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COMPARISON OF TWO METHODS FOR DEMODULATION OF PULSE SIGNALS – APPLICATION IN CASE OF CENTRAL SLEEP APNEA

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Abstract. In the field of 24/7 human health monitoring, pervasive computing makes possible the continuous analysis of physiological parameters from an ambulatory device with a great acceptability. This paper presents two methods for obtaining cardiac and respiratory rates from a single arterial pressure signal: AM-FM demodulation and Singular Spectrum Analysis (SSA). With the aim to monitor sleep apnea, two simulated central sleep apnea were performed and recorded with Biopac reference system. The results showed a good evaluation of the cardiac rate with Singular Spectrum Analysis and bad results with AM-FM demodulation. For the respiration rate, some other signals were tested with average results for both methods. Further experiments will deal with real sleep apnea cases and algorithm improvements.

Keywords: optical pulse signal, AM-FM demodulation, Singular Spectrum Analysis (SSA), Heart and Respiratory Rates.

Classification numbers: 4.8.4, 4.9.3, 4.1.4.

1. INTRODUCTION

In the coming years, there will be a strong development of non-invasive physiological devices for monitoring health conditions. Telecare and telemedicine are going to be more and more employed, especially in developed countries. In our case, we have designed a device comprised of an optical arterial pulse and mechanical sensors, worn at the wrist for a great acceptability, which can be used to monitor people conditions (mainly heart and respiratory rates). Our aim applications are physiological monitoring for elderly persons and sleep apnea detection.

Central sleep apnea is a sleep disorder in which the brain doesn't send regular signals to breathe, causing the breathing to pause and restart repeatedly during sleep. Methods for survey are the important points in this case. Estimation of respiratory rate from physiological signals has been investigated by many authors using various types of methods.

Among methods listed below, we will only focus on two different types of methods: methods using demodulation techniques and methods using Principal Component Analysis (PCA) techniques (or similar techniques).

Several methods exist for respiration rate recovery from photoplethysmogram (PPG) signal, for instance in [1 - 3], a basic way to extract breathing rate is carried out by signal filtering, Addison *et al*. [4 - 11] use wavelet transform and Bruno *et al*. [12] propose AM-FM demodulation.

While these methods are focusing on finding only the respiratory information from PPG, other techniques need other physiological signals:

- Iamratanakul *et al.* [13] propose a complex model that is used to recover respiration rate thanks to 3 different signals: impedance between two electrocardiogram (ECG) leads, arterial blood pressure and heart rate. The performance of a linear model combining the three estimators (additive, AM, FM) is then evaluated.
- Orphanidou [14] introduce a fusion method of ECG and PPG using Empirical Mode Decomposition (EMD).
- McNames and Aboy [15] describe an Extended Kalman Filter using a state model with several parameters which is built to track information such as cardiac fundamental frequency and higher harmonics, respiratory fundamental frequency and higher harmonics, cardiac components harmonic amplitudes and phases, pulse pressure variation, etc. Another Kalman model is used by Foussier *et al*. [16].
- Jafari *et al*. [17] use MEMS-sensors combined with ICA and PCA methods.

Even if some of the above methods reached good achievements, these methods were not tested specifically in case of sleep apnea, except the study of Fang *et al.* [18], where a microphone is used with a smartphone in order to investigate cases of obstructive sleep apnea.

We are interested in finding a way to recover both heart rate and respiratory information from a single arterial blood pressure signal. We also want to preserve a trade-off between complexity and robustness.

In this paper, we study two different ways to obtain both heart and respiratory rates. These parameters are parts of the arterial pulse signal thanks to the phenomenon called the Respiratory Sinus Arrhythmia (RSA). The two investigated methods are:

- The AM-FM demodulation
- The Singular Spectrum Analysis

Each algorithm is detailed and is able to provide both cardiac and respiratory frequency information. We compare the two algorithms in terms of mean error and standard deviation of the cardiac frequency thanks to a reference signal obtained from another device. For the respiration rate, due to the fact of the lack of reliable reference (motion noise), we have evaluated over another free available database which provides reliable respiratory recorded signals.

2. MATERIALS AND METHODS

2.1. Materials

We used the Biopac Systems Inc. MP100 device as the reference device with following sensors and actuators:

- Optical photoplethysmography (PPG);
- Electrocardiogram (ECG);
- Respiration belt.

In order to assess the respiratory-induced changes in PPG and ECG signals when simulating apnea, the subject held his breathing for approximately 70 seconds. Two trials were performed and signals were recorded using Acknowledge Biopac Systems software. The sampling rate was 500 Hz during the 12 minutes total duration of measurements. Data processing was performed off-line using MatLab or C programs and libraries.

Figure 1 displays part of the 4 recorded signals for the study: 2 PPGs, 1 ECG and 1 respiration signals. ECG signal was used by AcqKnowledge software to calculate the reference heart rate.

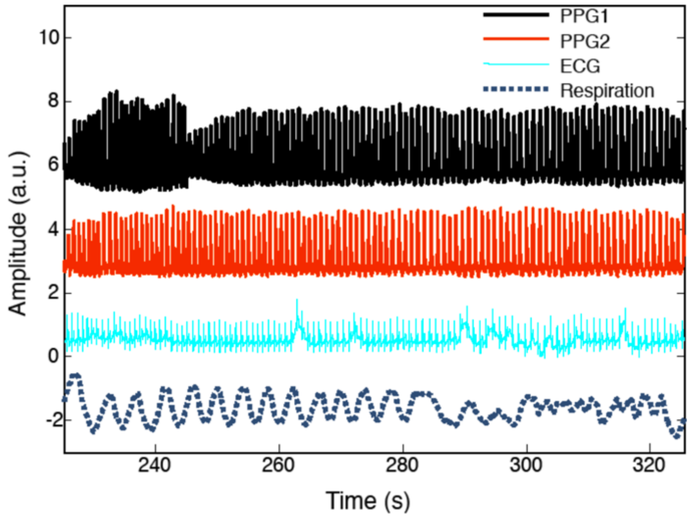

Figure 1. PPGs, ECG and respiration signals.

2.2. Analytical methods

2.2.1. AM-FM demodulation

In [12], cardiac and respiratory rhythms have been successfully extracted from a single arterial pulse signal during sleep thanks to the AM-FM demodulation. In order to apply the method to signals acquired in our labs, we used the algorithm described in Figure 2 to extract heart and respiratory rates.

The pulse signal is first filtered through a second order Tchebychev filter. The AM-FM demodulation is performed to get the instantaneous frequency composed of cardiac frequency and an image of respiration signal. Then this signal is filtered and Fourier transform is calculated. The frequency of maximum amplitude is attributed to respiratory rate. Moreover, mean estimation in each RR interval (time duration between two consecutive peaks of the signal) of the pulse signal is calculated in order to also get heart rate information.

AM-FM demodulation is performed using Teager energy operator [12, 19, 20]. Depending on some constraints fulfilments, the discrete-time Teager energy operator, ψ, applied to the discrete signal $x[n]$, using Discrete-time Energy Separation Algorithm-1a (DESA-1a) proposed by Maragos *et al*. [20], is simply expressed by:

$$\Psi[x[n]] = \frac{x^2[n-1] - x[n]\,x[n-2]}{T_s^2} \tag{1}$$

and gives directly the amplitude envelope $|a[n]|$ and frequency component $f_i[n]$ of $x[n]$:

$$|a[n]| \approx \frac{\Psi[x[n]]}{\sqrt{\Psi[\dot{x}[n]]}} \tag{2}$$

$$f_i[n] \approx \frac{1}{2\pi}\sqrt{\frac{\Psi[\dot{x}[n]]}{\Psi[x[n]]}} \tag{3}$$

where T_s is the sampling period and $x;^{\cdot}[n]$ denotes the numerical differentiation of $x[n]$.

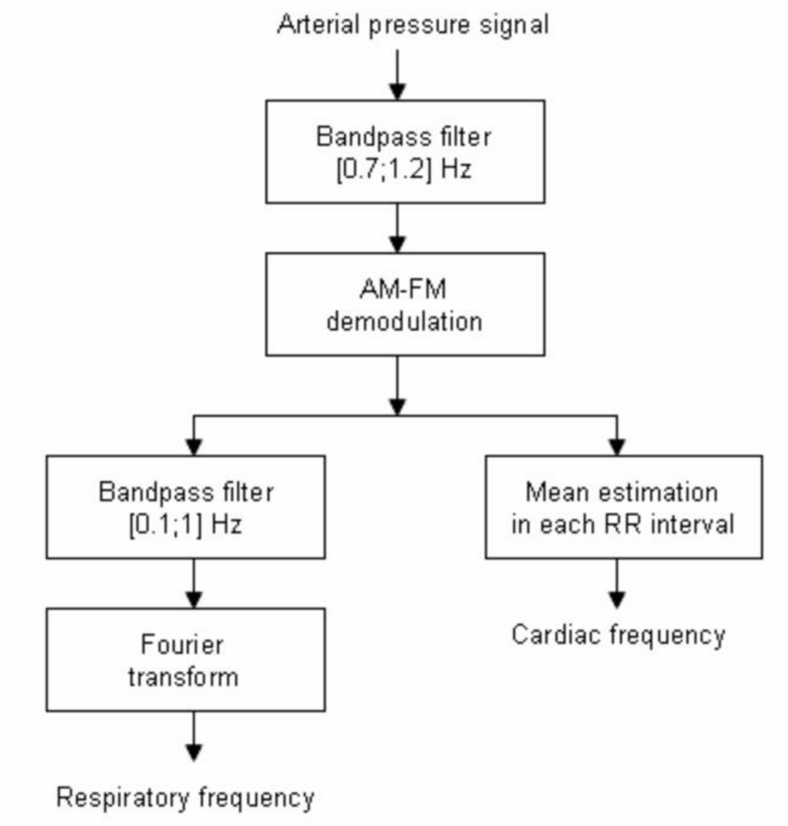

Figure 2. AM-FM demodulation diagram.

An example of cardiac frequency estimation is given on Figure 3 with results obtained: from PPG signal itself (RR intervals counting), from Biopac AcqKnowledge software (beat by beat estimation) and from AM-FM demodulation (mean estimation in each RR interval); the respiration signal is also drawn from respiration belt. During the simulated apnea, the heart rate first decreases and then increases. We can notice that the AM-FM demodulation tends to generally overestimate the heart rate and that the errors increase during the simulated apnea episode.

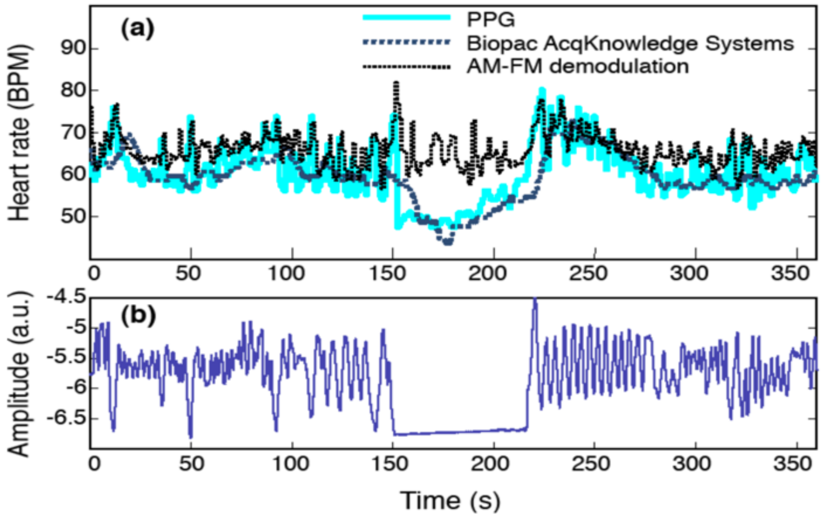

Figure 3. (a) Results for cardiac rate estimation obtained from PPG signal (continuous line), from AcqKnowledge Biopac Systems (thick dotted line) and from AM-FM demodulation (thin dotted line); (b) Respiration signal from respiration belt.

2.2.2. Empirical SSA method

Singular Spectrum Analysis is a technique used for analyzing climatic time series [21-25]. The SSA is often used to enhance the Signal-to-Noise Ratio (SNR) or extract in the time series the trends or oscillations in order to understand the inner dynamics or predict the system future behavior. Among some interesting properties of SSA, we can also mention an iterative algorithm that can be applied in signal with missing data [26]. SSA was recently used as a denoising pre-processing stage for Heart Rate estimation from PPG [27 - 30].

We develop an empirical method for cardiac and respiratory rates estimation using only a single arterial pressure signal [31]. The algorithm used can be set as in Figure 4, with several optional processing like denoising, single phase rectification (cf. Figure 5) or iterative procedure when needed.

The original pulse signal $x[n]$ of length N is cut in overlapping portion of length M. In other words; we reshape the original signal into the trajectory matrix A, whose rows are vectors of length M (sliding window over the signal $x[n]$).

SSA performs a Karhunen-Loève decomposition of an estimate of the correlation matrix based on M lagged copies of the signal.

$$\mathrm{A} = \begin{pmatrix} x[1] & x[2] & \dots & x[M] \\ x[2] & x[3] & \dots & x[M+1] \\ \dots & \dots & \dots & \dots \\ x[N\text{-}M+1] & x[N\text{-}M+2] & \dots & x[N] \end{pmatrix} \tag{4}$$

A key point is the choice of the length window M, for instance if we want to catch an oscillation pattern whose period is L samples, then we should try a $M > L$. There is also a trade-off with the calculation cost with large **Error!**. According to Vautard and Ghil [23], the value of

M has to be chosen in the interval [1;N] (no optimal choice exists, so the value of M has to be tested over a reasonable range).

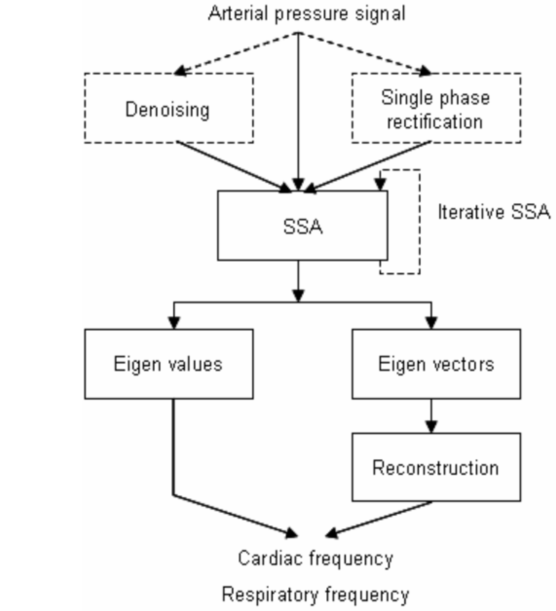

Figure 4. Empirical algorithm with SSA for retrieving cardiac and respiratory information from an arterial pressure signal, optional stages are represented with dashed line.

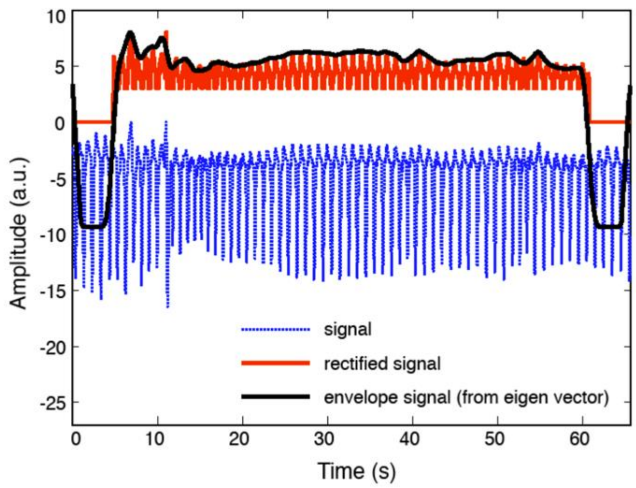

Figure 5. Original signal and single phase rectifier procedure, beginning and end of the upper signal are set to zero in order to avoid side effects; the extracted envelope is also drawn.

The unbiased estimator of the lag covariance matrix C is calculated:

$$C_i^{(M)} = \frac{1}{N\text{-}i} \sum_{j=1}^{N\text{-}i} x[j]\, x[j{+}1] \tag{5}$$

where i belongs to [0; M - 1].

Then a Singular Value Decomposition (SVD) is performed in order to obtain a diagonal matrix of eigen values D sorted in decreasing order and a matrix of the associated eigen vectors V.

$$C = UDV \tag{6}$$

These eigen vectors are called Empirical Orthogonal Functions (EOFs) [22] or Direction of Principal Components or Singular Vectors [32]. Reconstruction of the signal based upon a few selected eigen vectors can be applied (see [24] for details). Usually, the first eigen value (or vector) is associated with the AM modulation, the next two eigen values can generally be associated with heart rate.

Another fact to be mentioned is that there are only a few eigen values with great value and a lot of eigen values with small one (parsimonious representation), only a few eigen vectors contain the majority of the signal energy, the others can be considered as noise contribution. It is then possible to denoise by reconstructing the signal with the biggest eigen values while leaving the meaningless ones.

Figures 6 and 7 show examples of SSA eigen values and eigen vectors extracted from a pulse signal and compared to original signal and associated respiration signal. It can be seen that respiration and first eigen value reconstructed signal are clearly highly correlated and it is the same for the original arterial pulse signal with the second eigen value reconstructed signal.

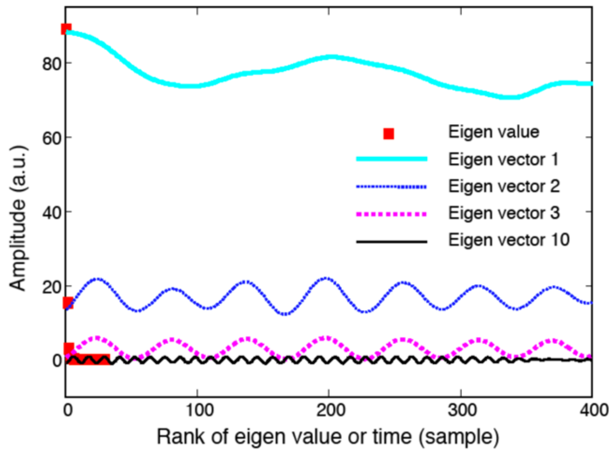

Figure 6. Eigen values of SSA: only a few number of eigen values collect the major part of the signal energy, and a few associated eigen vectors.

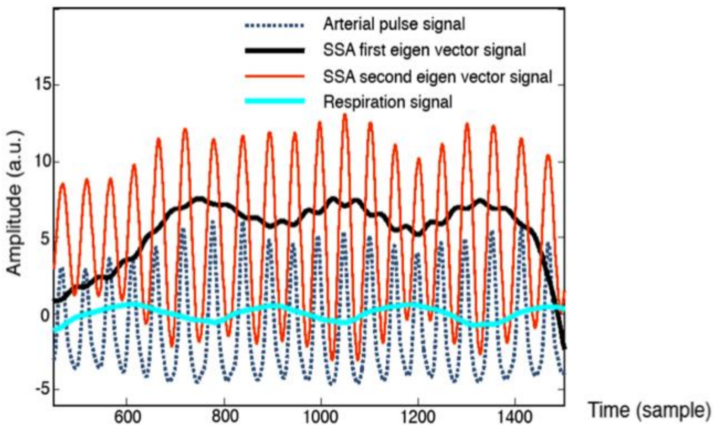

Figure 7. An example of demodulation performed by Singular Spectrum Analysis.

2.2.3. Heart rate and respiratory rate extraction

We have identified at least three possibilities for extracting heart rate and respiratory information after an SSA transform: a) Reconstruction of the signal based upon the basis of a few selected eigen vectors (one or two values), then find each peak in this signal and then calculate the Heart Rate [33]; b) Perform a Fast Fourier Transform (FFT) of the reconstructed vector and find the frequency value of the spectrum maximum; c) Directly use the eigen values with a scale factor correlated to *M* [23].

Figure 8 shows the processing results of one case in which respiration was held during several seconds.

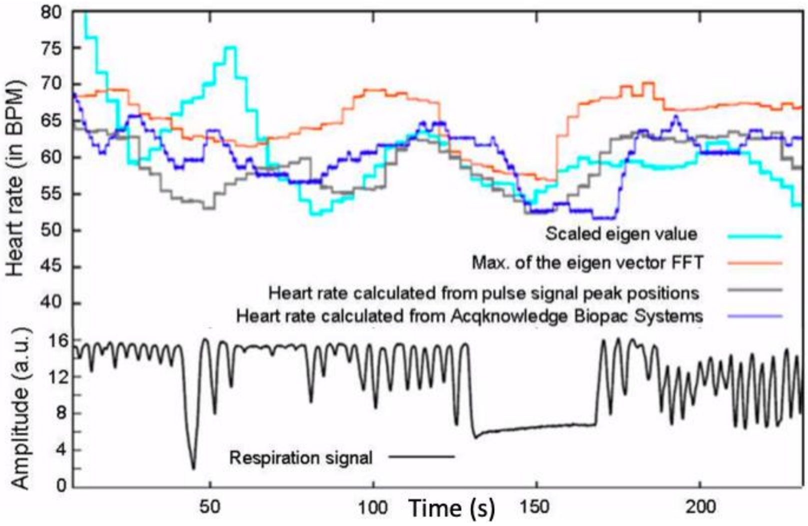

Figure 8. Heart rate estimation calculated with 3 methods thanks to SSA, heart rate reference given by Acqknowledge Biopac software (top); Respiration signal with simulated apnea (bottom).

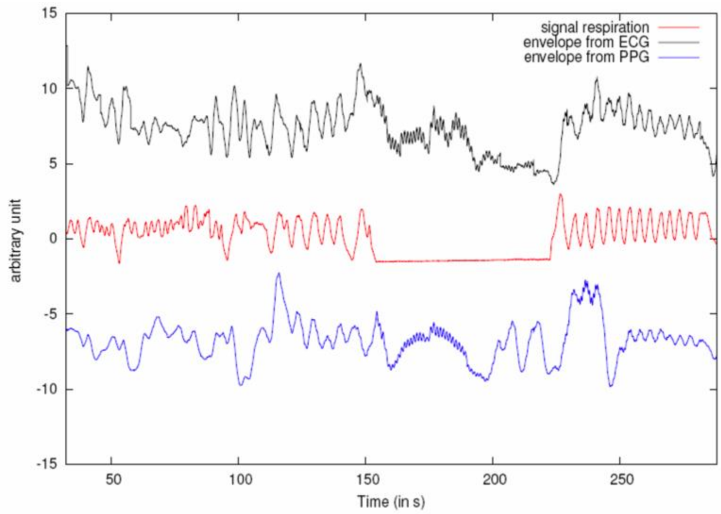

Figure 9. Examples of envelope extracted signals obtained from ECG (top) and PPG (bottom); chest respiration signal reference (middle).

We have calculated the heart rate with the pre-cited methods involving SSA algorithm, they are presented together with the heart rate calculated with Biopac. Even if some differences are observed, the results seem to be robust enough, even during the apnea episode.

Figure 9 displays demodulated respiratory components extracted from ECG and PPG signals compared to the respiration signal reference. It is proved that the SSA algorithm performs also very well on ECG signals with very few modifications. It is also clear that during the apnea episode, due to the lack of respiratory components, cardiac components are fully represented in the first eigen vector.

3. RESULTS AND DISCUSSION

We compared both methods: SSA and AM-FM demodulation, using the two simulated sleep apnea trials. Due to the fact of the lack of reliable respiratory reference, we are only able to assess results with cardiac reference.

3.1. Comparison results for cardiac frequency

The Figure 10 shows an example of heart rate estimation using SSA scaled eigen value method and AM-FM demodulation, where we can see that the AM-FM estimation of cardiac frequency is not robust during the simulated sleep apnea episode.

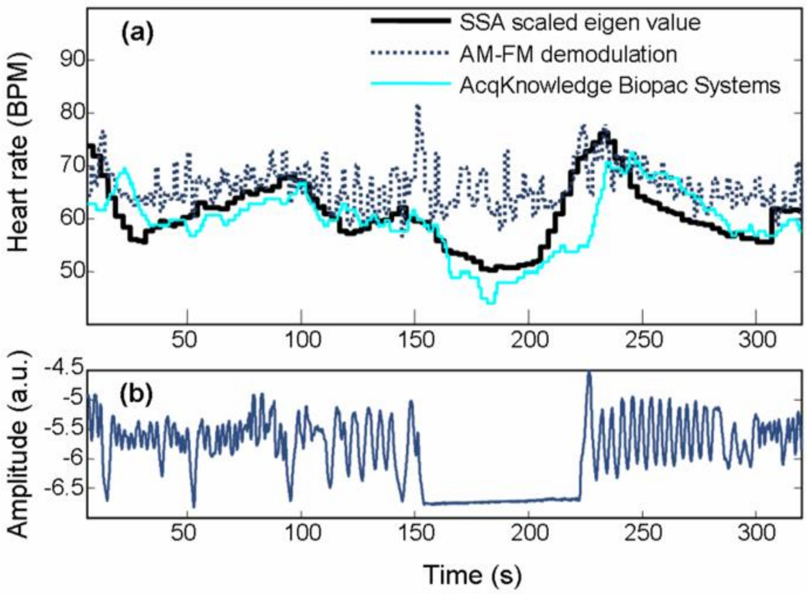

Figure 10. (a) Cardiac rate estimation obtained from SSA scaled eigen value method (thick continuous line), from AM-FM demodulation (dotted line) and from AcqKnowledge Biopac Systems (thin continuous line); (b) Respiration signal with simulated apnea.

The results comparing AM-FM demodulation and SSA methods (FFT, scaled eigen value, peak counting) for two trials are displayed in the Table 1. The mean error and the associated standard deviation are calculated using the Acknowledge Biopac heart rate as a reference.

The heart rate estimation is respectively better with the SSA scaled eigen value, the SSA eigen vector peak counting, the AM-FM demodulation, the SSA eigen vector FFT. The associated standard deviation is roughly the same for all methods.

During the time of simulated apnea, the heart rate estimation from AM-FM demodulation gave wrong results, this is due to the fact that there is no respiratory contribution in the pulse signal during this time. We can conclude that this kind of algorithm is not suitable for monitoring people prone to central sleep apnea. Furthermore, all constraints defined in [12, 19, 20] are not always satisfied. So, it will not be possible to obtain an AM-FM demodulation of arterial signals without errors.

Since the respiration evaluation was not concluded with our signals, we have tried it over another free available database.

Table 1. Heart rate estimation obtained from AM-FM demodulation and SSA methods and compared with reference (mean error and standard deviation in BPM (Beats Per Minute)).

Method	Mean error		Standard deviation	
	Signal 1	Signal 2	Signal 1	Signal 2
AM-FM demodulation	5.5	4.0	5.0	4.3
Eigen vector FFT	4.4	4.3	4.5	4.6
Scaled eigen value	0.2	-0.6	4.8	5.2
Eigen vector peak counting	0.3	-2.5	3.8	5.4

3.2. Test upon another database for respiration evaluation

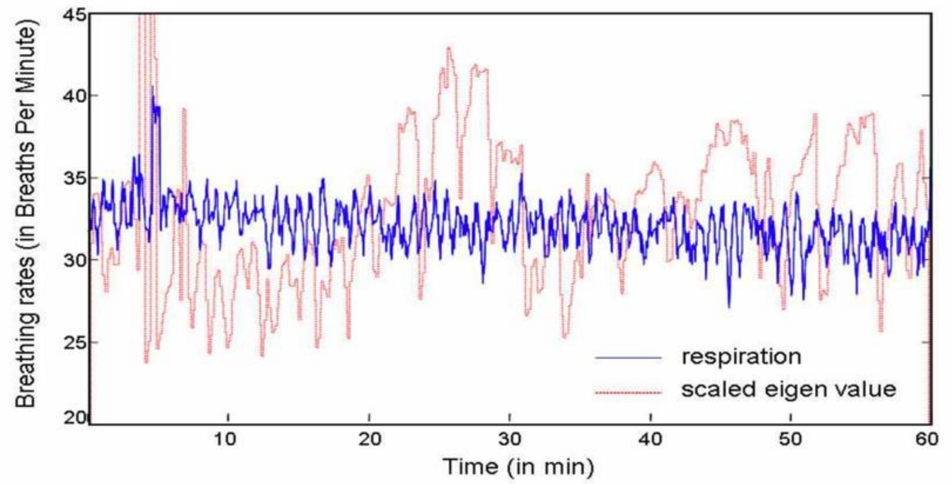

Figure 11. Breathing rate obtained from respiration signal (peaks counting) and with SSA scaled eigen value.

We have found a database which provides reliable respiratory recorded signals provided by a pediatric intensive care center. The pediatric patient was suffering from a traumatic brain injury. These signals can be found at the following url: http://bsp.pdx.edu/. Among the data we have chosen 2 parameters only:

• Respiration: zipped text file, sampled at 125 Hz, units are scaled in the range [0;1023], respiration signal sampling frequency 125 Hz,

• Pulse oximetry: one of the two signals from a pulse oximeter sampled at 125 Hz, units are scaled in the range [0;1023].

These data are characterized by high rates: cardiac around 200 beats per minute and respiratory around 32 breaths per minute. The record lasted about 6 hours, we tested upon the first hour only. The results for the respiration rates are drawn in Figures 11-13.

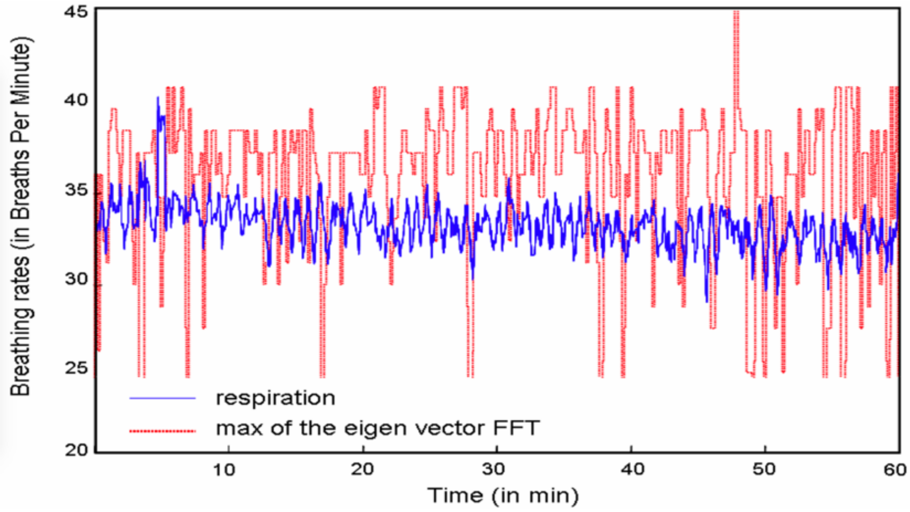

Figure 12. Breathing rate obtained from respiration signal (peaks counting) and with maximum value of the FFT of the reconstructed signal.

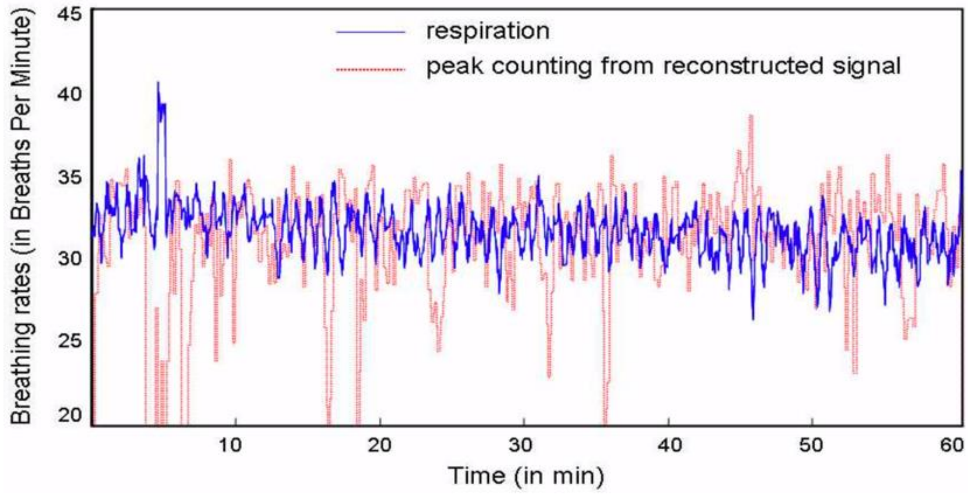

Figure 13. Breathing rate obtained from respiration signal (peaks counting) and with peak counting from reconstructed signal.

In fact, the SSA can extrapolate data [26], so some extra respiratory peaks are sometimes been added while they should had not been, sometimes two are merged together. Some examples of encountered difficult situations are illustrated on Figures 14 and 15.

The mean error and standard deviation for respiration rate was computed using a peak counting over the respiration signal as a reference. These results are presented in Table 2, where the AM-FM demodulation presents the lower standard deviation, as oppose to the SSA scaled eigen value that presents the highest standard deviation.

However, if we calculate the coefficient of variation (standard deviation/mean error i.e. 1.3/2.2= 0.6), we obtain a value inferior to 1 which indicates low variance, so maybe, if we can compensate the error mean bias (which is not proven and need further investigation), then AM-FM demodulation could be a suitable method when there is no apnea.

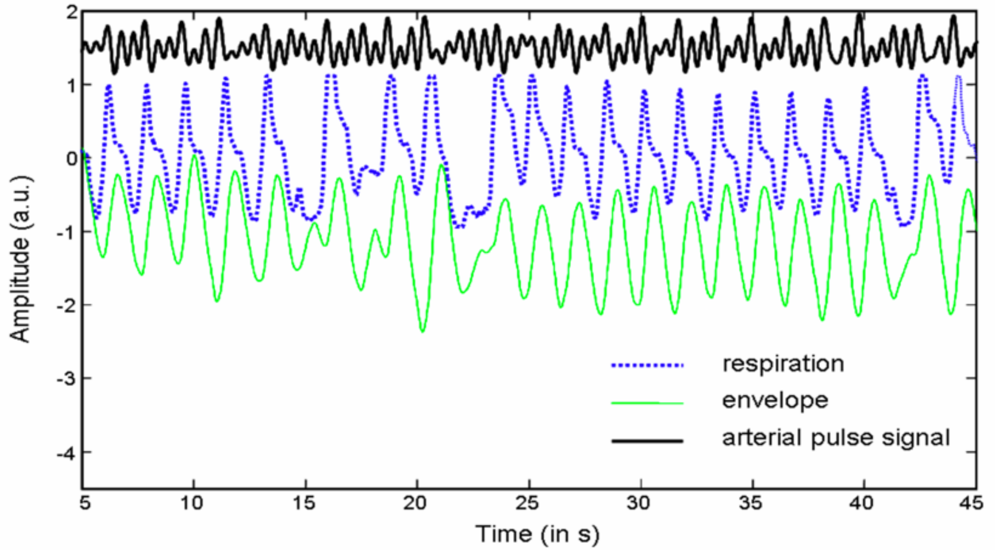

Figure 14. Examples of added extra peaks in reconstructed signal between 15 and 20 s.

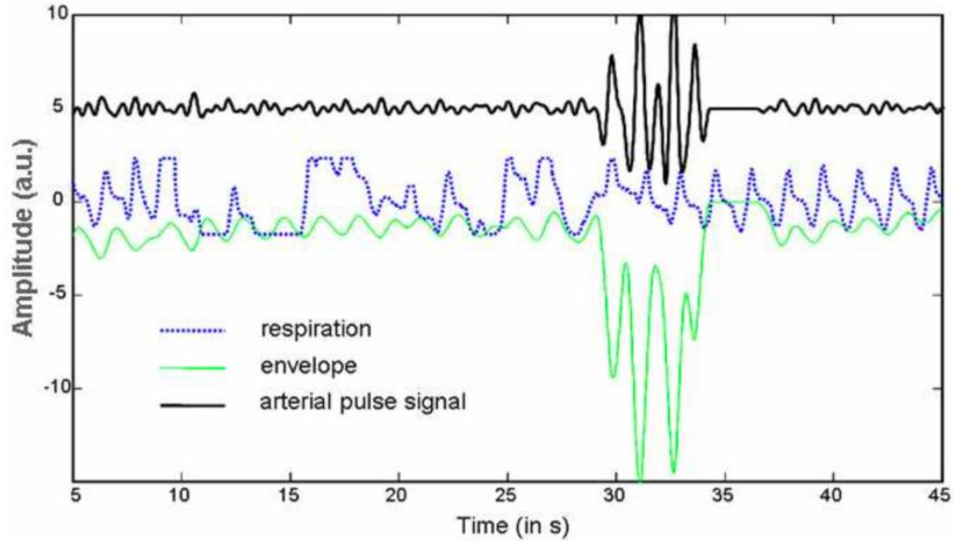

Figure 15. Examples of problems occurring with saturated parts for the respiration and artifacts for the arterial pulse signal.

Table 2. Respiratory rate estimation obtained from AM-FM demodulation and SSA methods and compared with reference (mean error and standard deviation in BPM (Beats Per Minute)).

Method	Mean error	Standard deviation
AM-FM demodulation	2.2	1.3
Eigen vector FFT	3.5	3.7
Scaled eigen value	0.5	4.9
Eigen vector peak counting	-1.4	3.0

The complexity comparison of two methods is presented in Table 3. Although the SSA method has a higher complexity than the AM-FM demodulation, it is not a limitative factor in our application since the calculation power of embedded devices like smart wrist-watches or smartphones has increased over the years and permits this calculation cost. Another drawback is that eigen values have to be properly scaled in order to be fitted to the correct range of values. In this case, eigen vectors can also be further exploited and can be used to help to set this scale factor properly. This problem is currently under investigation since it can be solved in many ways.

Table 3. Complexity comparison of two methods to physiological signal processing.

Method	Complexity
AM-FM demodulation	$O(N \log_2 N)$
SSA	$O(M^2 (N - M + 1))$

SSA appears to be a powerful and promising tool for physiological signal processing with great potential applications in monitoring, denoising or processing signals with missing data [26]. SSA was also tested with ECG signals and showed performances similar to pulse signals.

4. CONCLUSIONS

The context of this study was to perform 24/7 ambulatory monitoring by developing a device worn at the wrist with a great acceptability and which is able to retrieve heart and respiration rate from a single arterial pressure signal. In this paper, we have compared two methods for obtaining cardiac and respiratory rates from the single arterial pressure signal: SSA and AM-FM demodulation. It was proved that the SSA method was more robust to retrieve cardiac rate during apnea. We have adapted the SSA technique used in climatic time series analysis in the case of arterial pulse rate analysis by adding some pre-processing stages which are mandatory in order to succeed in catching the desired information: first a single phase rectification had to be done, then a zero padding procedure at both extremities of the signal window for a better matrix conditioning and computing.

Both methods were evaluated using 2 trials including a simulated sleep apnea episode: it was shown that the AM-FM demodulation failed for estimating the heart rate during the apnea episode. Respiration rate was evaluated using one hour of data from a pediatric patient with traumatic brain injury. SSA has the advantages that it does not need a priori hypothesis about the type of the modulation in the signal (unlike the AM-FM demodulation) and then can efficiently perform heart rate estimation during central sleep apnea.

The cardiac rate estimation using SSA eigen value is an interesting new way for obtaining directly physiological frequency estimation without using FFT. Due to its flexibility, improvements of the SSA method are expected in signals with missing data or noise.

Developing more complex algorithms is necessary to improve the accuracy of the actual programs. Further trials will be employed to assess the methods with data including central, obstructive or mixed sleep apnea cases.

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REFERENCES

1. Prasetiyo R. B., Choi K. S., Yang G. H. - Design and implementation of respiration rate measurement system using an information filter on an embedded device, Sensors **18** (2018) 4208. DOI: 10.3390/s18124208.
2. Charlton P. H., Bonnici T., Tarassenko L., Alastruey J., Clifton D. A., Beale R., Watkinson P. J. - Extraction of respiratory signals from the electrocardiogram and photoplethysmogram: technical and physiological determinants, Physiol Meas, IOP Publishing **38** (5) (2017) 669–90. DOI: 10.1088/1361-6579/aa670e.
3. Nilsson L., Johansson A. and Kalman S. - Respiration can be monitored by photoplethysmography with high sensitivity and specificity regardless of anaesthesia and ventilatory mode, Acta Anaesthesiologica Scandinavica **49** (8) (2005) 1157-1162.

   DOI: 10.1111/j.1399-6576.2005.00721.x.
4. Addison P. S. and Watson J. N. - Secondary Wavelet Feature Decoupling (SWFD) and its use in detecting patient respiration from the photoplethysmogram, Engineering in Medicine and Biology Society, 2003. Proceedings of the 25th Annual International Conference of the IEEE, 2003, pp. 2602-2605. DOI: 10.1109/IEMBS.2003.1280448.
5. Addison P. S. and Watson J. N. - Secondary Transform Decoupling of Shifted Nonstationary Signal Modulation Components: Application to Photoplethysmography, International Journal of Wavelets, Multiresolution and Information Processing **2** (1) (2004) 43-57. DOI: 10.1142/S0219691304000329.
6. Clifton D., Douglas J. G., Addison P. S. and Watson J. N. - Measurement of Respiratory Rate From the Photoplethysmogram in Chest Clinic Patients, Journal of Clinical Monitoring and Computing **21** (1) (2007) 55-61. DOI: 10.1007/s10877-006-9059-3.
7. Leonard P. A., Beattie T. F., Addison P. S. and Watson J. N. - Standard pulse oximeters can be used to monitor respiratory rate, Emergency Medicine Journal **20** (6) (2003) 524-525. DOI: 10.1136/emj.20.6.524.
8. Leonard P. A., Beattie T. F., Addison P. S. and Watson J. N. - Wavelet analysis of pulse oximeter waveform permits identification of unwell children, Emergency Medicine Journal **21** (1) (2004) 59-60. DOI: 10.1136/emj.2003.004887.
9. Leonard P. A., Clifton D., Addison P. S., Watson J. N. and Beattie T. - An automated algorithm for determining respiratory rate by photoplethysmogram in children, Acta Paediatrica **95** (9) (2006) 1124-1128. DOI: 10.1080/08035250600612280.

10. Leonard P. A., Douglas J. G., Grubb N. R., Clifton D., Addison P. S. and Watson J. N. - A Fully Automated Algorithm for the Determination of Respiratory Rate from the Photoplethysmogram, Journal of Clinical Monitoring and Computing **20** (1) (2006) 33-36. DOI: 10.1007/s10877-005-9007-7.

11. Leonard P. A., Grubb N. R., Addison P. S, Clifton D. and Watson J. N. - An Algorithm for the Detection of Individual Breaths from the Pulse Oximeter Waveform, Journal of Clinical Monitoring and Computing **18** (5) (2004) 309-312. DOI: 10.1007/s10877-005-2697-z.

12. Bruno S. and Scalart P. - Estimation of cardiac and respiratory rhythms based on AM-FM demodulation and an adaptive eigen vector decomposition, 13th European Signal Processing Conference, 2005, pp. 05. HAL Id: inria-00482656.

13. Iamratanakul S., McNames J. and Goldstein B. - Estimation of respiration from physiologic pressure signals, Engineering in Medicine and Biology Society, 2003. Proceedings of the 25th Annual International Conference of the IEEE, 2003, pp. 2734-2737. DOI: 10.1109/IEMBS.2003.1280482.

14. Orphanidou C. - Derivation of respiration rate from ambulatory ECG and PPG using ensemble empirical mode decomposition: Comparison and fusion, Computers in biology and medicine **81** (2017) pp. 45–54 . DOI: 10.1016/j.compbiomed.2016.12.005

15. McNames J. and Aboy M. - Statistical Modeling of Cardiovascular Signals and Parameter Estimation Based on the Extended Kalman Filter, IEEE Transactions on Biomedical Engineering **55** (1) (2008) 119-129. DOI: 10.1109/TBME.2007.910648.

16. Foussier J., Teichmann D., Jia J., Misgeld B., and Leonhardt S. - An adaptive kalman filter approach for cardiorespiratory signal extraction and fusion of non-contacting sensors, BMC medical informatics and decision making **14** (1) (2014) p. 37.

    DOI: 10.1186/1472-6947-14-37.

17. Jafari Tadi M., Lehtonen E., Teuho J., Koskinen J., Schultz J., Siekkinen R., Koivisto T., Pänkäälä M., Teräs M., and Klén R. - A computational framework for data fusion in mems-based cardiac and respiratory gating, Sensors **19** (19) (2019) p. 4137.

    DOI: 10.3390/s19194137.

18. Fang Y., Jiang Z., and Wang H. - A novel sleep respiratory rate detection method for obstructive sleep apnea based on characteristic moment waveform, Journal of healthcare engineering, vol. 2018, 2018. DOI: 10.1155/2018/1902176.

19. Maragos P., Kaiser J. and Quatieri T. - Energy separation in signal modulations with application to speech analysis, IEEE Transactions on Signal Processing **41** (10) (1993) 3024-3051. DOI: 10.1109/78.277799.

20. Maragos P., Kaiser J. and Quatieri T. - On amplitude and frequency demodulation using energy operators, IEEE Transactions on Signal Processing **41** (4) (1993) 1532-1550.

    DOI: 10.1109/78.212729.

21. Broomhead D. S. and King P. - Extracting qualitative dynamics from experimental data, Physica D: Nonlinear Phenomena **20** (2-3) (1986) 217-236. DOI: 10.1016/0167-2789(86)90031-X.

22. Ghil M., Allen M. R., Dettinger M. D., Ide K., Kondrashov D., Mann M. E., Robertson A. W., Saunders A., Tian Y., Varadi F. and Yiou P. - Advanced spectral methods for climatic time series, Rev. Geophys **40** (1) (2002) 1003. DOI: 10.1029/2000RG000092.
23. Vautard R. and Ghil M. - Singular spectrum analysis in nonlinear dynamics, with applications to paleoclimatic time series, Physica D: Nonlinear Phenomena **35** (3) (1989) 395-424. DOI: 10.1016/0167-2789(89)90077-8.
24. Vautard R., Yiou P. and Ghil M. - Singular-spectrum analysis: a toolkit for short, noisy chaotic signals, Physica D: Nonlinear Phenomena **58** (1-4) (1992) 95-126.

    DOI: 10.1016/0167-2789(92)90103-T.
25. Yiou P., Sornette D. and Ghil M. - Data-adaptive wavelets and multi-scale singular-spectrum analysis, Physica D: Nonlinear Phenomena **142** (3-4) (2000) 254-290.

    DOI: 10.1016/S0167-2789(00)00045-2.
26. Schoellhamer D. H. - Singular spectrum analysis for time series with missing data, Geophysical Research Letters **28** (16) (2001) 3187-3190. DOI: 10.1029/2000GL012698
27. Zhang Z. - Photoplethysmography-based heart rate monitoring in physicalactivities via joint sparse spectrum reconstruction, IEEE transactions on biomedical engineering **62**(8) (2015) pp. 1902–1910. DOI: 10.1109/TBME.2015.2406332.
28. Zhang Z., Pi Z., and Liu B. - Troika: A general framework for heart rate monitoring using wrist-type photoplethysmographic signals during intensive physical exercise, IEEE Transactions on biomedical engineering **62** (2) (2014) 522–531.

    DOI: 10.1109/TBME.2014.2359372.
29. Zhang Z. - Heart rate monitoring from wrist-type photoplethysmographic (ppg) signals during intensive physical exercise, in 2014 IEEE Global Conference on Signal and Informat IEEE, 2014, pp. 698–702. DOI: 10.1109/GlobalSIP.2014.7032208.
30. Zhang Y., Liu B., and Zhang Z. - Combining ensemble empirical mode decomposition with spectrum subtraction technique for heart rate monitoring using wrist-type photoplethysmography, Biomedical Signal Processing and Control, vol. 21, pp. 119–125, 2015. DOI: 10.1016/j.bspc.2015.05.006
31. Nguyen Q. V., Le Page R., Goujon J. M., Guyader P. and Billon M. - Pulse rate analysis in case of central sleep apnea: A new algorithm for cardiac rate estimation, Engineering in Medicine and Biology Society, 2009. EMBC 2009, 31th Annual International Conference of the IEEE, 2009, pp. 5490-5493. DOI: 10.1109/IEMBS.2009.5334561.
32. Schreiber T., Kantz H., Kurths J. and Mayer-Kress G. - Nonlinear analysis of physiological data, Springer, 1998. DOI: 10.1007/978-3-642-71949-3
33. Carter T. A., Tapia R. A. and Papakonstantinou A. - An introduction to linear algebra for pre-calculus students. Technical report, Rice University, May 1995.