Vietnam Journal of Science and Technology ***56*** *(4) (2018) 503-514*

DOI: 10.15625/2525-2518/56/4/10671

EFFICIENT OPTIMIZATION OF PRESSURE REGULATION IN WATER DISTRIBUTION SYSTEMS USING A NEW-RELAXED PRESSURE REDUCING VALVE MODEL

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Received: 26 July 2017; Accepted for publication: 3 June 2018

Abstract. Water loss is a global problem. Water loss in leakages in water distribution systems (WDSs) can be reduced by regulation of the average excessive pressure through optimizing operations of Pressure Reducing Valves (PRVs) placed in WDSs. This optimization task can be formulated as a nonlinear program (NLP). Since the pressure settings of PRVs are control variables determining to overall pressure of the system, the PRV model should be accurate and can account all scenarios occurring in WDSs in practice. The PRV models, having been used until now, either cannot describe complete operation modes of real PRVs or is complicated. In this paper, we develop a mathematical model for PRVs capable of describing complete operation modes of PRVs in practice. In addition, we show that the PRV model equation can be relaxed so that it is useful for formulation of NLP. With the proposed PRV model, the formulated NLP can be solved by NLP algorithms based gradient methods to high quality solution and in less computation time. Two benchmarks of WDSs are taken to demonstrate the efficiency of the proposed PRV model.

Keywords: leakage reduction; nonlinear optimization; EPANET, water distribution systems.

Classification numbers: 5.2.1; 5.2.2; 5.2.3

1. INTRODUCTION

Excessive water loss is a global problem and it reaches a significant part of total water production. Water losses in water distribution systems (WDSs) comprise real losses and apparent losses. The real losses are mainly caused by leakages at network fittings, pipe joints, breaks and/or bursts in pipes while the apparent losses are due to inaccurate meters and/or unauthorized consumption [1-4]. Pressure management by reducing of excessive pressure in a WDS is one of the most cost-effective measures to control water leakages. Water leakages can be considered as additional demands at nodes and mathematically modeled as proportional relation to the nodal pressure [1-3]. The leakage amount in a WDS increases significantly when operating at an excessive pressure. By lowering the pressures in WDSs to appropriate levels, water leakages can be decreased, the probability of creating new leakages is minimized, and the amount of energy wasted by pumping at unnecessarily high levels of pressure is also reduced

[5]. Pressure reducing valves (PRVs) are commonly installed in WDSs for regulation of pressure management [1-5]. The task of operators is to determine optimal operations of PRVs (i.e., pressure settings) so as to minimize excessive pressure in a WDS. This engineering issue can be casted as a nonlinear program (NLP) [1, 3-5].

There are many solution approaches having been used to solve the NLP. Ghaddar et al. [4] developed a new model of PRV in the NLP for optimal pressure management based on the fact that when a PRV is placed in one link, it will increase the headloss across the link. Although this model does not require additional variable needed for representing PRV openings, it can only describe two operation modes of real PRVs. Such the model is not appropriate for the case where PRV acts as check valve to prevent reverse flows. In order to reach globally optimal solution, the authors also transformed the NLP into a polynomal form and it can be efficiently solved by using hierarchy of semidefinite (SDP) relaxations.

To enhance regulation of pressure and reduce more water leakage, authors in [5] presented an approach to optimize operation schedules of both boundary and internal PRVs for 24 hours to minimize the leakage flows in the Domestic Metter Areas (DMAs) by formulating and solving a NLP. As a result, the control flow modulation curves for the PRVs, i.e., the relations between the flows and the pressure settings which are essential for online PRV control, were deduced. Vairavamoorthy and Lumbers [6] formulated a NLP for optimal pressure regulation to minimize the leakage flow in WDSs, in which the nodal pressures are allowed to be lower than their minimum values by a minor violation in order to achieve a higher decrease of leakage amount in a WDS. The sequential quadratic programming (SQP) approach was used to solve the NLP. The disadvantage of this approach is due to the non-smooth model of PRV. Genetic algorithms (GAs) combined with a hydraulic simulator, EPANET 2, were also used to determine optimal outlet pressures of PRVs for optimal pressure regulations in WDSs [7-9]. From the optimal solution, outlet pressures of a PRV can be adjusted continuously by flow modulating schedule. The method of sequential convex programming (SCP) proposed in [10] was also used to determine optimal pressure settings for PRVs so as to regulate pressure of DMAs. The idea of this method lies in the fact, in each iteration, the nonlinear and non-convex equality constraints are linearized to obtain a linear program (LP) and it can be solved efficiently by LP solver.

The quality and accuracy of the NLP solution highly depends the optimization model where the PRV model is essentially important. The PRV models, having been used until now, either cannot describe complete operation modes of real PRVs or is complicated.

Recently, the author of this paper developed a PRV model which can describe full operation modes of real PRVs [3]. However, this model is strongly nonlinear due to the introduction of variables (i.e., representing PRV openings) which makes NLP solvers difficult to solve the formulated optimization problem or requires a huge computation time. A PRV model combining characteristics in [3] and [4] should be developed.

In this paper, our contribution is to develop a new-relaxed mathematical model for PRVs which is capable of describing complete behaviors of PRVs in practice (*fully opened*, *normal*, and *check valve* modes). In addition, we show that the equality of PRV model can be relaxed into an inequality one allowing the formulated NLP to be solved by available NLP algorithms in less computation time and resulting optimal solution has high quality.

The remainder of the paper is organized as following: In Section 2, the nonlinear optimization problem for optimal pressure management will be formulated. The derivation of a new- relaxed PRV model will be presented in Section 3. In Section 4, two case studies are taken to demonstrate the efficiency of our new relaxed-PRV model. Conclusion is presented in section 5.

2. FORMULATION OF OPTIMIZATION PROBLEM FOR OPTIMAL PRESSURE MANAGEMENT

The objective function to be minimzed is defined as the excessive pressure at all nodes in the WDS in the optimization time horizon T [3]

$$\min_{\mathbf{H},\mathbf{Q},v} F = \sum_{i=1}^{NJ}\sum_{k=1}^{T}\left(H_{i,k} - H_{i,k}^{L}\right) \tag{1}$$

where NJ is the total number of nodes and T=24 hours is the time horizon. We consider a WDS with NP pipes, NR reservoirs, and $NPRV$ pressure reducing valves (PRVs); $H_{i,k}$ is head at node i, at time interval k; $H_{i,k}^{L}$ is the minimum allowable head at node i.

The equality constraints consist of the following equations:

The continuity equation at node i [2]

$$\sum_{j,k} Q_{j,i,k} - d_{i,k} - l_{i,k} = 0; i = 1,...,NJ \tag{2}$$

The leakage amount, $l_{i,k}$, associated to node i [1].

$$l_{i,k} = C_L L_{t,i} p_{i,k}^{\gamma} \tag{3}$$

$$p_{i,k} = H_{i,k} - E_i; L_{t,i} = 0.5\sum_{j} L_{i,j} \tag{4}$$

where $p_{i,k}$ and E_i are pressure and elevation of node i, respectively; $L_{i,j}$ is length of pipe connecting node i to node j; C_L is given leakage coefficient; γ is leakage exponent [1]; $Q_{j,i,k}$ is the flow rate from node j to node i at time interval k; $d_{i,k}$ is the demand at node i, at time interval k

The energy equation for the pipe connecting node i to node j

$$H_{i,k} - H_{j,k} - \Delta H_{i,j,k} = 0;\ ij = 1,...,NP \tag{5}$$

where $H_{i,k}$ is the head at node i; $\Delta H_{i,j}$, is the head loss which can be computed either by the Hazen-Williams equation [1, 2]

$$\Delta H_{i,j.k} = \frac{10.67 L_{i,j}}{D_{i,j}^{4.87} C_{i,j}^{1.852}} Q_{i,j,k} \left|Q_{i,j,k}\right|^{0.852} \tag{6}$$

or by the Darcy-Weisbach equation [2]

$$\Delta H_{i,j,k} = \frac{8 L_{i,j} f}{g\pi^2 D_{i,j}^5} \left|Q_{i,j,k}\right| Q_{i,j,k} \tag{7}$$

where f is the roughness of the pipe which is implicitly calculated from the Colebrook-White equation [2]

$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{2.51}{\mathrm{Re}\sqrt{f}} + \frac{k}{3.71D}\right) \tag{8}$$

In equations (6) and (7) L, D, and C are the length, diameter, and the Hazen-Williams coefficient of the pipe, respectively; ε in (8) is the pipe roughness; V and ν are the velocity of flow through the pipe and the viscosity of the water, respectively. To employ equation (7) for formulation of optimization problem, we use a smooth form proposed in [11].

The energy equation for pressure reducing valve placed in link *i,j* proposed in [3].

$$\max\left(0, H_{i,k} - H_{j,k}\right) - v_{i,j,k}^{-1} R_{i,j} Q_{i,j,k}^2 = 0 \tag{9}$$

$$0 < v_{i,j,k} \le 1;\ 0 \le Q_{i,j,k};\ ij = 1,\ldots,NPRV \tag{10}$$

$$R_{i,j} = \frac{8K_{i,j}}{\pi^2 g D_{i,j}^4} \tag{11}$$

where $R_{i,j}$ is resistance of the PRV; $K_{i,j}$ is the head loss coefficient of the PRV when it operates at fully opended mode.

Bound constraints for flows, and heads ensure the WDS operated properly

$$H_i^L \le H_{i,k} \le H_i^U \tag{12}$$

$$Q_{i,j}^L \le Q_{i,j,k} \le Q_{i,j}^U \tag{13}$$

The resulting NLP problem has $\left(NJ + NP + NPRV + NR\right) \times T$ optimization variables. It can be seen that equation model of PRVs in Eq.9 is very nonlinear since there are fractional terms of $v_{i,j,k}^{-1} Q_{i,j,k}^2$ or billinear terms $v_{i,j,k}\left(H_{i,k} - H_{j,k}\right)$ which may make NLP solver difficult to solve the NLP problem or converged to low quality optimal solution.

3. A NEW-RELAXED PRESSURE REDUCING VALVE MODEL

Real PRVs can operate in one of three operation modes, namely *fully opened* (mode 1), *normal* (mode 2), and *check valve mode* (mode 3) [3]. Our contribution here is to develop a new-relaxed PRV model, which is capable of describing complete operation modes in practice while preserving relaxation property of PRV model in [4] (i.e., without the need of $v_{i,j,k}$).

In the *normal mode* (mode 1), the PRV adjusts its resistance to maintain the pressure on the downstream node to a preset value, model equation of the PRV is an inequality [2]

$$H_i - H_j > R_{i,j} Q_{i,j}^2 \tag{14}$$

when the PRV operate in the *fully opened mode* (mode 2) (i.e., the PRV cannot maintain preset value of pressure on the downstream node due to the pressures on the upstream and downstream side are less than the preset pressure), and model equation of the PRV is [2]

$$H_i - H_j = R_{i,j} Q_{i,j}^2 \tag{15}$$

In mode 3, the PRV acts as a *check valve* to prevent water flow in a reverse direction (i.e., when the pressure on the downstream side is higher than the one on the upstream side, $H_i < H_j$). The model equation of the PRV in this operation mode ensures following relation [2]

$$Q_{i,j} = 0 \tag{16}$$

It can be seen from Eq.14 and Eq.15 that when the PRV operates in *normal mode* or *fully opened mode*, it tends to increase the headloss across link *ij* where it is placed. Therefore, instead of introducing a coefficient to represent PRV opening $v_{i,j,k}$ as in Eq. 9 [3], the PRV model in these operation modes (*normal mode and fully opened mode*) can be represented by a one inequality which is so-called relaxed PRV model as following

$$H_i - H_j \geq R_{i,j} Q_{i,j}^2 \tag{17}$$

However, this model does not represent the check valve mode of PRVs. In particular, when $H_i < H_j$ (i.e., check valve mode), we cannot find $Q_{i,j}$ to satisfy equality (17). To tackle this issue, we extend (17) to incorporate *check valve mode* by using *max* function as

$$\max\left(0, H_i - H_j\right) \geq R_{i,j} Q_{i,j}^2 \tag{18}$$

To use this equality in formulation of NLP problem which can then be solved by NLP algorithms based gradient methods, we employ interior-point approximation method discussed in [12] to approximate the left-hand side of (18) into the smooth one as

$$\max\left(0, H_i - H_j\right) \simeq \frac{\left(H_i - H_j + \sqrt{\left(H_i - H_j\right)^2 + \varepsilon^2}\right)}{2}$$

The new- relaxed PRV model now is

$$H_i - H_j + \sqrt{\left(H_i - H_j\right)^2 + \varepsilon^2} \geq 2R_{i,j} Q_{i,j}^2 \tag{19}$$

where ε is chosen as $1.e-6$.

The newly proposed PRV model (Eq. 19) is different to the model equation reported in [3] in that it does not require introducing a coefficient $v_{i,j,k}$ for representing the opening of PRV while the PRV model [3] used it (Eq. 21). Therefore, our formulated NLP will have less optimization variables than the NLP formulated with the PRV model in [3] which explains the reason why using our new PRV model, the computation time can be significantly decreased. In addition, according to the use of $v_{i,j,k}$ the PRV model in [3] is less accurate because $v_{i,j,k}$ cannot reach to a zero value when PRV operates at fully closed mode. In particular, the NLP formulated with the new PRV model has $\left(NJ + NP + NR\right) \times T$ optimization variables while it is $\left(NJ + NP + NPRV + NR\right) \times T$ for the NLP formulated with the model in [3]. In next section, we will demonstrate the efficiency of our new PRV model with two case studies.

4. CASE STUDIES

4.1. Case study 1

We consider the WDS, as shown in Fig. 1 comprising of 37 links, 22 nodes, 3 reservoirs, which was used as a case study for the PRV localization in [1]. The data for links, the demand pattern, and reservoir heads for 24 hours, the discharge coefficient C_L and the leakage exponent parameter γ are taken from [1, 3]. Our objective is to compare the performance of the NLP problem formulated with our new proposed PRV model in (19) and the one formulated with the existing PRV model in (9) as reported in [3] in terms of objective function value and computation time. We consider scenarios on number of PRVs placed in the WDS. A nonlinear programming solver, IPOPT in [13], is employed to solve the formulated NLP problem. All computation experiments are accomplished on CPU-Pentium (R) Dual-Core 2.8 GHz, 3.0 GB RAM. Optimal results are given in Table 1, respectively. Except the case of 4 PRVs, both formulated NLP problems gives the same objective function values. However, it can be seen that for all cases, IPOPT took much less computation time for solving the NLP formulated with our new PRV model as compared with the one using PRV model in [3]. The reason for the reduction of computation time is due to the fact that using the new PRV model the formulated NLP has less optimization variables.

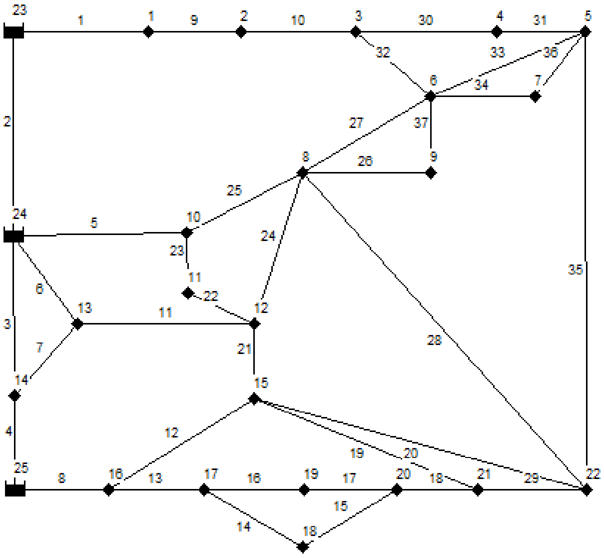

Figure 1. Water distribution system.

Table 1. Comparison of optimal solutions.

		New PRV model Eq.19		PRV model in Eq. 9 in [3]	
No. of PRVs	Links	Objective function value (m)	Computation time (s)	Objective function value(m)	Computation time (s)
2	11, 21	1937.450	0.272	1937. 450	1.590
3	11,21,20	1462.375	0.494	1462.425	3.660
4	11,21,20,1	1367.000	0.694	1431.174	3.545
5	11,21,20,1,5	944.196	0.739	945.179	5.054

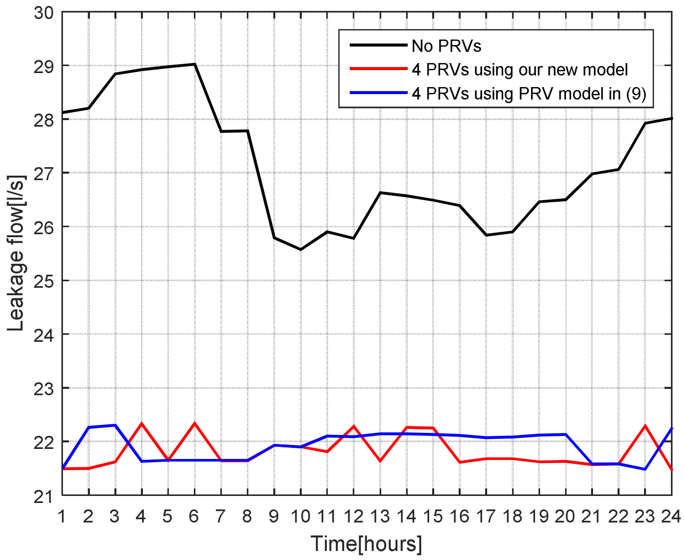

Figure 2. Water leakage flows.

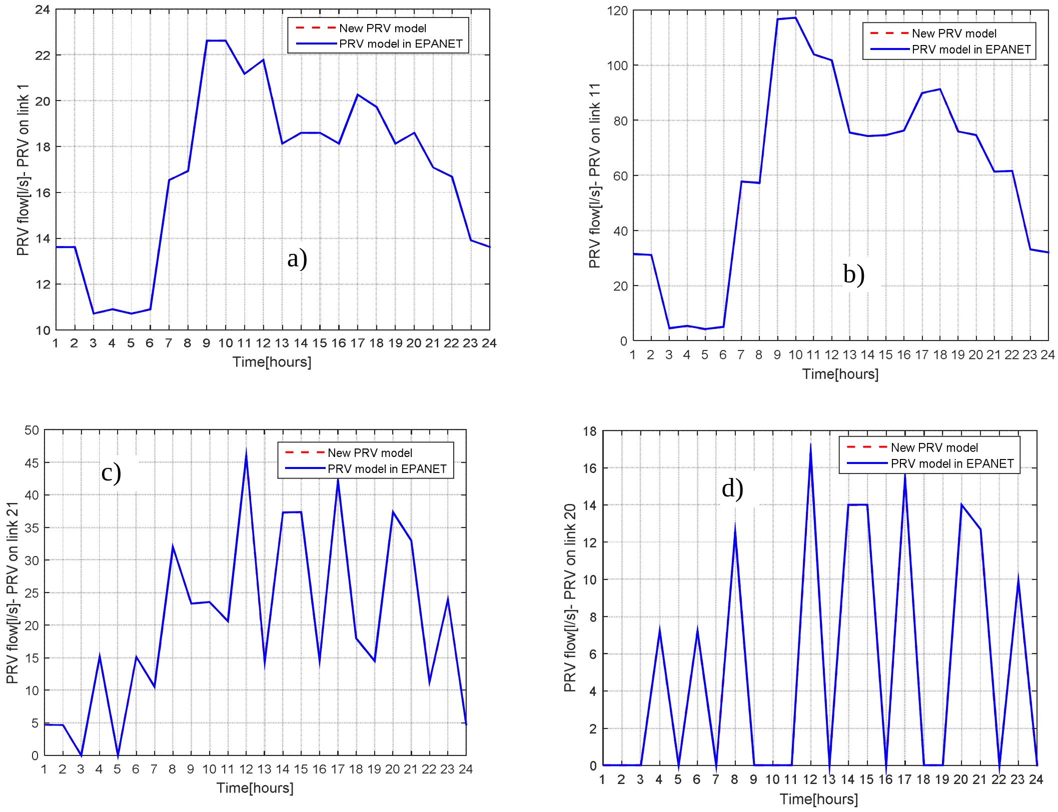

Figure 3. Comparisons of flows of 4 PRVs (Fig. 3(a) to 3(d) optimal flows with simulation flows).

In the case where 4 PRVs are placed in the WDS, using our new PRV model, the optimized pressure settings of PRVs result in an objective function value of 1367.000 (m) while it is 1431.174 (m) for the PRV model in [3] (Eq. 9).

The leakage flows for 24 hours in this case given in Fig. 2 imply that more water leakage is saved when our PRV model is used. In particular, water leakage amount per day is 523.42(L) while it is 526.10(L) when PRV model in [3] is used. The differences of optimal solutions from two NLPs are due to the fact that PRV model in Eq. 9 in [3] significantly depends on variable $v_{i,j,k}$ and its equation is highly nonlinear. This may make IPOPT to be converged to bad and inaccurate optimal solution.

To demonstrate the accuracy of the new PRV model with the most accurate, but non-smooth PRV model in a well-known hydraulic simulator, EPANET 2 [9], we compare the optimal flow rates through PRVs with their flow rates resulted by simulating the WDS using EPANET 2. The optimal flows (using our new PRV model) are plotted in Fig.3 in red lines while the flows resulted from simulation with EPANET 2 are blue lines. Once again, it can be seen that these two flows are almost the same implying that our new PRV model is indeed as accurate as non-smooth PRV model in EPANET 2. The advantage of the new model over the model in EPANET 2 is that it is smooth and can be used in formulation of NLP which can be solved by NLP algorithms based gradient method.

4.2. Case study 2

To demonstrate the efficiency and accuracy of the new PRV model, we consider the largest benchmark of water distribution system, EXET as shown in Fig. 4, with 2165 pipes, and 1892 nodes for optimal pressure management [14]. The data of the WDS as well as locations of PRVs are given in [3, 14]. The demand pattern values for T = 24 hours is given in Table 2.

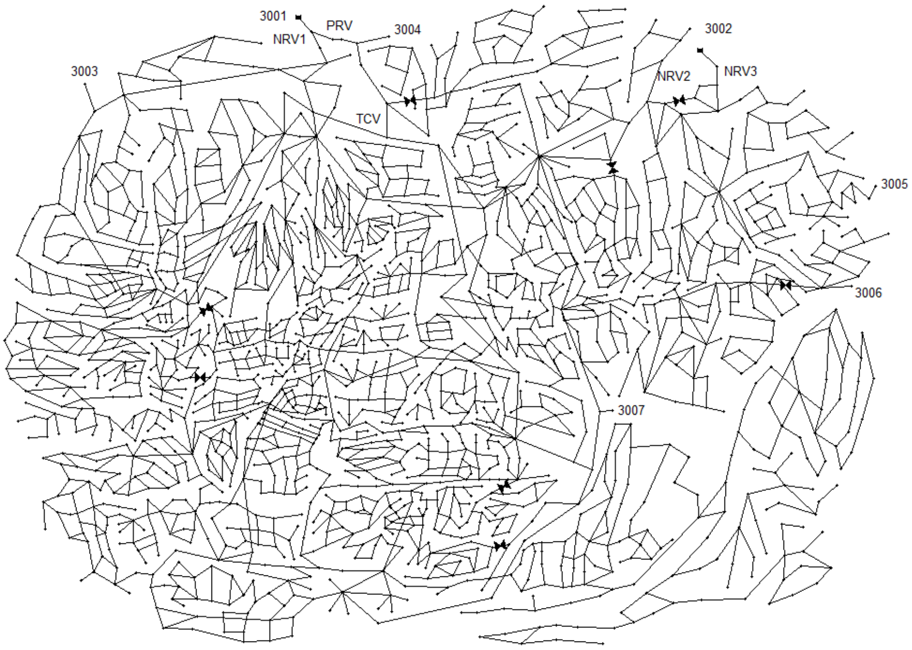

Figure 4. EXNET network.

The optimal pressure regulation problem is formulated as a large- scale NLP with more than 96000 optimization variables. Similar to case study 1, we compare the efficiency of using our new PRV model (Eq. 19) with the existing one in Eq. 9 in [3]. Two scenarios on number of PRVs are taken. The NLPs are solved to a highly accurate solution by setting the tolerance parameter in IPOPT to 1.e-7 and maximum number of iterations is set to 5000. Optimal results for the two scenarios and for two PRV models are given in Table 3, respectively.

It can be seen that using our new PRV model, IPOPT [13] solves NLPs for all scenario with computation time less than that required for solving the NLP formulated using existing PRV model in [3]. Moreover, using our PRV model, it results in an accurate solution with lower objective function values.

Table 2. Demand pattern values for 24 hours.

Time [hour]	Demand	Time [hour]	Demand
1:00	0.251	13:00	0.586
2:00	0.175	14:00	0.538
3:00	0.147	15:00	0.536
4:00	0.143	16:00	0.541
5:00	0.148	17:00	0.595
6:00	0.199	18:00	0.690
7:00	0.444	19:00	0.830
8:00	0.731	20:00	0.933
9:00	0.763	21:00	1.000
10:00	0.656	22:00	0.971
11:00	0.627	23:00	0.735
12:00	0.613	24:00	0.456

Table 3. Comparison of optimal solutions.

	New PRV model Eq.19		PRV model in Eq.9 in [3]	
No. of PRVs	Objective function value	Computation time (s)	Objective function value	Computation time (s)
3	2199559.437	395.3	2220930.959	687.8
8	1934641.424	283.4	2198714.117	482.5

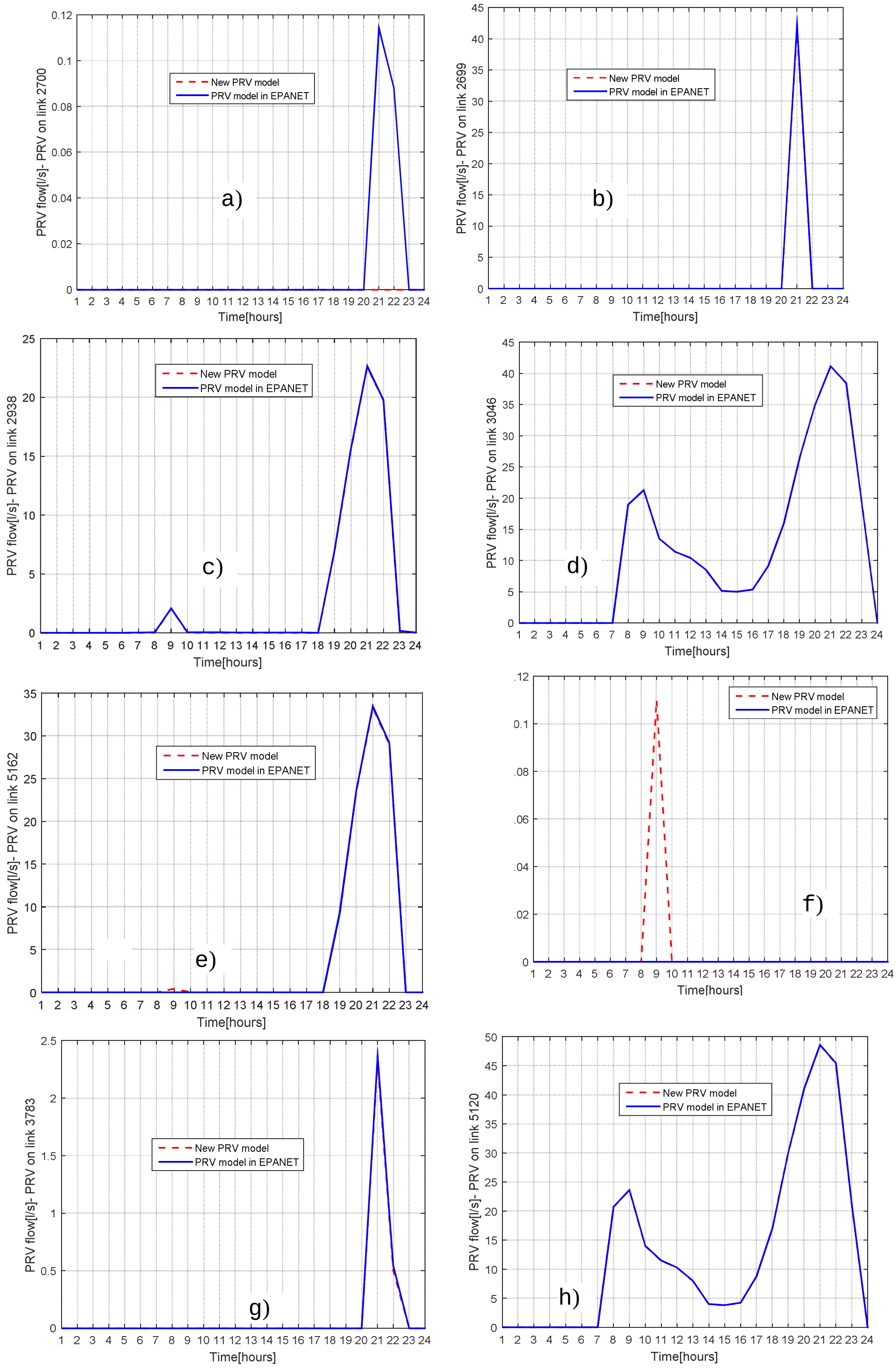

Figure 5. Comparisons of flows of 8 PRVs (Fig 5(a) to 5(h) optimal flows with simulation flows.

To evaluate the accuracy of the new PRV model in Eq.19, again we simulate the EXNET network using EPANET 2 [9] with optimal pressure settings of PRVs. Comparisons of PRV flows are demonstrated in Fig. 5(a) to 5(h). It can be seen that our developed PRV model is indeed accurate enough for employing in model based optimization for optimal pressure regulation. In particular, flows through all PRVs for 24 hours are nearly the same as flows resulted from simulation using EPANET 2 [9] except very small flows on PRV 4186 and 2700 (i.e., 0.105 L/s at time interval 9 in Fig. 5(a) and 5(f)).

In the two studied benchmarks, we have carried out the comparison of optimal solutions (objective function values and computation time) resulted by solving the two NLPs in which one NLP is formulated with the new PRV model and the other one is formulated with the existing PRV model in [3]. Resulting comparison given in Table 1 and 3 revealed that the new PRV model outperforms the PRV model in [3].

5. CONCLUSIONS AND FUTURE WORKS

This paper proposed a new-relaxed mathematical model for pressure reducing valves (PRVs) which on one hand describes complete operation modes for PRV in practice, and does not require additional variables for representing PRV openings as in existing PRV models on other hand. Two benchmarks of water distribution systems are taken to optimize their pressure management. The results have demonstrated that using our new- relaxed PRV model, the computation time for solving the formulated NLP for optimal pressure management can be significantly reduced. In addition, optimal solutions are achieved with high accuracy leading to lower objective function values. In future works, we will concentrate on simulation and optimization operations of WDSs using smoothing models for regulating devices including pumping stations.

Acknowledgement. This work was supported by the science and technology project " Study on operational optimization for large-scale water supply systems. Code: B2017-TNA-31" in 2017.

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