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A UNIFIED PORT-HAMILTONIAN APPROACH FOR MODELLING AND STABILIZING CONTROL OF ENGINEERING SYSTEMS

Ngoc-Ha Hoang[1, 2, *], Phuong-Quyen Le[2], Chi-Thuan Nguyen[3]

[1]*Institute of Research and Development, Duy Tan University, Da Nang, 550000, Viet Nam*

[2]*Faculty of Electrical-Electronic Engineering, Duy Tan University, Da Nang, 550000, Viet Nam*

[3]*Research and Development Center for Radiation Technology, HCM City, Viet Nam*

[*]Email*: hoangngocha2@duytan.edu.vn, ngocha.h@gmail.com*

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Abstract. This work deals with systems whose dynamics are affine in the control input. Such dynamics are considered to be significantly differentially expressed in a canonical form, namely the quadratic (pseudo) port-Hamiltonian representation, in order to explore further some structural properties usable for the tracking-error passivity-based control design without the (generalized) canonical transformation. Different kinds of linear and nonlinear engineering systems including an open isothermal homogeneous system and a continuous biochemical fermenter are used to illustrate the approach.

Keywords: engineering systems, quadratic port-Hamiltonian representation, passivity, tracking-error control.

Classification numbers: 3.7.1, 4.10.2, 4.10.4, 5.4.2.

1. INTRODUCTION

This paper deals with the port-based modelling of general engineering systems [1] whose dynamics are described by a set of Ordinary Differential Equations (ODEs) and affine in the control input u as follows:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u},\ \ \boldsymbol{x}(\boldsymbol{t} = \boldsymbol{0}) = \boldsymbol{x_0} \tag{1}$$

where $x = x(t)$ is the state vector contained in the operating region $\mathrm{D} \in \mathbb{R}^n$, $f(x) \in \mathbb{R}^n$ expresses the smooth (nonlinear) function with respect to x. The input-state map and the control input are denoted by $g(x) \in \mathbb{R}^{n\times m}$ and $u \in \mathbb{R}^m$, respectively. It is worth noting that many industrial applications in the fields - physical, mechanical, electrical, and biochemical, etc. belong to this kind of systems [2 - 5].

In addition to the Bond graph modelling [6, 7], the port-based modelling [8, 9] leads to the so-called port-Hamiltonian (PH) systems. It is important to transform the dynamic equation (1) into the PH representation prior to developing state feedback laws for stabilizing control purposes [10 - 14]. In this work, we focus our attention on a particular class of the PH systems, called the quadratic PH systems, where the Hamiltonian function is of the quadratic form [8, 15]. In other words, once the quadratic PH representation of the system dynamics is derived, then the tracking-error passivity-based control approach can be advantageously applied to show stabilization properties despite abnormal behaviours (for example, combined input-output multiplicities [16]). This is the main contribution of this study.

The paper is organized as follows. Section 2 gives a brief overview of the PH representation of affine dynamical systems, including motivating examples. Section 3 is devoted to two case studies. The first case study focusses on an open, isothermal homogeneous system while the second one is a continuous biochemical fermenter system. The design of an error-tracking-based dynamic controller together with the implementation of numerical simulations for the purpose of comparison is then included. We end the paper with some concluding remarks in Section 4.

Notations: The following notations are considered throughout the paper:

- $\mathbb{R}$ is the set of real numbers.
- T stands for the matrix transpose operator.
- m and n $(m \leq n)$ are positive integers.
- x_0 is the initial value of the state vector x.

2. THE QUADRATIC (PSEUDO) PH REPRESENTATION

Assume that the drift vector field $f(x)$ of the dynamics (1) verifies the so-called separability condition [17 - 19], that is, $f(x)$ can be decomposed and expressed as the product of some (interconnection and damping) structure matrices and the gradient of a potential function with respect to the state variables, i.e. of the co-state variables:

$$f(x) \triangleq [J(x) - R(x)]\frac{\partial H(x)}{\partial x} \tag{2}$$

where $J(x)$ and $R(x)$ are the $n \times n$ skew-symmetric interconnection matrix (i.e. $J(x) = -J(x)^{\mathrm{T}}$) and the $n \times n$ symmetric damping matrix (i.e. $R(x) = R(x)^{\mathrm{T}}$), respectively while $H(x) : \mathbb{R}^n \to \mathbb{R}$ represents the Hamiltonian storage function of the system (possibly related to the total energy of the system) and if the damping matrix $R(x)$ is positive semi-definite

$$R(x) \geq 0 \tag{3}$$

then the dynamic model (1) with (2) is said to be a PH representation with dissipation [8, 9]. It is then completed with the output $y \triangleq g(x)^{\mathrm{T}}\frac{\partial H}{\partial x}$ and rewritten as follows[1]:

$$\begin{cases} \frac{dx}{dt} = [J(x) - R(x)]\frac{\partial H(x)}{\partial x} + g(x)u, \\ y = g(x)^{\mathrm{T}}\frac{\partial H(x)}{\partial x}. \end{cases} \tag{4}$$

It can be shown for the PH representation (4) that the time derivative of the Hamiltonian $H(x)$ satisfies the energy balance equation below [8, 9]

[1]We shall not elaborate any further on the PH representation here (for example, the concepts related to the cyclo-passive and passive property or Dirac structure, etc.) and refer the reader to [8, 9, 19] for more details and applications.

$$\frac{dH(x)}{dt} = -\left[\frac{\partial H(x)}{\partial x}\right]^{\mathrm{T}} R(x) \frac{\partial H(x)}{\partial x} + u^{\mathrm{T}} y. \quad (5)$$

With (3), Eq. (5) becomes:

$$\underbrace{\frac{dH(x)}{dt}}_{stored\ power} \leq \underbrace{u^T y}_{supplied\ power} \quad (6)$$

From a physical point of view, inequality (6) implies that the total amount of energy supplied from external source is always greater than the increase in the energy stored in the system. Also, equality in (6) holds if and only if the damping matrix $R(x)$ that is strongly related to the dissipation term is equal to 0. Thus, the PH system (4) is said to be passive with the input u and the output y corresponding to the Hamiltonian storage function $H(x)$ [20].

Remark 1. *If the damping matrix* $R(x)$ (3) *is negative semi-definite or indefinite then the energy balance equation* (5) *might lose its physical meaning. In other words, inequality* (6) *is not met. In that case, the structure* (4) *is called a pseudo PH system* [19].

Motivated by the recent work of Monshizadeh and coauthors [15], the (pseudo) PH representation (4) is considered here with the Hamiltonian given by

$$H(x) \triangleq \frac{1}{2} x^{\mathrm{T}} R_{di} x, \quad (7)$$

where the constant square matrix R_{di} is symmetric positive definite. The PH form (4) with (7) then reduces to the affine quadratic PH representation that enables the tracking-error passivity-based control design for the stabilization of the state x at a desired set-point x^* [21, 22] without the (generalized) canonical transformation as done in [14]. To highlight our motivation, the quadratic PH representation of linear electrical and mechanical systems will be provided next (extracted from literature, see e.g. [2, 9, 23]).

Motivating example 1. Consider the linear time-invariant circuit consisting of the series connection of a resistor (with resistance R), an inductor (with inductance L), a capacitor (with capacitance C), and a voltage source V [23], as sketched in Fig. 1.

$$\begin{cases} \text{For the inductor L: } \phi_L = Li \text{ and } u_L = \frac{d\phi_L}{dt}; \\ \text{For the capacitor C: } i = \frac{dq_C}{dt} \text{ and } q_C = Cu_C; \\ \text{For the resistor R: } u_R = Ri. \end{cases}$$

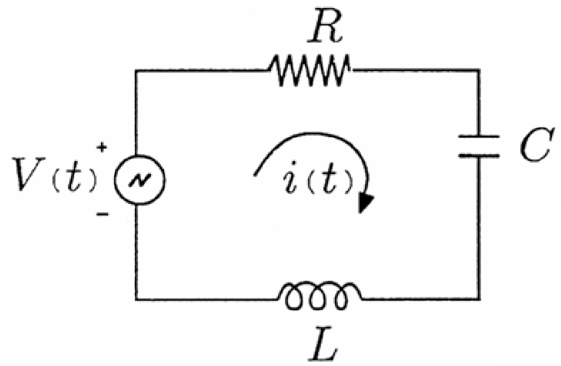

Figure 1. A series RLC circuit.

On the basis of electric circuit theory [2, 24], the following constitutive equations are derived:

$$\begin{cases} \text{For the inductor L: } \phi_L = Li \text{ and } u_L = \frac{d\phi_L}{dt}; \\ \text{For the capacitor C: } i = \frac{dq_C}{dt} \text{ and } q_C = Cu_C; \\ \text{For the resistor R: } u_R = Ri. \end{cases} \tag{8}$$

where q_C and ϕ_L are the charge stored in the capacitor C and the magnetic flux through the inductor L, respectively, while i is the electric current passing through the circuit and u_L is the voltage of the inductor L (and similarly for u_R and u_C). By considering Kirchhoff's voltage law (i.e., the second law [24]), one obtains:

$$V = u_R + u_C + u_L. \tag{9}$$

Using (8), Eq. (9) becomes:

$$V = Ri + \frac{q_C}{C} + \frac{d\phi_L}{dt} \tag{10}$$

From Eqs. (8) and (10), the following equations hold:

$$\begin{pmatrix} \frac{dq_C}{dt} \\ \frac{d\phi_L}{dt} \end{pmatrix} = \begin{pmatrix} \frac{\phi_L}{L} \\ -\frac{q_C}{C} - R\frac{\phi_L}{L} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} V. \tag{11}$$

Let x be the vector consisting of the charge q_C and the magnetic flux ϕ_L, i.e. $x = (x_1, x_2)^T \equiv (q_C, \phi_L)^T$, Eq. (11) therefore becomes Eq. (1) with:

$$f(x) = \begin{pmatrix} \frac{x_2}{L} \\ -\frac{x_1}{C} - R\frac{x_2}{L} \end{pmatrix} \tag{12}$$

$$g(x) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{13}$$

and

$$u = V. \tag{14}$$

On the other hand, Eq. (12) can be rewritten as follows:

$$f(x) = \begin{pmatrix} 0 & 1 \\ -1 & -R \end{pmatrix} \begin{pmatrix} \frac{x_1}{C} \\ \frac{x_2}{L} \end{pmatrix} \tag{15}$$

This, combined with (2), yields:

$$J(x) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{16}$$

$$R(x) = \begin{pmatrix} 0 & 0 \\ 0 & R \end{pmatrix}, \tag{17}$$

and the Hamiltonian function H(x) is given by Eq. (7) with

$$R_{di} = \frac{1}{2} \begin{pmatrix} \frac{1}{C} & 0 \\ 0 & \frac{1}{L} \end{pmatrix}. \tag{18}$$

Hence, the dynamics (11) give rise to a quadratic PH representation where the output y is expressed as

$$y = \frac{x_2}{L} \equiv i \text{ (the current)} \tag{19}$$

It is important to note that $R(x) = R(x)^T \geq 0$ and the Hamiltonian $H(x)$ (7) with (18) is equal to the total energy of the system (i.e., it characterizes the amount of energies stored in capacitor and inductor, respectively). Consequently, it has the unit of energy.

Motivating example 2. Consider an ideal mass-spring-damper system as shown in Fig. 2 [23].

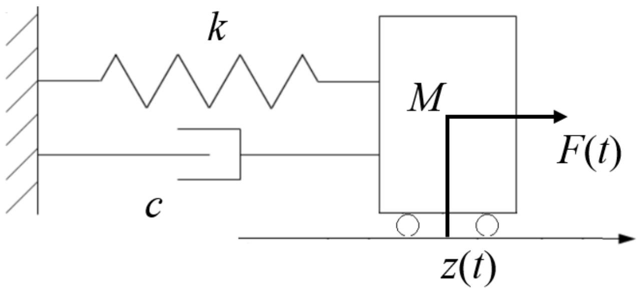

Figure 2. A mass-spring-damper system.

The following equation is derived using Newton's second law [25][2]:

$$M\frac{d^2z(t)}{dt^2} = F - kz(t) - c\frac{dz(t)}{dt}, \tag{20}$$

where:

- M is the mass of body;
- F is the external force;
- k is the stiffness constant of the linear spring;
- c is the damping constant;

Let x be the vector consisting of the movement z(t) and the momentum $M\frac{dz(t)}{dt}$ of the body, i.e. $x = (x_1, x_2)^T \equiv \left(z(t), M\frac{dz(t)}{dt}\right)^T$, Eq. (20) can be rewriten as follows:

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -C \end{pmatrix} \begin{pmatrix} kx_1 \\ \frac{x_2}{M} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} F. \tag{21}$$

Similarly to the previous motivating example, the system dynamics (21) lead to a quadratic PH representation (4) with:

$$J(x) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{22}$$

$$R(x) = \begin{pmatrix} 0 & 0 \\ 0 & c \end{pmatrix}, \tag{23}$$

$$g(x) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, u = F, \tag{24}$$

and the Hamiltonian function H(x) given by Eq. (7) with

[2]Equation (20) belongs to the (generalized) Euler-Lagrange equations of classical mechanics [2, 9].

$$\mathrm{R_{di}} = \frac{1}{2}\begin{pmatrix} \mathrm{k} & 0 \\ 0 & \frac{1}{\mathrm{M}} \end{pmatrix}. \tag{25}$$

Finally, the output y is derived as

$$\mathrm{y} = \frac{\mathrm{x_2}}{\mathrm{M}} \equiv \frac{\mathrm{dz(t)}}{\mathrm{dt}} \text{ (the velocity)} \tag{26}$$

In this example, the Hamiltonian H(x) (7) with (25) is also equal to the total energy of the system (i.e., it characterizes the amount of the elastic potential energy of the spring and the kinetic energy of the body, respectively). Consequently, it has the unit of energy. The damping matrix R(x) (23) is symmetric positive semi-definite.

In what follows, we shall illustrate the derivation of the quadratic (pseudo) PH representation of nonlinear chemical and biological systems. This is the main contribution of this work.

3. CASE STUDIES

3.1. Case study 1: An open isothermal homogeneous system with internal transformation

We consider next the transformations described by Van de Vusse mechanism taking place in an isothermal continuous stirred tank reactor to produce products from raw materials

$$\begin{array}{c} S_1 \overset{k_1}{\rightarrow} S_2 \overset{k_2}{\rightarrow} S_3 \\ 2\,S_1 \overset{k_3}{\rightarrow} S_4 \end{array} \tag{27}$$

where S_i stands for species i. The species S_1 and S_2 are the reactant and main product, respectively. The main product S_2 is of most interest to practitioners while the two other undesired products are S_3 and S_4. A typical example of the Van de Vusse mechanism is the synthesis of cyclopentenol from cyclopentadiene by sulfuric acid-catalyzed addition of water in a dilute solution. Based on the material balance equations, the mathematical model of the system is given as follows [26- 29]:

$$\begin{cases} \frac{dx_1}{dt} = -k_1x_1 - 2k_3x_1^2 + (x_{10} - x_1)u \\ \frac{dx_2}{dt} = k_1x_1 - k_2x_2 - x_2u \end{cases} \tag{28}$$

where:

- x_1 and x_2 are the concentrations of S_1 and S_2, respectively;
- x_{10} is the concentration of S_1 in the inlet;
- u is the dilution rate and considered as the control input;
- k_i, i = 1, 2, 3, are the (constant) isothermal reaction kinetics and $k_1 = k_2$ (see e.g., [26, 28]).

Let us state the following proposition.

Proposition 1. *The system dynamics (28) admit a quadratic PH representation (4) where* $x = (x_1, x_2)^{\mathrm{T}}$ *and the Hamiltonian is of the form (7)*[3] *with*

[3]In this case, the Hamiltonian $H(x)$ has a clear physical meaning and is strongly related to the inventories-based storage function of chemical processes [30].

$$R_{di} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{29}$$

and

$$J(x) = \begin{pmatrix} 0 & -k_1/2 \\ k_1/2 & 0 \end{pmatrix}, \tag{30}$$

$$R(x) = \begin{pmatrix} k_1 + 2k_3x_1 & -k_1/2 \\ -k_1/2 & k_2 \end{pmatrix}, \tag{31}$$

$$g(x) = \begin{pmatrix} x_{10} - x_1 \\ -x_2 \end{pmatrix}, \tag{32}$$

$$y = (x_{10} - x_1)x_1 - x_2^2. \tag{33}$$

Proof. First of all, the dynamics (28) are rewritten as Eq. (1) with $f(x) = \begin{pmatrix} -k_1x_1 - 2k_3x_1^2 \\ k_1x_1 - k_2x_2 \end{pmatrix}$ and $g(x)$ (32). Let $M(x)$ be the square matrix given by $\begin{pmatrix} -k_1 - 2k_3x_1 & 0 \\ k_1 & -k_2 \end{pmatrix}$, it follows that $f(x) = M(x)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. It can easily be checked that the separability condition (2) is met for $f(x)$ above where $H(x)$ is of the quadratic form (7) with R_{di} given by (29). Using the fact that any square matrix can uniquely be written as sum of a symmetric and a skew-symmetric matrix thanks to the Toeplitz decomposition of linear algebra, one may write $J(x) = \frac{M(x) - M(x)^{\mathrm{T}}}{2}$ and $R(x) = -\frac{M(x) + M(x)^{\mathrm{T}}}{2}$ that lead to Eqs. (30) and (31), respectively. Finally, the damping matrix $R(x)$ (31) is symmetric positive definite because all the principal minors of $R(x)$ are (strictly) positive due to the fact that $k_1 = k_2$. The latter completes the proof.

3.2. Case study 2: A continuous biochemical fermenter system

We consider next the dynamic model of a second order continuous biochemical fermenter described by the equations (see Section 4 in [3])

$$\begin{cases} \frac{dc_x}{dt} = \mu(c_S)c_x - \frac{q}{V}c_x \\ \frac{dc_S}{dt} = -\frac{\mu(c_S)}{Y}c_x + \frac{q}{V}\left(S_f - c_S\right) \end{cases} \tag{34}$$

where:

- c_x and c_s denote the cell and substrate concentrations, respectively;
- The term $\mu = \mu(c_s)$ denotes the specific cell growth rate;
- q is the volumetric inflow rate of the reactor and is equal to the outflow rate;
- V is the total reactor volume and is assumed to be constant;
- S_f is the feed of substrate entering the reactor;
- Y is the biomass/substrate yield coefficient. Let us state the following proposition.

Let us state the following proposition.

Proposition 2. *The system dynamics (34) are a quadratic pseudo PH representation (4) where* $x = (x_1, x_1)^{\mathrm{T}} \equiv (c_x, c_s)^{\mathrm{T}}$ *and the Hamiltonian storage function is of the form (7) with*

$$R_{di} = \tfrac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{35}$$

and

$$J(x) = \begin{pmatrix} 0 & \mu(c_s)/_{2Y} \\ -\mu(c_s)/_{2Y} & 0 \end{pmatrix}, \tag{36}$$

$$R(x) = \begin{pmatrix} -\mu(c_s) & \mu(c_s)/_{2Y} \\ \mu(c_s)/_{2Y} & 0 \end{pmatrix}, \tag{37}$$

$$g(x) = \begin{pmatrix} -c_x \\ S_f - c_S \end{pmatrix}, u = \tfrac{q}{V}, \tag{38}$$

$$y = (S_f - c_S)c_s - c_x^2. \tag{39}$$

Proof. Equations in (34) are rewritten as

$$\begin{pmatrix} \frac{dc_x}{dt} \\ \frac{dc_S}{dt} \end{pmatrix} = \underbrace{\begin{pmatrix} \mu(c_S) & 0 \\ -\frac{\mu(c_S)}{Y} & 0 \end{pmatrix}}_{M(x)} \begin{pmatrix} c_x \\ c_S \end{pmatrix} + \begin{pmatrix} -c_x \\ S_f - c_S \end{pmatrix} \frac{q}{V}. \tag{40}$$

From this, the proof immediately follows by using the same arguments as done in the previous case study. Note that the symmetric matrix $R(x)$ (37) is indefinite (i.e. neither positive definite nor negative definite).

3.3. Further discussions

Two of the main advantages of the quadratic (pseudo) PH representation are summarized as follows, (i) it circumvents the passivation design of the dynamics by input coordinate transformations [14] and (ii) it enables the control design via tracking-error approach with specific control benefits compared to the interconnection and damping assignment passivity-based control (IDA-PBC) approach [10, 12], that is, no need to solve matching equations that are expressed by partial differential equations.

In the quadratic (pseudo) PH framework, the key idea of the tracking-error passivity-based control approach consists in guaranteeing that the system trajectory x globally exponentially tracks some reference trajectory x_d when time goes to infinity while x_d is of the form

$$\frac{dx_d}{dt} = [J(x) - R(x)]\frac{\partial H(x_d)}{\partial x_d} + R_I(x)\frac{\partial \mathcal{H}(e)}{\partial e} + g(x)u, \tag{41}$$

where the damping injection $R_I(x)$ is a symmetric positive definite matrix to be appropriately chosen such that[4]

$$R(x) + R_I(x) > 0, \tag{42}$$

and $\mathcal{H}(e) = \frac{1}{2}e^{\mathrm{T}}R_{di}e$ with $e = x - x_d$ the error state vector. At the control design stage, only m components of the reference trajectory x_d are chosen in such a way that their time evolutions converge globally asymptotically or exponentially to the corresponding m-values of the desired

[4]We refer the reader to [21, 22] for a complete proof.

constant set-point x^*, that is, $\frac{dx_{d,i}}{dt} = K_i(x_i^* - x_{d,i})$, $i = 1, \ldots, m$, provided that the corresponding $m \times m$ submatrix obtained from $g(x)$ is full rank.

As a matter of illustration, we reconsider the **Case study 2** (Subsection 3.2) where the specific cell growth rate $\mu(c_s)$ is assumed given by the Monod-kinetics with an additional substrate overshoot term [3]

$$\mu(c_s) = \frac{\mu_{max}\, c_s}{d_1 + c_s + d_2\, c_s^2}, \tag{43}$$

where the scalars μ_{max}, d_1 and d_2 are positive. The continuous fermenter system exhibits the combined input-output multiplicities behaviour [3, 16] which is very challenging but interesting for the stabilizing control design. A three-step design procedure is provided below with the tracking-error passivity-based control approach.

Step 1 (the damping injection): From the damping matrix $R(x)$ (37) and the stabilization condition (42), the damping injection element $R_I(x)$ can be chosen as

$$R_I(x) = \begin{pmatrix} \mu(x_2) + \delta_1 & -\frac{\mu(x_2)}{2Y} \\ -\frac{\mu(x_2)}{2Y} & \delta_2 \end{pmatrix}, \tag{44}$$

where δ_1 and δ_2 are positive.

Step 2 (the reference trajectory): From Proposition 2 and Eqs. (41) and (44), the reference trajectory is given by:

$$\frac{dx_{d,1}}{dt} = \mu(x_2)x_{d,1} + (\mu(x_2) + \delta_1)(x_1 - x_{d,1}) - \frac{\mu(x_2)}{2Y}(x_2 - x_{d,2}) - x_1 u, \tag{45}$$

$$\frac{dx_{d,2}}{dt} = -\frac{\mu(x_2)}{Y}x_{d,2} - \frac{\mu(x_2)}{2Y}(x_1 - x_{d,1}) + \delta_2(x_2 - x_{d,2}) + (S_f - x_2)u. \tag{46}$$

Step 3 (the control design): First, the dynamics of $x_{d,1}$ is chosen to be assigned, that is, $\frac{dx_{d,1}}{dt} \triangleq K(x_1^* - x_{d,1})$ where the scalar K is positive while x_1^* is the first component of the desired set-point $x^* = (x_1^*, x_2^*)^{\mathrm{T}}$. The state feedback law is then derived from (45) as

$$u = \frac{1}{x_1}\left(-K(x_1^* - x_{d,1}) + \mu(x_2)x_{d,1} + (\mu(x_2) + \delta_1)(x_1 - x_{d,1}) - \frac{\mu(x_2)}{2Y}(x_2 - x_{d,2})\right). \tag{47}$$

The simulation parameters can be found in Tables 1 and 2. Figure 3 shows that the convergence of the system state x to the desired set-point x^* is guaranteed with the corresponding control input u (see Fig. 4).

Table 1. Simulation parameters of the fermenter model [3].

Quantity	Value	Unit
μ_{max}	1	1/s
d_1	0.03	mol/m^3
S_f	10	mol/m^3 s
Y	0.5	mol/kg BM
d_2	0.5	m^3/mol
x^*	(4.80, 0.40)	

Table 2. Control parameters and initial conditions.

Quantity	Value
K	0.1
$\delta_1 = \delta_2$	100000
IC_1	(2, 0.1)
IC_2	(1.5, 4)

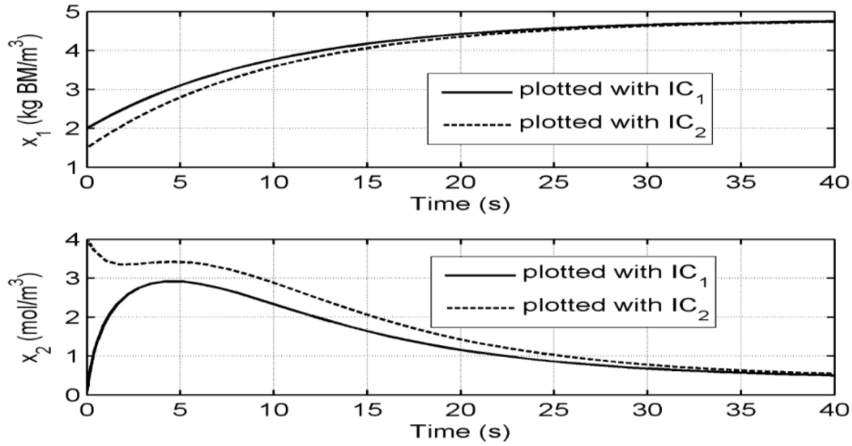

Figure 3. The time evolution of the system states under controller (47).

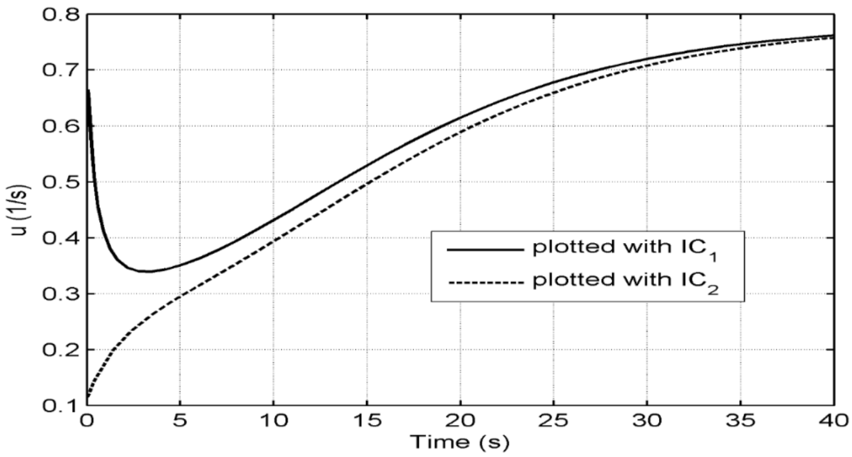

Figure 4. The control input computed from (47).

In order to assess the performance of the proposed controller, we consider next the interconnection and damping assignment passivity-based control (IDA-PBC) approach [3, 10, 12] for the purpose of comparison. Indeed, for the case study we are concerned with here, a qualified state feedback control law can be derived as [3]

$$u = \mu(x_2) - \frac{K_P}{x_1^2}\left\{-x_1(x_1 - x_1^*) + \left(S_f - x_2\right)(x_2 - x_2^*)\right\}. \tag{48}$$

Figure 5 shows the time evolution of the system states under controller (48) with the control gain K_P equal to K, that is, $K_P = 0.1$ has been used. As indicated, despite the oscillations at the beginning of the operation the convergence of the system states to the desired set-point is in about 20 seconds, i.e. the settling time is two times faster than the one with controller (47) (see Figure 3). Nevertheless, if no input constraint (i.e. the input saturation or $u(t) \geq 0$) is imposed, this feature could be paid to the admissibility of the control input due to its negative value which is physically inacceptable as seen in Figure 6. In other words, the fermenter system under controller (47) may be operated with better performance (i.e. avoiding a very fast settling time provided by a larger domain of validity for operating conditions and initial conditions).

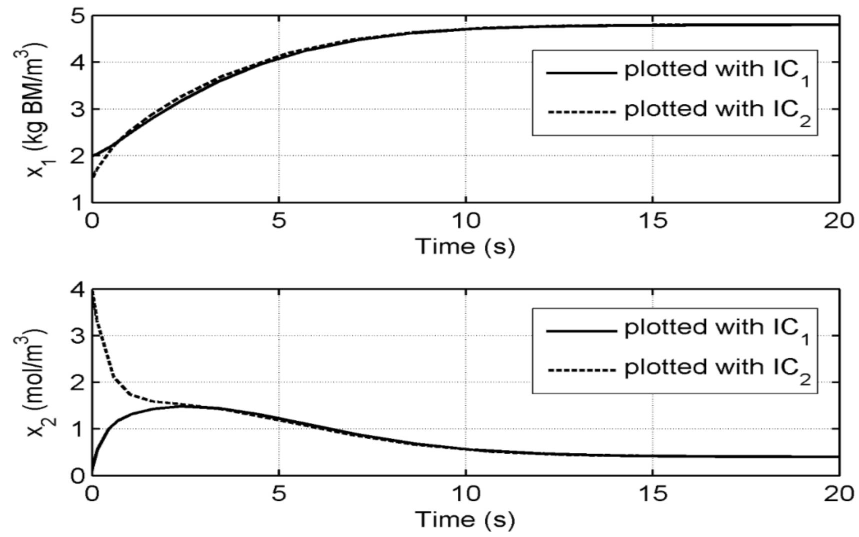

Figure 5. The time evolution of the system states under controller (48).

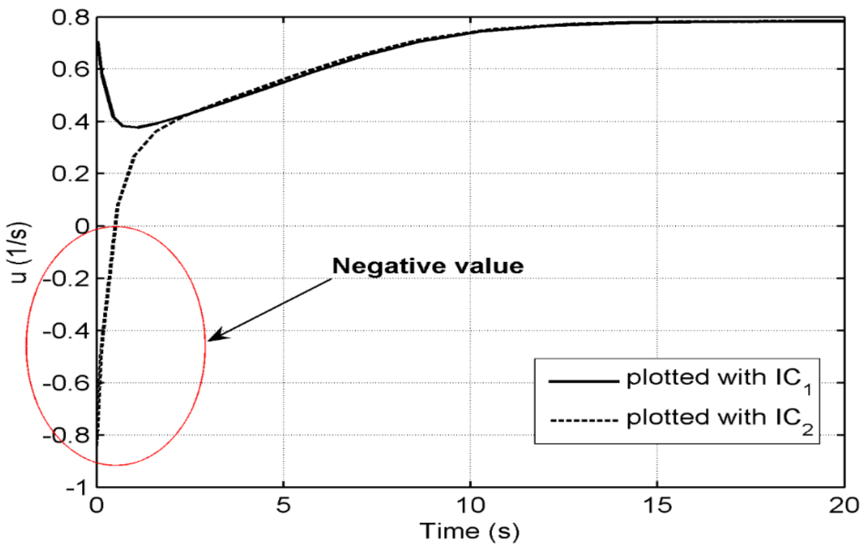

Figure 6. The control input computed from (48).

4. CONCLUSION

In this work, an introductory survey of the port Hamiltonian-based modelling of linear electrical and mechanical systems is given. This modelling framework can be adapted for nonlinear chemical and biological systems leading to a unified quadratic (pseudo) PH representation. The resulting presentation enables the tracking-error passivity-based control approach with specific control benefits. It remains now to extend the proposed approach to large dimensional engineering systems.

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