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Analysis of wave spectrum by Blackman-Tukey method and fast fourier method

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Abstract
Energy of surface waves is almost dissipated as propagating from shallow water to muddy flat and in mangrove forest. The study aims to analyze energy spectral density by using Blackman-Tukey (BT) and Fast Fourier Transform (FFT) methods in order to analyze the wave energy in mangrove areas in Cu Lao Dung (Soc Trang province). BT method is easy to use especially in short time series. Selection of lag number m is very important to determine the energy amount and number of peaks. Whereas, FFT method helps us to analyze the shift of spectral energy as waves propagate into shallower water. The results show that wave energy is dissipated from shallow water to muddy flat and into mangrove forests. The spectral energy shifts from low frequency to higher frequency as propagating into mangrove forests. This can prove the non-linear characteristics of waves in mangrove forests and the complicated hydrodynamic processes in mangrove forests.

Keywords: Wave spectrum, Blackman-Tukey method, FFT method, mangrove forests, Cu Lao Dung.

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INTRODUCTION

Surface waves from the sea to mangrove areas are an important impact factor for mangrove environments. The wave energy is almost dissipated as propagating from shallow water to mangrove forests. The studies on wave energy dissipation in mangrove forests are very numerous, but are not popular in mangrove forests in Vietnam, especially in Southern Vietnam. In reality, there are many methods for spectral analysis, such as Maximum Entropy Method (MEM), Multi-Taper Method (MTM), Auto Regressive (AR), Multiple Signal Classification (MUSIC), Blackman-Tukey (BT), Fast Fourier Transform (FFT),... [1].

In this study, Blackman-Tukey BT and Fast Fourier Transform FFT methods will be used for wind wave spectral analysis [2]. Based on the measured data in Cu Lao Dung (Soc Trang province) [3], the methods can prove their strong points as well as their limits in applications. Wind wave energy dissipation in mangrove forests at the study site is analyzed and discussed.

RESEARCH METHODOLOGY AND DATA COLLECTION

Research methodology

The simplest and the most natural representation of the confused sea surface would be the linear superposition of many harmonics travelling in various directions [4]. Spectral analysis is used to determine the partition of the variance of a time series as a function of frequency. For stochastic wind wave time series, contributions from the different frequency components are expressed in terms of the frequency spectral density. In practice, the term spectrum is applied to all spectral functions such as auto spectrum for one time series, or cross-spectrum for the two time series. Frequency spectra are usually estimated by either of two methods. The first is based on the Wiener-Khintchine theorem and is called the Blackman-Tukey procedure. The Wiener-Khintchine relations link variance functions in the time domain to those in the frequency domain. In the second method, called the Cooley-Tukey method, the direct Fast Fourier Transformation is used [4].

Data collection

The data for analysis of energy spectral density is used from wave data collection of two field measurements in Cu Lao Dung (Soc Trang province) in March, 2014 and September, 2015 [4]. Every field measurement lasted 14 days and wave instruments were set up in three or four stations: one in shallow water, one in muddy flat and one or two in mangrove forests (fig. 1). The setup information for wave gauges is shown in table 1.

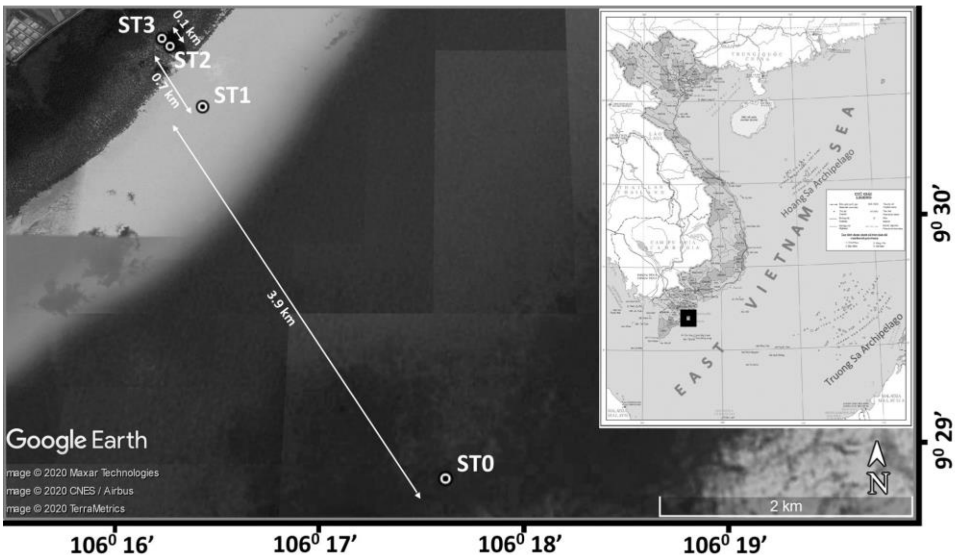


Figure 1. Locations of stations in the study site

Table 1. Information of instrumental settings

Stations ST	Wave types	Settings
Shallow water station ST0 10°22'46.4"N 106°51'57.1"E	Valeport MIDAS DWR 27111 (UK)	Burst: 30 minutes Rate: 4 Hz Wave burst: 2048
Muddy flat station ST1 10°23'26.8"N 106°52'48.0"E	RBR (USA)	Burst: 30 minutes Rate: 4 Hz Samples: 2048
Mangrove forest stations ST2, ST3 10°23'27.8"N 106°52'49.0"E	AWH-USB (JP)	Burst: 30 minutes Rate: 10 Hz Samples: 4800

RESEARCH RESULTS

Blackman-Tukey methods

Choosing the lag number m for calculation of autocorrelation function

In order to calculate the frequency spectral density by using BT method, the autocorrelation function must be determined [5]. The high correlation depends on the chosen value of lag number in the record. Usually, the m value used is not more than n/5 where n is the number of collected samples [6]. However, it is necessary to choose the lag number m for wave spectral analysis.

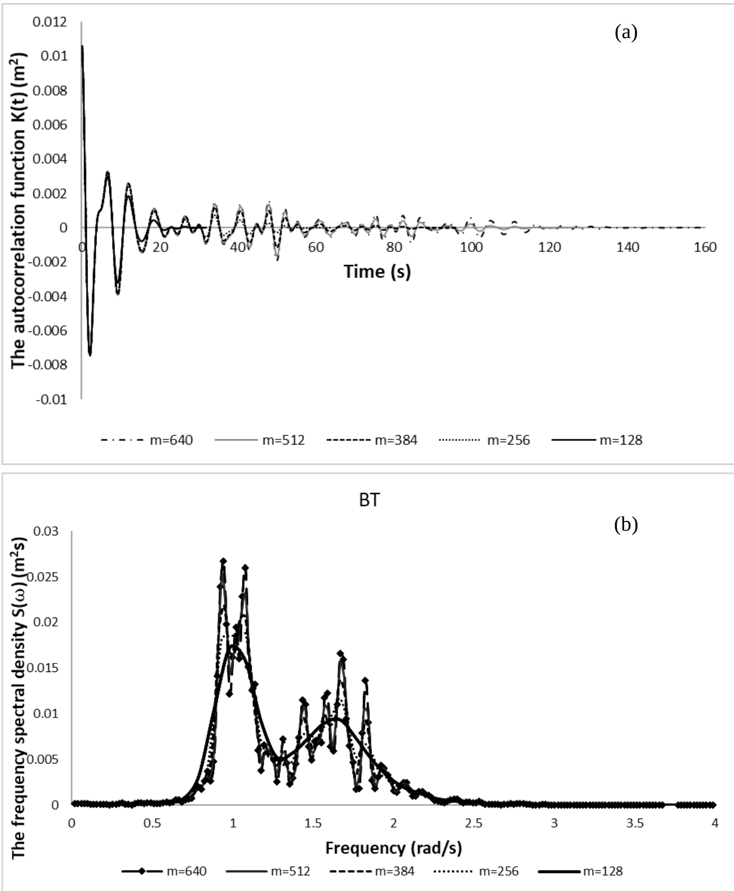


Figure 2. Autocorrelation function (a) and corresponding frequency spectral density (b) with different m values: 640, 512, 384, 256 and 128

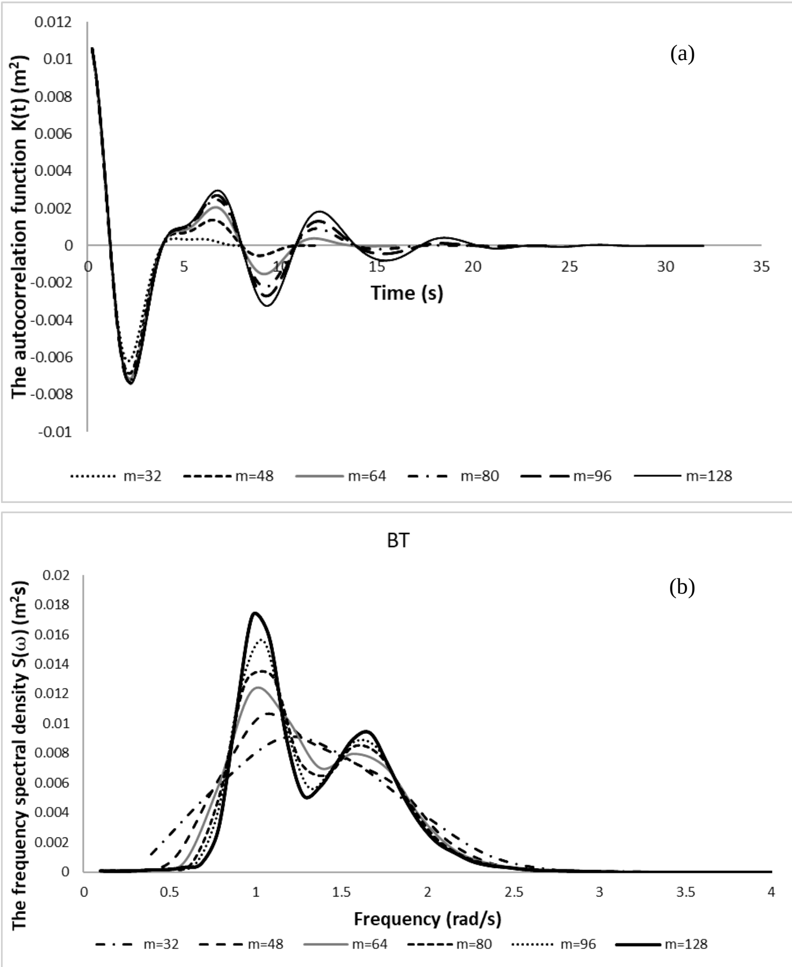


Figure 3. Autocorrelation function (a) and corresponding frequency spectral density (b) with different m values: 32, 48, 64, 80, 96, 112 and 128

According to wave instrumental settings in three different stations (table 1), there are at least 2048 samples in every record in collected data series. Therefore with n = 2048, the lag number m is chosen about 409 (i.e. not more than n/5).

The autocorrelation function in time with different m = 640, 512, 384, 256 and 128 is given in figure 2a. Results shows that the correlations are irregular and unstable when m is larger than 128. As a result, the corresponding frequency spectral density (figure 2b) has two lobes and the spectra get more noises as m is larger. It can be said that the spectra are the most stable when m is 128.

Similarly, the different correlation functions are considered with different m of 32, 48, 64, 80, 96, 112 and 128 (fig. 3). Figs. 3a and 3b show the autocorrelation function and corresponding spectral density with different m values. It can be seen that those figures give the high and stable correlation. The smaller m value gets, the weaker the correlation is. Therefore, the peaks of spectral energy density also decrease in accordance with m value. When m > 64, the spectra has two lobes: The higher peak in lower frequency (f = 1.0 rad/s) and the lower peak in higher frequency (f = 1.8 rad/s). When m is smaller, the lobe of spectral energy will get lower. When m < 64, the spectrum with one lobe is observed and the peak frequency is 1.2 rad/s, which is in the range of two peak frequencies (f = 1.0 rad/s and f = 1.8 rad/s) in case m > 64.

The influence of sampling number n

From the data analysis, it is necessary to choose the suitable lag number m for wind wave spectral calculations. Especially in shallow waters and coastal zones, waves behave with the characteristic of nonlinearity. Therefore, in the wave data analysis in mangrove forest, m = 64 is chosen in spectral density function by BT method. It can be seen that the spectral density function by using BT method does not change so much with different n and m from m = 64 to m = 128. Figure 4 shows the spectral density in BT method with n = 512 and different m. The main difference is that the value of spectral energy is higher for more samples n (fig. 5). From the result, the BT method is advised to apply in case the samples are limit.

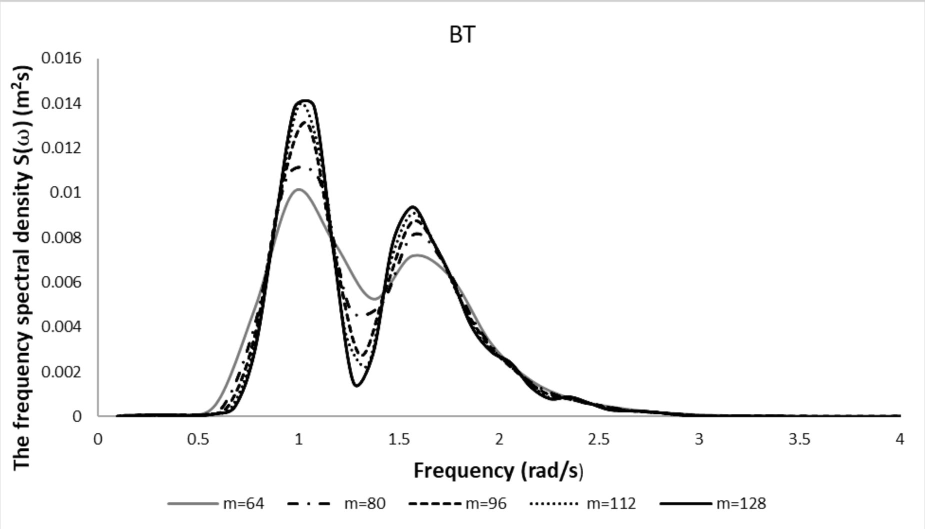


Figure 4. The spectral density function with n = 512 and different m

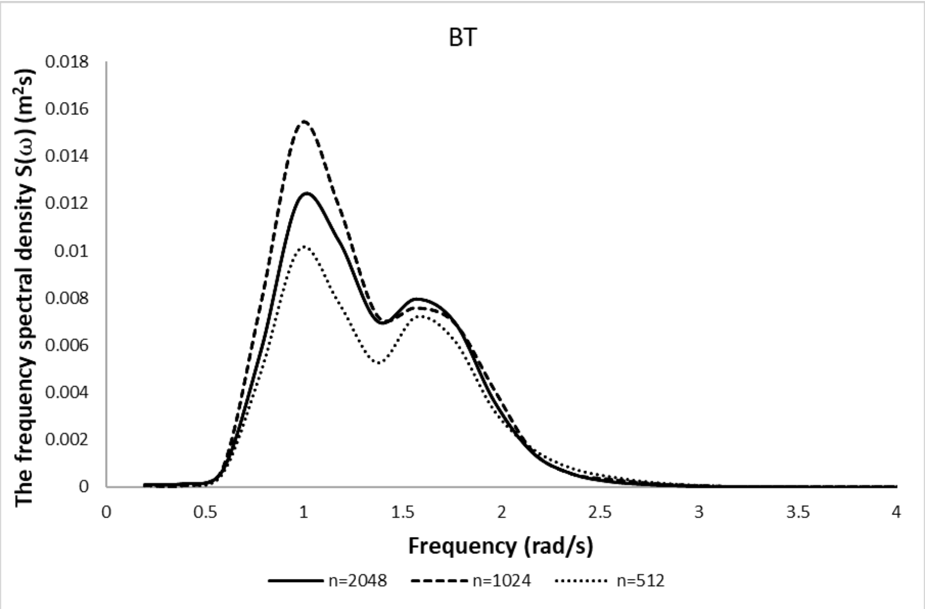


Figure 5. The spectral density function by using Blackman-Tukey method with m = 64 and different n

FFT method

Selection of the length L

Publications on FFT algorithms are very numerous such as Cooley and Tukey (1965), Otnes and Enochons (1972), Bendat and Piersol (1986), Emery and Thomson (1997), and many others [5]. For practical calculations, the observed record is usually divided into *K* segments, each of length $L\Delta t$. The length *L* will get influence on frequency resolution and spectral shape. With the large *L*, despite better frequency resolution, the spectral shape gets unclear due to the small number of average spectral function. Contrastingly, with the small *L*, the spectrum gets better and less noise because the average spectral function is higher. However, the small *L* can make the spectra length less accurate due to low frequency resolution.

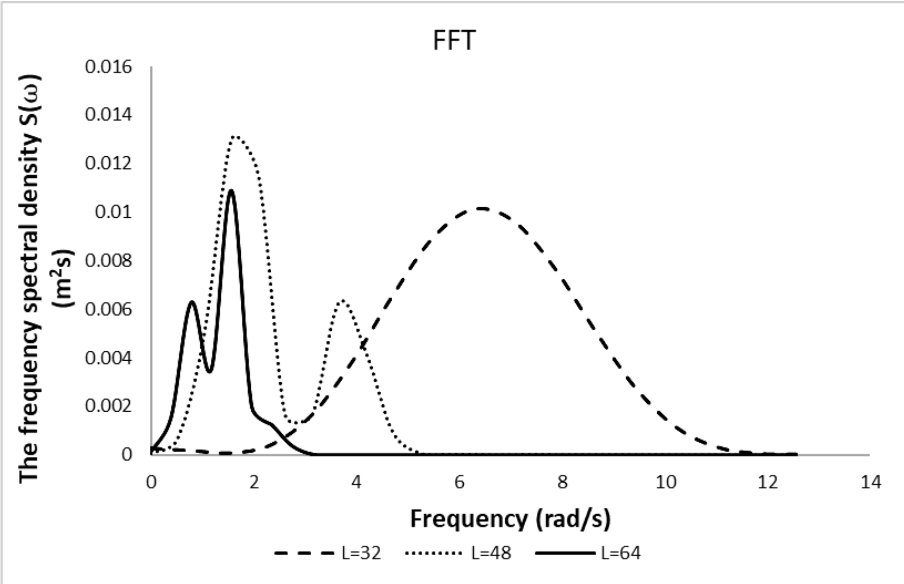


Figure 6. The energy spectral density function by FFT method with different L of 32, 48 and 64

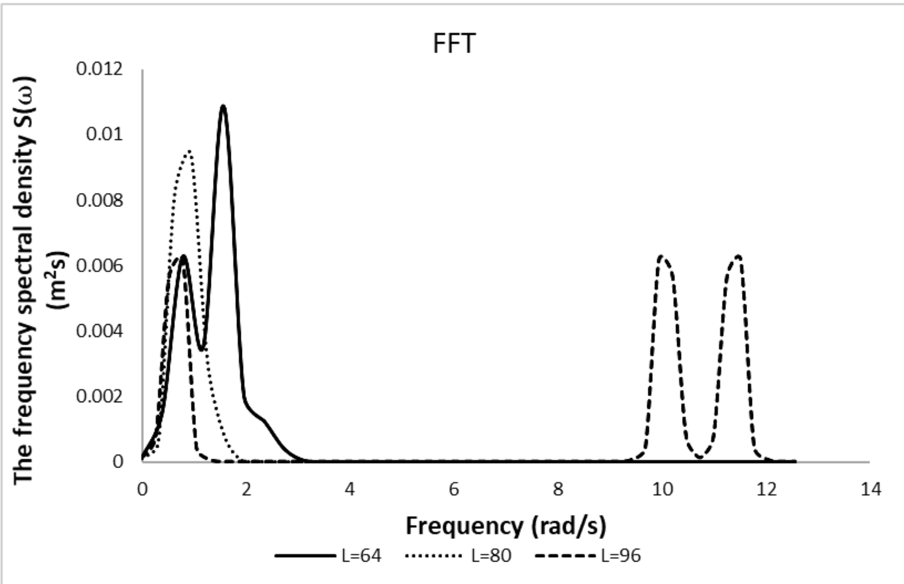


Figure 7. The energy spectral density function by FFT method with different L of 64, 80 and 96

Figs. 6 and 7 prove that with different L, the spectral density changes to frequency: main frequency is at 2–12 rad/s for L = 32; at 0.5–5 rad/s for L = 48 and 0–3 rad/s for L = 64. When L is larger than 80, the spectral shape gets more noises and trends to separate into different distribution parts. Therefore, the length L of 64 is considered the most suitable choice for wave spectral density analysis by FFT method.

Influence of samples n

The number of samples n get influence on spectra shape in FFT method. Fig. 8 shows that when n is large enough, the spectral function is stable with frequency. In contrast, when n is small enough, the spectral function can be separated into different frequency energies. Therefore, for wind wave spectral analysis, FFT method is advised to apply when samples are large enough. Or else in case of small samples, wave spectral analysis by BT method is more suitable. More illustrations are shown in figure 9.

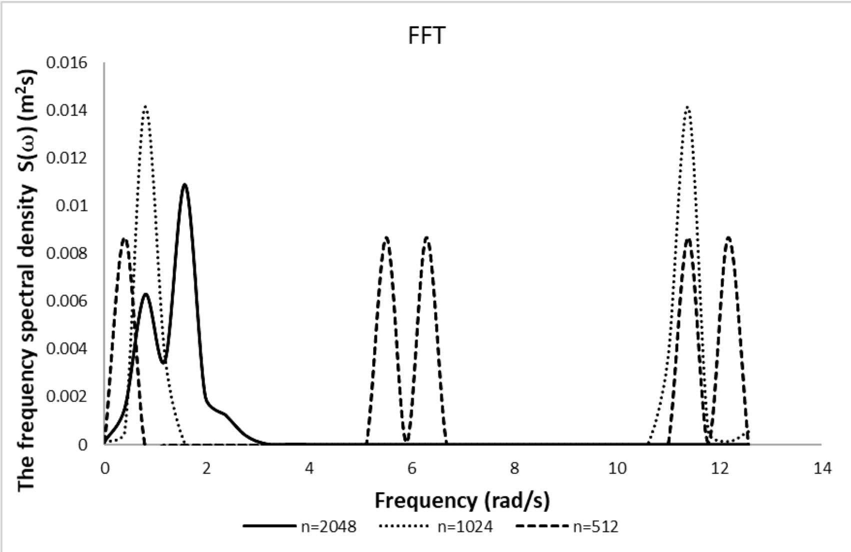


Figure 8. Spectral density function by FFT method with L = 64 and different n

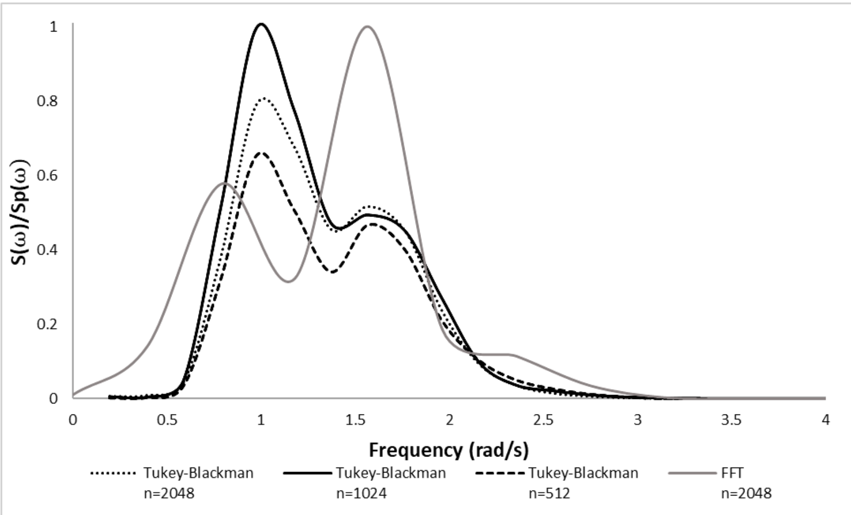
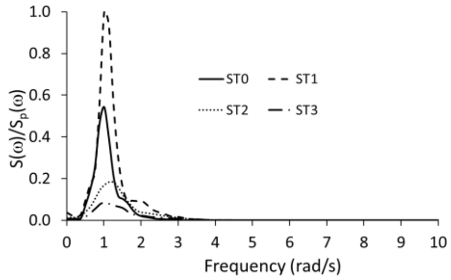


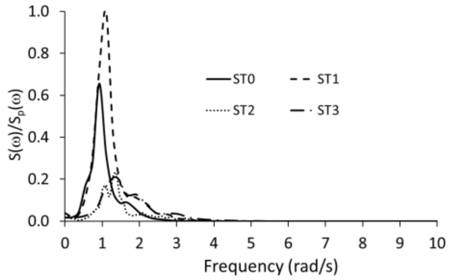
Figure 9. Spectral results from BT and FFT methods

The shift of energy in mangrove areas

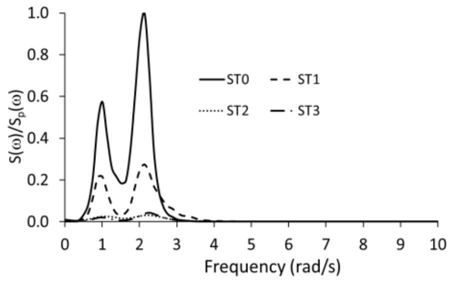
Fig. 10 illustrates the energy spectral changes in some observations in Cu Lao Dung (Soc Trang province) by using BT and FFT methods. Both methods give the suitable results that the higher significant wave height gets the higher wave spectra. The spectral energy decreases 40–60% as waves propagate from shallow water to muddy flat and remains 10% energy in mangrove forest. Waves are almost dissipated as propagating into mangrove forest. Wave energy dissipation is mainly due to influence of bottom fiction, wave breaking and wave-trunk interaction [6]. In general, the spectra in two methods are similar in shapes but different in magnitude. However, the peak frequencies of spectra in two methods are almost the same as well.



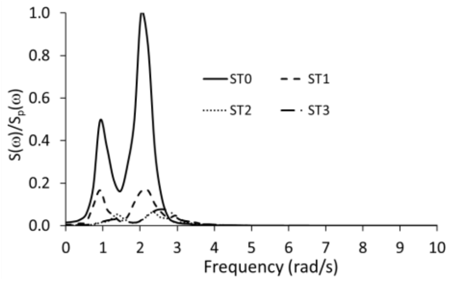
(a) At 02:00 on 23 Sept., 2014 by BT method



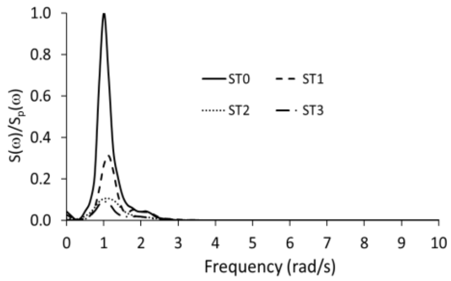
(b) At 02:00 on 23 Sept., 2014 by FFT method



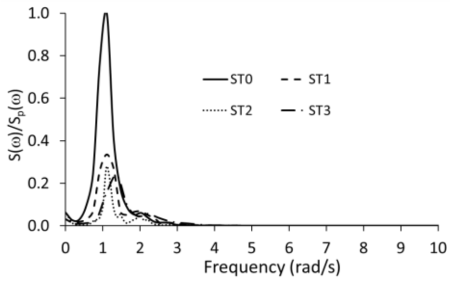
(c) At 15:00 on 25 Sept., 2014 by BT method



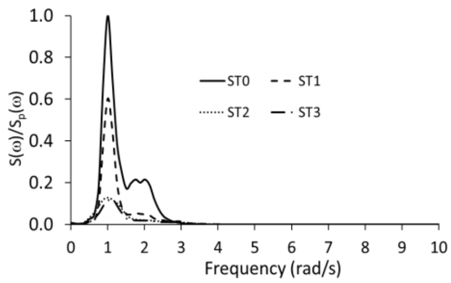
(d) At 15:00 on 25 Sept., 2014 by FFT method



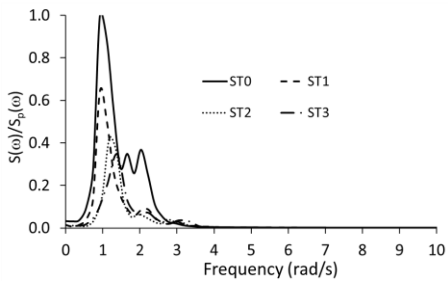
(e) At 03:30 on 26 Sept., 2014 by BT method



(f) At 03:30 on 26 Sept., 2014 by FFT method



(g) At 04:00 on 27 Sept., 2014 by BT method



(h) At 04:00 on 27 Sept., 2014 by FFT method

Figure 10. Wave spectra in stations in Cu Lao Dung in September 2014 by BT method (a, c, e, g) and FFT method (b, d, f, h)

Results from two methods show that the energy of spectral peak frequency is shifted from low frequency to high frequency from ST0 to ST3. When propagating into mangrove forests, the wave frequency decreases i.e. wave period increases: 3–4 s wave period in shallow water and 1–2 s wave period in mangrove forest. Examples of wave spectral peak shift in Cu Lao Dung, Soc Trang in two methods are shown in fig. 10. The energy shift in FFT method is shown more obviously than that in BT method. In the stations in mangroves (ST2 and ST3), side lobe occurs next to main lobe. The side lobe peak in FFT method is shown more clearly than that in BT method. Furthermore, the spectral energies in ST2 and ST3 by FFT method provide higher values and more distinct shifts than those in BT method. It is very significant for nonlinear problems of wave energy in mangrove forests [7].

CONCLUSIONS

The results of wave energy analysis in mangrove areas by BT and FFT methods showed that the wave energy decreased strongly when propagating from shallow water to the mudflat and in mangrove forests. Using BT and FFT methods, we recognize that every method has its own advantages and disadvantages. Therefore, it is necessary to study and to consider every specific case in application.

The outstanding advantage of the BT method is popular, easy to calculate, especially for short monitoring time series. The choice of the lag number m is very important to determine the energy and the number of spectral peaks. However, the BT method could not remove the weakened main lobes by strong side lobes and the frequency resolution is limited by the data series.

The greatest advantage of FFT method is the frequency shift as waves propagate into shallower water. It is very significant for nonlinear problems of wave energy in mangrove forests. However in FFT method, the number of samples should be sufficient and the choice of the length L should be suitable for every specific problem.

It can be proved that both methods give good results, the higher the significant wave height, the larger the spectrum and vice versa. The spectral energy decreases 40–60% as waves propagate from shallow water to muddy flat and remains 10% in mangrove forest in Cu Lao Dung (Soc Trang). Waves are almost completely dissipated when propagating into mangrove forest. Wave energy dissipation is mainly due to influence of bottom fiction, wave breaking and wave-trunk interaction. Spectral energy shift and nonlinearity are clearly shown in the mangrove forest. This result is very significant and important for the study of nonlinear wave energy in the mangrove areas.

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