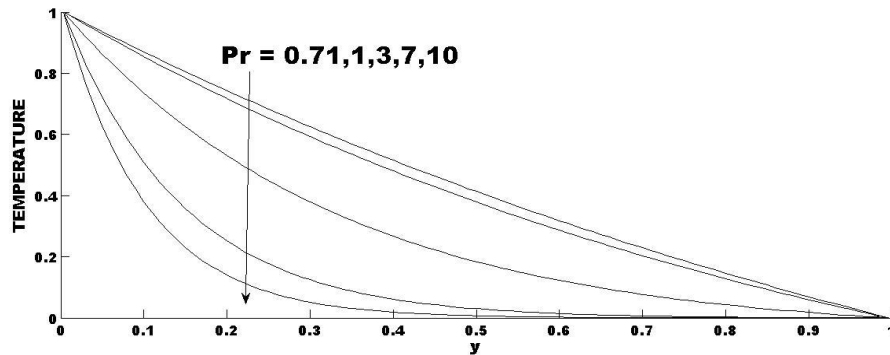


Figure 8. Temperature profile for different values of Pr.

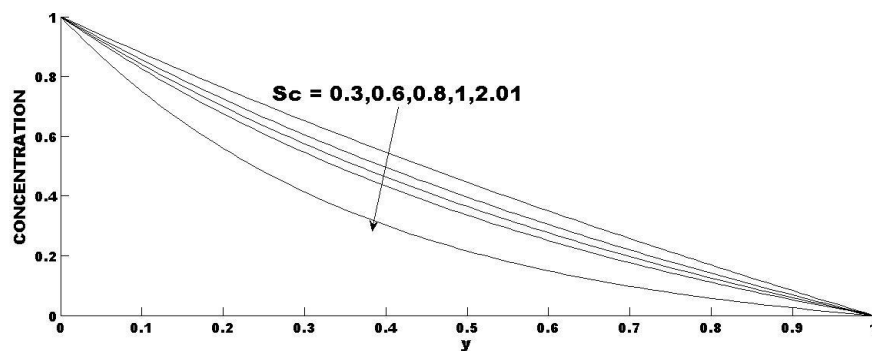


Figure 9. Concentration profile for different values of Sc.

#### 4. CONCLUSIONN

In this paper, the governing equations for variable viscosity effects on magnetic heat and mass transfer equations over a vertical plate have been studied. The resulting equations have been analytically solved using the perturbation method. The results displayed illustrate the flow characteristics with respect to velocity, temperature, and concentration. It is found that when the Grashof number increases, the concentration buoyancy effect is enhanced and thus, the fluid velocity increases. The velocity as well as the temperature decrease with an increase in the Prandtl number Pr. It is revealed that, with the increase of chemical reaction parameters, the velocity decreases as well. The velocity decrease with the increase in the Schmidt numbers. An increase in buoyancy force decreases the skin friction, whereas there is no effect on the Nusselt number and Sherwood number. An increase in the magnetic field increases the skin friction and a reverse trend is seen for higher time on the Nusselt number. The Prandtl number has a sign on the Nusselt number. The Sherwood number decreases due to higher values of the Schmidt number.

**CRedit authorship contribution statement.** Hamza Abubakar: Methodology, Investigation, Formal analysis, editing and restructuring. Sani Isah: Formal analysis, Supervision. I.J. Uwanta: Supervision.

**Declaration of competing interest.** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## REFERENCES

1. Hassan A., Lawal O., and Amurawaye F. - The Effect of Variable Viscosity on a Reactive Heat Generating Fluid Flow over a Convective Surface, *Journal of the Nigerian Mathematical Society* **39** (1) (2020) 21-38.
2. Chinyoka T. and Makinde O. D. - Computational dynamics of unsteady flow of a variable viscosity reactive fluid in a porous pipe, *Mechanics Research Communications* **37** (3) (2010) 347, 353.
3. Sparrow E. M. and Cess R. D. - Radiation heat transfer, Hemisphere, Washington, DC, 1978.
4. Mbeledogu I. U., and Ogulu A. - Heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous flat plate in the presence of radiative heat transfer, *International journal of heat and mass transfer* **50** (9-10) (2007) 1902-1908.
5. Hassan A. R. Maritz, R. and Gbadeyan J. A. - A reactive hydromagnetic heat generating fluid flow with thermal radiation within porous channel with symmetrical convective cooling, *International Journal of Thermal Sciences* **122** (1) (2017) 248-256.
6. Hassan A. R. and Gbadeyan J. A. - A reactive hydromagnetic internal heat generating fluid flow through a channel, *International Journal of Heat and Technology* **33** (3) (2015) 43-50.
7. Nield D. A. and Bejan A. A. - Convection in porous media, 2<sup>nd</sup> edition, Springer, Newyork, 1999.
8. Ingham D. B. and Pop I. - Transport Phenomena in Porous Media II, Pergamon, Oxford, 2002.
9. Ingham D. B. and Pop I. - Transport Phenomena in porous Media I, Pergamon, Oxford, 1998.
10. Bejan A. Khair K. R. - Heat and Mass Transfer by Natural Convection in Porous Media, *International Journal Heat and Mass Transfer* **28** (1995) 09-9118.
11. Trevisan O. V. and Bejan A. - Combined Heat and Mass Transfer by Natural Convection in Porous Medium, *Advnce Heat Transfer* **20** (1990) 315-352.
12. Abd El-aziz M. - Radiation of effect on the flow and heat transfer over an unsteady stretching sheet, *International Communication in Heat and Mass Transfer* **6** (5) (2008) 521-524.
13. Mukhopadyay S. - Effect of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium, *International Journal of Heat and Mass Transfer* **52** (2009) 3261-3265.
14. Seddeek M. - A Effects of radiation and variable viscosity on a MHD free convection flow past a semi-infinite flat plate with an aligned magnetic field in the case of unsteady flow, *International Journal of Heat and Mass Transfer* **45** (4) (2002) 931-935.
15. Seddeek M. A., Darwish A. A., and Abdelmeguid M. S. - Effects of chemical reaction and variable viscosity on hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous mediawith radiation, *Communications in Nonlinear Science and Numerical Simulation* **12** (2) (2007) 195-213.

16. Dandapat B. S., Santra B., and Vajravelu K. - The effects of variable fluid properties and thermocapillarity on the flow of a thin film on an unsteady stretching sheet, *International Journal of Heat and Mass Transfer* **50** (5-6) (2007) 991-996.
17. Hayat T., Khan M. I., Farooq M., Gull N., and Alsaedi A. - Unsteady three-dimensional mixed convection flow with variable viscosity and thermal conductivity, *Journal of Molecular liquids* **223** (2016) 1297-1310.
18. Siddiqa S., Begum N., and Hossain M. A. - Radiation effects from an isothermal vertical wavy cone with variable fluid properties, *Applied Mathematics and Computation* **289** (2016) 149-158.
19. Nadeem S., Malik M. Y., and Abbas N. - Heat transfer of three-dimensional micropolar fluid on a Riga plate, *Canadian Journal of Physics* **98** (1) (2020) 32-38.
20. Krishna C. M., ViswanathaReddy G., Souayah B., Raju C. S. K., Rahimi-Gorji M., and Raju S. S. K. - Thermal convection of MHD Blasius and Sakiadis flow with thermal convective conditions and variable properties, *Microsystem Technologies* **25** (10) (2019) 3735-3746.
21. Cherkasov S. G., Anan'ev A. V., and Moiseeva L. A. - Limitations of the Boussinesq model on the example of laminary natural convection of gas between vertical isothermal walls, *High Temperature* **56** (6) (2018) 878-883.
22. Animasaun I. L. - Melting heat and mass transfer in stagnation point micropolar fluid flow of temperature dependent fluid viscosity and thermal conductivity at constant vortex viscosity, *Journal of the Egyptian Mathematical Society* **25** (1) (2017) 79-85.