

AN APPLICATION OF WAVELET THEORY TO ELECTROCARDIOGRAMS SINGULAR POINTS ANALYSIS

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ABSTRACT

In recent years, wavelet transform-based algorithms have been developed and successful in the signal processing in many areas since the wavelet transform is a useful and powerful tool for analysing signals in both transient and stationary intervals. In this paper, a report is made on an application of the mentioned capability of wavelet transform for grasping subtle changes and discontinuities in electrocardiogram (ECG) signals. From the signal processing point of view, it shows that the diagnostic of ECG signals is carried out by excluding any recognised abnormality as singular points or irregular structures. For analyzing purpose, wavelet footprints are characterized by scale-space vectors in a discrete model by discontinuing signals in piecewise polynomial. And a focus is made on applications of fetal ECG detection from maternal ECG.

1. INTRODUCTION

Electrocardiography (ECG) still plays a basic role in cardiology being a effective, simple, non-invasive graphy with low cost procedures for carrying out diagnoses of cardiovascular disorders (high epidemiologic incidence). Pathological alterations observable by electrocardiography can be divided into three main areas, namely cardiac rhythm disturbances (arrhythmia), dysfunction of myocardial blood perfusion (cardiac ischemia) and chronic alteration of the mechanical structure of the heart (for example, left ventricular hypertrophy) [1].

ECG signal represents the variation of electrical potential during the cardiac cycle as recorded between surface of electrodes on the body of patient. Characteristic shape of this signal is resulted from an action potential propagating within the heart and causing a constraction of various portions of the cardiac muscle. This internal excitation starts at the sinus node which acts as a pacemaker, and then spreads to the atria creating characteristic P wave in the ECG. The cardiac excitation then reaches to the ventricles (ventricular depolarization) giving rise to the characteristic QRS complex. Once the ventricles have been completely stimulated (ST segment of the ECG), they repolarize corresponding to the T wave of the ECG. The detection and timing of these waves is very important for diagnostic purposes [2].

From the signal processing point of view, the diagnostic of the ECG is made by excluding any recognised abnormality as singular points or irregular structures as shown in Fig.1 [1].

Recently, researchers in applied mathematics and signal processing have developed powerful wavelet methods for multiscale representation and analysis of signals. Unlike the traditional Fourier techniques, wavelet methods localize the information in time-frequency

plane; in particular those methods are capable of grasping subtle changes and discontinuities in signals [3, 4].

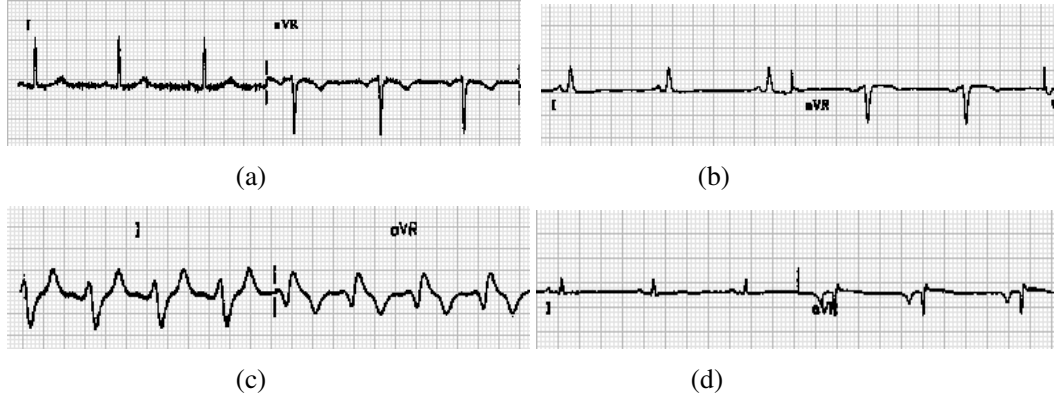


Figure 1. (a) Normal adult 12-lead ECG (Lead I, aVR shown only)
 (b) Left ventricular hypertrophy (Lead I, aVR shown only);
 (c) Hyperkalaemia (Lead I, aVR shown only);
 (d) Right atrial hypertrophy (Lead I, aVR shown only).

In the second paragraph, an introduction is briefly made on wavelet transform and discontinuities analysis which gives rise to reported applications and proposed algorithm in the third one for period detection, discontinuous points in ECG signals in terms of wavelet footprints. The last paragraph is for discussions and directions for further researches to be carried out.

2. A BRIEF ON WAVELET TRANSFORM AND DISCONTINUITIES ANALYSIS

2.1. Continuous and Discrete Wavelet transform

A fundamental property of the wavelet transform is that the time and frequency resolutions vary in the time-frequency plane [3]. Wavelet transform is a linear transform and it has non-fixed *mother waveforms*. Wavelet transform permits one to choose suitable basic functions in its transforming to separate intended and unintended components in the analytic signal [4].

Definition of continuous wavelet transform [4]: A function $\psi \in L^1(\mathfrak{R}) \cap L^\infty(\mathfrak{R})$ with $\int_{-\infty}^{\infty} \psi(t) dt = 0$ is called a *wavelet*. For every $s \in L^p(\mathfrak{R}), 1 \leq p \leq \infty$, defines *Continuous wavelet transform* as follows:

$$W_\psi s(a, \tau) = \int_{-\infty}^{\infty} s(t) \frac{1}{\sqrt{a}} \psi^* \left(\frac{t - \tau}{a} \right) dt = \langle s, \psi_{a, \tau} \rangle_{L^2} \quad \text{For all } a, \tau \in \mathfrak{R}_+ \times \mathfrak{R} \quad (1)$$

where, a and τ are respective the dilation and translation parameter, and $\psi_{a, \tau} = \frac{1}{\sqrt{a}} \psi \left(\frac{t - \tau}{a} \right)$.

Function ψ is called *mother wavelet* and is chosen so that it is localized at $t = 0$, and some frequencies $f = fo > 0$ (and/or $f = -fo$). Mother wavelet has a property that the set $\{\psi_{a,\tau}\}$ forms an orthonormal basis in $L^2(\mathfrak{R})$. That is, the mother wavelet can generate any function in $L^2(\mathfrak{R})$. $\psi(t)$ must satisfy the admissibility condition [5] to ensure not to have any direct current (DC) component in daughter wavelet:

$$\int_{-\infty}^{\infty} |\Psi(\omega)|^2 |\omega|^{-1} d\omega < \infty. \quad (2)$$

In order to analyze discrete signal, the scale and shift parameters are discretized as:

$$a = 2^m; \tau = n.2^m, \text{ where } n, m \text{ are integers.} \quad (3)$$

The set of dilated and shifted versions of wavelets:

$$\{\psi_{m,n}(t)\}_{m,n \in \mathbb{Z}} = \{2^{-m/2} \psi(2^{-m}t - n)\} \quad (4)$$

forms a basis of **Discrete Wavelet Transform** in $L^2(\mathfrak{R})$. The discrete wavelet transform is a unique and stable decomposition of any finite energy signal $x(t)$ in terms of $\{\psi_{m,n}(t)\}_{m,n \in \mathbb{Z}}$ [6]:

$$x(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} d_{m,n} \psi_{m,n}(t) \quad (5)$$

where, wavelet coefficients $d_{m,n}$ in their terms are given by:

$$d_{m,n} = \langle x(t), \psi_{m,n}(t) \rangle \quad (6)$$

Theoretically, a complete representation of $x(t)$ requires a discrete wavelet family $\{\psi_{m,n}(t)\}$ of an infinite number of functions. However, a low-pass-natured complementary scaling function $\phi(t)$ play a key role in multiresolution analysis theory.

The wavelet function has zero average and each term $d_{m,n}$ measures a local variation of $x(t)$ at resolution of 2^m and the partial sum:

$$x_{J+1}(t) = \sum_{m=J+1}^{\infty} \sum_{n=-\infty}^{\infty} d_{m,n} \psi_{m,n}(t) \quad (7)$$

represents an approximation of $x(t)$ at resolution 2^{J+1} .

Approximation function $x_{J+1}(t)$ can be expressed in terms of shifted versions of a function called the scaling function as mentioned above:

$$x_{J+1}(t) = \sum_{n=-\infty}^{\infty} y_{J,n} \phi_{J,n}(t) \quad (8)$$

where, $\phi_{J,n}(t) = \frac{1}{2^{J/2}} \phi(t/2^J - n)$. Hence, any function in $L^2(\mathfrak{R})$ can be completely represented by using J -finite resolution of wavelet and scaling function as:

$$x(t) = \sum_{n=-\infty}^{\infty} y_{J,n} \varphi_{J,n}(t) + \sum_{m=-\infty}^J \sum_{n=-\infty}^{\infty} d_{m,n} \psi_{m,n}(t). \quad (9)$$

Scaling coefficients $y_{J,n} = \langle x(t), \varphi_{J,n}(t) \rangle$ tend to measure the local regularity of $x(t)$ at scale 2^J . Therefore, the first term of (9) represents a coarse version of $x(t)$ as opposed to the detail version provided by the second term of (9).

The wavelet function and the scaling function have a relationship but their link does not reduce to the expansion shown in (9) which describes a multiresolution structure of wavelet transform [7].

2.2. Periodicity detection

Theorem for periodicity detection by Haar wavelet: Given a T -periodic signal $x(t) \in L^2(\mathfrak{R})$, then $W_{\psi}x(a, \tau)$, continuous wavelet transform of $x(t)$ is T -periodic in time and $2T$ -periodic in scale with $\psi(t)$ is the Haar Wavelet.

Haar wavelet and scaling functions are given respectively by:

$$\psi(t) = 1_{[0, \frac{1}{2})} - 1_{[\frac{1}{2}, 1)}, \quad \varphi(t) = 1_{[0, 1)} \quad (10)$$

where $1_{[a,b)}$ denotes the characteristic function equaling to 1 on $[a,b)$ and zero everywhere else.

Haar wavelet has one vanishing moment and finite support. That is: $\int_{-\infty}^{\infty} \varphi(t) dt = 0$.

A wavelet coefficient is obtained by correlating signal with daughter wavelet at a specific time and scale. Since Haar wavelet is piecewise constant wavelet degree two resulting to above theorem. An example is illustrated in Fig. 2a with continuous wavelet transform carrying out for a simple periodic signal (sine). In the case where the period of signal is slowly varying, the effect is shown in Fig. 2b. It is useful in sinus arrhythmia diagnosing.

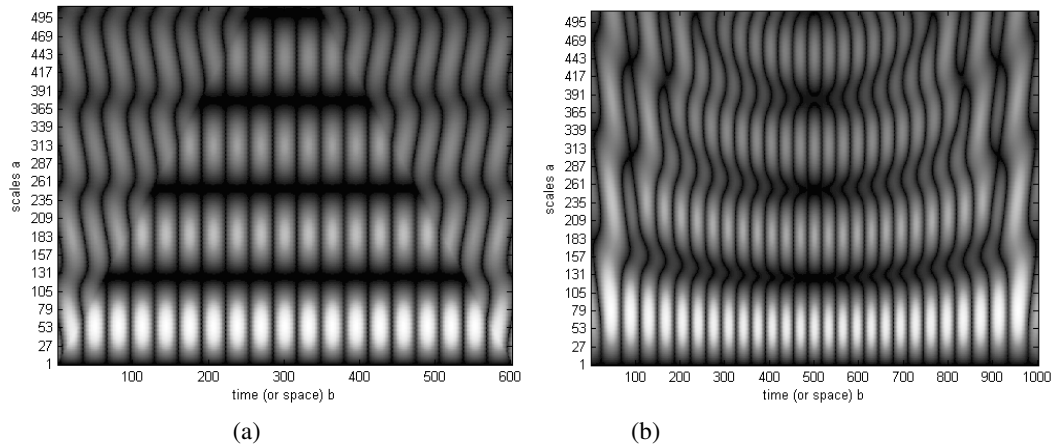


Figure 2. Lattice structure of Wavelet transform of periodic signal (a), slowly varying in period (b)

2.3. Discontinuities analysis

Singular points and irregular structures often carry important information in ECG signal for diagnosing purpose [9]. In this paper, we restrict our attention to Haar wavelets because it is well suited for discontinuities analysis.

Consider a signal made of a single Dirac at t_0 , $x(t) = a\delta(t - t_0)$, J level wavelet decomposition of this signal as:

$$x(t) = \sum_{n=-\infty}^{\infty} y_{J,n} \varphi_{J,n}(t) + \sum_{m=-\infty}^J \sum_{n=-\infty}^{\infty} d_{m,n} \psi_{m,n}(t). \quad (11)$$

Move from discontinuous- to discrete-time presentation for signal $x[n]$ with $n = 0 \dots N - 1$ and use Haar wavelets, one has:

$$x[n] = \sum_{l=0}^{N/2^J-1} y_l \varphi_{J,l}[n] + \sum_{j=1}^J \sum_{l=0}^{N/2^j-1} d_{j,l} \psi_{j,l}[n]. \quad (12)$$

Haar wavelet has one vanishing moment and finite support, therefore only a limited number of wavelets overlapping the location t_0 is influenced by this Dirac. The set of points that t_0 is included in the support of wavelet is called *cone of influence* [3]. The cone of influence is illustrated in Fig. 3.

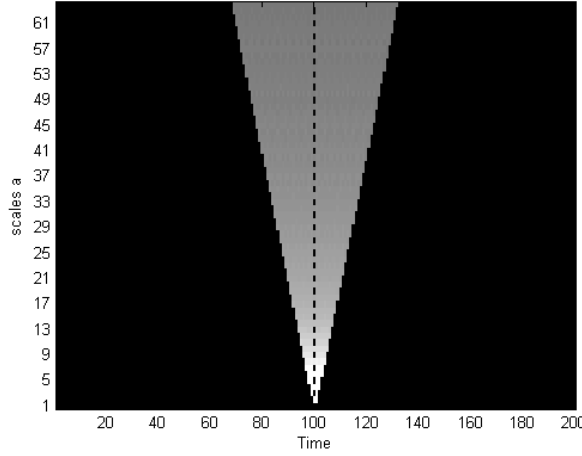


Figure 3. The cone of influence of a Dirac using CWT with Haar wavelet

The only non-zero wavelet coefficients of (12) are coefficients in this cone of influence. Thus (12) becomes:

$$x[n] = \sum_{l=0}^{N/2^J-1} y_l \varphi_{J,l}[n] + \sum_{j=1}^J d_{j,k_j} \psi_{j,k_j}[n], \text{ with } k_j = \lfloor k / 2^j \rfloor \quad (13)$$

this leads to the definition of footprints.

Definition of Wavelet footprint [8]: Given a signal with only one Dirac at position k , a scale-space vector obtained by gathering together all the wavelet coefficients in the cone of influence of k , then imposing its norm equal to 1 is called a footprint $f_k[n]$.

In term of wavelet basic, this footprint can be written as:

$$f_k[n] = \sum_{j=1}^J c_{j,k_j} \psi_{j,k_j}[n] \text{ with } c_{j,k_j} = d_{j,k_j} / \sqrt{\sum_{j=1}^J d_{j,k_j}^2}. \quad (14)$$

A signal with step discontinuity at k can be expressed in terms of the scaling functions and footprint as follows:

$$x[n] = \sum_{l=0}^{N/2^J-1} y_l \phi_{J,l}[n] + \alpha f_k[n] \text{ with } \alpha = \langle x, f_k[n] \rangle = \sum_{j=1}^J c_{j,k_j} d_{j,k_j} \quad (15)$$

3. APPLICATIONS

3.1. Period detection and sinus arrhythmia

Sinus arrhythmia shown in Fig. 4 is characterised by variations in heart rate from beat to beat which are greater than that would be expected from normal respiratory variation. It is irregular due to fluctuations of autonomic tone resulting in phasic changes of discharge rate. During inspiration, the parasympathetic tone falls and the heart rate quickens, on expiration the heart rate falls.



Figure 4. Sinus Arrhythmia

Modulus maxima of wavelet transform of the periodic signal form a lattice in time-frequency plane. Measuring the distance between vertices will disclose periodicities in the signal. Due to the variation in period of the signal, the lattice patterns of modulus maxima will not be squares.

3.2. Fetal ECG detection

Accurate detection of fetal ECG signal during pregnancy has the potential to provide important informations in diagnosing fetal cardiac diseases, especially fetal arrhythmias. It is not possible to recognize the fetal ECG signal from that of a pregnant woman using standard ECG leads. However, when using a wrist leads placed over the maternal abdomen on both sides of the

uterine fundus, the amplitude of maternal QRS complex will be reduced by 90% as compared to that of the standard case, meanwhile the fetal QRS will be easier to detect [10].

Since 1960, many different methods have been developed for detecting the fetal ECG. Most of those focus on multi-channel mixtures of signals [11]. A direct method subtract a thoracic maternal ECG from the abdominal composite ECG, and other more recent employs Independent Component Analysis, which extracts fetal ECG by assuming independently statistical sources [12].

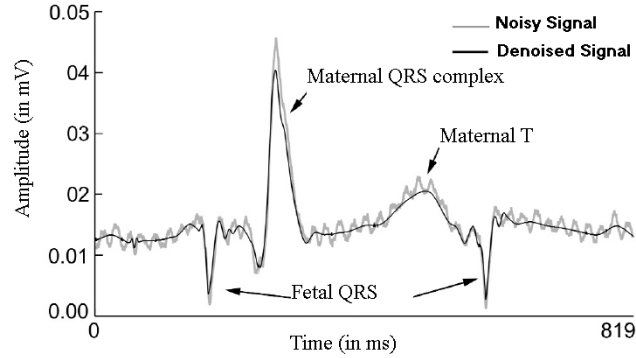


Figure 5. The fetal QRS complex in maternal ECG signal considered as discontinuity

As shown in Fig. 5, the fetal QRS complex can be clearly seen in the maternal ECG signal. The period of fetal QRS is faster than that of maternal. It appears once or twice and orientation of the fetal heart is opposite to that of the maternal heart. The Fetal QRS complex can be seen as a discontinuous locations in the maternal ECG signal. Hence, an algorithm to estimate the location of these discontinuities is proposed as follows:

1. Estimate the period of signal by using theorem for periodicity detection by Haar wavelet.
2. Compute discrete Harr wavelet transform coefficients within a period signal:

$$\text{Set } c_i^0 = x[i]; i = 1, \dots, N - 1;$$

$$\text{Compute scaling function coefficients: } c_i^j = \frac{c_{2i}^{j-1} + c_{2i+1}^{j-1}}{\sqrt{2}};$$

$$\text{Compute wavelet function coefficients: } d_i^j = \frac{c_{2i}^{j-1} - c_{2i+1}^{j-1}}{\sqrt{2}}.$$

3. Set universal threshold $T = \sigma\sqrt{2 \ln N}$ (due to noise with variance σ).
4. Compute $D = \sum_j d_{k_j}^j$ with $k_j = \lfloor k / 2^j \rfloor$, then if $D \geq T$, a discontinuity is found at k .
5. Eliminate all discontinuities of the ECG signal with the standard lead (no fetal influence), a discontinuity coressponding to the fetal QRS complex is obtained.
6. Reconstruct signal corresponding to the obtained footprints (the fetal QRS complex).

4. CONCLUSION

Wavelet footprints form an overcomplete basis and efficient at analysing and representing the singular structures of a signal. In this paper, a proposed method of detecting the singular points like fetal QRS complex in maternal ECG signal with wavelet footprints has been reported. The method is found to be simple and efficient in diagnosing and treating fetal arrhythmias.

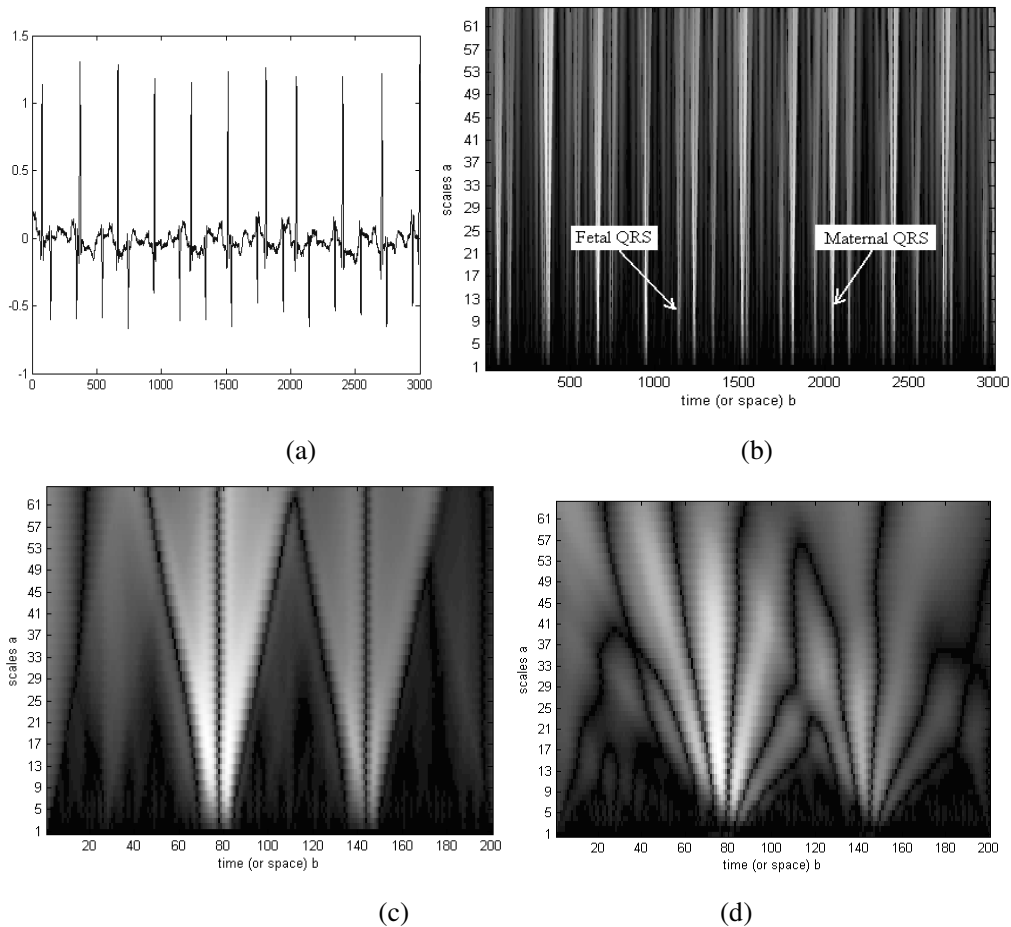


Figure 6. The fetal influence (weaker and faster) on ECG signal from pregnant woman (a)
Wavelet footprints to recognize (by coefficients) the fetal influence (b)
Fetal wavelet footprint and maternal wavelet footprint (c)
in a period comparing to CWT of ECG with the fetal influence using db4 wavelet (d)

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TÓM TẮT

ÁP DỤNG BIẾN ĐỔI SÓNG CON PHÂN TÍCH TÍN HIỆU ĐIỆN TIM DỰA TRÊN CÁC ĐIỂM KÌ DỊ

Trong những năm gần đây, các thuật toán dựa vào biến đổi wavelet đã được phát triển và áp dụng thành công vào việc xử lý tín hiệu ở nhiều lĩnh vực khác nhau vì biến đổi wavelet có khả năng phân tích các thuộc tính dừng và quá độ ở tín hiệu, nhất là khả năng phân biệt các thay đổi đột biến ở các tín hiệu. Trong bài báo này, theo điểm xử lý tín hiệu, việc chuẩn đoán thực hiện bằng cách ghi nhận các bất thường ở tín hiệu điện tim (ECG) như các điểm kì dị hay các cấu trúc bất thường.

Bài báo gồm 04 phần ngoài phần giới thiệu. Trong phần thứ hai, biến đổi wavelet, giải tích các điểm kì dị (gãy) và các vết trên cơ sở biến đổi wavelet Haar được giới thiệu một cách ngắn gọn làm cơ sở để áp dụng vào mô hình các đột biến nhằm đề xuất thuật toán phân tích tín hiệu ECG cũng như các kết quả trong ứng dụng tách tín hiệu điện tim của thai nhi từ tín hiệu điện tim của người mẹ trong phần thứ ba. Cuối cùng là các nhận xét và định hướng nghiên cứu tiếp.

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