FINITE ELEMENT MODELLING FOR ELECTRIC FIELD DISTRIBUTION AROUND POSITIVE STREAMERS IN OIL

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Abstract. Electric field distribution of positive streamers during propagation was determined with the finite element method by using COMSOL multiphysics. Modelling was performed at 210 kV and 270 kV. The geometrical shape of streamers was modelled with cylinder and sphere for the case of 210 kV while a growing cylinder was used for streamer propagation at 270 kV. In addition, a spherical model was used for determining the relationship between the branching of streamers and the electric field at the tip of branches. It is obtained from the simulation results that the 2nd mode streamers has the electric field at channel tips of about 0.1 MV/cm while 8.3 MV/cm was received for the 4th mode streamers. The simulation results also reveal that the shielding effect resulting from streamer branching significantly reduces the electric field at the channel tips, and the shielding effect disappears with the angle $\alpha$ between channels is about $30^\circ$-$60^\circ$ depending on the size of streamer envelope. The hypothesis on correlation among velocity, streamer branching and electric field is suggested.

Keywords: streamers, electric field, branching, velocity, finite element modelling.

Classification numbers: 2.3.1, 2.8.3, 2.10.1.

1. INTRODUCTION

Streamers in mineral oil have been investigated for a long time to understand prebreakdown phenomena, i.e. streamer initiation and propagation, occurring in oil [1-7]. Based on this understanding, the new insulating liquids can be designed and testing standards for high voltage equipment can be amended. However, the full understanding of mechanism behind streamer propagation has not been achieved yet. Therefore, many investigations were performed with streamer propagation in model oils with and without aromatic typed additives [6, 7]. It was reported that streamers behave in different modes with increasing applied voltage in a type of paraffinic oil, e.g. Exxsol oil, as seen in Fig. 1 and Fig. 2 [6]. Similar results were reported in other types of model oils and mineral oil [3, 7]. In these figures, streamer structure and velocity change with different modes. As seen in Fig. 1, there is a threshold value, $V_a$, and streamers switch to fast mode from slow mode if the applied voltage exceeds $V_a$. At the slow mode, i.e. the
2\textsuperscript{nd} mode, streamers have a multifilament structure with low velocity of about 1-3 km/s (Fig. 2a) and become more branching with increasing applied voltage (Fig. 2b). At the fast mode, \emph{i.e.} the 4\textsuperscript{th} mode, streamers have tree-like structure and propagate with the speed of about 100 km/s (Fig. 2d). The 4\textsuperscript{th} mode streamers become more branching with a slight increase in velocity (Fig. 2e) when the applied voltage is much higher than \( V_a \). The 3\textsuperscript{rd} mode streamers were transition ones with the velocity of about 4-10 km/s (Fig. 2c) and appear for a period of time during the transition process when streamers switch from the 4\textsuperscript{th} mode to the 2\textsuperscript{nd} mode.

The low velocity of the 2\textsuperscript{nd} mode streamers is possibly due to the effect of more branching, \emph{i.e.} the shielding effect, which results in low electric field at the streamer channel tips [4]. By contrast, high velocity of the 4\textsuperscript{th} mode streamers was explained by high electric field at streamer channel tips due to the less branched structure of streamers, \emph{i.e.} tree-like [4]. The electric field at the streamer channel tips was calculated in previous studies [1, 2, 4]. However, these studies investigated the electric field of the 2\textsuperscript{nd} and 3\textsuperscript{rd} mode streamers at different applied voltages as well as at different experiments, and there is a lack of determination of the electric field at the tips of branches of the streamers at the 4\textsuperscript{th} mode. Moreover, the influence of the shielding effect on the electric field at the channel tips were not yet determined, and the correlation between

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Positive streamer velocity versus applied voltage (redrawn from [6]).}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Streamer shape versus applied voltage [6].}
\end{figure}
streamer branching, velocity and electric field was also not yet established. In this paper, therefore, the electric field at the streamer channel tips at a magnitude of applied voltage, which results in streamers in the 2nd, 3rd and 4th modes in one experiment, was determined with the finite element method by using COMSOL multiphysics software, which was also used to simulate the influence of shielding effect on the electric field at the streamer channel tips. In addition, based on the simulation results, the relationship among the branching of streamers, velocity and electric field was also discussed.

2. FEM MODEL FOR STREAMER PROPAGATION

Figure 3 presents a 2D axisymmetric model that represents the experimental setup of the test cell, which is used in [6], for simulation of the electric field distribution during the propagation of streamers in the gap of the point-plane electrode system. The high voltage point electrode has a diameter of 0.15 mm while the diameter of the plane electrode is 340 mm. The electrode system was made by stainless steel and was installed vertically in a borosilicate test cell, which contains Exxsol oil. The geometrical model of streamers changes with different stages of streamer propagation in oil gap as well as different values of applied voltages.

![Figure 3. The 2D axial symmetry model for simulation of electric field distribution.](image-url)
For simulation of the electric field distribution in the electrode gap with the presence of streamers, images of streamers during propagation at 210 kV (Fig. 4) and 270 kV (Fig. 5) were used to determine the shape and size of streamer envelope. The velocity of streamers is calculated from framing image sequences. At 210 kV, streamers first start with the 4th mode (Fig. 4a) followed by the 3rd mode (Fig. 4b) and terminate with the 2nd mode (Fig. 4d). In addition, at this value of applied voltage, it was observed that the streamer structure has either cylindrical or spherical shapes corresponding to different periods of propagation time. Thus, conductive cylinder was used to simulate electric field distribution for streamers in Fig. 4a while cylindrical and spherical models with the voltage drop along streamer channel of approximately 10 kV/cm [4] were used for other cases. Both cylindrical and spherical models are shown in Fig. 6.

Figure 4. Framing images of streamers during propagation at 210 kV [6].

Figure 5. Images of the fast mode streamers (the 4th mode) in oil at 270 kV [6].

Figure 6. Models for simulation of electric field; (a) \( l = 16 \text{ mm} \) and \( \phi_m = 0.2 \text{ mm} \) for streamers in Fig. 4a, \( l = 37 \text{ mm} \) and \( \phi_k = 23 \text{ mm} \) for streamers in Fig. 4b; (b) \( \phi_k = 50 \text{ mm} \) and \( 65 \text{ mm} \) for streamers in Fig. 4c and d.
The size of cylindrical and spherical models was determined from streamer envelope in Fig. 4 with the use of the known dimension of the needle electrode as a benchmark. At 270 kV, streamers appear only in the 4th mode during propagation time and have a tree-like shape, which consists of many short branches encircling the main channel. The diameter of the main channel is about 0.15 mm measured from the images shown in Fig. 5. Because the fast mode streamers, i.e. the 4th mode, has high conductivity as suggested in a previous reference [4], the growing cylinder model, which is conductive, is used to determine the distribution of electric field around streamers (Fig. 7). An increment in steps of 10 mm is used to simulate the length \( l \) of the growing cylinder. For the sake of simplicity, it is assumed that there are no space charges around streamers.

For investigating the influence of streamer branching on the electric field at streamer channel tips, the image the slow mode streamers, i.e. the 2nd mode streamers, in Exxsol oil at applied voltage of 210 kV shown in Fig. 8a was used to determine the shape and size of streamer envelope. It is observed that the overall shape of streamer is nearly spherical, and streamers comprise of many thin and long channels. It is considered that the channels of streamers are distributed around the \( z \) axis and in the \( r-z \) plane (Fig. 8b). This indicates that the modelling of the so called “shielding effect” formed by streamer branching is really a 3D problem. For simplicity, the angle \( \beta \) between surrounding channels is considered to be 0°, i.e. hollow cones encircle the main channel and the shielding effect of channels around \( z \) axis is considered to be maximum. Therefore, a 2D axial symmetry model shown in Fig. 3 was reused with a more detail in geometrical of streamers structure (Fig. 9). The main channel (\( \phi_m = 0.1 \) mm) coincides with the \( z \) axis. Thickness, \( t \), of the hollow cones is 0.05 mm, which is equal to the diameter of side branches. Both the tips of the main channel and surrounding channels lie on an imaginary spherical surface. Diameter of the sphere (\( \phi \)) was chosen to be 40 mm and 75 mm, which are positions of streamers that cross 50 % and 93.8 % of the electrode gap. For each diameter of the sphere, the angle \( \alpha \) between channels was increased in steps from 0° to 60°. Again, for simplified simulation, both the main channel and surrounding channels are considered to be conductive, and there are no space charges around streamers.
Finite Element Modelling for Electric Field Distribution around Positive Streamers in Oil

Figure 8. The image of the 2nd mode streamers [6] and the distribution of streamer branches at 210 kV.

Figure 9. The 2D axial symmetry model for simulation of shielding effect with spherical model.

3. FEM METHOD OF ANALYSIS

The correlation between the electric field $E$ and the electric potential $V$ is expressed by equation (1).

$$ E = -\nabla V $$

From Maxwell’s equation,

$$ \nabla E = \frac{\rho}{\varepsilon_0 \varepsilon_r} $$

where $\varepsilon_0$ is the permittivity of air or vacuum ($8.854 \times 10^{-12}$ F/m), $\varepsilon_r$ is the relative permittivity of insulator and $\rho$ is the volume density of charges. The Poisson’s equation shown in (3) can be established from equation (1) and equation (2)
\[ \nabla^2 V = -\frac{\rho}{\varepsilon_0 \varepsilon_r} \]  
\[ (3) \]

As the charge \( \rho = 0 \), the Poisson’s equation can be converted into Laplace’s equation as follows

\[ \nabla^2 V = 0 \]  
\[ (4) \]

For 2D axial symmetry problem, the distribution of potential is dependent upon the coordinate \( 0 \). Thus, the equation (4) is rewritten as follows

\[ \nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{\partial V}{\partial z} \right) = 0 \]  
\[ (5) \]

The FEM method reported in references [8-9] is used to solve equation (5) as all boundary conditions are known. An open domain is applied to the point-plane gap problem, i.e. the electric field is zero at infinity. For simplification of simulation, the outermost boundaries are at infinity. Due to these assumptions, the conditions of boundary are set as bellows.

\( V = V_{\text{applied}} \) on the point electrode (high voltage); \( V = 0 \) on the plane electrode (ground); \( nD = 0 \) on outermost boundaries

Figure 10 shows the typical mesh of one case of simulation with elements of triangles. The density of elements is higher while its size is smaller for regions around electrodes and streamer branches. Similar results were observed for other cases.

\[ \text{Figure 10. The mesh of the model for simulation of shielding effect } (\phi = 75 \text{ mm}, \alpha = 5^\circ) . \]

4. RESULTS AND DISCUSSION

4.1. The electric field distribution in the electrode gap

Figure 11 shows the distribution of electric field around streamers at 210 kV derived from simulation results with mesh parameters shown in Table 1. The electric field reaches the maximum value \( (E_t) \) at the surface of streamer envelope in the direction of gap axis, and decreases with an increase in distance away from a streamer region. From Fig. 11, \( E_t \) is

62
determined and plotted with increasing streamer extension as presented in Fig. 12. It is observed that $E_t$ reduces gradually with increasing streamer length due to an increase in diameter of streamer envelope until it reaches the minimum value at about 60% of gap crossing. Then, $E_t$ increases again because of the approaching of streamers to the plane electrode. Similar results were reported in the 2nd mode and 3rd mode streamers by other researchers \[1, 2, 4\]. Apparently, $E_t$ obtains the value of about 8.3 MV/cm, which is higher than the electric field of about 7 MV/cm at the tip of the point electrode, for the 4th mode streamers (Fig. 4a) and drops to about 0.16 MV/cm for the 3rd mode streamers (Fig. 4b) and reduces to the minimum value of 0.1 MV/cm for the 2nd mode streamers (Fig. 4c). Fig. 12 also shows the propagation velocity of streamers exhibited in Fig. 4. It seems that there is a correlation between the velocity and the electric field $E_t$ during streamer propagation. Streamers with high electric field at their tips propagate with high velocity, and vice versa. With the electric field of about 8.3 MV/cm at their tips, streamer velocity reaches the value of about 45 km/s. However, when the electric field at streamer tips drops to approximately 0.1-0.2 MV/cm, the streamer velocity reduces to 2-4 km/s. Therefore, it is inferred that if the electric field at streamer tips exceeds a value of about 8.3 MV/cm, streamers will travel at high speed of approximately 45 km/s over the entire electrode gap.

![Figure 11. Plots of electric field. The letter symbols referred to streamer images shown in Fig. 4.](image)

**Table 1. Mesh statistics (210 kV).**

<table>
<thead>
<tr>
<th>No</th>
<th>Parameters</th>
<th>Value (Fig. 11a)</th>
<th>Value (Fig. 11b)</th>
<th>Value (Fig. 11c)</th>
<th>Value (Fig. 11d)</th>
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<td>4970</td>
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<td>0.07562</td>
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<td>0.8159</td>
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<td>$4.325 \times 10^{-6}$</td>
<td>$4.325 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Figure 13 shows the surface plots of the electric field for the 4th mode streamers at 270 kV. Again, the electric field gets the maximum value at the streamer tips. From these plots, the
maximum electric field $E_t$ was obtained, and $E_t$ versus streamer growth is shown in Fig. 14. It is observed that $E_t$ gradually increases from 8 MV/cm to 25.5 MV/cm when streamers propagate across the electrode gap distance with the speed of about 100 km/s at 270 kV. The growth of streamer channels leads to a phenomenon that resembles the extension of the point electrode resulting in electrode gap reduction and thus an increase in $E_t$. The high magnitude of $E_t$ (8 - 25.5 MV/cm) could be used to explain why the 4th mode streamers (Fig. 5) propagate with very high velocity ($\sim$ 100 km/s). Compared to the value of $E_t$ in Fig. 12, it was observed that if $E_t$ increase to the value of about 10 MV/cm after initiation, streamers will keep travel with high velocity. Otherwise, streamers will propagate with a decrease in velocity. Thus, the critical value of approximately 10 MV/cm can be considered as a threshold value to convert low mode streamers into fast mode streamers. However, it is aware that this threshold value is estimated without regard to the existence of charges surrounding the tips of branches. Thus, the real value of the threshold electric field at the channel tips could be lower close to the tips and could increase further away from the tips. The mesh parameters for FEM simulation of this case is presented in Table 2.

**Figure 12.** Velocity versus electric field at the channel tips of streamers at 210 kV.

**Figure 13.** Distribution of the electric field around streamer channel tip at 270 kV.
Figure 14. Channel tip field $E_t$ versus streamer growth at 270 kV.

Table 2. Mesh statistics (270 kV).

<table>
<thead>
<tr>
<th>No</th>
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<th>Value $(l = 60 \text{ mm})$</th>
<th>Value $(l = 78 \text{ mm})$</th>
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<tr>
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<td>Average element quality</td>
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<td>0.8219</td>
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</tr>
<tr>
<td>4</td>
<td>Element area ratio</td>
<td>$4.325 \times 10^{-6}$</td>
<td>$4.325 \times 10^{-6}$</td>
<td>$4.325 \times 10^{-6}$</td>
<td>$4.325 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

4.2. The influence of the shielding effect on electric field at channel tips of streamers

Figure 15 shows some typical simulation results for the spherical model ($\phi = 75 \text{ mm}$). The modelling results shows that the maximum electric field ($E_t$) was found at the main channel tip and edges of hollow cones. However, $E_t$ at the tip of main channel is much higher than that of edges of hollow cones. Outside the channel tip and cone edges, the electric field significantly reduces. From simulation results with varying the value of angle $\alpha$, $E_t$ is determined and plotted as shown in Fig. 16. It is found that $E_t$ significantly increases with less branching, i.e. higher angle $\alpha$ between channels, and become saturated with $\alpha$ of about $30^\circ$ and $60^\circ$ for streamer envelope diameter of 75 mm and 40 mm, respectively. This means that an increase in streamer branching raises the shielding effect resulting in lower $E_t$ and vice versa. The similar results are obtained between two cases. However, $E_t$ of the bigger diameter of streamer envelope ($\phi = 75 \text{ mm}$) with higher branching degree still higher than that of the smaller diameter ($\phi = 40 \text{ mm}$) with lower degree of branching. This indicates that the influence of the shielding effect on $E_t$ possibly reduces as streamers approach the plane electrode. The mesh parameters for FEM simulation of this case is presented in Table 3.
Figure 15. The distribution of electric field for $\phi = 75$ mm.

Figure 16. The channel tip field versus angle between branches.
4.3. The relationship among electric field, branching and velocity

From Fig. 4, Fig. 5, Fig. 12, Fig. 14 and Fig. 16, it is observed that the 2nd mode streamers consisting of numerous filamentary branches propagate with the velocity of about 1-2 km/s, and the electric field at the channel tips of streamers is estimated to be about 0.13-0.19 MV/cm. It is also obtained that the 3rd mode streamers propagating with velocity of about 4-10 km/s has the electric field at their tips of about 0.2 MV/cm. The 4th mode streamers with few branches travel with velocity of 50 km/s-100 km/s and reach the estimated field at the streamer tips of about 8-25.5 MV/cm. This indicates that more branching, which is manifested with high number of branches, is associated with low velocity (1-2 km/s) and low electric field (~ 0.2 MV/cm) at the streamer channel tips and vice versa. Therefore, the relationship between branching and velocity of positive streamers is suggested as follows. Streamers initiating with the speed of about 1-2 km/s allow the development of branches, i.e. more branching. Due to branching, the macroscopic field of streamers becomes lower leading to a reduction in streamer velocity. By contrast, when streamers start with the high velocity (~ 50 km/s), the chance for streamer branches to develop is low. Therefore, the electric field in front of the dominating branches raises greatly, which further increases the speed of streamers. The relationship among velocity, branching and electric field is summarized as shown in Fig. 17. This suggested hypothesis is also supported by experimental results that streamer propagation across the electrode gap was observed to be controlled by the macroscopic electric field of streamers [5], and guiding tubes that suppressed branching accelerated streamers [4, 10].

![Diagram of the relationship between branching and velocity of streamers](image-url)
4. CONCLUSIONS

Simulation of electric field distribution for positive streamers during propagation and the influence of the shielding effect on the electric field were performed. The simulation results show that the electric field reduces with streamer extension and reaches the minimum value at the position of about 60% of the electrode gap before increases again due to streamer proximity to the plane electrode. The channel tip field of streamers at the 2nd mode is determined to be about 0.1 MV/cm while 8.3 MV/cm was received for the 4th mode streamers. It was also observed that the shielding effect formed by streamer branching greatly reduces the electric field at streamer channel tips. The shielding effect reduces with increasing the angle α between channels and has a tendency of saturation at the angle α of about 60° and 30° for 40 mm and 75 mm of diameters of streamer envelope, respectively. The hypothesis on the relationship among electric field, velocity and branching of streamers is proposed as follows. If starting with high electric field at the tips (~10 MV/cm), streamers will propagate with very high velocity which results in less branching and thus high electric field (~10 MV/cm) which further raises the velocity, and vice versa. However, the accuracy of the hypothesis should be further checked with simulation results from higher applied voltage, e.g. 540 kV, in next study.

REFERENCES
