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PROGRAM MOTION OF UNLOADING MANIPULATORS

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Abstract. In the paper, the program motion of an unloading manipulator which is treated as a first integral of the considered system, is investigated. Currently, the popular way to solve such problem is the method of Lagrange multipliers only. In the paper, the authors use another approach, the Principle of Compatibility, in which the required program is treated as one of motion equations of the system. In the particular case, the program is considered as one of first integrals of the system. For illustrating the proposed method, the motion of an unloading manipulator of three degrees of freedom is considered.

Keywords: first integral, transmission matrix method, unloading manipulator, the principle of compatibility.

Classification numbers: 5.3.5; 5.3.6

1. INTRODUCTION

Consider a manipulator whose grippers must move a load along a prescribed trajectory. Such problem has been studied in many works in [1-4] and still come into much attention by many researchers. Up to now, the problem is solved by the Lagrange multipliers method only. However, the method owns some inconveniences due to adding more variables, Lagrange multipliers, to equations of motion. It is important that by using this method the opportunity of controlling the manipulator will be taken away. In this paper the proposed method overcome such inconveniences by not using the Lagrange multipliers and therefore the motion of the considered system is described in terms of generalized coordinates only.

For solving the stated problem it is used the method proposed in [5], in that work, the method of transmission matrix is applied to derive the system equations of motion and the equation of trajectory of the required program is treated as the first integral [6].

2. DYNAMICAL MODEL

Let consider a scleronomous holonomic system, whose position is defined by the Lagrange coordinates $q_j\,(j=\overline{1,n})$ and the generalized forces noted as $Q_j\,(j=\overline{1,n})$, respectively. From now on the symbols are used: matrices in bold letters, vectors considered as column matrices, the letter T at upper right corner denotes the matrix transposition.

Assume that (nxn) inertia matrix is denoted by **A**, (nx1) matrix of generalized forces is denoted by $\mathbf{Q}^{(0)}$, the equations of mechanical systems are written in matrix form [5-8]:

$$\boldsymbol{A\ddot{q}} = \boldsymbol{Q}^{(0)} + \boldsymbol{Q}^{(1)} + \boldsymbol{Q}^{(2)} \tag{1}$$

where: A=$\left[a_{ij}\right]_{i,j=\overline{1,n}}$ is a (nxn) regular matrix called the inertia matrix; $\ddot{\mathbf{q}}$ is a (nx1) matrix of generalized acelerations, $\ddot{\mathbf{q}} = \begin{bmatrix} q_1 & q_2 & . & . & q_n \end{bmatrix}^T$; $\mathbf{Q}^{(0)}$-a (nx1) matrix of generalized forces derived from the system's potential, the active and dissipation forces; $\mathbf{Q}^{(1)}$, $\mathbf{Q}^{(2)}$- **(**nx1**)** matrices, which are determined by the inertia matrix as described in [5-8].

As known, the motion of the system subjected to the program motion can be written as follows [5-8]:

$$g_\alpha(t, q_1, q_2, .., q_n) = 0 \quad ; \quad \alpha = \overline{1, r} \tag{2}$$

There exist two approaches to solve the stated problem as follows:

Method 1. The required program is treated as ideal constraints and the method of Lagrange multipliers is used. As known, the method is not simple because it requires to calculate the Lagrange multipliers.

Method 2. The stated problem is solved by means of the Principle of Compatibility. According to this, the program is considered as part of the motion equations of the system. Thus, it is neccessary to add some forces as the control inputs on the considered system. According to this method the motion equations are written as follows:

$$\boldsymbol{A\ddot{q}} = \boldsymbol{Q}^{(0)} + \boldsymbol{Q}^{(1)} + \boldsymbol{Q}^{(2)} + \boldsymbol{U} \tag{3}$$

where **U** is the (nx1) matrix of the form:

$$\mathbf{U} = \begin{bmatrix} U_1 & U_2 & . & . & U_n \end{bmatrix}^T \tag{4}$$

Its components are defined by means of following equations [6]:

$$\boldsymbol{GA}^{-1}\boldsymbol{U} + \mathbf{G}^{(*)} = 0 \tag{5}$$

where:

$$\begin{gathered} \boldsymbol{G} = \left[\, g_{\alpha j} \,\right]; g_{\alpha j} = \frac{\partial g_\alpha}{\partial q_j} \;; \boldsymbol{G}^* = \boldsymbol{G}^{(0)} + \boldsymbol{GA}^{-1}(\boldsymbol{Q}^{(0)} + \boldsymbol{Q}^{(1)} - \boldsymbol{Q}^{(2)})_{\alpha=\overline{1,n}} (\alpha = \overline{1,r}; j = \overline{1,n}) \\ \mathrm{G}^{(0)} = \left[\left(\sum_{i,j=1}^{n} \frac{\partial^2 g_\alpha}{\partial q_i \partial q_j} \dot{q}_i \dot{q}_j + \sum_{j=1}^{n} \frac{\partial^2 g_\alpha}{\partial q_j \partial t} + \frac{\partial^2 g_\alpha}{\partial^2 t} \right) \right]_{\alpha=\overline{1,r}} \end{gathered} \tag{6}$$

and $\mathbf{Q}^{(1)}$, $\mathbf{Q}^{(2)}$ are (nx1) matrices of inertia forces defined by the inertia matrix A [5-8].

In this paper the following method is proposed

By using the method in [6], the required program is treated as a first integral of the system. Originally, the program motion is not described by the system's equations of motion (1). Therefore, it is necessary to act some control forces on the system. By doing that, the motion equations of the system are written as follows:

$$\boldsymbol{A\ddot{q}} = \boldsymbol{Q}^{(0)} + \boldsymbol{Q}^{(1)} + \boldsymbol{Q}^{(2)} + \boldsymbol{U} + \boldsymbol{R} \tag{7}$$

for the given program being the first integral of the system, it is neccessary to realize the condition.

$$\boldsymbol{DR = 0}$$

where **D** is (kxn) matrix whose elements are the coefficients to express all of the generalized acceleration $\ddot{q}_i\,(i=\overline{1,n})$ in terms of the independent generalized accelerations $\ddot{q}_\sigma$.

($(\sigma=\overline{1,k}=n-r)$ .Therefore, we have:

$$\begin{gathered} \boldsymbol{D(A\ddot{q} - Q^{(0)} - Q^{(1)} - Q^{(2)} - U) = 0} \\ g_\alpha(t, q_j, \dot{q}_j) = 0 \end{gathered} \tag{8}$$

It is noted that the control forces U_j are the forces acting on the system of interest. In the particular case, they may be components of the force $\boldsymbol{Q}^{(0)}$.

3. MOTION INVESTIGATION OF AN UNLOADING MANIPULATOR

Consider the motion of three-link unloading manipulator: link OA has length l_1, mass m_1 with the mass center at the rotatory joint O. Link AD, a cylinder rotating about the axis A, has mass m_2, and the mass center at C_2 ($AC_2=c_2$). Piston BC has mass m_3, and the mass center C_3 ($BC_3=c_3$). The inertia moment of the link OA about the rotatory axis O denoted by J_1. The inertia moments of the links AD and BC about the its mass center denoted by J_2, J_3, respectively. The links OA and AD are exerted by the couples M_1 and M_2, respectively. Cylinder B is subjected to the expulsive force F. The friction forces in the articulated joints and the slip joint are neglected. The load is treated a point mass. The required program is of the form:

$$y - 2x + 2(l_1 + l_2) = 0 \tag{9}$$

where y and x are the coordinates of the load. It means that the load must be moved along the trajectory described by the equation (9), i.e. the inclined line KL (Fig.1).

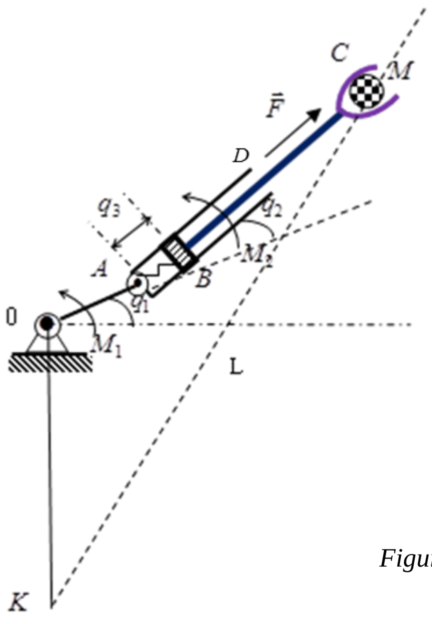

Figure 1. The Investigated unloading manipulator.

where

$$OK = 2(l_1 + l_2);\ OL = l_1 + l_2$$

The considered manipulator of interest is of 3 degrees of freedom. Let choose q_1, q_2, q_3 as the generalized coordinates, where q_1 is the position angle of the link OA with respect to the fixed axis Ox, q_3 is the position angle of the link AD with respect to the link OA, and q_3 is the displacement of the piton BC with respect to the cylinder AD (see Fig. 1).

The motion equations of the manipulator are rewritten from (8) as:

$$\mathbf{DA\ddot{q} = D(Q^0 + Q^{(1)} + Q^{(2)} + U)} \tag{10}$$

where **A**-the (3x1) inertia matrix, $\mathbf{Q}^{(0)}$–the (3x1) matrix of the potential forces and dissipative forces, $\mathbf{Q}^{(1)}$, $\mathbf{Q}^{(2)}$–the (3x1) inertia forces, **U**-the (3x1) control forces, which are of the form: $\mathbf{U} = \begin{bmatrix} M_1 & M_2 & F \end{bmatrix}^T$

In order to calculate the above matrices, we use the method of transmission matrix [7,8].

For this aim, let us introduce the symbols:

$$q_4 \equiv \frac{dq_1}{dt};\ q_5 \equiv \frac{dq_2}{dt};\ q_6 \equiv \frac{dq_3}{dt};\ q_7 \equiv \frac{d^2 q_1}{dt^2};\ q_8 \equiv \frac{d^2 q_2}{dt^2};\ q_9 \equiv \frac{d^2 q_3}{dt^2} \tag{11}$$

and develop the matrices:

$$t_1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix};\quad t_2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & l_1 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix};t_3 = \begin{bmatrix} 1 & 0 & q_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

$$t_{11} = \begin{bmatrix} -\sin q_1 & -\cos q_1 & 0 \\ \cos q_1 & -\sin q_1 & 0 \\ 0 & 0 & 0 \end{bmatrix};t_{21} = \begin{bmatrix} -\sin q_2 & -\cos q_2 & 0 \\ \cos q_2 & -\sin q_2 & 0 \\ 0 & 0 & 0 \end{bmatrix};t_{31} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$r_1 = \begin{bmatrix} c_1 \\ 0 \\ 1 \end{bmatrix};\ r_2 = \begin{bmatrix} c_2 \\ 0 \\ 1 \end{bmatrix};r_3 = \begin{bmatrix} c_3 \\ 0 \\ 1 \end{bmatrix};\ r = \begin{bmatrix} l_1 \\ 0 \\ 1 \end{bmatrix};\ P_1 = \begin{bmatrix} 0 \\ -m1g \\ 0 \end{bmatrix};P_2 = \begin{bmatrix} 0 \\ -m_2 g \\ \ \end{bmatrix};P_3 = \begin{bmatrix} 0 \\ -m_3 g \\ 0 \end{bmatrix};P = \begin{bmatrix} 0 \\ mg \\ 0 \end{bmatrix} \tag{12}$$

The coordinates of the mass centers C_1, C_2, C_3 and of the load are defined by following formulas:

$$\boldsymbol{r_{01} = t_1 r_1;\quad r_{02} = t_1 t_2 r_2;\quad r_{30} = t_1 t_2 t_3 r_3;\quad r_0 = t_1 t_2 t_3 r} \tag{13}$$

Potential energy π can be written as:

$$\begin{aligned} \pi &= \mathbf{r}_{01}^T \mathbf{P}_1 + \mathbf{r}_{02}^T \mathbf{P}_2 + \mathbf{r}_{03}^T \mathbf{P}_3 + \mathbf{r}_0^T \mathbf{P} \\ &= -[m_1 c_1 + (m_2 + m_3 + m) l_1] g \sin q_2 - [m_2 c_2 + m_3 c_3 + m l_3 + (m_3 + m) q_3] g \sin(q_1 + q_2); \end{aligned} \tag{14}$$

Therefore, generalized forces $\mathbf{Q}^{(0)}$ and the matrices of control forces can be expressed as:

$$\mathbf{Q}^{(0)} = \begin{bmatrix} -\dfrac{\partial \pi}{\partial q_1} - \alpha_1 q_4 \\ -\dfrac{\partial \pi}{\partial q_2} - \alpha_2 q_5 \\ -\dfrac{\partial \pi}{\partial q_3} - \alpha_3 q_6 \end{bmatrix} ; \mathbf{U} = \begin{bmatrix} M_1 \\ M_2 \\ F \end{bmatrix} \tag{15}$$

where $\alpha_1, \alpha_2, \alpha_3$ are viscous resistance coefficients of moments and forces acting on links OA, AD, and BC, respectively.

To calculate matrix **D**, we substitute the expression of y, and x in terms of the generalized coordinates:

$$x = l_1 \cos q_1 + (l_3 + q_3)\cos(q_1 + q_2) \; ; \quad y = l_1 \sin q_1 + (l_3 + q_3)\sin(q_1 + q_2) \tag{16}$$

into the expressions (9), we obtain now:

$$f \equiv l_1 \sin q_1 + (l_3 + q_3)\sin(q_1 + q_2) - 2(l_1 \cos q_1 + (l_3 + q_3)\cos(q_1 + q_2)) + 2(l_1 + l_2) = 0 \tag{17}$$

and we have:

$$\begin{aligned} f_1 \equiv \frac{df}{dt} &= [(cos(q_1 + q_2) + 2sin(q_1 + q_2)(l_3 + q_3) + (2sinq_1 + cosq_1)l_1]q_4 \\ &+ [(l_3 + q_3)[cos(q_1 + q_2) + 2sin(q1 + q2)]q5 + [sin(q1 + q2) - 2cos(q1 + q2)]q_6 \end{aligned} \tag{18}$$

Hence, matrix **D** is given by:

$$\boldsymbol{D} = \begin{bmatrix} 1 & 0 & D_{13} \\ 0 & 1 & D_{23} \end{bmatrix} ; D_{13} = d_{31} / d_{33} \;\; ; \;\; D_{23} = d_{32} / d_{33}; d_{31} = \frac{\partial f}{\partial q_1}; d_{32} = \frac{\partial f}{\partial q_2}; d_{33} = \frac{\partial f}{\partial q_3} \tag{19}$$

where:

$$\begin{aligned} d_{31} &= [(cos(q_1 + q_2) + 2sin(q_1 + q_2))(l_3 + q_3) + (cosq_1 + 2sinq_1)l_1]; \\ d_{32} &= [cos(q_1 + q_2) + 2sin(q_1 + q_2))(l_3 + q_3)]; \\ d_{33} &= [sin(q_1 + q_2) - cos(q_1 + q_2)] \end{aligned} \tag{20}$$

The $\mathbf{Q}^{(1)}$,$\mathbf{Q}^{(1)}$ – generalized forces of inertia forces are calculated by means of the inertia matrix **A**. By means of the method of the transmission matrix, the elements of inertia matrix are follows [6-8]:

$$\begin{aligned} a_{11} &= m_1 r_1^T t_{11}^T t_{11} r_1 + m_2 r_2^T t_2^T t_{11}^T t_{11} t_2 r_2 + m_3 r_3^T t_3^T t_2^T t_{11}^T t_{11} t_2 t_3 r_3 + m r^T t_2^T t_{11}^T t_{11} t_2 r + J_1 + J_2 + J_3 = m_2 (l_1^2 + c_2^2) \\ &+ m_3 (l_1^2 + c_3^2 + q_3^2 + 2c_3 q_3 + 2l_1 \cos q_2 (c_3 + q_3) + m(l_1^2 + l_3^2 + c_1^2 + q_3^2 + 2q_3 l_3 + 2(l_1 \cos q_2 (l_3 + q_3)) \\ &+ J_1 + J_2 + J_3 \end{aligned}$$

$$\begin{aligned} a_{12} &= m_2 r_2^T t_{21}^T t_1^T t_{11} t_2 r_2 + m_3 r_3^T t_3^T t_{21}^T t_1^T t_{11} t_2 t_3 r_3 + m r^T t_3^T t_{21}^T t_1^T t_{11} t_2 t_3 r + J_2 + J_3 = m_2 (c_2^2 + c_2 l_1 \cos q_2) \\ &+ m_3 (c_3^2 + q_3^2 + c_3 q_3 + l_1 \cos q_2 (c_3 + q_3)) + m(l_3^2 + q_3^2 + q_3 l_3 + l_1 \cos q_2 (l_3 + q_3)) + J_2 + J_3 \end{aligned}$$

$$a_{13} = m_3 r_3^T t_{31}^T t_2^T t_1^T t_{11} t_2 t_3 r_3 + m r_3^T t_{31}^T t_2^T t_1^T t_{11} t_2 t_3 r_3 = (m + m_3) l_2 \sin q_2$$

$$\begin{aligned} a_{22} &= m_2 r_2^T t_1^T t_{21}^T t_1^T t_1 t_{21} r_2 + m_3 r_3^T t_3^T t_{21}^T t_1^T t_1 t_{21} t_3 r_3 + m r_3^T t_3^T t_{21}^T t_1^T t_1 t_{21} t_3 r = m_2 c_2^2 + m_3 (c_3^2 + q_3^2 + 2c_3 q_3) \\ &+ m(l_3^2 + q_3^2 + 2l_3 q_3) + J_2 + J_3 \end{aligned}$$

$$a_{23} = m_3 r_3^T t_{31}^T t_2^T t_1^T t_1 t_{21} t_3 r_3 + m r_3^T t_{31}^T t_2^T t_1^T t_1 t_{21} t_3 r = m l_1 \sin q_2$$
$$a_{33} = m_3 r_3^T t_{31}^T t_2^T t_1^T t_1 t_2 t_{31} r_3 + m r^T t_{31}^T t_2^T t_1^T t_1 t_2 t_{31} r = (m + m_3) \qquad (21)$$

$\boldsymbol{Q}^{(1)}, \boldsymbol{Q}^{(*)}$ - (3x1) matrices can be written as:

$$\mathbf{Q}^{(1)} = \begin{bmatrix} Q_1^{(1)} & Q_2^{(1)} & Q_3^{(1)} \end{bmatrix}^T;$$
$$Q_1^{(1)} = 0.5\mathbf{q}^T \mathbf{D_1 A q}; \qquad Q_2^{(1)} = 0.5\mathbf{q}^T \mathbf{D_2 A q}; \qquad Q_3^{(1)} = 0.5\mathbf{q}^T \mathbf{D_3 A q} \qquad (22)$$
$$\mathbf{Q}^{(2)} = \mathbf{D}_1 \mathbf{A} \mathbf{q}_1^* + \mathbf{D}_2 \mathbf{A} \mathbf{q}_2^* + \mathbf{D_3 A} \mathbf{q}_3^*;$$

where $\boldsymbol{q}, \boldsymbol{q}^*$ are (3x1) matrices and $\boldsymbol{D_i A}(i = \overline{1, n})$ is a (3x3) matrix:

$$\boldsymbol{q} = \begin{bmatrix} q_4 & q_5 & q_6 \end{bmatrix}^T; \ \boldsymbol{q}_1^* = \begin{bmatrix} q_4^2 & q_5 q_4 & q_6 q_4 \end{bmatrix}; \ \boldsymbol{q}_2^* = \begin{bmatrix} q_4 q_5 & q_5^2 & q_6 q_5 \end{bmatrix}; \ \boldsymbol{q}_3^* = \begin{bmatrix} q_4 q_6 & q_5 q_6 & q_6^2 \end{bmatrix}$$

$$\mathbf{D}_1 \mathbf{A} = \begin{bmatrix} \frac{\partial a_{11}}{\partial q_1} & \frac{\partial a_{12}}{\partial q_1} & \frac{\partial a_{13}}{\partial q_1} \\ \frac{\partial a_{12}}{\partial q_1} & \frac{\partial a_{22}}{\partial q_1} & \frac{\partial a_{23}}{\partial q_1} \\ \frac{\partial a_{13}}{\partial q_1} & \frac{\partial a_{23}}{\partial q_1} & \frac{\partial a_{33}}{\partial q_1} \end{bmatrix} = \mathbf{0}; \ \mathbf{D}_2 \mathbf{A} = \begin{bmatrix} \frac{\partial a_{11}}{\partial q_2} & \frac{\partial a_{12}}{\partial q_2} & \frac{\partial a_{13}}{\partial q_2} \\ \frac{\partial a_{12}}{\partial q_2} & \frac{\partial a_{22}}{\partial q_2} & \frac{\partial a_{23}}{\partial q_2} \\ \frac{\partial a_{13}}{\partial q_2} & \frac{\partial a_{23}}{\partial q_2} & \frac{\partial a_{33}}{\partial q_2} \end{bmatrix}; \mathbf{D_3 A} = \begin{bmatrix} \frac{\partial a_{11}}{\partial q_3} & \frac{\partial a_{12}}{\partial q_3} & \frac{\partial a_{13}}{\partial q_3} \\ \frac{\partial a_{12}}{\partial q_3} & \frac{\partial a_{22}}{\partial q_3} & \frac{\partial a_{23}}{\partial q_3} \\ \frac{\partial a_{13}}{\partial q_3} & \frac{\partial a_{23}}{\partial q_3} & \frac{\partial a_{33}}{\partial q_3} \end{bmatrix}; \qquad (23)$$

The equations of motion for robotic arm are of the form:

$$\begin{bmatrix} 1 & 0 & D_{31} \\ 0 & 1 & D_{32} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} q_7 \\ q_8 \\ q_9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & D_{31} \\ 0 & 1 & D_{32} \end{bmatrix} \left\{ \begin{bmatrix} Q_1^{(0)} \\ Q_2^{(0)} \\ Q_3^{(0)} \end{bmatrix} + \begin{bmatrix} Q_1^{(1)} \\ Q_2^{(1)} \\ Q_3^{(1)} \end{bmatrix} + \begin{bmatrix} Q_1^{(*)} \\ Q_2^{(*)} \\ Q_3^{(*)} \end{bmatrix} + \begin{bmatrix} M_1 \\ M_2 \\ F \end{bmatrix} \right\} \qquad (24)$$

which can be written as:

$$(a_{11} + D_{31})q_7 + (a_{12} + D_{31}a_{23})q_8 + (a_{13} + D_{31})q_9 = Q_1^{(0)} + Q_1^{(1)} + Q_1^* + D_{31}(\mathrm{Q}_3^{(0)} + Q_3^{(1)} + Q_3^*) + M_1 + D_{31}F$$
$$(a_{12} + D_{32})q_7 + (a_{22} + D_{32}a_{23})q_8 + (a_{23} + D_{32})q_9 = Q_2^{(0)} + Q_2^{(1)} + Q_2^* + D_{32}(\mathrm{Q}_3^{(0)} + Q_3^{(1)} + Q_3^*) + M_2 + D_{32}F \qquad (25)$$

The system of equations (25) and (18) to solve the problem is a system of differential equations.

In the work [5], a solution to the problem is proposed by solving the secondary differential equations when the equations of motion are expressed as:

$$f_1 \equiv d_{31}q_4 + d_{32}q_5 + d_{33}q_3 = 0 \qquad (26)$$

The problem is solved by the system of equations (25) and (27) now

$$f_2 \equiv \frac{df_1}{dt} = d_{31}q_7 + d_{32}q_8 + d_{33}q_9 + \left(\frac{\partial d_{31}}{\partial q_1} q_4 + \frac{\partial d_{31}}{\partial q_2} q_5 + \frac{\partial d_{31}}{\partial q_3} q_6\right) q_4)$$
$$+ \left(\frac{\partial d_{32}}{\partial q_1} q_4 + \frac{\partial d_{32}}{\partial q_2} q_5 + \frac{\partial d_{32}}{\partial q_3} q_6\right) q_5 + \left(\frac{\partial d_{33}}{\partial q_1} q_4 + \frac{\partial d_{33}}{\partial q_2} q_5 + \frac{\partial d_{33}}{\partial q_3} q_6\right) q_6 \qquad (27)$$

To solve these equations, it is possible to use software. Here the Maple software is used.

Results of numerical simulation

Numerical simulation of robotic arm is performed with the following parameters:

l_1 = 1 m, l_3 = 0.5 m, m_1 = 1 kg, m_2 = 1 kg, m_3 = 1 kg, m = 5 kg, α_1 = 0 rad, α_2 = 0 rad, α_3 = 0.1 rad, c_1 = 0.5 m, c_2 = 0.25 m, c_3 = 0.25 m, J_1 = 0.02 kgm^2, J_2 = 0.01 kgm^2, J_3 = 0.05 kgm^2, g = 10 m/s^2, l_0 = 0.01 m, M_1 = 25 Nm, M_2 = 0.1 Nm, F = 0.05 N.

The initial conditions are:

$q_1(0)$ = 0 rad, $q_2(0)$ = 0 rad, $q_3(0)$ = 0 m, $q_4(0)$ = 0.03 rad/s, $q_5(0)$ = -0.01 rad/s, $q_6(0)$ = 0 m/s,

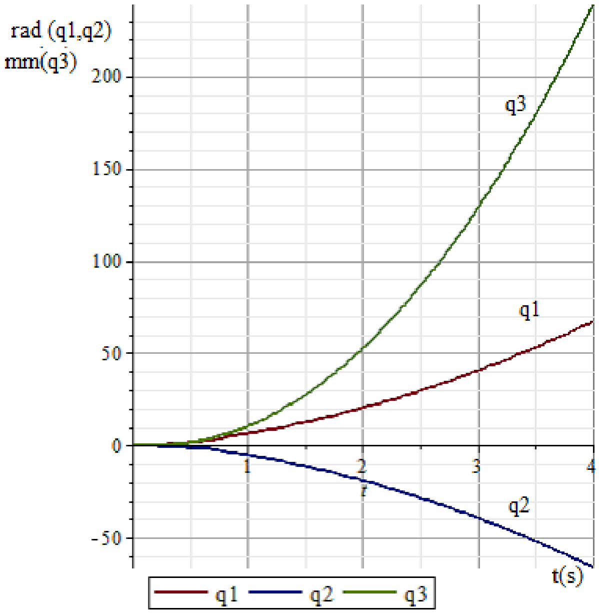

Figure 2. Graph of rotation angle q_1 and q_2 and the displacement q_3 of the plunger.

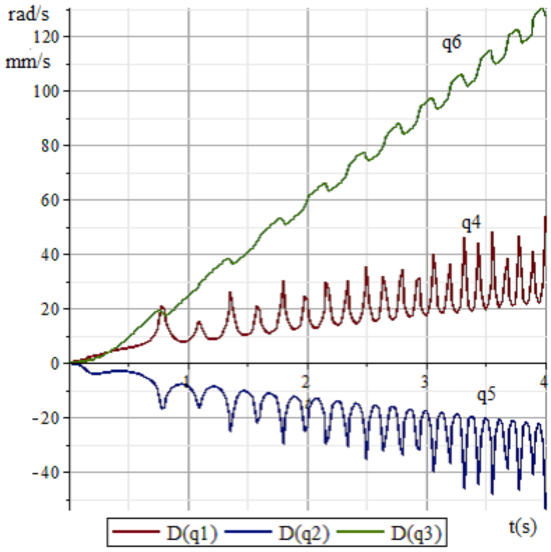

Figure 3. Graph of angular velocities q_4, q_5 and velocity q_6.

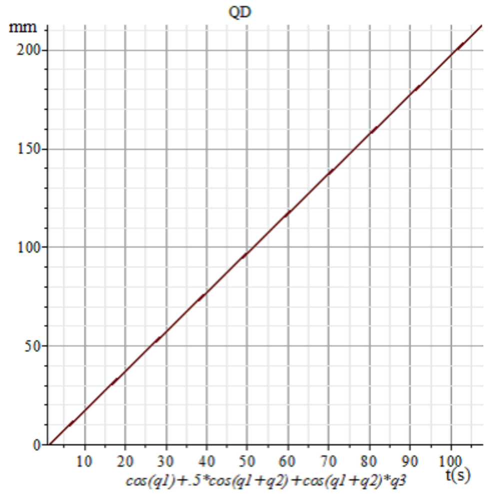

Figure 4. Graph of orbital motion.

4. CONCLUSIONS

In the paper a new method for controlling the manipulator to move a load along a required trajectory is considered. Such problem belongs to type of controlling the program motion. This is one of the most important problems in controlling manipulators. This paper proposes a method for the problem of interest based on of a new point of view that the program motion can be treated as a first integral of the considered system.

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