Journal of Marine Science and Technology; Vol. 17, No. 4B; 2017: 138-144
DOI: 10.15625/1859-3097/17/4B/13001
http://www.vjs.ac.vn/index.php/jmst

DETERMINATION OF THE CONSTANT W_O FOR LOCAL GEOID OF VIETNAM AND IT'S SYSTEMATIC DEVIATION FROM THE GLOBAL GEOID

Nguyen Ngoc Truong*, Tran Van Nhac

Geophysics Department, Physics and Engineering Physics Faculty, University of Science, Vietnam National University-Ho Chi Minh city
*E-mail: nntruong@hcmus.edu.vn

Received: 9-11-2017

ABSTRACT: Constant W_o, defining the geoid, has important applications in the area of physical geodesy. With the development of artificial Earth satellite, constant W_o for the global geoid approximating the oceans on Earth can be calculated from an expansion of spherical harmonics - Stokes constants determined by observation of perturbations in artificial satellite's orbits. However, the Stokes constants are limited, therefore the geoid constant W_o could not be calculated for local geoid (state geoid) from the mentioned expansion of spherical harmonics. In this paper, we present a method to determine the constant W_o for local geoid of Vietnam, using generalized Bruns formula and Neyman boundary problem. The initial data used are Faye gravity anomalies surveyed on land and sea of Southern Vietnam. The constant W_o is then used to calculate the systematic deviation of the local geoid of Vietnam from the global geoid EGM - 96.

Keywords: The geoid, Stokes constants, Bruns fomular, Neyman boundary problem.

INTRODUCTION

The subject of the paper in the field of geodetic physics, related to gravity potential, and gravity anomalies are the original data to determine the geoid, which is the equatorial surface coinciding with the calm ocean surface, no wave, no wind, no tides, and no currents. The geoid shape is considered to be the shape of the Earth. Geoid is the standard surface for determining the standard elevation of territorial topography [1]. The elevation ζ of the geoid surface was determined against the reference ellipsoid surface, it is referred to as the height anomaly. In this paper, we use the spheroid, that is approximative ellipsoid, which is normal potential $U(\rho, \varphi)$ extracted from the serial of gravity potential W with spherical harmonics n and centrifugal potential [2].

The global geoid is approximately the ocean surface on Earth, determined by satellite method that does not approximate the sea surface of each country, including Vietnam. The traditional Stokes integral formula is used to determine the local geoid by using ground-based gravity anomalies. Since 1991, Lan P. H. has identified the local geoid for Viet Nam with accuracy of 1.5 - 2.0 m [3]. In 1998, Vo D. H. used the EGM-96 gravity model combination to build the geoid VN 2003, with details from 0.2 m to 0.5 m.

However, the Stokes formula considers the standard reference surface to calculate the geoid height as a sphere, not an ellipsoid, so the Stokes formula does not contain the constant U_o of the reference ellipsoid and the constant W_o of the local geoid [4]. To determine the systematic deviation (displacement) between the local geoid of Vietnam and the global geoid, it is necessary to know the local geoid constant $W_o^{'}$ and the global geoid constant W_o. However, Pham Hoang Lan postulated that the local geoid constant $W_o^{'}$ cannot be determined [5]. This is a problem that this paper deals with.

To solve this problem, we used 3738 Faye gravity anomaly data in Southern Vietnam and sea of Southern Vietnam, at coordinates of $8.16^o \rightarrow 17^o$ latitude North, $104.5^o \rightarrow 112^o$ longitude East, to transform into ground-based potential anomalies *T*, by applying the Neyman boundary problem. In addition, we measure GPS at 20 specific locations along the coast of Vietnam to determine the standard geoid heights in Vietnam. Since then, we have determined the geoid constant $W_o^{'}$ for the local geoid of Vietnam by using the general Bruns formula.

Local geoid constant W_o is important for determining the local geoid height ζ of Vietnam relative to any reference ellipsoid surface with the equation $U(\rho, \varphi) = U_o$ and determining the systematic deviation of the local geoid of Vietnam from the global geoid EGM - 96 as we described in this paper.

THE GENERAL BRUNS FORMULA, NEYMAN BOUNDARY PROBLEM, GEOID CONSTANT AND THE SYSTEMATIC DEVIATION BETWEEN TWO GEOIDS

The general Bruns formula

The general Bruns formula has the form [6]:

$$\zeta = \frac{T}{\gamma} + \frac{U_o - W_o}{\gamma} \qquad (1)$$

With: ζ- the geoid height relative to the reference ellipsoid has an equation $U(\rho, \varphi) = U_o$; *T*- the disturbed potential is potential anomaly of satellite gravity method, random variation, depending on latitude and longitude:

$$T(\rho,\varphi,\lambda) = W(\rho,\varphi,\lambda) - U(\rho,\varphi) \qquad (2)$$

γ- normal gravity values change slowly in latitude φ.

Formula (1) is the general Bruns formula, where T/γ is the fast variable component, set:

$$\zeta_o = \frac{U_o - W_o}{\gamma} \qquad (3)$$

ζ_o- the component changes slowly with normal gravity (latitude φ).

This is the deviation of approximately optimal spheroid surface, which is determined by equation $U(\rho, \varphi) = W_o$ ($U_o = W_o$, also known as the common spheroid), with reference ellipsoid surface U_o.

Neyman boundary problem

The Neyman boundary problem [7]: There is derivative V_z of the gravitational potential *V* for *z*-dimension (V_z - gravitational force), distributed on the plane of observation Oxy. We need to find the potential *V* in out space that satisfies the equation Laplace and the boundary conditions, mentioned above, and is regular in infinity.

Applying the Poisson formula (in the Oxyz coordinate system, with the upward axis Oz) for the derivative V_z, that is identical to the gravity anomaly Δg:

$$V_z(x,y,z) = \frac{z}{2\pi}\iint_S \frac{V_z(\xi,\eta,\zeta)\,d\xi d\eta}{\left[(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2\right]^{\frac{3}{2}}} \qquad (4)$$

To multiply the two sides with - *dz*, and integrate by *z*, $z \rightarrow \infty$, $\zeta = 0$ (on the plane of observation Oxy):

$$-\int_z^\infty \frac{\partial V(x,y,z)}{\partial z} dz = -\frac{1}{2\pi}\iint_S V_z(\xi,\eta,0)\int_z^\infty \frac{zdz}{\left[(x-\xi)^2 + (y-\eta)^2 + z^2\right]^{\frac{3}{2}}} d\xi d\eta \qquad (5)$$

$$\Leftrightarrow -V(x,y,\infty)+V(x,y,z)=\frac{1}{2\pi}\iint_S \frac{V_z(\xi,\eta,0)\,d\xi d\eta}{\left[(x-\xi)^2+(y-\eta)^2+z^2\right]^{\frac{1}{2}}} \quad (6)$$

Since $V(x, y, \infty) = 0$, regular in infinity, we have the Neyman boundary problem, with $z = 0$:

$$V(x,y,0)=\frac{1}{2\pi}\iint_S \frac{V_z(\xi,\eta,0)\,d\xi d\eta}{\left[(x-\xi)^2+(y-\eta)^2\right]^{\frac{1}{2}}} \quad (7a)$$

Applying (7a) with $V = T$, the disturbed potential (potential anomaly) and V_z is gravity anomaly Δg. We have the formula to calculate disturbed potential T from gravity anomalies Δg:

$$T(x,y,0)=\frac{1}{2\pi}\iint_S \frac{\Delta g(\xi,\eta,0)\,d\xi d\eta}{\left[(x-\xi)^2+(y-\eta)^2\right]^{\frac{1}{2}}} \quad (7b)$$

Geoid constant W_o

When the spheroid satisfies the equation $U(\rho, \varphi) = W_o$ (the geoid constant W_o instead of U_o), we obtain the equation of the approximately optimal spheroid of geoid [8].

Then, reference ellipsoid will duplicate with approximately optimal spheroid of geoid and geoid will fluctuate around approximately optimal spheroid of geoid, geoid heights obtain negative values and positive values, according to traditional Bruns formula:

$$\zeta=\frac{T}{\gamma} \quad (8)$$

After transforming the observed gravity anomaly to the potential anomaly, T combines with geoid height h, measured by GPS in the coast of Vietnam as a boundary condition. We determine the local geoid constant W_o in formula (1). At the coast, the standard height H = 0, so $\zeta = h$ - GPS receiver.

Appling (1) to local geoid by re-symbolizing: $W_o \equiv W_o'$, $T \equiv T'$, $\zeta \equiv \zeta$, so that :

$$\zeta'=\frac{T'}{\gamma}+\frac{U_o-W_o'}{\gamma} \quad (9)$$

From (9): $W_o'=T'-\gamma\zeta'+U_o$ (10)

The local geoid constant W_o' is calculated by the values T', γ, U_o, ζ'; U_o= 62636851.71 - ellipsoid constant of normal gravity WGS - 84; γ- normal gravity formula of normal gravity WGS - 84.

$$\gamma=\frac{9.7803267714\left(1+0.001931851386\sin^2\varphi\right)}{\sqrt{1-0.0066943799013\sin^2\varphi}} \quad (11)$$

ζ'- obtained from GPS observation to measure geodetic height in the coastal area of Southern Vietnam, we have: $\zeta' = h$; T'- calculated from gravity anomalies by integral method (7b) (solution of Neyman boundary problem).

The systematic deviation between two geoids

Apply the formula (1) to the global geoid and local geoid: $\zeta=\frac{T}{\gamma}+\frac{U_o-W_o}{\gamma}$ (12)

$$\zeta'=\frac{T'}{\gamma}+\frac{U_o-W_o'}{\gamma} \quad (13)$$

T: Disturbed potential with global geoid (disturbed potential of satellite method); T': Disturbed potential with local geoid (ground-based potential anomaly).

Set $\Delta\zeta = \zeta' - \zeta$, $\Delta T = T' - T$, we have:

$$\Delta\zeta=\left(\frac{T'}{\gamma}+\frac{U_o-W_o'}{\gamma}\right)-\left(\frac{T}{\gamma}+\frac{U_o-W_o}{\gamma}\right)=\frac{\Delta T}{\gamma}+\frac{W_o-W_o'}{\gamma} \quad (14)$$

Symbol:

$$\Delta\zeta_o = \frac{W_o - W_o^{'}}{\gamma} \qquad (15)$$

$\Delta\zeta_o$: the systematic deviation between two geoids, systematically varies with γ.

$\Delta\zeta_o = \frac{W_o - W_o^{'}}{\gamma}$ is the systematic deviation between two spheroids, that are approximately optimal spheroids of geoids (dotted line) as fig. 1.

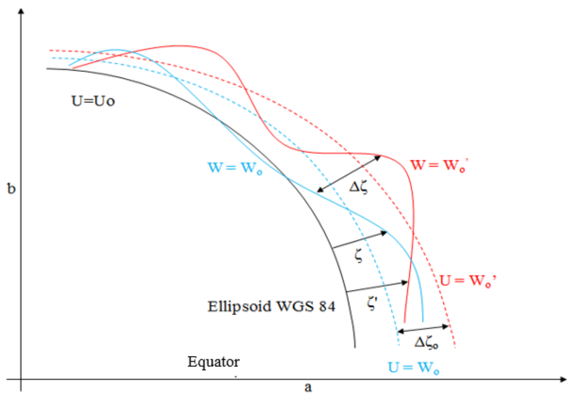

Fig. 1. The systematic deviation between two geoids is the systematic deviation between two spheroids that are approximately optimal spheroids of geoids

CALCULATION RESULTS

Faye gravity anomaly map

The data used to process in this paper is the Faye gravity anomaly data in Southern Vietnam and sea of Southern Viet Nam, at coordinates of $8.16^o \rightarrow 17^o$ latitude North, $104.5^o \rightarrow 112^o$ longitude East, with 3738 points. These include ground-based gravity data and satellite sea-based gravity, provided by Southern Vietnam Geological Mapping Division.

Use the Surfer to interpolate data and Matlab to calculate data.

Data are interpolated by Surfer with size-grid 0.9’ × 0.9’, i.e. 1.6 km × 1.6 km. The size-grid is (0.9’ × 0.9’) to retain the real data at the sea in the interpolation data (fig. 2).

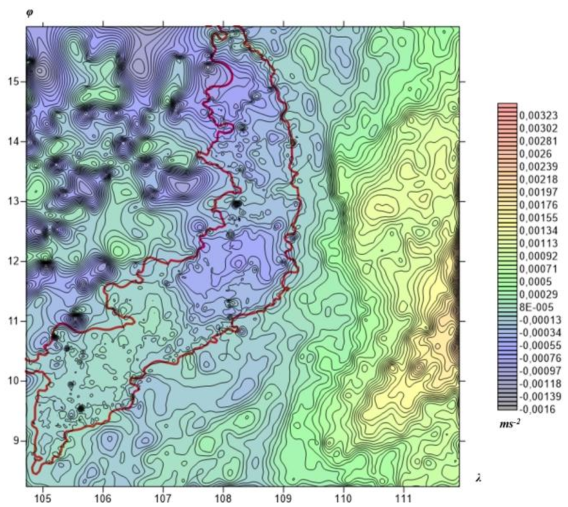

Fig. 2. Contour lines of gravity anomaly, interpolated with size-grid 0.9’ × 0.9’ (contour lines are separated with 4 mGal)

Determining the local disturbed potential from gravity anomaly Δ*g* according to the Neyman problem

Applying formula (7b) to calculate the local disturbed potential *T’* from the gravity anomaly Δ*g* at 9409 points distributed on the grid in the study area. We establish the map of the contour lines of the local disturbed potential *T’* (fg. 3).

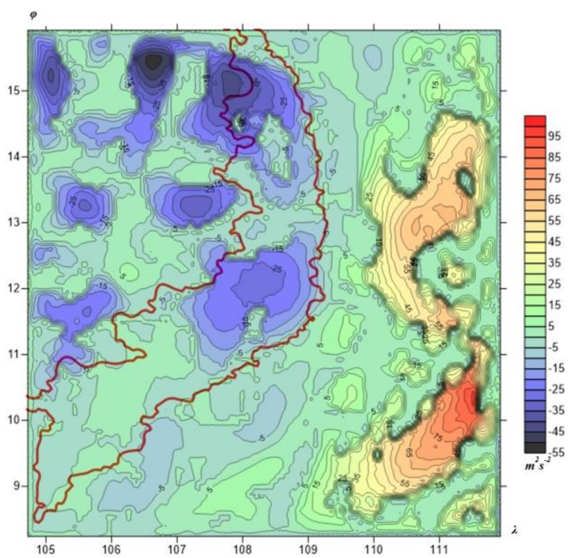

Fig. 3. Contour lines of local disturbed potential *T’* with 9409 data (contour lines are separated with 2 m^2s^{-2})

The contour lines of disturbed potential are smoother than contour lines of gravity anomaly, reflecting real geoid waves.

Determining the local geoid constant $W_o^{'}$

Selecting 20 location points on the coast of Thanh Hai, Bao Thuan commune, Ba Tri district, Ben Tre province to calculate the local geoid constant $W_o^{'}$. The coastal area measured in Bao island has a coastline of about 5 km, overlooking the East Sea.

Here local geoid height is determined by the Garmin Montana 650 GPS meter - on September 25, 2015 - (at the coast, we have elevation terrain H = 0 so the geodetic height is measured by GPS: h = ζ'- local geoid height). Measurement is operated on 2000 m long straight, linear north-south, line along the coastline in relatively flat terrain, interval between points is 100 m. Measurement is conducted at medium tide (water level between the highest and lowest tide from the coast) (table 1).

Table 1. Data of geoid height ζ' và disturbed potential *T'* at 20 points

Longitude (°)	Latitude (°)	ζ' (m)	*T'* (m^2s^{-2})	Longitude (°)	Latitude (°)	ζ' (m)	*T'* (m^2s^{-2})
106.6902	10.02133	3.5	-2.7443	106.6869	10.01225	2.0	-2.7411
106.69	10.0204	2.0	-2.7415	106.6864	10.01133	1.0	-2.742
106.6898	10.01952	1.0	-2.7405	106.6859	10.01043	1.0	-2.7431
106.6895	10.01863	2.0	-2.7402	106.6854	10.00958	3.0	-2.7444
106.6893	10.177	2.5	-2.7392	106.6849	10.0088	1.0	-2.7463
106.689	10.01685	3.0	-2.7389	106.6843	10.00795	1.0	-2.7473
106.6887	10.01598	2.0	-2.7393	106.6837	10.00715	1.0	-2.7492
106.6883	10.01508	3.0	-2.7396	106.6832	10.00633	1.0	-2.7512
106.6878	10.0139	2.0	-2.7395	106.6826	10.00548	2.0	-2.7524
106.6874	10.0131	2.5	-2.7407	106.6822	10.00495	2.5	-2.7535

Applying (10) with disturbed potential *T'*, ellipsoid constant U_o, normal gravity γ and geoid height ζ' at 20 points, we have 20 values of local geoid constants W_o' (table 2).

Table 2. Values of local geoid constants $W_o^{'}$

Longitude (°)	Latitude (°)	$W_o^{'}$ (m^2s^{-2})	Longitude (°)	Latitude (°)	$W_o^{'}$ (m^2s^{-2})
106.6902	10.02133	62636815	106.6869	10.01225	62636829
106.69	10.0204	62636829	106.6864	10.01133	62636839
106.6898	10.01952	62636839	106.6859	10.01043	62636839
106.6895	10.01863	62636829	106.6854	10.00958	62636820
106.6893	10.177	62636825	106.6849	10.0088	62636839
106.689	10.01685	62636820	106.6843	10.00795	62636839
106.6887	10.01598	62636829	106.6837	10.00715	62636839
106.6883	10.01508	62636820	106.6832	10.00633	62636839
106.6878	10.0139	62636829	106.6826	10.00548	62636829
106.6874	10.0131	62636825	106.6822	10.00495	62636825

To average $W_o^{'}$ we have result:

$$W_o^{'} \pm \Delta W_o^{'} = 62636830 \pm 7.8 \ \left(m^2 s^{-2}\right)$$

With $\Delta W_o^{'}$ is the accuracy of $W_o^{'}$.

Determining the systematic deviation $\Delta\zeta_o$ between two geoids

Both global geoid and local geoid are randomly variable, very complex in terms of latitude and longitude. If we want to investigate the systematic deviation between the two geoid surfaces, we must express two approximately optimal spheroids of geoids with the reference ellipsoid on one diagram, do not directly investigate two real geoid surfaces.

Choosing ellipsoid WGS-84 that has an ellipsoid constant U_o as a reference face for calculating the systematic deviation of two approximately optimal spheroids of two geoids (global geoid and local geoid). From (12) and (13) we have two formulas for the two systematic deviations between approximately optimal spheroids of geoids with the reference ellipsoid surface:

$$\zeta_o = \frac{U_o - W_o}{\gamma} \text{ và } \zeta_o^{'} = \frac{U_o - W_o^{'}}{\gamma}$$

In which: $\gamma(\varphi)$ is selected as γ = 9.7827 ms^{-2} at latitude φ = 12.5^o, (latitude φ varies between 8.16^o→16^o latitude North, corresponding to the latitude of the South pole and Central Vietnam). We have $W_o^{'}$ = 62636830 m^2s^{-2} (local geoid constant); W_o = 62636856.88 m^2s^{-2} (global geoid constant - EGM96); U_o= 62636851.71 m^2s^{-2} (ellipsoid WGS-84), instead of the above formulas, we have:

$$\zeta_o = -0.5\ m \text{ và } \zeta_o^{'} = 2.2\ m$$

Thus, the approximately optimal spheroid of local geoid is shifted upward relative to the ellipsoid WGS-84 about 2.2 m. Also, the approximately optimal spheroid of global geoid is shifted downward relative to the ellipsoid WGS-84 about 0.5 m. So, it is synonymous with the displacement of the two corresponding geoids, because the geoid bonds to the approximately optimal spheroid. The systematic deviation varies slowly in terms of γ (latitude φ). We find that ζ_o and $\zeta_o^{'}$ change very slowly in the study area.

Using the local value $W_o^{'}$ and the global value W_o (EGM-96) to (15), giving the systematic deviation between the two geoid surfaces.

$$\Delta\zeta_o = \zeta_o^{'} - \zeta_o = \frac{W_o - W_o^{'}}{\gamma}$$

The latitude φ in the formula γ (15) receives 8.16^o→16^o latitude North (corresponding to the latitude of the South pole and Central Vietnam), with step $\Delta\varphi$ = 0.5^o we find that the systematic deviation $\Delta\zeta_o$ varies slowly in terms of latitude φ (table 3).

Table 3. Values of $\Delta\zeta_o$ varies slowly in terms of latitude φ

Latitude (^{o})	8.6	9	9.5	10	10,5	11	11,5	12
$\Delta\zeta_o$(m)	2.748058	2.748019	2.747979	2.747937	2.747893	2.747846	2.747798	2.747747
Latitude (^{o})	12.5	13	13.5	14	14.5	15	15.5	16
$\Delta\zeta_o$(m)	2.747695	2.74764	2.747584	2.747525	2.747465	2.747403	2.747338	2.747272

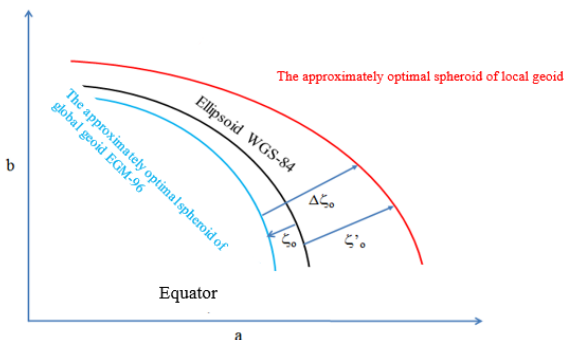

Fig. 4. The relative position of two approximately optimal spheroids of global geoid and local geoid is compared to the reference ellipsoid WGS-84

Because $\Delta\zeta_o$ varies slowly in terms of latitude φ, we can select $\Delta\zeta_o$ = 2.74 m as specific value of study area (fig. 4).

CONCLUSION

The local geoid constant $W_o^{'}$ for Vietnam is first determined by applying the Bruns formula and Neyman boundary problem for the local area with GPS measurement at the Vietnamese coast.

Calculating the constant $W_o^{'}$ for the local geoid of the Vietnamese state is important for geodetic physics such as:

Determining the systematic deviation between the local geoid surface of Vietnam and the global geoid surface. This quantity varies very slowly, gradually increasing to the equator, valued at over 2.74 m in the study area.

The relative position of two approximately optimal spheroids of global geoid and local geoid is compared to the reference ellipsoid WGS-84.

Open up the possibility to investigate systematic deviation between the local geoid in Vietnam and the global geoid nationally, from Hon Dau to Ca Mau.

Open up the possibility to establish exactly local geoid of Vietnam to interrelate any reference ellipsoid, which has real geoid waves.

REFERENCES

1. Kaula, W. M., 1967. Theory of Satellite Geodesy (Blaisdell, Waltham, MA, 1966). *Google Scholar*.
2. Kaula, W. M., 1987. 13. Satellite Measurement of the Earth's Gravity Field. In *Methods in Experimental Physics* (Vol. 24, pp. 163-187). Academic Press.
3. Lan, P. H., Neyman ,Yu. M., 2010. Local quasigeoid and global quasigeoid. *Journal of Geology, Vienam Academy of Science and Technology*.
4. Nghia H, H., 2003. Determination of geoid surface and deflection of plumb-line by gravity anomaly according to satellite data, *PhD Thesis, University of Science, Vietnam National University Ho Chi Minh City*.
5. Lan, P. H., Neyman, Yu. M., Sugaipova, I. S., 2013. Evaluating the displacement of local quasigeoid relative to global quasigeoid at the tide gauge Hon Dau. *Journal of Mining and Geology*, Vol. 41.
6. Bursa, M., and Pec, K., 1993. Gravity field and dynamics of the earth. *Berlin, New York: Springer, c1993*.
7. Vaníček, P., Kingdon, R., and Santos, M., 2012. Geoid versus quasigeoid: a case of physics versus geometry. *Contributions to Geophysics and Geodesy*, **42**(1), 101-118.
8. Shinbirev, B. P., 1975. Theory of the shape of the earth. *Moscow Izdatel Nedra*.