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A MATHEMATICAL MODEL OF INTERIOR BALLISTICS FOR THE AMPHIBIOUS RIFLE WHEN FIRING UNDERWATER AND VALIDATION BY MEASUREMENT

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Abstract. The paper is focused on study of the interior ballistics model of amphibious rifle when firing underwater based on the standard interior ballistics of automatic rifle using gas operated principle. The presented mathematical model is validated and experimentally verified for the 5.56 mm underwater projectile fired from the 5.56 mm amphibious rifle. The result of this research can be applied to design the underwater ammunition, underwater rifle and amphibious rifle.

Keywords: amphibious rifle, interior ballistics, underwater rifle, underwater ammunition, underwater projectile.

Classification numbers: 5.4.2, 5.4.4.

1. INTRODUCTION

One of the most serious problems important in the amphibious rifle and the underwater projectile design is research of the interior ballistic processes [1]. Comparison with the standard interior ballistics of automatic rifle in air which used gas operated principle [2], the interior ballistics under water is very different. In this case, the biggest difference is that the projectile must be impacted of the water inside barrels while the viscosity of water is much more important than those of air.

When the projectile is inside the barrel, a small amount of water is located in the gap between the projectile and the barrel. Under the effect of gas pressure, the amount of water is also moving. Because the specific gravity of water is not as the same as the specific gravity of the projectile, so the velocity of the water is different to the velocity of the projectile. On the other hand, theoretical studies of fluid dynamics have shown that this water itself also has different speed along the surface of the projectile and the inner of barrel. In fact, the water volume in gap is very small in comparison with the entire volume of water in the barrel bore. Therefore, for simplicity of calculation, this water can be considered as moving at the same velocity as the projectile. Thus, in the process of projectile movement through the barrel bore, the projectile's weight is calculated as the sum of the projectile weight and the actual weight of water in the barrel bore at the time. This weight will vary according to the distance of projectile motion.

In addition, the projectile was impacted of water pressure in the process of firing. This pressure consists of hydrostatic pressure and dynamic pressure. The dynamic pressure increases with quadrat of the projectile velocity creating drag force for projectile.

The above characteristics indicates that it is difficult to calculate the interior ballistics when firing underwater by the model of the standard interior ballistics in air. To solve this problem, the paper presents a developed mathematical model for investigation of the interior ballistics of the amphibious rifle firing the underwater ammunition. This mathematical model is derived from the standard interior ballistics in air. Besides, the developed mathematical model has been validated and experimentally verified.

2. MATHEMATICAL MODEL OF INTERIOR BALLISTICS FOR THE AMPHIBIOUS RIFLE WHEN FIRING UNDER WATER AMMUNITION

2.1. Basic assumptions

In order to build the mathematical model of interior ballistics for the amphibious rifle when firing under water ammunition, the assumptions are used as follows:

- The burning of the propellant according to geometric rules.
- Because the water is in the gap between projectile and bore, the gas passing through this gap is neglected and the water in the gap is not evaporated by the hot gases.
- The projectile's weight is calculated by the total actual weight of projectile and the weight of water ahead the projectile.
- Velocity of the water in front of projectile is calculated by the velocity of the projectile motion in bore.
- Ignoring the heat loss inside the barrel.
- Water is incompressible.
- The projectile can rotate about the axis of barrel because the diameter of projectile under water is smaller than diameter of barrel bore.
- Conditions for derivation of the interior ballistic process equation of underwater rifle are: the barrel is placed horizontally, and water is in static state (Fig.1).

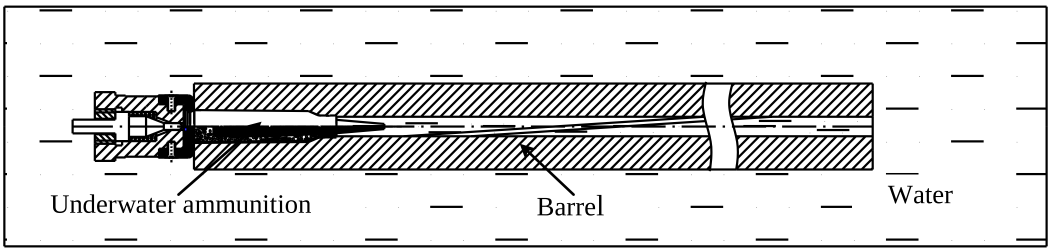

Figure 1. The brief models of underwater projectile move in the barrel.

- According to the above characteristics, the process of moving projectiles in the barrel can be divided into two phases (Fig. 2):

Phase I. Starting the projectile started to move until the tip of projectile to the cross section of the muzzle. In these phases, the projectile's weight is calculated by the total actual weight of the projectile and the weight of water in the barrel.

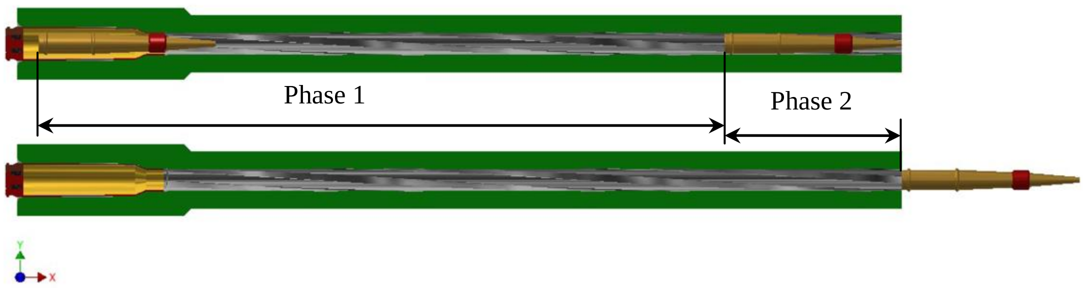

Figure 2. Schematic of the process of the underwater projectile move in the barrel.

Phase II. It starts when the projectile tip leaves the muzzle cross section and ends when the projectile bottom reaches the muzzle cross section. In this phase, the actual projectile's weight is considered only.

2.2. The system of differential equations for interior ballistic of the amphibious rifle when firing under water ammunition

In accordance with classical interior ballistics theory, the interior ballistics equations of automatic weapon when firing in air is [4]:

$$\begin{cases} \psi = \chi z(1+\lambda z) \\ \dfrac{dz}{dt} = \dfrac{p}{I_k} \\ Sp(l_\psi + l) = f\omega\psi - \dfrac{\theta}{2}\varphi m v \\ l_\psi = l_0\left(1 - \dfrac{\Delta}{\delta} - \Delta\left(\alpha - \dfrac{1}{\delta}\right)\psi\right) \\ v = \dfrac{dl}{dt} \end{cases} \tag{2.1}$$

where: ψ - the fraction of burned powder; χ, λ - the shape coefficient of powder; z - the relative thickness of burned powder; p - the average pressure of power gas in the barrel; I_k - the dynamite quantity coefficient; S - the cross section of barrel; l_ψ - the fictive length of free volume of charge chamber; l - the displacement of projectile inside of barrel; f - the force of powder; ω - the mass of powder charge; $\theta = k-1$, k - adiabatic constant; φ - the coefficient of projectile fictitious mass; m - the projectile mass; v - the velocity of projectile; Δ - the loading density of powder; δ - the powder density; α - the co-volume of powder.

The system of differential equations for interior ballistic of the amphibious rifle when firing under water ammunition is made by using the burning rate law equation, the rate of gas forming which as same in air as Eq. (2.2) (2.3) and developed equation of projectile translation motion and the fundamental equation of interior ballistics.

$$\frac{dz}{dt} = \frac{p}{I_k} \tag{2.2}$$

$$\psi = \chi z(1 + \lambda z) \tag{2.3}$$

2.2.1 The equation of projectile translation motion in the barrel bore when firing underwater

In order to describe the underwater projectile motion in the barrel, the 2D Descartes coordinates system has been established at the center of bottom gas chamber O as shown in Fig. 3.

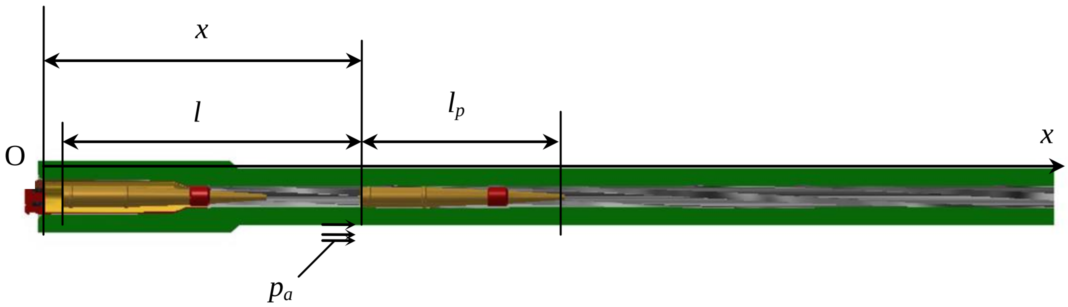

Figure 3. Coordinate system to study underwater interior ballistics.

Where: x - axis represent the horizontal axis of the projectile symmetry. It also is the horizontal axis of the barrel; l_b - the length of barrel; l_p - the length of underwater projectile; l - the displacement of projectile inside of barrel; p_a - the pressure behind the projectile bottom.

According to the third assumption and Newton's Second Law, we can describe the motion of underwater projectile in the barrel as bellow:

$$\begin{cases} \dfrac{dl}{dt} = v \\ m_t \dfrac{dv}{dt} = Sp_a - F_d \end{cases} \tag{2.4}$$

where: m_t - the total mass of underwater projectile and water in the barrel; m_p - the underwater projectile mass; m_w - the water mass in the barrel and it can be calculated by

$$m_w = \rho S\left(l_b - l_p - l\right) \tag{2.5}$$

ρ - the fluid density; F_d - the total drag force acting on the noise of underwater projectile when moving in the barrel.

The total drag force F_d acting on the noise of projectile consists of pressure drag force and friction drag force as bellow [5]:

$$F_d = F_p + F_f \tag{2.6}$$

where: F_p is the pressure drag force; F_f is the friction drag force.

The pressure drag force F_p include the drag force caused by hydrostatic pressure and the drag force caused by hydraulic pressure [6]. So, it can be calculated by:

$$F_p = \left(p_{atm} + \rho g h\right)S + \frac{1}{2}\rho v^2 S \tag{2.7}$$

where: p_{atm} - the atmospheric pressure; g - gravitational acceleration; h - the depth of firing.

The friction drag force F_f is given by formula [7]:

$$F_f = \frac{1}{2}C_f \rho v^2 \pi d\left(l_b - l_p - l\right) \tag{2.8}$$

where: d - the diameter of bore; C_f - the skin friction coefficient. It depends on the Reynolds number Re and is calculated according to relations introduced in Table 1 [8].

Table 1. The dependence of skin friction coefficient on the Reynolds number.

Reynolds number (Re)	Skin friction coefficient (C_f)
$0 < \mathrm{Re} < 2300$	$C_f = \dfrac{64}{\mathrm{Re}}$
$2300 \le \mathrm{Re} < 4000$	$C_f = \dfrac{2.7}{\mathrm{Re}^{0.53}}$
$\mathrm{Re} \ge 4000$	$C_f = \dfrac{1}{\left[1.8\times\log\left(\mathrm{Re}\right) - 1.5\right]^2}$

In Tab. 1, the Reynolds number is given by formula $R_e = \dfrac{vd}{\mu}$, where μ is the kinematic viscosity of the fluid.

From Eq. (2.4) to Eq. (2.8), we can rewrite the system of equations describing the motion of the underwater projectile in bore as bellow:

$$\begin{cases} \dfrac{dl}{dt} = v \\ \left[m_t + \rho S\left(l_b - l_p - l\right)\right]\dfrac{dv}{dt} = Sp_a - \left\{\left[\left(p_{atm} + \rho g h\right)S + \dfrac{1}{2}\rho v^2 S\right] + \dfrac{1}{2}C_f \rho v^2 \pi d\left(l_b - l_p - l\right)\right\} \end{cases} \tag{2.9}$$

or

$$\begin{cases} \dfrac{dl}{dt} = v \\ \dfrac{dv}{dt} = \dfrac{Sp_a}{m_t \varphi_H} \end{cases} \tag{2.10}$$

where

$$\varphi_H = \frac{1}{1-\dfrac{\left(p_{atm}+\rho gh\right)S+\dfrac{1}{2}\rho v^2 S+\dfrac{1}{2}C_f \rho v^2 \pi d\left(l_b-l_p-l\right)}{Sp_a}} \tag{2.11}$$

In addition, depending on the phase of motion, the water mass in bore and the total drag force are changed. This change is shown in Tab. 2.

Then, we must determine the pressure behind the projectile bottom p_a. In accordance with classical interior ballistics theory, we can describe the pressure distribution at a distance x from the bottom of the cartridge chamber by Eq. (2.11) [9]. At the moment, the projectile bottom is in the position l and its acceleration is $\frac{dv}{dt}$.

$$\frac{1}{\rho_x}\frac{\partial p_x}{\partial x} = -\frac{x}{l_{cb}+l}\frac{dv}{dt} \tag{2.12}$$

where $\rho_x = \rho = \frac{\omega}{gS\left(l_{cb}+l\right)}$ with l_{cb} is the length of gas chamber.

Table 2. The change of the water mass in bore and the total drag force during projectile motion in bore.

Phase of motion	Total mass of underwater projectile and water	Total drag force
Phase I $\left[0 \le l \le \left(l_b-l_p\right)\right]$	$m_t = m_p + \rho S\left(l_b-l_p-l\right)$	$F_d = \left(p_{atm}+\rho gh\right)S+\frac{1}{2}\rho v^2 S$ $+\frac{1}{2}C_f \rho v^2 \pi d\left(l_b-l_p-l\right)$
Phase II $\left[\left(l_b-l_p\right) < l = l_b\right]$	$m_t = m_p$	$F_d = F_p = \left(p_{atm}+\rho gh\right)S+\frac{1}{2}\rho v^2 S$

From the Eq. (2.12) and the Eq. (2.4), we can rewrite Eq. (2.11) as bellow:

$$\frac{\partial p_x}{\partial x} = -\left(\frac{x}{l_{cb}+l}\right)\frac{dv}{dt} \tag{2.13}$$

So, substituting the Eq. (2.11) into the Eq. (2.13) we have formula as

$$\frac{\partial p_x}{\partial x} = -\frac{\omega}{\varphi_H g m_t}\frac{x}{\left(l_{cb}+l\right)_2} p_a \tag{2.14}$$

Integral Equation (2.14) from x to $l_{cb}+l$ we get the equation describing the pressure distribution as follows:

$$p_x = p_a\left[1+\frac{\omega}{2\varphi_H g m_t}\left(1-\frac{x^2}{\left(l_{cb}+l\right)^2}\right)\right] \tag{2.15}$$

Thus, we can determine the average pressure of power gas in the barrel p as

$$p = \frac{1}{l_{cb}+l}\int_0^{l_{bd}+l} p_x dx = p_a\left(1+\frac{1}{3}\frac{\omega}{\varphi_H g m_t}\right) \tag{2.16}$$

According to the Eq. (2.16), Eq. (2.11) and equation system (2.9), we can rewrite the system of equations describing the motion of underwater projectile in the barrel as bellow:

$$\begin{cases} \dfrac{dl}{dt} = v \\ \dfrac{dv}{dt} = \dfrac{S\dfrac{p}{\left(1+\dfrac{1}{3}\dfrac{\omega}{\varphi_H g m_t}\right)} - F_d}{m_t} \end{cases} \tag{2.17}$$

2.2.2. The energy conservation equation of interior ballistics for the amphibious rifle when firing under water ammunition

Based on the fundamental equation of interior ballistics in air [10], we can rewrite this equation in case firing underwater as bellow:

$$Sp\left(l_\psi + l\right) = f\omega\psi - \theta\sum_{i=1}^{n} W_i \tag{2.18}$$

where $\sum_{i=1}^{n} W_i$ is total energy conversion of gas and it is divided into 6 parts as follows:

- Energy pushes the underwater projectile move:

$$W_1 = \frac{1}{2} m_p v^2 \tag{2.19}$$

- Energy pushes the water in bore move:

$$W_2 = \frac{1}{2} m_t v^2 = \frac{1}{2}\rho\left(l_b - l_p - l\right)v^2 \tag{2.20}$$

- Energy to eject the water out of muzzle barrel:

$$W_3 = \int_0^l \frac{\rho v^2 S}{2} dl \tag{2.21}$$

- Energy to prevent the friction between water and bore:

$$W_4 = \int_0^l \frac{C_f \rho \pi d\left(l_b - l_p - l\right)v^2}{2} dl \tag{2.22}$$

- Energy to push the product of burn and powder not burned moving in the space after the bottom of the projectile:

$$W_5 = \frac{\omega v^2}{6} \tag{2.23}$$

- Energy to prevent the hydrostatic pressure at h depth:

$$W_6 = \left(p_{atm} + \rho g h\right) S l \tag{2.24}$$

Combining equations Eq. (2.2), Eq. (2.3), Eq. (2.17), Eq. (2.18), we build the system of differential equations for interior ballistic of the amphibious rifle when firing underwater ammunition as follows:

$$
\begin{cases}
\dfrac{dz}{dt} = \dfrac{p}{I_k} \\
\psi = \chi z\left(1+\lambda z\right) \\
\dfrac{dl}{dt} = v \\
\dfrac{dv}{dt} = \dfrac{S\dfrac{p}{\left(1+\dfrac{1}{3}\dfrac{\omega}{\varphi_H g m_t}\right)} - F_d}{m_t} \\
Sp\left(l_\psi + l\right) = f\omega\psi - \theta\sum_{i=1}^{6} W_i
\end{cases}
\tag{2.25}
$$

3. INTERIOR BALLISTIC CALCULATION

The mathematical model of interior ballistics built above is applied for the 5.56 mm underwater cartridge which is firing from the 5.56 mm amphibious rifle. The parameters of 5.56 mm under water cartridge is shown as in Fig. 4. In order to validate the mathematical model, we will calculate with the different barrel length, different projectile mass (different materials) and different powder mass. The cases of investigation are shown as in Tab. 3.

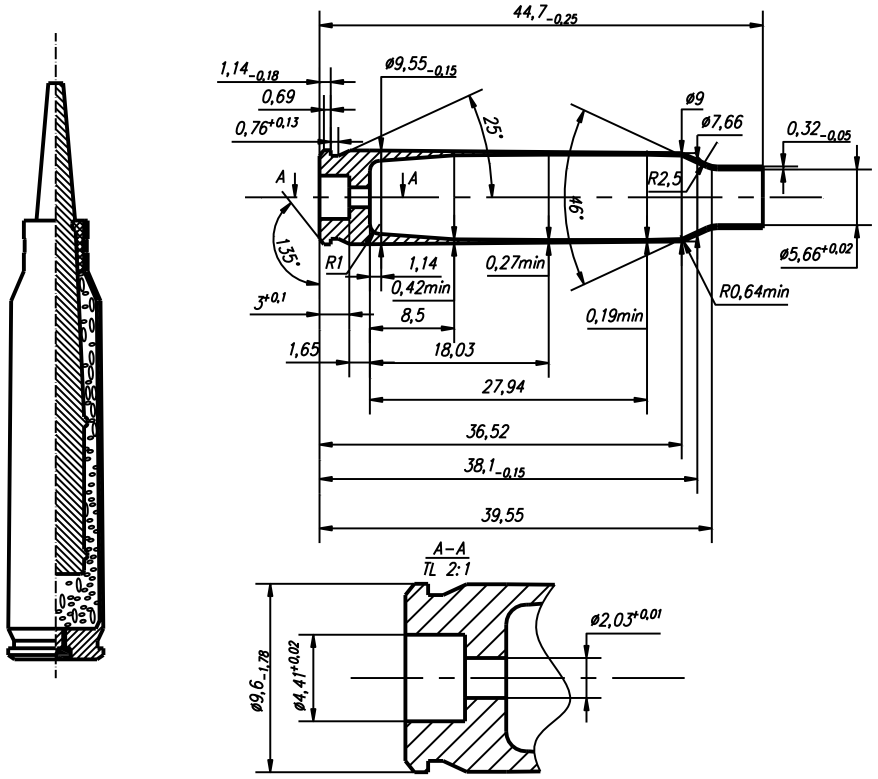

Figure 4. The parameters of 5.56 mm underwater cartridge.

Table 3. The cases of investigation.

Cases of investigation	Material of projectile	Mass of projectile (g)	Length of barrel (mm)	Mass of powder (g)	
Case 1	Bronze	6.8	376	Type A	0.5
				Type B	0.55
				Type C	0.6
				Type D	0.65
Case 2	Bronze	6.8	415	Type A	0.5
				Type B	0.55
				Type C	0.6
				Type D	0.65
Case 3	Tungsten carbide	13.7	376	Type A	0.5
				Type B	0.55
				Type C	0.6
				Type D	0.65
Case 4	Tungsten carbide	13.7	415	Type A	0.5
				Type B	0.55
				Type C	0.6
				Type D	0.65

The main input parameters to solve the mathematical model of interior ballistics are given in Tab. 4.

Table 4. The main input parameters to solve.

Notation	**Parameters**	**Value**
d	Caliber of gun	0.0556 dm
	Chamber volume	0.00165 dm^3
l_p	Length of projectile	50 mm
g	Acceleration of gravity	9.81 m/s^2
ρ	Density of water	1000 kg/m^3
h	Depth of the firing	1m
p_{atm}	Atmospheric pressure	101325 Pa
μ	Kinematic viscosity of the water	0.00089 Pa s

The system of differential equations for underwater interior ballistic (Eq. (2.25)) has been solved using the Runge-Kutta of the 4th order integration method and the MATLAB programming environment. Selected results of solution are presented in graphs from Fig. 5 to Fig. 8. The maximum of pressure and muzzle velocity are shown in Tab. 5.

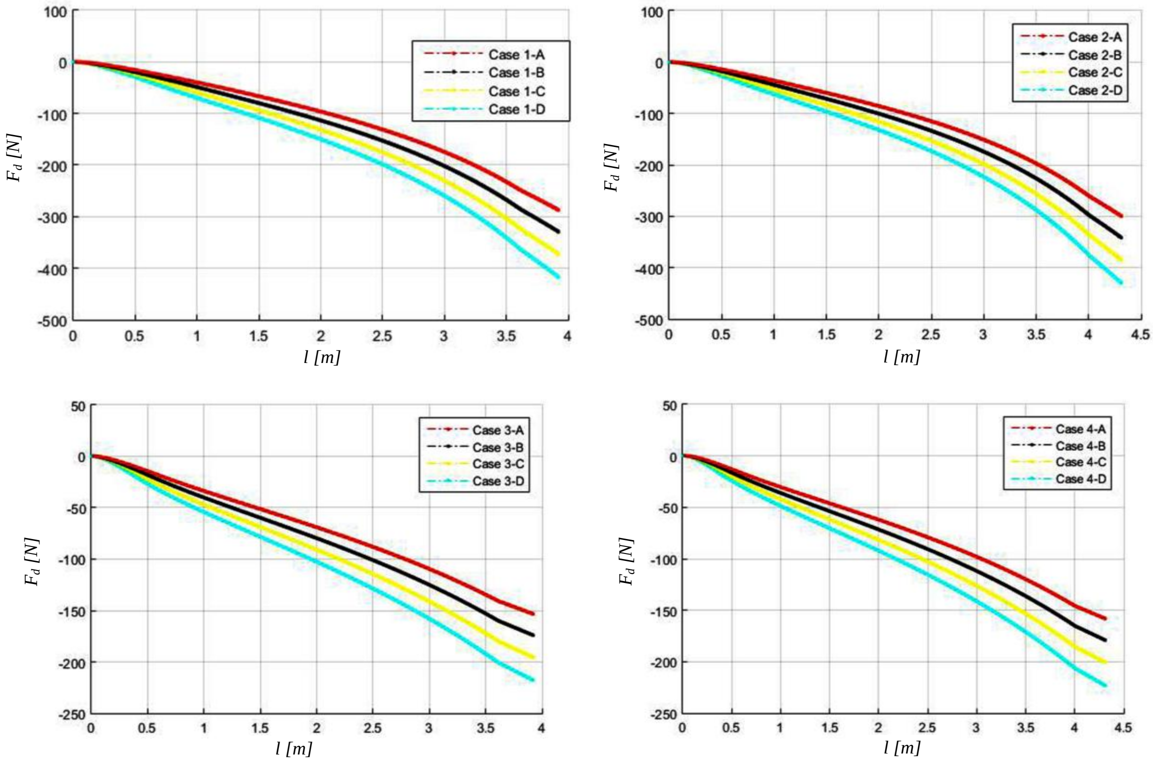

Figure 5. The total drag force vs. trajectory of projectile.

Table 5. The results of solution about the maximum of pressure and muzzle velocity

Cases of investigation		Maximum of pressure (MPa)	Muzzle velocity (m/s)
Case 1	Type A	158.3543	478.8050
	Type B	194.3549	512.6540
	Type C	236.8182	545.2861
	Type D	287.0130	577.1616
Case 2	Type A	166.1994	489.1870
	Type B	204.2270	522.4373
	Type C	249.1320	554.6195
	Type D	302.2899	586.0122
Case 3	Type A	212.2827	350.0405
	Type B	262.3105	372.8633
	Type C	321.7631	395.1657
	Type D	392.7289	417.0046
Case 4	Type A	219.5277	355.4646
	Type B	271.4436	378.1959
	Type C	333.1954	400.3838
	Type D	406.9934	422.2009

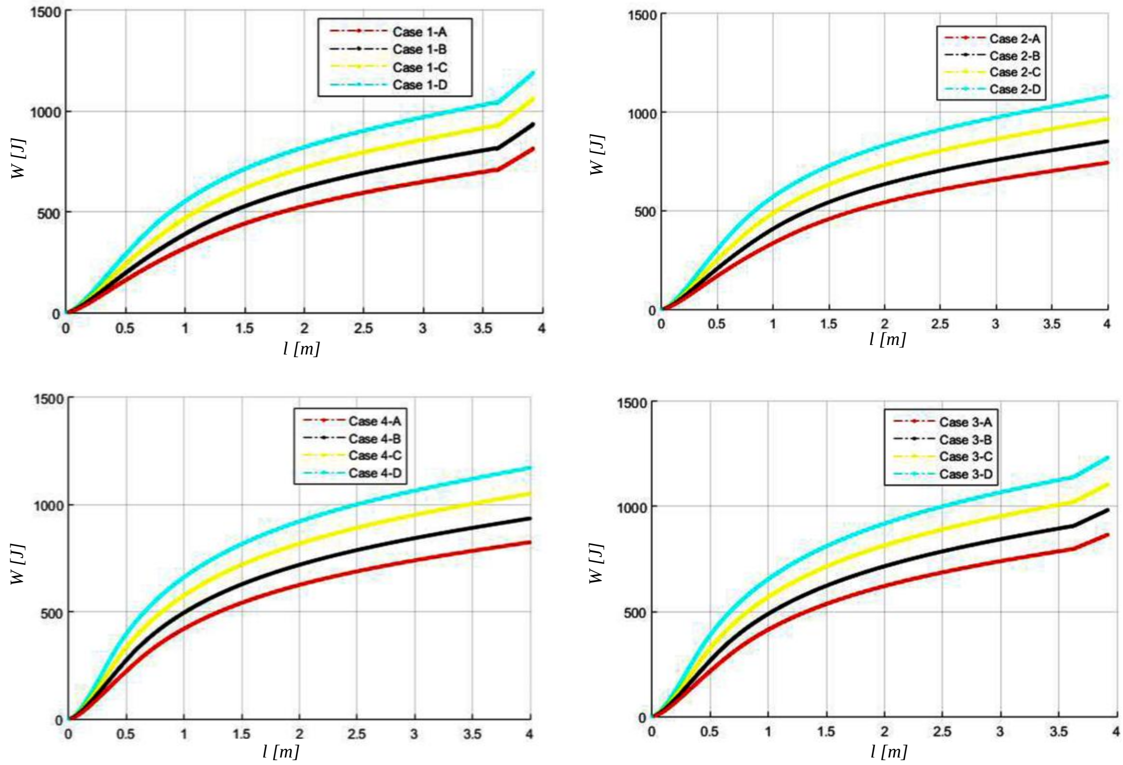

Figure 6. The total energy conversion vs. trajectory of projectile.

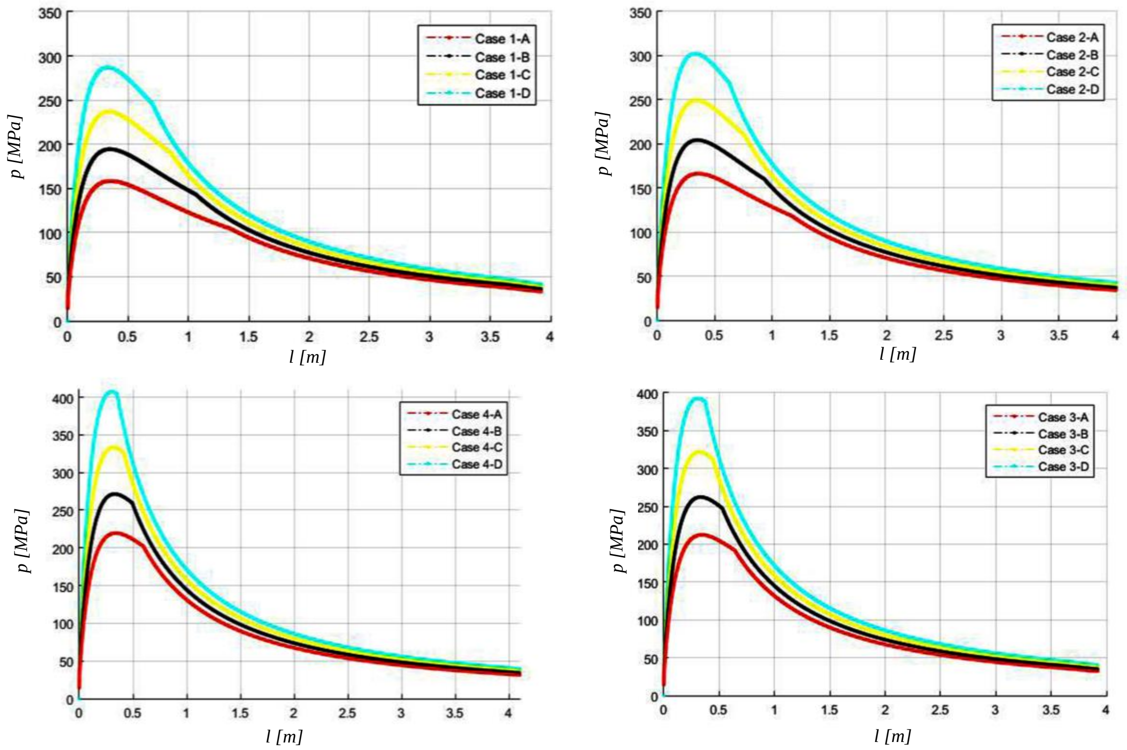

Figure 7. The pressure vs. trajectory of projectile.

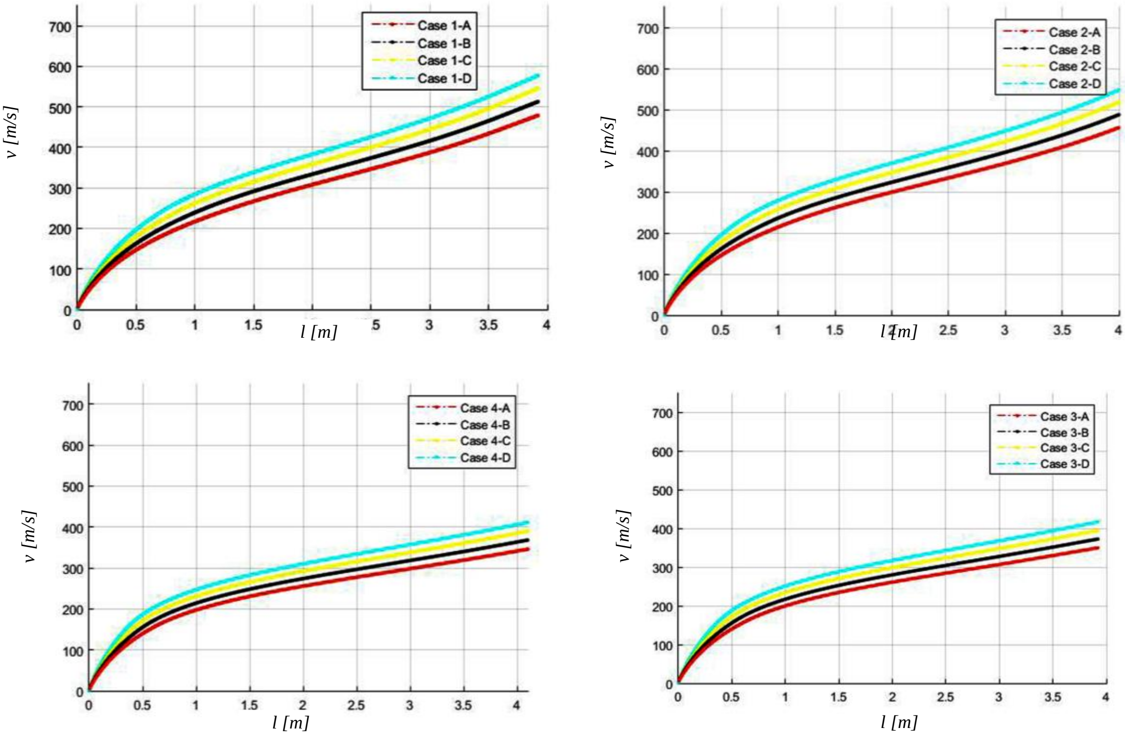

Figure 8. The muzzle velocity vs. trajectory of projectile.

4. THE EXPERIMENTAL MEASUREMENTS AND DISCUSSION

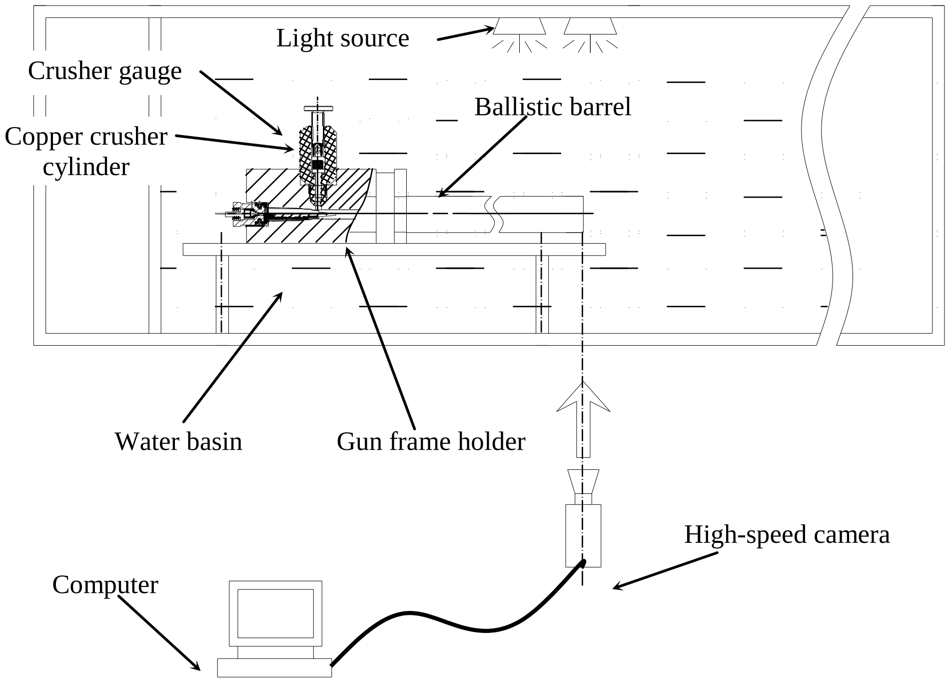

Figure 9. Schematic of the experimental setup.

In order to verification of the mathematical model, computation results of the maximum of pressure and muzzle velocity are compared with the measured values by experimental investigation. Experiments were held in the Weapon Technology Center of the Le Quy Don Technical University in Hanoi. The Crusher gauge is used to determine the maximum of pressure, while the high-speed camera system is used to measure the muzzle velocity. The schematic of the experimental setup is shown in Fig. 9 and the photograph of the experimental setup with the ballistic barrel is shown in Fig. 10.

Experiment results obtained and the comparison with theoretically calculated are shown in Tab. 6.

Table 6. The maximum of pressure and muzzle velocity.

Cases of investigation		Maximum of pressure			Muzzle velocity		
		Model (MPa)	Experiment (MPa)	Difference	Model (m/s)	Experiment (m/s)	Difference
Case 1	Type A	158.3543	157.21	0.72 %	478.8050	473.37	1.14 %
	Type B	194.3549	193.05	0.67 %	512.6540	508.06	0.90 %
	Type C	236.8182	234.50	0.98 %	545.2861	539.23	1.11 %
	Type D	287.0130	285.00	0.70 %	577.1616	571.54	0.97 %
Case 2	Type A	166.1994	165.21	0.60 %	489.1870	484.00	1.06 %
	Type B	204.2270	202.85	0.67 %	522.4373	517.21	1.00 %
	Type C	249.1320	247.13	0.80 %	554.6195	549.02	1.01 %
	Type D	302.2899	300.37	0.64 %	586.0122	580.12	1.01 %
Case 3	Type A	212.2827	210.15	1.00%	350.0405	346.21	1.09 %
	Type B	262.3105	260.13	0.83 %	372.8633	368.86	1.07 %
	Type C	321.7631	320.16	0.50 %	395.1657	391.00	1.05 %
	Type D	392.7289	390.00	0.69 %	417.0046	412.65	1.04 %
Case 4	Type A	219.5277	217.72	0.82 %	355.4646	351.87	1.01 %
	Type B	271.4436	270.00	0.53 %	378.1959	374.97	0.85 %
	Type C	333.1954	330.05	0.94 %	400.3838	396.69	0.92 %
	Type D	406.9934	403.12	0.95 %	422.2009	418.12	0.97 %

According to the comparison of the experimental results with the theoretical calculated obtained in these cases of investigation, the difference between the maximum of pressure values is approximately 0.75 % and between the muzzle velocity values is approximately 1.01 %. These differences indicate that the mathematical model of interior ballistics built in this article is reliable.

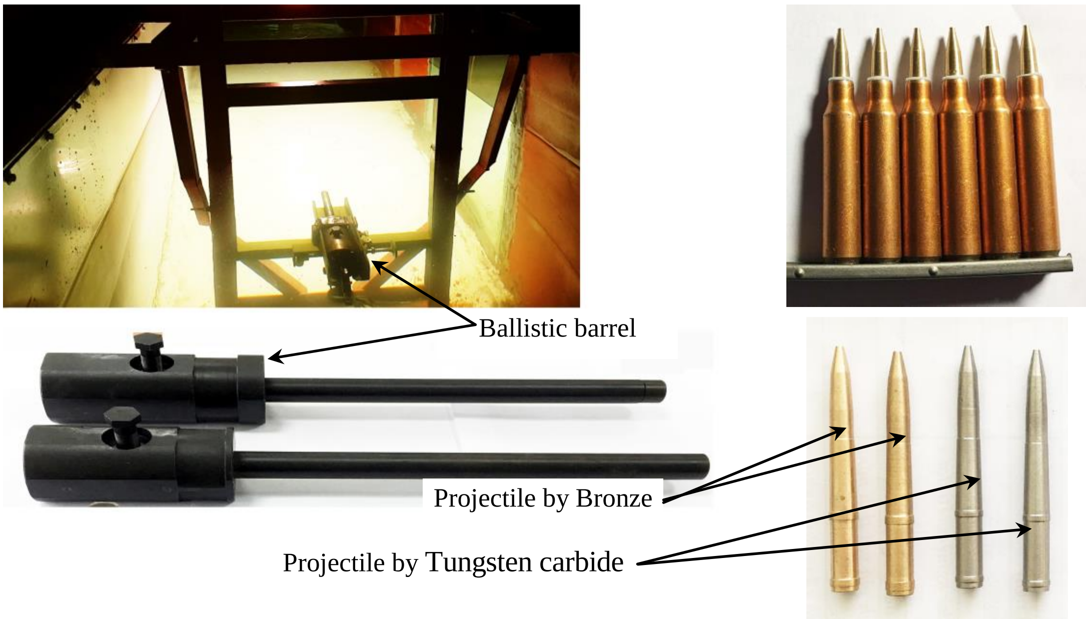

Figure 10. Schematic of the experimental setup with the ballistic barrel.

4. CONCLUSIONS

The article gives the arranged mathematical model of interior ballistics for the amphibious rifle when firing the ammunition under water. The reliability and valiability of this model were verified by experiments (Tab. 6).

This research clearly has some limitations. It has only investigated the interior ballistic in the bore barrel without the thermodynamics problem in the gas chamber of gas block. So, further research will focus on the combining between the interior ballistic in bore and the thermodynamics problem in the gas chamber of gas block.

Nevertheless, we believe our study could be a starting point and the new method to approach the interior ballistic of the amphibious rifle. In addition, the interior ballistics model of amphibious rifle when firing underwater can be used as powerful tools for designing the underwater ammunition, underwater rifle and amphibious rifle.

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