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OUTPUT TRACKING CONTROL FOR TRMS BASED ON TIME RECEDING OPTIMAL OBSERVATION OF DISTURBANCES

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Abstract. The paper presents an approach to design the adaptive output tracking controller for disturbed Twin Rotor Multi-Input Multi-Output System (TRMS) by using a time receding observer of functional disturbances for compensative control purpose, without using conventional methods as a neural network. To do this, first the disturbed Euler-Lagrange model of TRMS is converted to an equivalent bilinear form. And then, secondly an optimal estimator for this disturbed bilinear system is constructed based on time receding minimizing their effect. The complete output tracking controller for TRMS is created then by combining an exact linearization controller with the proposed disturbances estimation mechanism. Simulation results show that the here suggested controller meet completely the expected output tracking performances

Keywords: Euler-Lagrange, bilinear systems, TRMS, optimal estimation, output disturbances.

Classification numbers: 5.3.7, 5.8.

1. INTRODUCTION

The Twin Rotor Multi-Input Multi-Output System (TRMS) is known as a standard model of a helicopter [1]. Not only that it is attached to a tower as the helicopter construction, but also their position and velocity are controlled by the rotor velocity variation, which are similar to helicopter controller principle. Therefore, TRMS is often used to verify the performance of a new designed controller for helicopters. While the TRMS is operating there are many inevasible disturbances effects badly to the system dynamic behavior. The main reasons are due to unexpected air currents and vibrations by system structure. Some investigations for trying to eliminate these effects are published last decade in [2 - 6]. These publications propose mainly the disturbance compensation algorithm for continuous-time multi input multi output (MIMO) nonlinear plants under parametric uncertainties and external disturbances with quantized output signal. The auxiliary loop approach is used for estimation of disturbance function. The proposed algorithm guarantees that the output of the plant tracks the reference output with the required accuracy. In [4] and [5] a robust tracking controller for TRMS by using an integral sliding mode controller is proposed. To eliminate the discontinuity in the control signal, the controller is augmented by a sliding mode disturbance observer (DOB) [4]. The actuator dynamics is handled using a backstepping approach, which is applicable due to the continuous chattering free nature of the command signals generated using the disturbance observer based controller. The proposed controller is validated by simulation and experiment. A novel disturbance observer-based trajectory tracking controller based on integral back-stepping approach are presented in [6]. To avoid the complexity of analytically calculated derivatives of virtual control signals in the standard backstepping technique, a command filtered backstepping approach is utilized. Some experimental as well as simulation results in virtually real environment on TRMS similar prove the robustness of these controllers to model uncertainties and external disturbances. The method presented in [6] uses a simple form of DOB, which does not need to solve the plant model inverse, and uses H∞ control method using LMIs to design the Q-filter in the DOB. The obtained results confirm that the DOB is successful to estimating the disturbance and then to eliminate the disturbance occurred in TRMS.

Quite different to all these presented adaptive and robust control methods for TRMS, this paper will propose another control approach based on exact linearization control, which is often used for mechanical systems presented in [7], and the time receding optimal DOB presented theoretically in [8], for the purpose of eliminating adaptively the input disturbances. The here proposed approach is simple to implement and effective as shown by a numerical simulation afterward.

2. CONTROLLER DESIGN

2.1. TRMS modeling

In this paper the following Euler-Lagrange model of TRMS will be used consistently, which was established by using Lagrange equation for mechanical systems [9] and has been given similarly in [2]:

$$M(\underline{q})\underline{\ddot{q}}+C(\underline{q},\underline{\dot{q}})\underline{\dot{q}}+\underline{g}(\underline{q})=\underline{u}+\Delta(\underline{q},\underline{\dot{q}},\underline{\ddot{q}},t)+\underline{\eta}(t) \tag{1}$$

where

- $\underline{q}=\left(\alpha_h,\alpha_v\right)^T$ are the outputs of TRMS
- $M(\underline{q})=\left(m_{ij}(\underline{q})\right)$ and $C(\underline{q},\underline{\dot{q}})=\left(c_{ij}(\underline{q},\underline{\dot{q}})\right),\ i,j=1,2$,

with

$$m_{11}(\underline{q})=J_1\cos^2\alpha_v+J_2\sin^2\alpha_v+h^2\left(m_{T_1}+m_{T_2}\right)+J_3$$

$$m_{12}(\underline{q})=m_{21}(\underline{q})=h(m_{T_1}l_{T_1}\sin\alpha_v-m_{T_2}l_{T_2}\cos\alpha_v),\ m_{22}(\underline{q})=J_1+J_2$$

$$c_{11}(\underline{q},\underline{\dot{q}})=2\dot{\alpha}_v\left(J_2-J_1\right)\sin\alpha_v\cos\alpha_v,\ c_{12}(\underline{q},\underline{\dot{q}})=\dot{\alpha}_v h\left(m_{T_1}l_{T_1}\cos\alpha_v+m_{T_2}l_{T_2}\sin\alpha_v\right)$$

$$c_{21}(\underline{q},\underline{\dot{q}})=\dot{\alpha}_h\left(J_1-J_2\right)\sin\alpha_v\cos\alpha_v \quad \text{and} \quad c_{22}=0$$

as well as:

$$\underline{g}(\underline{q})=\begin{pmatrix}0\\ g\left(m_{T_1}l_{T_1}\cos\alpha_v+m_{T_2}l_{T_2}\sin\alpha_v\right)\end{pmatrix},$$

$\underline{u}=\left(M_{prop.h}\ ,\ M_{prop.v}\right)^T$ is the vector of control signals, $\underline{\eta}(t)=(\eta_1,\eta_2)^T$ are the vector of system input noises, and $\Delta(\underline{q},\underline{\dot{q}},\underline{\ddot{q}},t)$ expresses the model error, respectively [2].

It is obviously that $M(\underline{q})$ is a positive definite matrix. Both

$$u_1+\eta_1=\sum_i M_{ih}=M_{prop.h}-M_{fric.h}-M_{cable}+k_m\dot{\omega}_v\cos\alpha_v$$
$$u_2+\eta_2=\sum_i M_{iv}=M_{prop.v}-M_{fric.v}+k_t\dot{\omega}_h+M_{gyro}$$

are the sum of applied torques in the horizontal and vertical movements [2].

The vector of all unknown functions (the disturbances):

$$\underline{n}(t)=\Delta(\underline{q},\underline{\dot{q}},\underline{\ddot{q}},t)+\underline{\eta}(t)$$

represents both model uncertainties and system noises of the TRMS.

The EL model (1) of the TRMS above can be now rewritten accordingly in a so called input disturbed bilinear state equation as follows:

$$M(\underline{q})\underline{\ddot{q}}+C(\underline{q},\underline{\dot{q}})\underline{\dot{q}}=\underline{u}+\underline{d}(t) \tag{2}$$

with a new summarized system disturbances:

$$\underline{d}(t)=\underline{n}(t)-\underline{g}(\underline{q})\ . \tag{3}$$

2.2. Exact linearization controller for undisturbed system

The following exact linearization of undisturbed nonlinear system (2) is recommended by [7]. In the case that all uncertainties and gravity of (3) could be ignored, then the initial state equation (2) of TRMS will be altered to:

$$M(\underline{q})\underline{\ddot{q}}+C(\underline{q},\underline{\dot{q}})\underline{\dot{q}}=\underline{u}\ . \tag{4}$$

By using the following state-feedback controller:

$$\underline{u}=M(\underline{q})\left[\underline{\ddot{r}}+K_1\underline{e}+K_2\underline{\dot{e}}\right]+C(\underline{q},\underline{\dot{q}})\underline{\dot{q}} \tag{5}$$

the closed loop system, including the plant (4) and the controller (5), becomes exactly linear as follows:

$$\begin{pmatrix}\underline{\dot{e}}\\ \underline{\ddot{e}}\end{pmatrix}=\begin{pmatrix}\mathbf{0} & I\\ -K_1 & -K_2\end{pmatrix}\begin{pmatrix}\underline{e}\\ \underline{\dot{e}}\end{pmatrix} \text{ or } \underline{\dot{x}}=A\underline{x} \text{ with } A=\begin{pmatrix}\mathbf{0} & I\\ -K_1 & -K_2\end{pmatrix} \text{ and } \underline{x}=\begin{pmatrix}\underline{e}\\ \underline{\dot{e}}\end{pmatrix}. \tag{6}$$

In the controller (5) above the vector

$$\underline{r}(t)=\left(r_1(t),r_2(t)\right)^T$$

contains required references, which the outputs of TRMS has to track asymptotically, and

$$\underline{e}=\underline{r}-\underline{q} \tag{7}$$

is the vector of tracking errors, as well as $K_1,\ K_2$ are two arbitrarily chosen matrices.

Theorem 1: *If both* $K_1,\ K_2$ *are so chosen that*

$$K_1=\mathrm{diag}(k_{1i}),\ K_2=\mathrm{diag}(k_{2i}) \text{ with } k_{2i}^2>k_{1i}>0 \tag{8}$$

then $\underline{x}(t)$ *is bounded and converges asymptotically to the origin* $\underline{0}$.

Proof:

It is obviously that with the assumptions (8) both matrices:

$$Q = 2\begin{pmatrix} K_1^2 & \mathbf{0} \\ \mathbf{0} & K_2^2 - K_1 \end{pmatrix} \text{ and } P = \begin{pmatrix} 2K_1K_2 & K_1 \\ K_1 & K_2 \end{pmatrix} \tag{9}$$

are positive definite. Moreover, they satisfy the following Lyapunov equation:

$$A^T P + PA = -Q\,. \tag{10}$$

Hence, the matrix A given in (6) is Hurwitz, and it implies that $\underline{x}(t) = e^{At}\underline{x}(0)$ is bounded and tends asymptotically to zero. ■

2.3. Disturbances compensation based on time receding minimization

According to Theorem 1, the asymptotical tracking performance of the linearized controller (5) applied to the TRMS (2) can be obviously satisfied only if $\underline{d}(t) = \underline{0}$. In the case that we force to apply it to control the input disturbed system (2), i.e. in the case of $\underline{d}(t) \neq \underline{0}$, we evidentially have to eliminate the disturbances $\underline{d}(t)$ first before applying it.

The main idea here in this paper to eliminate the disturbances $\underline{d}(t)$ from the system (2) is that: (I) the disturbance $\underline{d}(t)$ at the time instant $t_k = kT_s$ will be approximated optimality $\widehat{\underline{d}}_k \approx \underline{d}(t_k)$ so that:

$$\left|\widehat{\underline{d}}_k - \underline{d}(t_k)\right| \to \min!$$

and then (II) giving it back to the system with:

$$\underline{u} - \widehat{\underline{d}}(t)$$

for compensating $\underline{d}(t)$ as illustrated in Fig. 1.

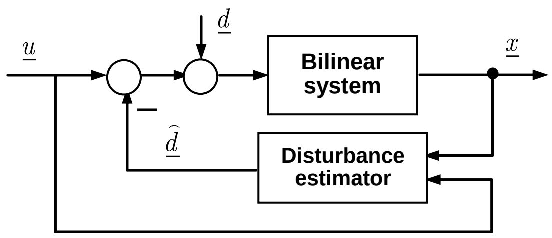

Figure 1. Disturbance estimator for compensation control purpose.

For a time receding construction of the disturbance estimator, the derivation of $\underline{x} = (\underline{q}, \dot{\underline{q}})^T$ in (2) will be replaced approximately first by the following Euler equation:

$$\dot{\underline{x}}(t_k) \approx \frac{\underline{x}(t_{k+1}) - \underline{x}(t_k)}{T_s} \tag{11}$$

with a chosen time receding horizon:

$$T_s = t_{k+1} - t_k \text{ for all } k .$$

This implies that the system model (2) of disturbed system after eliminating the disturbance $\underline{d}(t)$ by $\underline{\hat{d}}(t)$ at the time instant t_k will be approximated with:

$$\frac{\underline{x}_{k+1} - \underline{x}_k}{T_s} \approx \begin{pmatrix} \mathbf{0} & I \\ \mathbf{0} & -M^{-1}(\underline{x}_k)C(\underline{x}_k) \end{pmatrix} \underline{x}_k + \begin{pmatrix} \mathbf{0} \\ M^{-1}(\underline{x}_k) \end{pmatrix} \left[\underline{u} + \underline{d}_k - \underline{\hat{d}}_{k-1} \right] \tag{12}$$

or

$$\underline{x}_k \approx \left(I + T_s A_k^x \right) \underline{x}_{k-1} + T_s B_k \left[\underline{u} + \underline{d}_k - \underline{\hat{d}}_{k-1} \right] \tag{13}$$

where

$$A_k^x = \begin{pmatrix} \mathbf{0} & I \\ \mathbf{0} & -M^{-1}(\underline{x}_{k-1})C(\underline{x}_{k-1}) \end{pmatrix}, \; B_k = \begin{pmatrix} \mathbf{0} \\ M^{-1}(\underline{x}_{k-1}) \end{pmatrix}. \tag{14}$$

Now we compare the disturbed model (13) with a corresponding undisturbed one:

$$\underline{z}_k = \left(I + T_s A_k^z \right) \underline{z}_{k-1} + T_s B_k \left[\underline{u} - \underline{\hat{d}}_{k-1} \right] \tag{15}$$

where

$$A_k^z = \begin{pmatrix} \mathbf{0} & I \\ \mathbf{0} & -M^{-1}(\underline{z}_{k-1})C(\underline{z}_{k-1}) \end{pmatrix} \tag{16}$$

in the sense of determining the model difference:

$$\underline{x}_k - \underline{z}_k \approx \left(I + T_s A_k^x \right) \underline{x}_{k-1} - \left(I + T_s A_k^z \right) \underline{z}_{k-1} + T_s B_k \underline{d}_k$$

or

$$\underline{0} \approx \underline{b}_k - T_s B_k \underline{d}_k \tag{17}$$

with

$$\underline{b}_k = \underline{x}_k - \underline{z}_k + \left(I + T_s A_k^z \right) \underline{z}_{k-1} - \left(I + T_s A_k^x \right) \underline{x}_{k-1} . \tag{18}$$

Denote the right site of (17), which could be considered as model errors, by $\underline{\delta}_k$, then we have:

$$\underline{\delta}_k = \underline{b}_k - T_s B_k \underline{d}_k \approx \underline{0} \tag{19}$$

and this value $\underline{\delta}_k$ at the time instant t_k is obviously depended only on the disturbance $\underline{d}_k$, which is to estimate now.

The disturbance $\underline{d}_k$ at the time instant t_k will be hereafter approximated optimally according to minimize the following objective function:

$$J_k = \underline{\delta}_k^T \underline{\delta}_k = \left(\underline{b}_k - T_s B_k \underline{d}_k \right)^T \left(\underline{b}_k - T_s B_k \underline{d}_k \right) = \underline{d}_k^T \left(T_s^2 B_k^T B_k \right) \underline{d}_k - 2 \underline{b}_k^T T_s B_k \underline{d}_k + \underline{b}_k^T \underline{b}_k \tag{20}$$

It means that we have to determine the optimal value $\underline{\hat{d}}_k$ of $\underline{d}_k = \underline{d}(t_k)$ according to:

$$\underline{\hat{d}}_k = \arg\min J_k . \tag{21}$$

Since the optimization problem (21) is quadratic and unconstraint, this solution is easily obtained immediately as follows:

$$\underline{\hat{d}}_k = \left(T_s B_k^T B_k\right)^{-1} B_k^T \underline{b}_k . \tag{22}$$

Theorem 2: *If the input disturbances are detectable by state feedback, the approximation* (11) *is exact and* $\operatorname{rank} B_k = \dim \underline{u}$, *then:*

$$\underline{\hat{d}}_k = \underline{d}_k . \tag{23}$$

Proof: See [8]. ■

Figure 2 exhibits the principle of time receding optimal calculation of disturbances $\underline{d}(t)$ at all time instants $t_k,\ k=0,1,\ \ldots$ based on (14)-(16), (18) and (22), where $t_0 = 0$ and $\underline{z}_0$ is chosen arbitrarily.

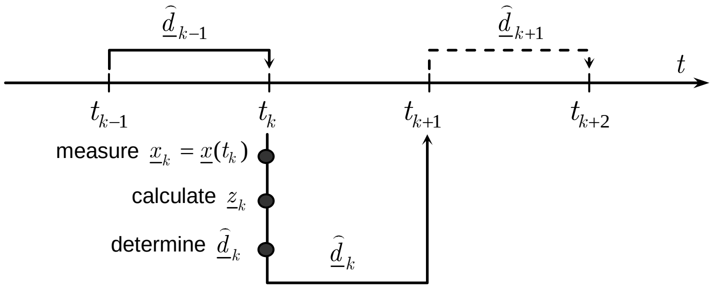

Figure 2. Scheme of time receding disturbance estimation [8].

2.4. Implementing the adaptive controller

For a convenient implementation purpose of the proposed adaptive controller, all calculations (14)-(16), (18), (22) given above will be now summarized in the following algorithm:

1. Convert the EL model (1) into bilinear form (2).
2. Choose an appropriate time receding horizon T_s. Assign arbitrarily $\underline{z}_{-1}$ and $\underline{\hat{d}}_{-1}$. Set $k=0$.
3. Measure $\underline{x}_k = \underline{x}(kT_s)$ from the controlled system and calculate:

   A_k^x, B_k, A_k^z according to (14) and (16).

   $\underline{z}_k$ according to (15), $\underline{b}_k$ according to (18) and $\underline{\hat{d}}_k$ according to (22).
4. Send $\underline{u} - \underline{\hat{d}}_k$ for a while of T_s to the controlled plan, where $\underline{u}$ is obtained from the state feedback controller (5) with two matrices K_1, K_2 chosen accordingly to (8).

5. Set $k := k+1$ and turn back to step 3.

Figure 3 illustrates the proposed adaptive controller based on time receding disturbance estimator.

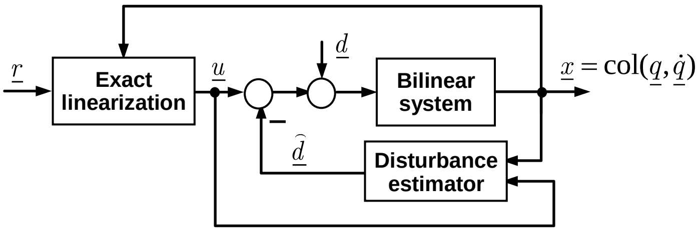

Figure 3. Adaptive control framework for output tracking disturbed TRMS.

2.5. Stability of closed loop system

The closed-loop control system exhibited in Fig. 3 has the disturbed EL model:

$$\frac{d^2\underline{e}}{dt^2} = -K_1\underline{e} - K_2\frac{d\underline{e}}{dt} + \underline{\phi}(\underline{q},t), \ \ \underline{e} = \underline{r} - \underline{q} \tag{24}$$

where

$$\underline{\phi}(\underline{q},t) = M(\underline{q})^{-1}\left[\underline{\hat{d}}(t) - \underline{d}(t)\right]$$

is the remaining estimation error, which based on Theorem 2 is bounded $\left|\underline{\phi}(\underline{q},t)\right| \le \mu$ with a very small constant μ, when $\underline{d}(t)$ is assumed to be continuous.

Converted equivalently the EL model (24) in:

$$\underline{\dot{x}} = A\underline{x} + B\underline{\phi}, \ B = \begin{pmatrix} \mathbf{0} \\ I \end{pmatrix}$$

with $A, \underline{x}$ are defined in (6), then with Q, P given in (9), we obtain for the candidate Lyapunov's function $V(\underline{x}) = (\underline{x}^T P\underline{x})/2$, the following inequality:

$$\begin{aligned} \frac{dV}{dt} &= \underline{x}^T(A^TP + PA)\underline{x} + \underline{\phi}^T(B^TP + PB)\underline{x} \le \left(-\delta\left|\underline{x}\right|^2 + \lambda\left|\underline{\phi}\right|\left|\underline{x}\right|\right) \\ &\le \left(\lambda\mu - \delta\left|\underline{x}\right|\right)\left|\underline{x}\right| \end{aligned} \tag{25}$$

where the relation (10) had been used and:

$$\delta = \min_i\left(k_{1i}^2 \ , \ k_{2i}^2 - k_{1i}\right), \ \lambda = \max_i\left(k_{1i} \ , \ k_{2i}\right). \tag{26}$$

Hence, the closed-loop system is ISS-stable with a very small attractor:

$$\mathcal{O} = \left\{\underline{x} = \mathrm{col}(\underline{e}, \underline{\dot{e}}) \,\middle|\, |\underline{x}| < \lambda\mu/\delta\right\}.$$

3. SIMULATTING ON TRMS

3.1. Disturbed bilinear model of TRMS

From the EL model of TRMS given in [2] it is obtained correspondingly the input disturbed bilinear model (2) as follows:

$$\begin{pmatrix} \mathbf{0} & I \\ \mathbf{0} & -M^{-1}(\underline{q})C(\underline{q},\dot{\underline{q}}) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{pmatrix} \text{ and } B_k = \begin{pmatrix} \mathbf{0} \\ M^{-1}(\underline{q}) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{pmatrix},\ \underline{g}(\underline{q}) = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}$$

with

$$\begin{aligned} a_{33} = & \Big[2(J_1^2 - J_2^2)\dot{\alpha}_v \sin\alpha_v \cos\alpha_v + h(J_1 - J_2)\dot{\alpha}_h (l_{T1}m_{T1}\sin\alpha_v - l_{T2}m_{T2}\cos\alpha_v)\sin\alpha_v \cos\alpha_v - \\ & -h(J_1 - J_2)\dot{\alpha}_h l_{T2}m_{T2}\sin\alpha_v \cos^2\alpha_v \Big] \Big/ \Big[J_1^2\cos^2\alpha_v + J_2^2\sin^2\alpha_v - h^2 l_{T_1}^2 m_{T_1}^2 \sin^2\alpha_v - \\ & -h^2 l_{T_2}^2 m_{T_2}^2 \cos^2\alpha_v + 2h^2 l_{T_1} l_{T_2} m_{T_1} m_{T_2} \sin\alpha_v \cos\alpha_v + (J_1 + J_2)h^2(m_{T_1} + m_{T_2}) + \\ & + J_1J_2 + J_1J_3 + J_2J_3 \Big] \end{aligned}$$

$$\begin{aligned} a_{34} = & -(J_1 + J_2)h\dot{\alpha}_v \left(m_{T_1} l_{T_1} \cos\alpha_v + m_{T_2} l_{T_2} \sin\alpha_v \right) \Big/ \Big[J_1^2\cos^2\alpha_v + J_2^2\sin^2\alpha_v - h^2 l_{T_1}^2 m_{T_1}^2 \sin^2\alpha_v - \\ & -h^2 l_{T_2}^2 m_{T_2}^2 \cos^2\alpha_v + 2h^2 l_{T_1} l_{T_2} m_{T_1} m_{T_2} \sin\alpha_v \cos\alpha_v + \\ & + (J_1 + J_2)h^2(m_{T_1} + m_{T_2}) + J_1J_2 + J_1J_3 + J_2J_3 \Big] \end{aligned}$$

$$\begin{aligned} a_{43} = & \Big\{ 2h(J_2 - J_1)\dot{\alpha}_v (l_{T_1}m_{T_1}\sin\alpha_v - l_{T2}m_{T2}\cos\alpha_v)\sin\alpha_v \cos\alpha_v - (J_1 - J_2)\Big[J_1\cos^2\alpha_v + \\ & + J_2\sin^2\alpha_v + h^2\left(m_{T_1} + m_{T_2}\right) + J_3 \Big]\dot{\alpha}_h \sin\alpha_v \cos\alpha_v \Big\} \Big/ \\ & \Big/ \Big[J_1^2\cos^2\alpha_v + J_2^2\sin^2\alpha_v - h^2 l_{T1}^2 m_{T1}^2 \sin^2\alpha_v - \\ & -h^2 l_{T_2}^2 m_{T_2}^2 \cos^2\alpha_v + 2h^2 l_{T_1} l_{T_2} m_{T_1} m_{T_2} \sin\alpha_v \cos\alpha_v + \\ & +(J_1 + J_2)h^2(m_{T1} + m_{T2}) + J_1J_2 + J_1J_3 + J_2J_3 \Big] \end{aligned}$$

$$\begin{aligned} a_{44} = & h^2\dot{\alpha}_v (l_{T_1}m_{T_1}\sin\alpha_v - l_{T_2}m_{T_2}\cos\alpha_v)\left(m_{T_1} l_{T_1}\cos\alpha_v + m_{T_2} l_{T_2}\sin\alpha_v \right) \Big/ \\ & \Big/ \Big[J_1^2\cos^2\alpha_v + J_2^2\sin^2\alpha_v - h^2 l_{T_1}^2 m_{T_1}^2 \sin^2\alpha_v - h^2 l_{T_2}^2 m_{T_2}^2 \cos^2\alpha_v + \\ & + 2h^2 l_{T_1} l_{T_2} m_{T_1} m_{T_2} \sin\alpha_v \cos\alpha_v + (J_1 + J_2)h^2(m_{T1} + m_{T2}) + J_1J_2 + J_1J_3 + J_2J_3 \Big] \end{aligned}$$

$$\begin{aligned} b_{31} = & (J_1 + J_2) \Big/ \Big[J_1^2\cos^2\alpha_v + J_2^2\sin^2\alpha_v - h^2 l_{T_1}^2 m_{T_1}^2 \sin^2\alpha_v - h^2 l_{T2}^2 m_{T2}^2 \cos^2\alpha_v + \\ & + 2h^2 l_{T_1} l_{T_2} m_{T_1} m_{T_2} \sin\alpha_v \cos\alpha_v + (J_1 + J_2)h^2(m_{T_1} + m_{T_2}) + J_1J_2 + J_1J_3 + J_2J_3 \Big] \end{aligned}$$

$$\begin{aligned} b_{32} = & -h(l_{T_1}m_{T_1}\sin\alpha_v - l_{T_2}m_{T_2}\cos\alpha_v) \Big/ \Big[J_1^2\cos^2\alpha_v - h^2 l_{T_1}^2 m_{T_1}^2 \sin^2\alpha_v - h^2 l_{T_2}^2 m_{T_2}^2 \cos^2\alpha_v + \\ & + 2h^2 l_{T_1} l_{T_2} m_{T_1} m_{T_2} \sin\alpha_v \cos\alpha_v + (J_1 + J_2)h^2(m_{T_1} + m_{T_2}) + J_1J_2 + J_1J_3 + J_2J_3 \Big] \end{aligned}$$

$$b_{41} = -h(l_{T1}m_{T1}\sin\alpha_v - l_{T2}m_{T2}\cos\alpha_v) \Big/ \Big[J_1^2\cos^2\alpha_v + J_2^2\sin^2\alpha_v - h^2 l_{T_1}^2 m_{T_1}^2 \sin^2\alpha_v -$$
$$- h^2 l_{T2}^2 m_{T2}^2 \cos^2\alpha_v + 2h^2 l_{T_1} l_{T_2} m_{T_1} m_{T_2} \sin\alpha_v \cos\alpha_v +$$
$$+ (J_1 + J_2)h^2(m_{T_1} + m_{T_2}) + J_1J_2 + J_1J_3 + J_2J_3 \Big]$$
$$b_{42} = J_1 cos^2\alpha_v + J_2 sin^2\alpha_v + h^2\left(m_{T1} + m_{T2}\right) + J_3 \Big/ \Big[J_1^2\cos^2\alpha_v + J_2^2\sin^2\alpha_v -$$
$$- h^2 l_{T_1}^2 m_{T_1}^2 \sin^2\alpha_v - h^2 l_{T_2}^2 m_{T_2}^2 \cos^2\alpha_v + 2h^2 l_{T_1} l_{T_2} m_{T_1} m_{T_2} \sin\alpha_v \cos\alpha_v +$$
$$+ (J_1 + J_2)h^2(m_{T_1} + m_{T_2}) + J_1J_2 + J_1J_3 + J_2J_3 \Big]$$

$g_1 = 0$ and $g_2 = g\left(m_{T_1} l_{T_1} \cos\alpha_v + m_{T_2} l_{T_2} \sin\alpha_v\right)$

Note that for the latter application of the exact linearization controller (5) both g_1, g_2 will be considered as supplement disturbances.

3.2. Simulation and results

The proposed adaptive controller, including the exact linearization state feedback controller (5) and the time receding disturbance estimator (22), is now implemented on Matlab to verify the output tracking performance to the desired references:

$$\underline{r}(t) = \begin{pmatrix} r_1(t) \\ r_2(t) \end{pmatrix} = \begin{pmatrix} 0.3\sin(0.0628t) + 0.7\sin(0.1256t) \\ 0.5\sin(0.1256t) \end{pmatrix}. \tag{27}$$

Furthermore, it is assumed in the simulation that the exact linearization controller (5) for the TRMS has two parameter matrices $K_2 = 2K_1 = 2I_{2\times 2}$, and is disturbed by:

$$\underline{n}(t) = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = \begin{pmatrix} 0.1\sin(0.3t) + 0.2\cos(0.1t) \\ 0.3\cos(0.2t) + 0.2\sin(0.5t) \end{pmatrix}. \tag{28}$$

It means that the bilinear model (2) of it will be disturbed by the total $\underline{d}(t) = \underline{n}(t) - \underline{g}(\underline{q})$ as explained already in (3). Note that both, the desired references (27) and the disturbances (28) are defined arbitrarily by authors just for carrying out the simulation. These arbitrary definitions do not affect to the latter obtained control performance.

For a purpose of a virtually real simulation, the TRMS will be carried out by using model parameters given in [1], which have been obtained via identification technique presented in [2]. Table 1 exhibits these parameters.

After executing the simulation program given in Appendix we obtain results exhibited below in Fig. 4 - Fig.7. While both Fig. 4 and Fig. 5 show the real disturbances $d_1(t), d_2(t)$ in comparison with their estimated values $\widehat{d}_1(t_k), \widehat{d}_2(t_k)$, two other Fig. 6 and Fig. 7 illustrate the tracking performance of TRMS outputs $q_1(t), q_2(t)$ to desired trajectories $r_1(t), r_2(t)$.

Figure 4 and Figure 5 show the estimated values of yaw and pitch angle disturbances. They coincide after a short time interval at the beginning almost exactly with their real values. According to Theorem 2, the estimation errors are caused only by time discretizing the continuous model (2) into (13). The tracking performance of TRMS outputs, the yaw and pitch angle to their references are shown in Fig. 6 and Fig. 7. It is seen there that both system outputs $q_1(t), q_2(t)$ have converged asymptotically to the references $r_1(t), r_2(t)$ as expected.

Table 1. Simulation parameters (provided in [1]).

Parameters	Symbol	Value	Unit
Length of the pivoted beam	h	0.06	m
Total mass of the free beam	m_{T_1}	0.8250	kg
Total mass of the counter balance beam	m_{T_2}	0.0908	kg
Centre of gravity of the free beam	l_{T_1}	0.0186	m
Centre of gravity of the counter balance beam	l_{T_2}	0.2443	m
Moment of inertia of the free beam	J_1	0.0591	kgm^2
Moment of inertia of the counter balance beam	J_2	0.0059	kgm^2
Moment of inertia of the pivoted beam	J_3	1.68e-05	kgm^2
Acceleration of gravity	g	9.81	m/s^2

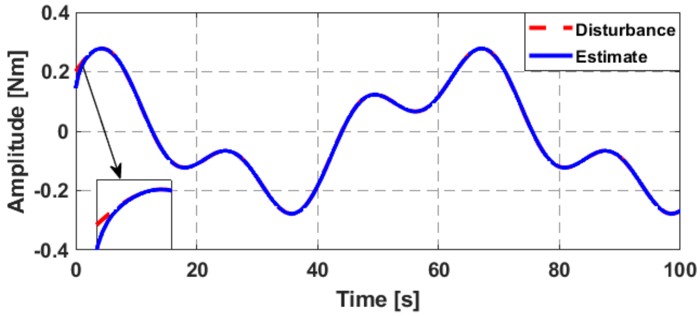

Figure 4. The first disturbance $d_1(t)$ in comparison with its estimated values.

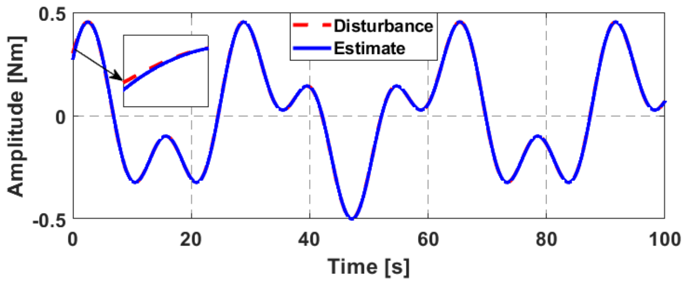

Figure 5. The second disturbance $d_2(t)$ in comparison with its estimated values.

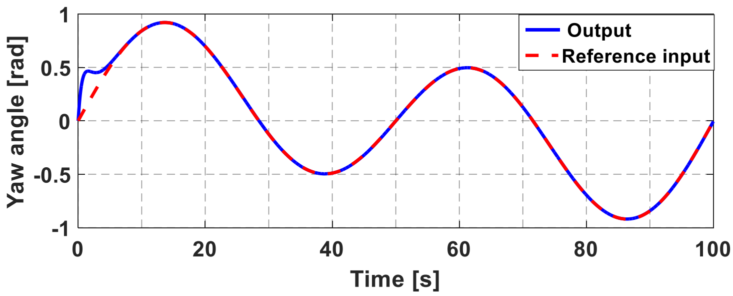

Figure 6. The yaw angle response of TRMS and the desired reference for it.

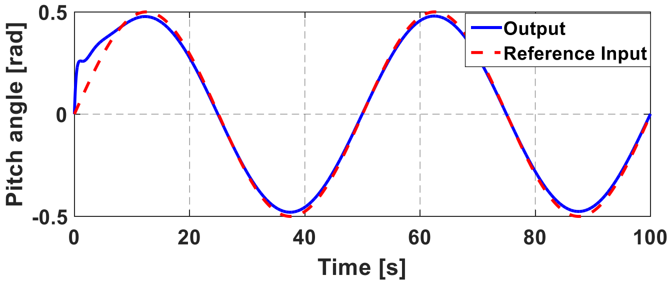

Figure 7. The pitch angle response of TRMS and the desired reference for it.

4. CONCLUSIONS

In this paper, the exact linearization controller and the time receding estimator for matched disturbances are combined to output tracking control the TRMS. The control scheme proposed in this paper is structurally simple to implement, but has conducted an excellent tracking performance. Moreover, as seen in the obtained inequality (25), together with the notes (26), the tracking behavior of closed-loop system, could be also improved easily by increasing k_{1i}, k_{2i} of the implemented controller. Finally, the numerical simulation in this paper shows, that the proposed controller can be now applied in practice.

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APPENDIX: PROGRAM SOURCE

TRMS.m

```
function dx = TRMS(t,x)
global g r r_d r_dd mT1 mT2 J1 J2 J3 h lT1 lT2 u Ax Bx dh d d_them
M1=J1*(cos(x(2)))^2 + J2*(sin(x(2)))^2 + (mT1+mT2)*(h^2)+J3;
M2=h*(mT1*lT1*sin(x(2))-mT2*lT2*cos(x(2)))+d_them;
M3=J1+J2; M=[M1 M2;M2 M3]; c11=2*x(4)*(J2-J1)*sin(x(2))*cos(x(2));
c12=h*x(4)*(mT1*lT1*cos(x(2))+mT2*lT2*sin(x(2)));
c21=x(3)*(J1-J2)*sin(x(2))*cos(x(2)); c22=0; C=[c11 c12;c21 c22];
d=[0.1*sin(0.3*t)+0.2*cos(0.1*t);0.3*cos(0.2*t)+0.2*sin(0.5*t)]%-[g1;g2];
r=[0.3*sin(0.0628*t)+0.7*sin(0.1256*t);0.5*sin(0.1256*t)];
r_d=[0.01884*cos(0.0628*t)+0.08792*cos(0.1256*t);0.0628*cos(0.1256*t)];
r_dd=[-0.001183*sin(0.0628*t)-0.01104*sin(0.1256*t);-0.0079*sin(0.1256*t)]; e=r-
[x(1);x(2)]; e_dot=r_d-[x(3);x(4)]; K1=eye(2); K2=2*eye(2);
u=M*(r_dd+K1*e+K2*e_dot)+C*[x(3);x(4)];
Ax=[0 0 1 0;0 0 0 1;zeros(2) -M\C]; Bx=[0 0;0 1;inv(M)];
dx=Ax*x+Bx*(u+d-dh);
```

runTRMS.m

```
global g r r_d r_dd mT1 mT2 J1 J2 J3 h lT1 lT2 u Ax Bx dh d d_them
g=9.81; mT1=0.825;mT2=0.0908;J1=0.0519;J2=0.0059;J3=1.68e-05;
h=0.06;lT1=0.0186;lT2=0.2443;x0=[0 0 1 3];d_them=1;
z0=x0'; t0=0; N=10000; Ts=0.01; dh=[0;0]; px=[]; ti=[]; pd=[]; pdh=[];pr=[];
for i=1:N+1
[t,x]=ode45(@TRMS,[t0 t0+Ts],x0);
k=length(t); t0=t(k); ti=[ti (i-1)*Ts]; px=[px;x0];
Mz1=J1*(cos(z0(2)))^2+J2*(sin(z0(2)))^2 +(mT1+mT2)*(h^2)+J3;
Mz2=h*(mT1*lT1*sin(z0(2))-mT2*lT2*cos(z0(2)))+d_them;
Mz3=J1+J2; Mz=[Mz1 Mz2;Mz2 Mz3];
cz11=2*z0(4)*(J2-J1)*sin(z0(2))*cos(z0(2));
cz12=h*z0(4)*(mT1*lT1*cos(z0(2))+mT2*lT2*sin(z0(2)));
cz21=z0(3)*(J1-J2)*sin(z0(2))*cos(z0(2));cz22=0;
Cz=[cz11 cz12;cz21 cz22]; Az=[0 0 1 0;0 0 0 1;zeros(2) -Mz\Cz];
B=Ts*Bx; A_x=eye(4)+Ts*Ax; A_z=eye(4)+Ts*Az;
z=A_z*z0+B*(u-dh); dh=((B'*B)\B')*(x(k,:)'-z+A_z*z0-A_x*x0');
z0=z; x0=x(k,:); pd=[pd d]; pdh=[pdh dh]; pr=[pr r];
end
figure(1);plot(ti,px(:,1),ti,pr(1,:)); legend('a_h','a_hr');
figure(2);plot(ti,px(:,2),ti,pr(2,:)); legend('a_v','a_vr');
figure(3); plot(ti,pd(1,:),ti,pdh(1,:)); legend('d1','dh1');
figure(4); plot(ti,pd(2,:),ti,pdh(2,:)); legend('d2','dh2');
figure(5); plot(ti,px(:,3),ti,px(:,4)); legend('a_hdot','a_vdot');
```