Vietnam Journal of Science and Technology ***57*** *(4) (2019) 513-523*

doi:10.15625/2525-2518/57/4/13167

FUZZY SLIDING MODE CONTROL OF A STRUCTURE BASED ON HEDGE ALGEBRAS CONTAINING INPUT TIME DELAY

Bui Hai Le[1, *], Le Tung Anh[1], Bui Van Binh[2], Nguyen Thai Tat Hoan[1]

[1]*School of Mechanical Engineering, Hanoi University of Science and Technology, No. 01 Dai Co Viet street, Hai Ba Trung district, Ha Noi*

[2]*Faculty of Mechanical Engineering, Electric Power University, No. 235 Hoang Quoc Viet street, Cau Giay district, Ha Noi*

[*]Email: *lebh79@gmail.com*

Received: 4 October 2018; Accepted for publication: 16 June 2019

Abstract. In this paper, the authors present the sliding mode control problem of a structure using hedge-algebras-based fuzzy controller considering the impact of time delay (de-sHAC). Controlled model is a structure subjected to earthquake excitations. Numerical simulations are implemented in order to show advantages of the proposed controller. Obtained results include: variation of maximum displacement and maximum absolute acceleration versus time delay; time responses of displacement, absolute acceleration and control force of the structure in the uncontrolled case, the controlled case using the hedge-algebras-based fuzzy controller with input time delay (de-HAC) and the de-sHAC.

Keywords: fuzzy control, sliding mode control, hedge-algebras, input time delay.

Classification numbers: 5.4.2, 5.4.3.

1. INTRODUCTION

Because of advantages of active vibration control in high effectiveness in control and high robustness for uncertain structures, it has been applied in a wide range of guaranteeing safety of structures and comfort of human. Especially, it has high performance in vibration control of civil engineering systems such as buildings and bridges.

First presented in 1965 by Zadeh, fuzzy set theory is a useful mathematical tool to model uncertain (unprecise) and vague data, and it has been reported in many real situations. Thus, we were provided a flexible and effective methodology to solve many practical control problems based on fuzzy rule [1]. Sliding mode control (SMC) has been effectively applied in process control in general and in vibration control of mechanical models because of its advantages in adaptiveness and robustness. However, a SMC can not avoid explicitly the chattering phenomenon, and as a result, time histories of control forces cannot be realized by real controllers. Combination of a SMC and a fuzzy logic controller (FLC) can reduce the impact of this phenomenon and achieve the advantages of SMC as well as of FLC [2].

Hedge-algebras (HA) theory was introduced in 1990 [3-10] and it has been actively developed since then in many works. The HA approach have a specific, fundamental and distinguished characteristic: the inherent semantics of linguistic terms formally generate their quantitative semantics called semantically quantifying mapping (SQM). In order to show applicability in practice, the theory was first applied to fuzzy control in 2008 and published in [11]. Following this research work, a series of articles in HA based fuzzy control were investigated and published with more satisfactory results in comparison with those of conventional FLC [12-16].

In fact, time delay is ubiquitous in control systems. Actuator and sensor dynamics do not permit the instantaneous generation of such forces, and hence the effective control gets delayed in time. Thus the presence of time delay in the control is inevitable when controlling structures subjected to dynamic loads, such as those caused by strong earthquake ground shaking [17].

The aforementioned reasons prompted the authors in this study to propose and design a sliding-mode hedge-algebras-based fuzzy controller containing input time delay (de-sHAC) to actively control a structure against earthquake.

2. HEDGE-ALGEBRAS-BASED FUZZY CONTROLLER

HA theory allows determining linguistic values of each linguistic variable by isomorphisms mapping called semantically quantifying mapping (SQM) based on few fuzziness parameters of each linguistic variable instead of using any fuzzy sets [3-10].

Consider a HA structure *AX* of a linguistic variable with its term-set *X* as follows:

$$AX = \left(X, G, C, H, \leq \right) \tag{1}$$

where G = (*Small*, *Big*) = (c^-, c^+) is the set of primary terms; C = ($\boldsymbol{0}$, $\boldsymbol{W}$, $\boldsymbol{1}$) where $\boldsymbol{0}$, $\boldsymbol{W}$ and $\boldsymbol{1}$ are specific constants called *absolutely Small*, *neutral* and *absolutely Big*, respectively; H = (*Very*, *Little*) = $H^+ \cup H^-$ is the set of hedges; and $\leq$ is a partially ordering relation on X.

All possible SQMs of linguistic values of the linguistic variable X can be calculated by using HA theory in [3-10]. Typical linguistic values with SQMs of X are arranged in Tab. 1 in the case of $fm(c^-)$ and $\mu(h^-) = 0.5$, in which $fm(c^-)$ and $\mu(h^-)$ are the fuzziness measure of c^- and h^-; *Bi*, *Sm*, *V* and *L* stand for *Big*, *Small*, *Very* and *Little*, respectively.

Table 1. Typical linguistic values with SQMs of *X*.

Linguistic values	*VSm*	*Sm*	*LSm*	***W***	*LBi*	*Bi*	*VBi*
SQMs	0.125	0.25	0.375	0.5	0.625	0.75	0.875

Next, a P-D hedge-algebras-based fuzzy controller (HAC) is designed with its control structure represented in Fig. 1, in which, x_1 and $\dot{x}_1$ are state variables and u is control variable [1]. It is assumed that the reference domains of the state variables and the control variable are given by $-a \leq x_1 \leq a$, $-b \leq \dot{x}_1 \leq b$ and $-c \leq u \leq c$. The linguistic values with SQMs of the variables are shown in Tab. 2 (denoted by "•").

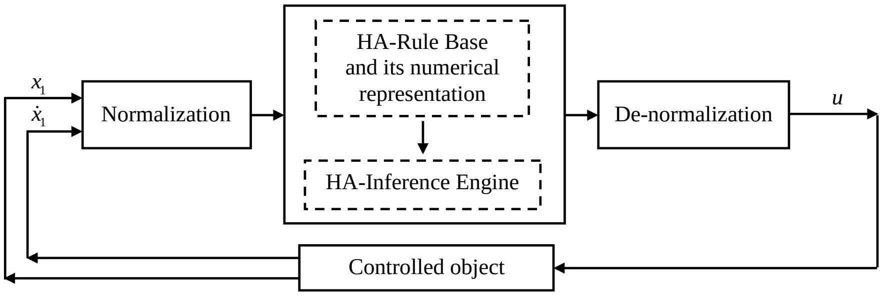

Figure 1. Control diagram of a HAC.

Table 2. Linguistic values with SQMs of the variables.

Linguistic values with SMQs	*VSm*: 0.125	*Sm*: 0.25	*LSm*: 0.375	***W***: 0.5	*LBi*: 0.625	*Bi*: 0.75	*VBi*: 0.875
x_1		•	•	•	•	•	
$\dot{x}_1$			•	•	•		
u	•	•	•	•	•	•	•

The normalization of the state variables x_1 and $\dot{x}_1$, which is to convert their reference domain into their SQMs domain and the de-normalization of the control variable u, which is to convert its SQMs domain into its reference domain, are presented in Fig. 2. A typical fuzzy rule base arranged in Tab. 3 with SQM values is used for HAC. By representing the HA rule base in Tab. 3 as a grid surface called semantically quantifying surface (SQS) as shown in Fig. 3 with grid nodes are the SQM values of the linguistic values occurring in Tab. 3.

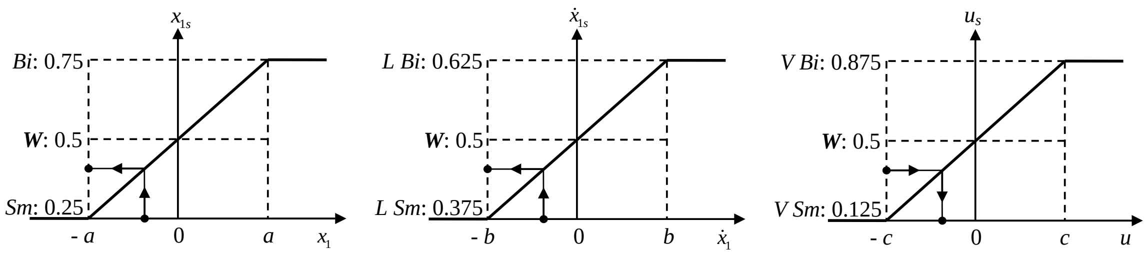

Figure 2. Normalization of the state variables and de-normalization of the control variable.

Table 3. HA-rule base with SQMs.

	$\dot{x}_1$ *LSm*: 0.375	***W***: 0.5	*LBi*: 0.625
Sm: 0.25	*VSm*: 0.125	*Sm*: 0.25	*LSm*: 0.375
LSm: 0.375	*Sm*: 0.25	*LSm*: 0.375	***W***: 0.5
W: 0.5	*LSm*: 0.375	***W***: 0.5	*LBi*: 0.625
LBi: 0.625	***W***: 0.5	*LBi*: 0.625	*Bi*: 0.75
Bi: 0.75	*LBi*: 0.625	*Bi*: 0.75	*VBi*: 0.875

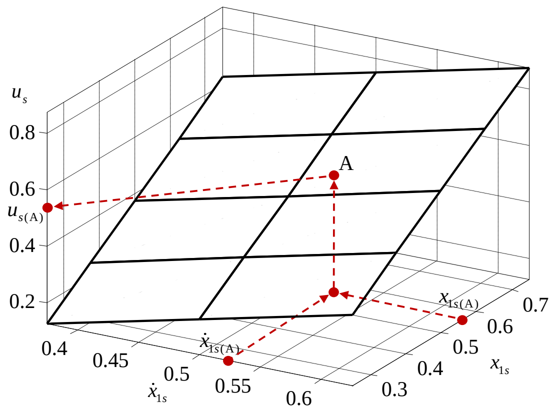

Figure 3. HA inference engine – SQS.

It can be seen from Fig. 1 and Tab. 3 that the HA inference engine can be operated as an explicit operation utilizing a very simple linear interpolation.

3. MOTION EQUATION OF THE STRUCTURE

As shown in Fig. 4, a single-storey model subjected to earthquake excitation $\ddot{x}_0$ is considered. The structural parameters are given as m = 1000 kg, c = 1.407 kNs/m, and k = 980 kN/m.

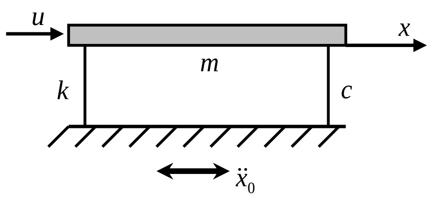

Figure 4. Model of SDOF subjected to earthquake.

The motion equation of SDOF model with actuator saturation is presented as:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = \text{sat}\left(u(t-\tau)\right) - m\ddot{x}_0(t) \tag{2}$$

In which, sat(u) depends on the limitation $u_{\lim}$ of the actuator [1]:

$$\text{sat}(u) = \begin{cases} u_{\lim} & \text{if} \quad u \geq u_{\lim} \\ u & \text{if} \quad -u_{\lim} \leq u \leq u_{\lim} \\ -u_{\lim} & \text{if} \quad u \leq -u_{\lim} \end{cases} \tag{3}$$

The control force u (the maximum value of the actuator saturation is 500 N) will be found out through the HAC control algorithms shown in Fig. 1, in which x_1 and $\dot{x}_1$ replaced by x and $\dot{x}$.

τ is the time delay (constant). This parameter shows the factors causing delay of the system controller. Eq. (2) also indicates that the control force of the HAC or sHAC with the input time delay τ (de-HAC or de-sHAC) is 0 when $t \leq \tau$ and $u(t - \tau)$ when $t > \tau$.

Eq. (2) can be rewritten under state-space form as below:

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{D}_1 u + \mathbf{D}_2 \ddot{x}_0 \tag{4}$$

where
$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -m^{-1}k & -m^{-1}c \end{bmatrix};\ \mathbf{D}_1 = \begin{bmatrix} 0 \\ m^{-1}b \end{bmatrix};\ \mathbf{D}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ \mathbf{z} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \tag{5}$$

4. SLIDING HEDGE-ALGEBRAS-BASED FUZZY CONTROLLER

In this section, the sliding hedge-algebras-based fuzzy controller (sHAC) will be presented in order to control the structure vibration.

Firstly, SMC is introduced and summarized based on transformations in [2, 18].

Secondly, HAC and SMC are combined in order to establish sHAC.

4.1. Sliding-mode controller

The non-linear control force in the SMC can be expressed as [2, 18]:

$$u(\mathbf{z},t) = u_{\text{eq}}(\mathbf{z},t) - \eta\,\text{sgn}(\sigma(\mathbf{z})) \tag{6}$$

where u_{eq} is a linear part of the control force, known as the equivalent control force, η is a constant design parameter and sgn is the signum function.

Consider sliding surface defined as

$$\sigma(\mathbf{z}) = \mathbf{S}\mathbf{z} \tag{7}$$

where the sliding surface coefficient matrix S (1×2*n*) is a design matrix, usually constant and the sliding surface is chosen to satisfy

$$\sigma(\mathbf{z}) = 0 \text{ and } \dot{\sigma}(\mathbf{z}) = 0 \tag{8}$$

According to the Utkin–Drazenovic method [2, 18], the equivalent control force can be determined as

$$u_{\text{eq}}(\mathbf{z},t) = -(\mathbf{S}\mathbf{D}_1)^{-1}\left[\mathbf{S}\mathbf{A}\mathbf{z} + \mathbf{S}\mathbf{D}_2\ddot{x}_0\right] \tag{9}$$

In Equation (9), neglecting the ground acceleration $\ddot{x}_0$ and, instead, we select a proper η parameter to compensate the uncertainties in the external excitation. In addition, the existence and the reachability of the sliding-mode must be guaranteed when choosing of η. Mathematically expressed, the condition that $\sigma(\mathbf{z})\dot{\sigma}(\mathbf{z}) < 0$, must be satisfied [2, 18].

This results in [2, 18]:

$$\sigma(\mathbf{z})\left[-\eta\,\text{sgn}(\sigma(\mathbf{z})) + (\mathbf{S}\mathbf{D}_1)^{-1}\mathbf{S}\mathbf{D}_2\ddot{x}_0\right] < 0 \tag{10}$$

and

$$\eta \geq \left|(\mathbf{S}\mathbf{D}_1)^{-1}\mathbf{S}\mathbf{D}_2\ddot{x}_0^*\right| \tag{11}$$

where $\ddot{x}_0^*$ is the maximum absolute value of $\ddot{x}_0$.

Thus, we can express the realizable control force as follows

$$u(\mathbf{z},t) = -(\mathbf{SD}_1)^{-1}\mathbf{SAz} - \eta\,\mathrm{sgn}(\sigma(\mathbf{z})) \tag{12}$$

The control force $u(\mathbf{z},t)$ must agree with the saturation condition in Eq. (6) because of the actuator limitation.

4.2. Sliding-mode hedge-algebras-based fuzzy controller

It can be observed from Eq. (11) and Eq. (12) that the control signals of SMC are changed suddenly in many times due to the signum function $\mathrm{sgn}(\sigma(\mathbf{z}))$. Hence, this phenomenon can be treated as a weak point of SMC.

In sHAC, instead of using Eq. (12), the control force is considered as below

$$u(\mathbf{z},t) = -(\mathbf{SD}_1)^{-1}\mathbf{SAz} - \eta u_{\mathrm{HAC}} \tag{13}$$

and this leads to changing the condition in Eq. (10)

$$\sigma(\mathbf{z})\left[-\eta u_{\mathrm{HAC}} + (\mathbf{SD}_1)^{-1}\mathbf{SD}_2\ddot{x}_0\right] < 0 \tag{14}$$

In which, u_{HAC} is obtained from HAC in subsection 4.1 when respectively replacing x_1 , $\dot{x}_1$ and u by σ, $\dot{\sigma}$ and u_{HAC}.

It can be emphasized that the HA-Rule Base with SQMs shown in Tab. 3 is appropriate for the condition $\sigma(\mathbf{z})\dot{\sigma}(\mathbf{z}) < 0$ or Eq. (14). In addition, the chattering phenomenon in sHAC will be softened when the signum function $\mathrm{sgn}(\sigma(\mathbf{z}))$ is replaced by u_{HAC}.

5. NUMERICAL SIMULATIONS

In this section, numerical simulations in active control of the structure mentioned in section 3 are performed in order to verify the effectiveness and robustness of the proposed controller with input time delay (de-sHAC). In which, the excitations of the 1940 El Centro earthquake is used with their peak ground accelerations are scaled to 0.112 g [19]. To evaluate the control system performance, two important evaluation criteria are described as follows:

* The peak storey drift J_1, which is related to structural safety:

$$J_1 = \max\left(\frac{|d(t)|}{d_{\max}}\right) \to \min \tag{15}$$

* The peak absolute acceleration J_2, which is related to human tolerance:

$$J_2 = \max\left(\frac{|\ddot{x}_a(t)|}{\ddot{x}_{a\max}}\right) \to \min \tag{16}$$

where, $d(t)$ and $d_{\max,}$ respectively are storey drift of the structure and peak storey drift in the controlled and uncontrolled responses, $\ddot{x}_a(t)$ and $\ddot{x}_{a\max}$, respectively are absolute acceleration of the structure and peak absolute acceleration in the controlled and uncontrolled responses.

Now, the control performance of the proposed controller for the nominal system will be investigated. For detailed comparison, normalized maximum values of responses of the structure are arranged in Tab. 4. It is found that the proposed controller produces better performance than the de-sHAC in terms of maximum acceleration reduction under the same maximum control force, where time delay $\tau = 0$ and 20 ms.

Table 4. Normalised maximum response values.

Control strategy	$\tau = 0$ ms		$\tau = 20$ ms	
	de-HAC	de-sHAC	de-HAC	de-sHAC
J_1	0.4615	0.4286	0.5708	0.3571
J_2	0.5842	0.5572	0.7468	0.5641

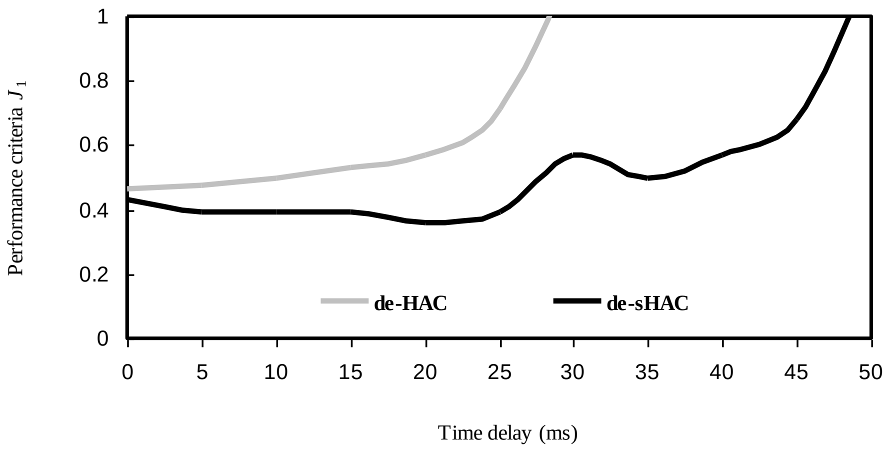

Figure 5. Performance criteria J_1 versus time delay.

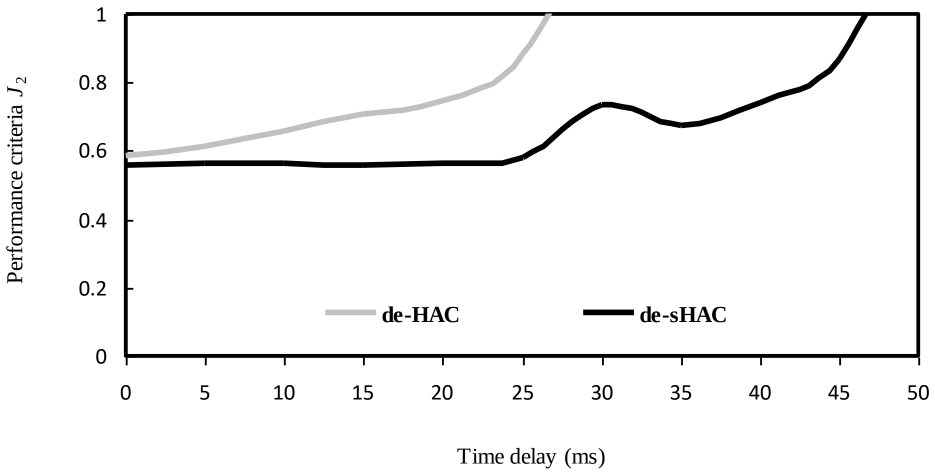

Figure 6. Performance criteria J_2 versus time delay.

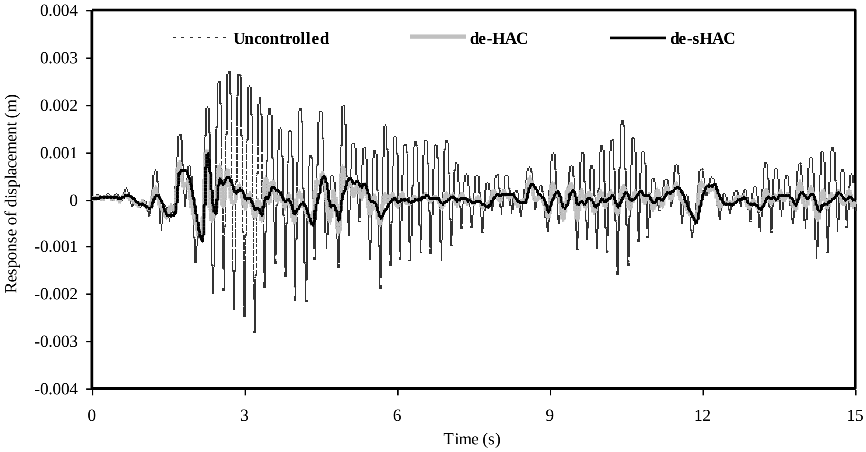

Figure 7. Response of displacement of the structure when $\tau = 0$ ms.

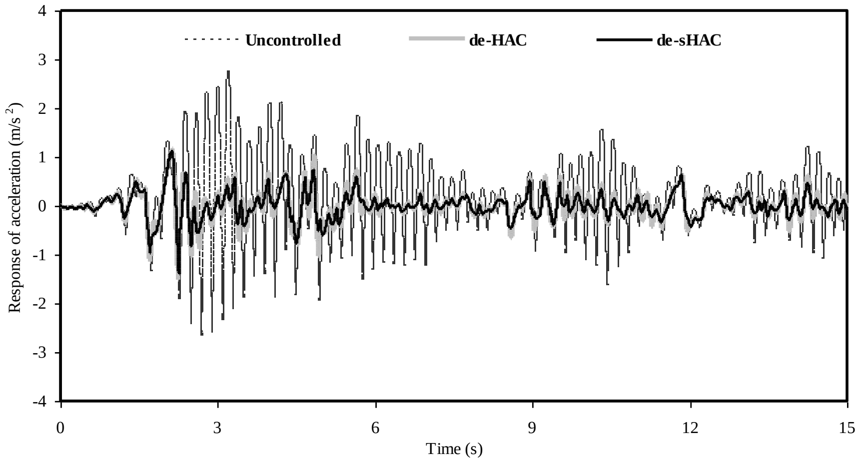

Figure 8. Response of acceleration of the structure when $\tau = 0$ ms.

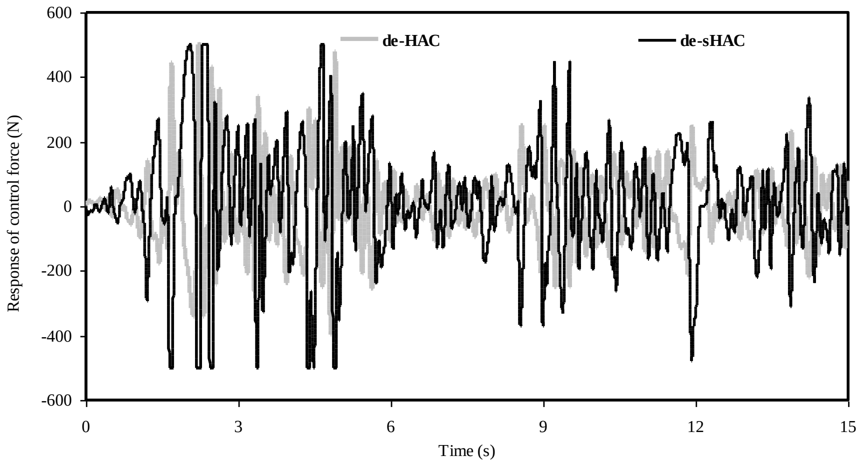

Figure 9. Response of control force when $\tau = 0$ ms.

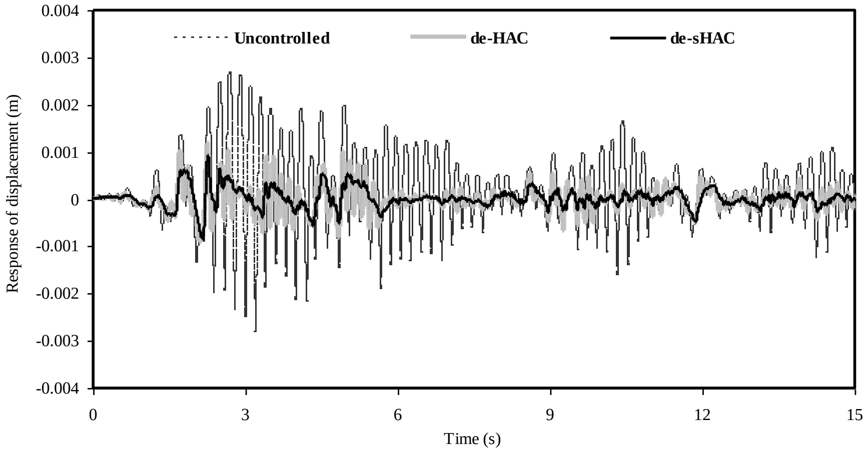

Figure 10. Response of displacement of the structure when $\tau = 20$ ms.

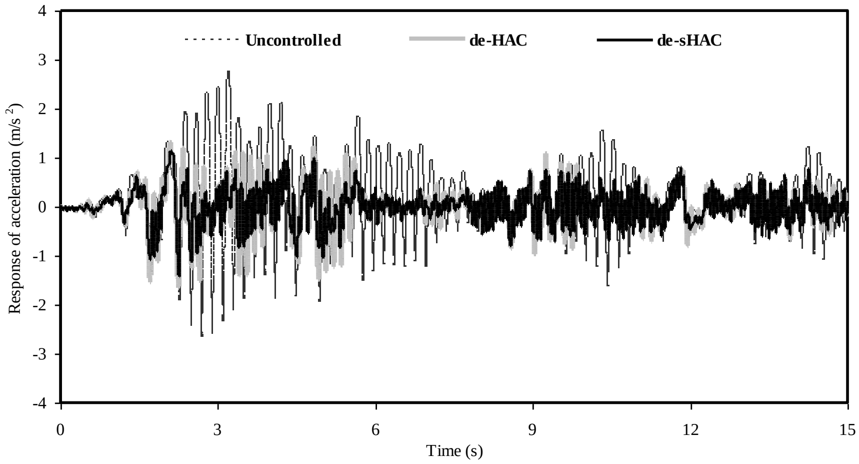

Figure 11. Response of acceleration of the structure when $\tau = 20$ ms.

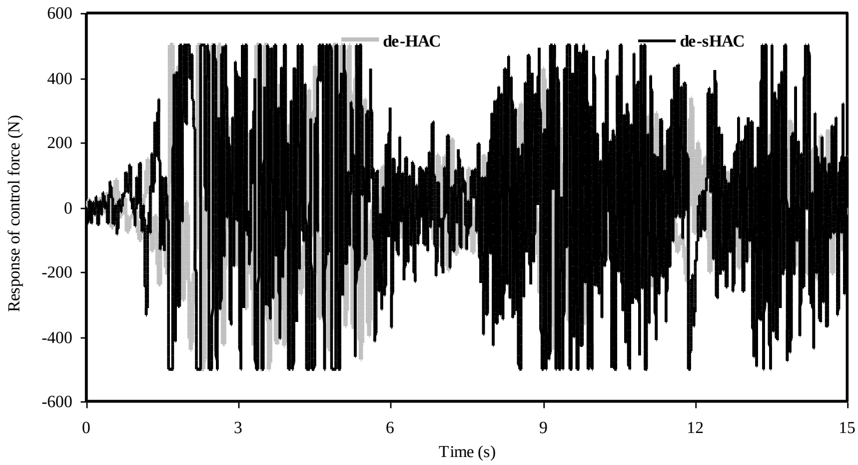

Figure 12. Response of control force when $\tau = 20$ ms.

To further study the effectiveness of the proposed controller with time delay, the influences of time delay on the responses of the structure are considered by calculating the values of J_1 and J_2 versus the time delay τ, as shown in Figs. 5-6. It can be seen from these figures that the de-sHAC performances are all better than the corresponding de-HAC performances and these good performances can be kept up to the maximum time delay (approximately τ = 25 ms) while the de-HAC is immediately destabilized.

When there is no time delay on input, *i.e.*, τ = 0 ms, the responses of the de-sHAC and the de-HAC controller are compared in Figs. 7-9, where only the displacement, the absolute acceleration of the structure and the control force are shown for clarity. It can be seen from Figs. 7-9 that better responses are obtained for the de-sHAC controller when τ = 0 ms.

When the time delay τ = 20 ms is included, with the same earthquake excitation, the responses of the displacement, the absolute acceleration of the structure and the control force are plotted in Figs. 10-12. The controller is feasible for the given maximum time delay τ = 20 ms. This means that the controller can stabilize with good results under parameter uncertainties as well as actuator saturation for any time delay within [0, 20] ms. Moreover, it can be seen from Figs. 10-12 that the de-sHAC is stable and the its performance is similar to that of no time delay in input case as shown in Figs. 7-9.

6. CONCLUSIONS

In this study, the de-sHAC in active control of a structure subjected to earthquake is proposed. Obtained results indicate that the proposed controller has robustness capacity, higher performance in control when comparing with that of the de-HAC controller. As a result, it can be emphasized that de-sHAC is an effective approach in vibration control of structures. However, considering different types of structures, such as active suspension systems, smart structures, flexible robots and so on, will provide further and interested researches in sliding mode control based on HA. In practical aspect, the controller de-sHAC could be installed into a real or experimental controlled system in order to evaluate its practical performance and an interesting study is the experimental assessment in structural control of de-sHAC on shaking table.

Acknowledgements. This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number “107.01-2017.306”.

REFERENCES

1. Anh N. D., Bui H. L., Vu N. L., Tran D. T. - Application of hedge algebra-based fuzzy controller to active control of a structure against earthquake, Structural Control and Health Monitoring **20** (4) (2013) 483-495.
2. Alli H., Yakut O. - Fuzzy sliding-mode control of structures, Engineering Structures **27** (2) (2005) 277-284.
3. Nguyen C. H., Tran T. S., Pham D. P. - Modeling of a semantics core of linguistic terms based on an extension of hedge algebra semantics and its application, Knowledge-Based Systems **67** (2014) 244-262.

4. Nguyen C. H., Pedrycz W., Duong T. L., Tran T. S. - A genetic design of linguistic terms for fuzzy rule based classifiers, International Journal of Approximate Reasoning **54** (1) (2013) 1-21.
5. Nguyen C. H., Pedrycz W. - A construction of sound semantic linguistic scales using 4-tuple representation of term semantics, International Journal of Approximate Reasoning **55** (3) (2014) 763-786.
6. Ho N. C., Long N. V. - Fuzziness measure on complete hedge algebras and quantifying semantics of terms in linear hedge algebras, Fuzzy Sets and Systems **158** (4) (2007) 452-471.
7. Ho N. C. - A topological completion of refined hedge algebras and a model of fuzziness of linguistic terms and hedges, Fuzzy Sets and Systems **158** (4) (2007) 436-451.
8. Ho N. C., Wechler W. - Extended hedge algebras and their application to fuzzy logic, Fuzzy Sets and Systems **52** (3) (1992) 259-281.
9. Ho N. C., Wechler W. - Hedge algebras: an algebraic approach to structure of sets of linguistic truth values, Fuzzy Sets and Systems **35** (3) (1990) 281-293.
10. Tran D. K., Huynh V. N., Nguyen C. H. - Hedge algebras, linguistic-value logic and their application to fuzzy reasoning, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems **7** (4) (1999) 347-361.
11. Ho N. C., Lan V. N. - Optimal hedge-algebras-based controller: Design and application, Fuzzy Sets and Systems **159** (8) (2008) 968-989.
12. Vukadinović D., Bašić M., Nguyen C. H., Vu N. L., Nguyen T. D. - Hedge-algebra-based voltage controller for a self-excited induction generator, Control Engineering Practice **30** (2014) 78-90.
13. Duc N. D., Vu N. L., Tran D. T., Bui H. L. - A study on the application of hedge algebras to active fuzzy control of a seism-excited structure, Journal of Vibration and Control **18** (14) (2012) 2186-2200.
14. Bui H. L., Nguyen C. H., Vu N. L., Nguyen C. H. - General design method of hedge-algebras-based fuzzy controllers and an application for structural active control, Applied Intelligence **43** (2) (2015) 251-275.
15. Bui H. L., Nguyen C. H., Bui V. B., Le K. N., Tran H. Q. - Vibration control of uncertain structures with actuator saturation using hedge-algebras-based fuzzy controller, Journal of Vibration and Control **23** (12) (2015) 1984-2002.
16. Bui H. L., Tran D. T., Vu N. L. - Optimal fuzzy control of an inverted pendulum, Journal of Vibration and Control **18** (14) (2012) 2097-2110.
17. Du H., Zhang N., Naghdy F. - Actuator saturation control of uncertain structures with input time delay, Journal of Sound and Vibration **330** (18) (2011) 4399-4412.
18. Adhikari R., Hiroki Y. - Sliding mode control of buildings with ATMD, Earthquake Engineering & Structural Dynamics **26** (1997) 409-422.
19. Lim C., Park, Y., Moon S. J. - Robust saturation controller for linear time-invariant system with structured real parameter uncertainties, Journal of Sound and Vibration **294** (2006) 1-14.