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OPTIMIZATION OF LONG-RANGE TRAJECTORY FOR AN UNPOWERED FLIGHT VEHICLE

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Abstract. This report presents problems of optimization of long-range trajectory for an unpowered flight vehicle at subsonic and transonic speed. The results may be recommended to have a new long range trajectory. The optimization problem is solved by numerical experiments while the normal load factor (normal acceleration) is used as optimization variables with compliance to flight constraints. The focus problem of this study is the investigation of the possibility of trajectory expansion according to the criteria of the maximum range in the first stage of the trajectory.

Keywords: long-range trajectory, maximum range, normal load factor, simulation.

Classification numbers: 5.6.2, 5.10.2.

1. INTRODUCTION

The problems of long-range trajectory for unpowered flight vehicles (UFVs) are the basic theoretical issues in order to be applied to airplanes and unmanned aerial vehicles (UAVs) with the engine fails or aerial gliding bodies [1] or smart bombs. Achieving maximum range is very important for UFVs, ensuring the safe flight and landing when the engine malfunctions and it's particularly important for possibility of combat aircrafts to drop smart bombs from a safe long distance to the enemy target. Regarding the problems of long-range trajectory optimization for UFVs, two basic methods are often used including direct and indirect methods. Studies on this issue can be listed as follows: Vinh [2] proposed an optimal solution for unpowered subsonic flight vehicle in the horizontal plane by applying Pontryagin's maximum principle. Lu [3] used an indirect method to study problems of optimal trajectory and obtained the path angle control law. As is known that the advantages of indirect methods are high accuracy, but the disadvantage of this method is the complex calculation and the difficulty in choosing the initial conditions. Nowadays, thanks to the advanced development of computer technology, direct methods are more used to calculate reference trajectories. For example, Rao's group [4] researched and developed a software, named GPOPS. This software has been programmed to solve optimal control problems for trajectory based on the application of numerical optimization method. Recently, Zhang *et al*. [5] also used a numerical method to provide an approximate optimal guidance scheme for UFVs at subsonic speeds based on the relationship between the maximum lift-to-drag ratio $(C_y/C_x)_{max}$ and dynamic pressure to obtain a new optimal trajectory. However, all previous studies have not mentioned the optimization problems of the first stage of trajectory that could be impacted to create a new maximum range. In this report, our study has obtained new results for the first phase, which is the new maximum range for UFV based on numerical experiment by piecewise linear approximation for the normal load factor variable (n_y). Then the combination between the first and second flight stages is simulated.

2. OPTIMAL TRAJECTORY AND CONSTRAINTS

2.1. Mathematical description of UFVs motion

The forces acting on the center of mass of UFV in the vertical plane are shown in Figure 1.

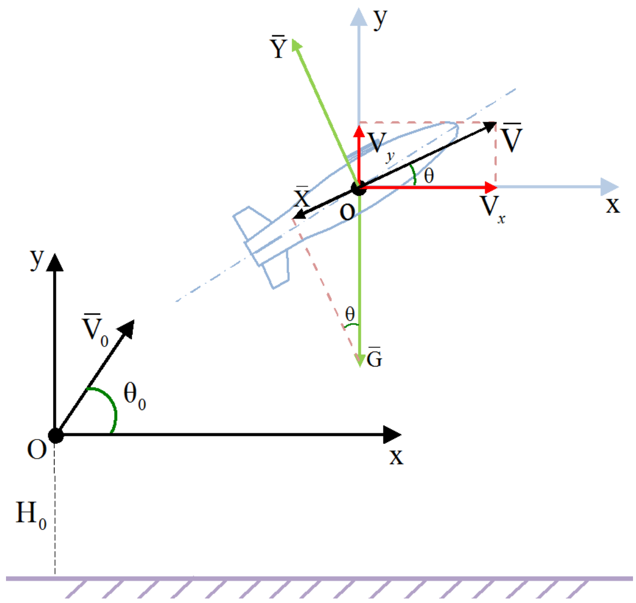

Figure 1. Diagram of forces acting on the center of mass of UFVs.

According to [5] and [6], the equations describing the motion of UFV in the vertical plane have the following form:

$$\dot{V} = -\frac{X + G\sin\theta}{m} \tag{1}$$

$$\dot{\theta} = \frac{Y - G\cos\theta}{mV} \tag{2}$$

$$V_x = \dot{D} = V\cos\theta \tag{3}$$

$$V_y = \dot{H} = V\sin\theta \tag{4}$$

where *H*, *V*, *θ*, *D*, *Y*, *X*, *G* and *m* are the altitude, velocity, flight path angle, range, lift, drag, gravitational acceleration and mass, accordingly. The lift, drag and gravity are defined as follows:

$$Y = C_y \frac{\rho V^2}{2} S_{ref}\,,\ X = C_x \frac{\rho V^2}{2} S_{ref}\,,\ G = mg \tag{5}$$

where C_y, C_x, S_{ref}, ρ and g are the lift coefficient, drag coefficient, reference area, density of atmosphere ($\rho = f(H)$) and gravitational acceleration, accordingly.

As mentioned in the introduction, the content of the study focuses on optimizing the first stage of the trajectory by choosing the optimal values of $n_y(\mathrm{t})$. Therefore, the variable n_y needs to be represented in the motion equations of UFVs. By definition $n_y = Y/G$, so from formula (2) we have:

$$\dot{\theta} = \frac{g\left(n_y - \cos\theta\right)}{V}. \tag{6}$$

2.2. Maximum range trajectory scheme for UFVs and constraints

2.1.1. Maximum range trajectory scheme for UFVs

Optimization of long-range trajectory is essentially a problem of maximum range trajectory. In general form, the maximum range trajectory scheme for UFVs can be described by three stages as shown in Figure 2. The first stage (P_0P_1) is trajectory extension section. The second stage (P_1P_2) is stage of steady gliding according to the maximum of the lift-to-drag ratio and the optimal flight path angle. The third stage (P_2P_3) is the trajectory of approaching to the target that depends on each special requirements, and is not considered in this study.

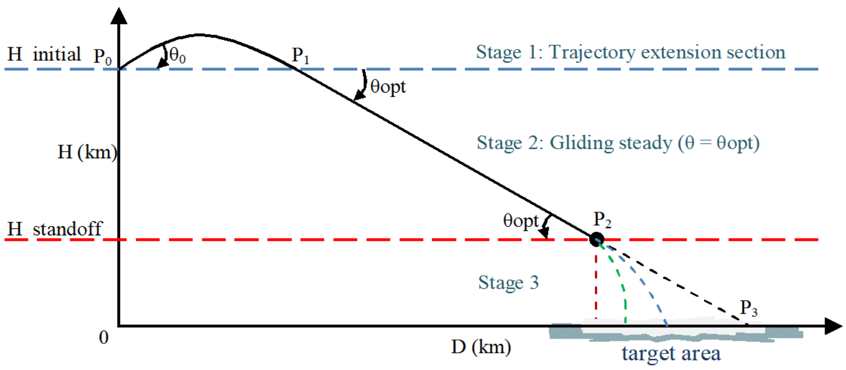

Figure 2. Maximum range trajectory scheme for UFVs.

2.1.2. Constraints

- *Velocity constraint*

The minimum velocity of UFV is always the most important reference parameter of the trajectory, which is a necessary constraint for UFVs. It means that the real velocity must always be more than the minimum velocity at all times, which is a basic requirement to ensure maneuverability of flight vehicle and avoids the stall of the UFVs. Minimum velocity is defined as follows:

$$V_{\min} = \sqrt{\frac{2Gn_y}{C_{y\max}\rho S_{ref}}} \tag{7}$$

where $C_{y\max}$ is the maximum lift coefficient of the flight vehicles, the G and S_{ref} are constant values. The rest, density of atmosphere (ρ) is altitude dependent function, n_y is a variable that is defined by piecewise linear approximation.

- *Flight path angle constraint the second flight stage*

Derived from the motion characteristics of the second flight stage, during the flight in order to achieve a maximum range needed to maintain the maximum lift-to-drag ratio, i.e. K= $(C_y / C_x)_{\max}$. This is also equivalent to maintaining a constant optimal flight path angle throughout the flight with stable velocity, it means $V = const$, $\theta = \theta_{opt} = const$. Then from equation (1) and (2) we have $X = -G\sin\theta$, $Y = G\cos\theta$, and the optimal flight path angle is defined as:

$$\theta_{opt} = \text{-}arctg\,\frac{1}{K_{\max}} \tag{8}$$

The coefficients K and $K_{\max}$ are determined according to formula:

$$K = \frac{C_y}{C_{x0} + AC_y^2} \quad \text{and} \quad K_{\max} = \frac{1}{2\sqrt{AC_{x0}}} \tag{9}$$

where C_{x0} and A are functions of Mach.

2.3. Dependency relationship of maximum range according to rules of normal load factor

The equation describing the motion of UFVs indicates that the range is a function that can be written in the form:

$$D = f(H, V, \theta, n_y) \tag{10}$$

where H, V, θ are functions of time and they are affected by value of regulating function $n_y(t)$. In contrast, $n_y(t)$ is the independent function (regulating parameter). Consequently, with each specific value of the initial condition H_0, V_0, θ_0, the maximum range ($D_{\max}$) only depends on the value of the regulating function $n_y(t)$.

As mentioned above, this study considers UFVs at subsonic and transonic speeds, where there is a complex aerodynamic change, so it is difficult to give an analytical formula for $D_{\max}$ in this case. In order to creation the maximum range trajectory that is combined between the first and second stages, we proposed a direct optimization solution which is optimized by a variable node of piecewise linear approximation of the function $n_y(t)$. That is to implement numerical experiments for choosing the node value of variable n_{yadj} (Figure 3).

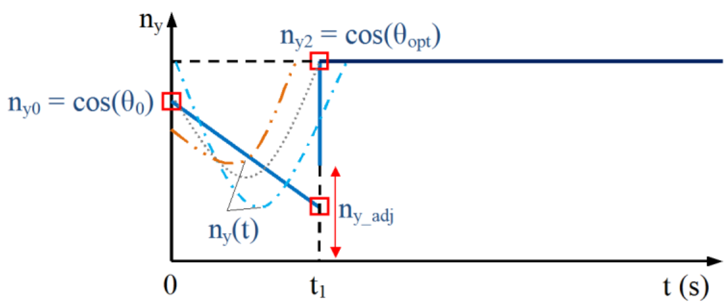

Figure 3. Choosing the node value of variable normal load factor approximation.

According to this way of choosing the node value of variable n_y in Figure 3, $D_{\max}$ can be written in the following general form:

$$D_{\max} = f(n_{y0}, n_{y\_adj}, n_{y2}) \tag{11}$$

where n_{y0}, n_{y_adj} and n_{y2} are node values at the moment $t = t_0 = 0$, $t = t_1$ and $t > t_1$. Node point n_{y0} is the initial value of the normal load factor variable which is defined $n_{y0} = \cos(\theta_0)$. And n_{y_adj} is the main node to determine the maximum range trajectory. According to the numerical experiment, we have found the value n_{y_adj} has a value between 0 and $\cos(\theta_0)$, i.e. $n_{y\_adj} \in (0;\ \cos(\theta_0))$. In addition, during the time the second flight stage of UFV ($t_1 < t \le t_2$), variable n_{y2} is imposed constant value and $n_{y2} = \cos(\theta_{opt})$.

Therefore, the approximation of function $n_y(t)$ is defined as follows:

$$n_y = \begin{cases} \dfrac{n_{y\_adj} - n_{y0}}{t_1} t + n_{y0} & 0 \le t \le t_1 \\ & \text{when} \\ \cos(\theta_{opt}) & t_2 \ge t > t_1 \end{cases} . \tag{12}$$

4. SIMULATION RESULTS AND DISCUSSION

The trajectories of UFV are simulated in the vertical plane. The system of equations are solved by the numerical method (the fourth-order Runge-Kutta method) with the specified initial conditions:

$$V = V_0;\ \theta = \theta_0;\ x = x_0 = 0;\ y = y_0 = H_0.$$

Conditions used in simulation. Motion of UFV is considered in the international standard atmospheric environment (ISA). Initial altitude is from 9 to 12 km, and initial velocity is transonic speed from 0.8 to 0.9 Mach.

Characteristics of the hypothetical UFV used in the simulation were shown in Table 1 and Figure 4.

Table 1. Characteristics of the hypothetical UFV.

m (kg)	$S_{ref}\,(m^2)$	C_{ymax}	$\theta_{opt}\,(^0)$
250	0.5	1.5	-8.5

And aerodynamic parameters include C_{x0} and A, which are shown in Figure 4.

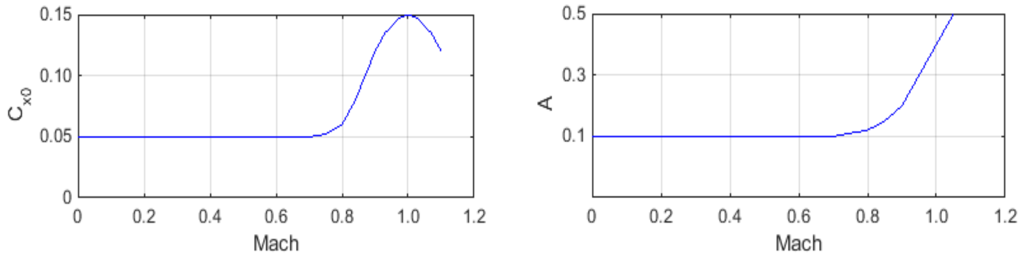

Figure 4. C_{x0} (a) and *A* (b).

Simulation results

Table 2. The trajectory parameters.

H_0 (km)	V_0 (m/s^2)	θ_0 $(^0)$	n_{y_adj}	t_1 (s)	D_{max} (m) *H standoff* = 4000(m)
9	270	5.0	0.8	48	42345.5
10	-	4.5	-	-	47826.7
11	-	4.0	-	-	53867.2
12	-	3.5	-	-	58301.7

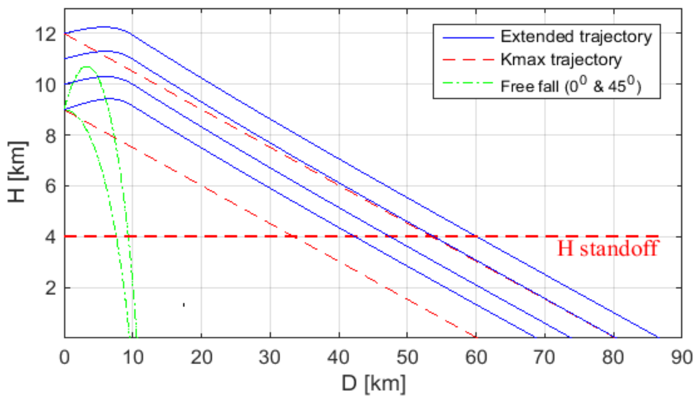

Figure 5. Results of simulation of the different trajectories.

The trajectory parameters are shown in Table 2, and simulation results of the different trajectories are shown in Figure 5.

The numerical experiments show that the trajectory extension section at the first stage made a new maximum range for UFVs which is larger than the old range about 7 to 10 km (Figure 5).

As shown in Table 2, the value of $n_{y\_adj} = 0.8$ is used. In fact, we investigated the hypothetical UFV with many different values of n_{y_adj} and found that: In the case of $n_{y\_adj} < 0.8$, the range of UFV is reduced because the curvature of the trajectory is quite large at the first stage. Conversely, the constraints are violated when the variable n_{y_adj} reaches 0.84 in the case of $n_{y\_adj} > 0.8$.

- *The motion parameters of hypothetical UFV*

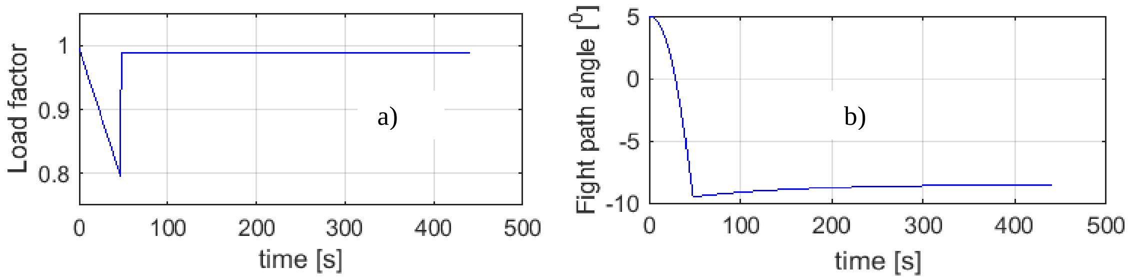

Figure 6. Normal load factor (a) and flight path angle (b).

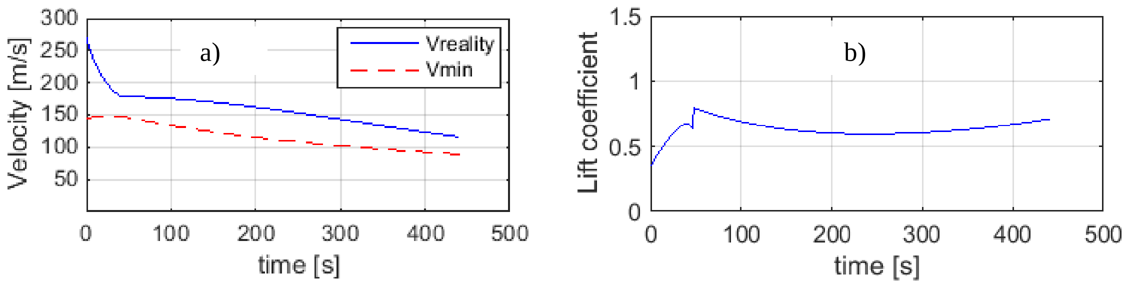

Figure 7. Velocity (a) and lift coefficient (b).

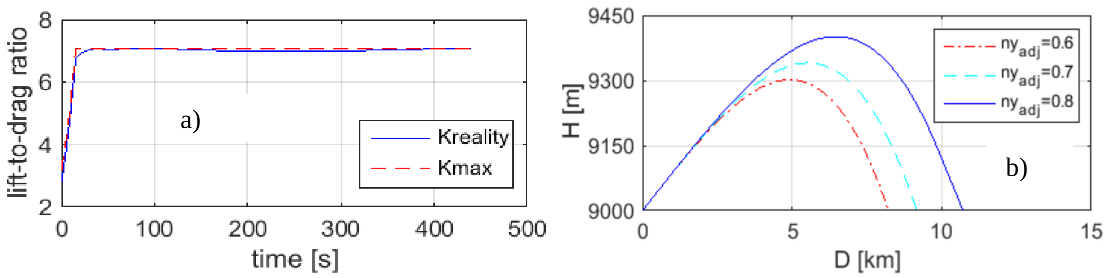

Figure 8. Lift-to-drag ratio (a) and trajectory expansion section (b).

Figures 6, 7 and 8 show the value of the motion parameters of UFV with initial conditions such as altitude 9000 (m), velocity 270 (m/s), flight path angle 5 (^{0}). The obtained results satisfy the constraints mentioned in the Section 2.2. The results shown in Figure 6.a and 6.b present the response of the flight path angle that depends on the impact of variable n_y. Figures 7 and 8.a show that the value of velocity and lift coefficient are in the allowable range, and lift-to-drag ratio K is always close to the value of K_{max}. Especially, Figure 8.b shows that the node values have an essential impact on the expansion ranges in the first stage of the trajectory.

5. CONCLUSION

Based on the results of numerical experiments, this study has proposed a new long-range trajectory for unpowered subsonic/transonic guidance flight vehicle. The content of the study combined two flight stages including the trajectory expansion stage and the stable gliding stage to create a new long-range trajectory for UFVs. Using the node values of piecewise linear approximation of the function of normal load factor $n_y(t)$ as optimisation variables and find the new function of normal load factor to solve the problems of optimizing long-range trajectory for UFVs. This solution can be flexibly applied to different independent initial conditions, as well as easily applicable to calculating for the similar UFVs.

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