Journal of Marine Science and Technology; Vol. 17, No. 4B; 2017: 14-20
DOI: 10.15625/1859-3097/17/4B/12987
http://www.vjs.ac.vn/index.php/jmst

# NONEQUIDISTANT TWO-DIMENSIONAL ANTENNA ARRAYS ARE BASED ON LATIN SQUARES FOR REGISTRATION OF COSMIC, ATMOSPHERIC AND LITHOSPHERIC RADIATION

**Nguyen Xuan Anh[1,4*], V. I. Lutsenko[2], Luo Yiyang[3], I. V. Popov[2]**

[1]*Institute of Geophysics, VAST, Vietnam*
[2]*A. Ya. Usikov Institute of Radio Physics and Electronics, Kharkov, Ukraine*
[3]*N. E. Zhukovsky National Aerospace University, Kharkov, Ukraine*
[4]*Graduate University of Science and Technology, VAST, Vietnam*
[*]E-mail: nxuananh05@gmail.com

Received: 11-9-2017

**ABSTRACT:** The possibility of using Latin squares for constructing two-dimensional non-equidistant antenna arrays that can be used for development of radio telescopes and systems for remote sensing of atmospheric and lithospheric radiation has been shown. An algorithm for calculating the coordinates of elements of a non-equidistant antenna array using the values of elements of Latin squares is proposed. It is shown that in this case, it is possible to obtain an almost complete coverage of the grid of spatial frequencies in the region of the arrangement of the elements at small filling coefficients of array. Directional patterns are studied and the side lobes levels are estimated for non-equidistant antennas obtained on the basis of Latin squares. The possibility of synthesizing of large antenna arrays on the basis of composite squares using the embedding of generative Latin squares is shown. The characteristics of the obtained arrays are studied. It is shown that using the shifts and mutual rotations of individual layers is involved in the synthesized array; its characteristics can be substantially improved.

**Keywords:** Latin squares, non-equidistant antenna arrays, spatial frequency coverage, lithospheric and atmospheric radiation.

## INTRODUCTION

Non-equidistant linear antenna arrays (AA) have long attracted attention [1]. Their main advantage is reduction in the number of elements of the antenna without significant loss of resolution, while maintaining a sufficiently low level of side lobes. Almost all large antennas of radio telescopes and long-range radar are arrays with non-equidistant arrangement of elements and having an unfilled aperture. Recently, non-equidistant antenna arrays are beginning to find application in the development of monitoring systems for seismic and thunderstorm activity in antennas for receiving lithospheric and atmospheric radiation. Many attempts to find a simple way of constructing a non-equidistant array with small side lobes were mostly unsuccessful. Therefore most often when creating non-equidistant arrays the statistical methods are used [1, 2].

Significant progress was made in 1978 [3] by David Garth Leeper, who proposed placing the elements of a linear antenna in some nodes of a uniform grid, where the nodes were selected in such a way that the sequence of their numbers formed a cyclic difference set (CDS). This approach was further developed in the papers of L. E. Kopilovich [4-6], by creating two-dimensional difference sets based on the use of the product of one-dimensional basis of sides, which allowed the construction of two-dimensional antenna arrays. In papers [7, 8] the possibility of two-dimensional non-equidistant antenna array using such mathematical structures as magic squares has been shown and their characteristics are examined. The present paper is the further logical continuation and is devoted to the investigation of the possibility of constructing two-dimensional lattices on the basis of Latin squares.

**Latin squares and their properties**

*A* Latin square of the n-th order is a table $L=(l_{ij})$ of size n × n filled with n elements of the set M in such a way that each element of *M* meets exactly ones in each row and in each column of the table. Example of a Latin square of the 3rd order:

$$\begin{pmatrix} A & B & C \\ C & A & B \\ B & C & A \end{pmatrix} \tag{1}$$

Let's consider some ways of formation of Latin squares.

If the set of natural numbers {1, 2,…, n} or the set {0, 1,…, n-1} is taken as the set **M**, then we obtain for the 3rd order the Latin square by the following formula (*A*=1, *B*=2, *C*=3):

$$S1 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix} \tag{2}$$

If a set *M* is taken as a set on the basis of the unit square of natural numbers (the unit matrix), namely:

$$I1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, A = I1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, B = 2*I1 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}, C = 3*I1 = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \tag{3a}$$

Then we get a square of the 9th order of the following form:

$$S1 = \begin{pmatrix} 1 & 0 & 0 & 2 & 0 & 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 3 \\ 3 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 & 0 & 0 & 2 \\ 2 & 0 & 0 & 3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 3 & 0 & 0 & 1 \end{pmatrix} \tag{3b}$$

If the set M is taken as the set of square of units *I*2, namely:

$$I2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, A = I2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, B = 2*I2 = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}, C = 3*I2 = \begin{pmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{pmatrix} \tag{4a}$$

Then we get a square of the 9th order of the following kind:

$$S3=\begin{pmatrix} 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\ 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\ 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\ 3 & 3 & 3 & 1 & 1 & 1 & 2 & 2 & 2 \\ 3 & 3 & 3 & 1 & 1 & 1 & 2 & 2 & 2 \\ 3 & 3 & 3 & 1 & 1 & 1 & 2 & 2 & 2 \\ 2 & 2 & 2 & 3 & 3 & 3 & 1 & 1 & 1 \\ 2 & 2 & 2 & 3 & 3 & 3 & 1 & 1 & 1 \\ 2 & 2 & 2 & 3 & 3 & 3 & 1 & 1 & 1 \end{pmatrix} \tag{4b}$$

If the set $M$ is taken as the set of the Magic square of natural numbers of the 3rd order $I3$ namely:

$$I3=\begin{pmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{pmatrix}, A=I3=\begin{pmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{pmatrix}, B=2*I3=\begin{pmatrix} 16 & 2 & 12 \\ 6 & 10 & 14 \\ 8 & 18 & 4 \end{pmatrix}, C=3*I3=\begin{pmatrix} 24 & 3 & 18 \\ 9 & 15 & 21 \\ 12 & 18 & 4 \end{pmatrix} \tag{5a}$$

Then we get a square of the 9th order (the same Latin) of the following kind:

$$S4=\begin{pmatrix} 8 & 1 & 6 & 16 & 2 & 12 & 24 & 3 & 18 \\ 3 & 5 & 7 & 6 & 10 & 14 & 9 & 15 & 21 \\ 4 & 9 & 2 & 8 & 18 & 4 & 12 & 18 & 4 \\ 24 & 3 & 18 & 8 & 1 & 6 & 16 & 2 & 12 \\ 9 & 15 & 21 & 3 & 5 & 7 & 6 & 10 & 14 \\ 12 & 18 & 4 & 4 & 9 & 2 & 8 & 18 & 4 \\ 16 & 2 & 12 & 24 & 3 & 18 & 8 & 1 & 6 \\ 6 & 10 & 14 & 9 & 15 & 21 & 3 & 5 & 7 \\ 8 & 18 & 4 & 12 & 18 & 4 & 4 & 9 & 2 \end{pmatrix} \tag{5b}$$

If the set $M$ is the set of the Magic square of natural numbers $I3$ for which the transpose operation is applied $I3^{'}$, $I3^{''}$ namely:

$$I3=\begin{pmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{pmatrix}, A=I3=\begin{pmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{pmatrix}, B=I3^{'}=\begin{pmatrix} 6 & 7 & 2 \\ 1 & 5 & 9 \\ 8 & 3 & 4 \end{pmatrix}, C=I3^{''}=\begin{pmatrix} 2 & 9 & 4 \\ 7 & 5 & 3 \\ 6 & 1 & 8 \end{pmatrix} \tag{6a}$$

Where, the sign' denotes rotation (transposition) of the matrix by $90^{o}$ in the counterclockwise direction. Then we get a square of the 9th order of the following kind.

The approaches discussed above provide ample opportunities for constructing of Latin squares of various types and non-equidistant antenna arrays on their basis.

$$S5=\begin{pmatrix} 8 & 1 & 6 & 6 & 7 & 2 & 2 & 9 & 4 \\ 3 & 5 & 7 & 1 & 5 & 9 & 7 & 5 & 3 \\ 4 & 9 & 2 & 8 & 3 & 4 & 6 & 1 & 8 \\ 2 & 9 & 4 & 8 & 1 & 6 & 6 & 7 & 2 \\ 7 & 5 & 3 & 3 & 5 & 7 & 1 & 5 & 9 \\ 6 & 1 & 8 & 4 & 9 & 2 & 8 & 3 & 4 \\ 6 & 7 & 2 & 2 & 9 & 4 & 8 & 1 & 6 \\ 1 & 5 & 9 & 7 & 5 & 3 & 3 & 5 & 7 \\ 8 & 3 & 4 & 6 & 1 & 8 & 4 & 9 & 2 \end{pmatrix} \qquad (6b)$$

## THE RULE FOR CALCULATING THE COORDINATES OF THE ELEMENTS OF A NON-EQUIDISTANT ARRAY ON THE BASIS OF LATIN SQUARES AND THE MAIN RESULTS

We will consider the elements $S_{lj}$ of the “Latin” square as the distances between adjacent elements of the antenna array (elements of the interferometer).This means, for example, that an element $S_{lj}$ of a square that has a value $S_{lj}$ is at distances $S_{lj}$ along the abscissa and ordinate axes from neighboring elements of the antenna array. Then the coordinates of the elements composing the antenna array can be written in terms of the values $S_{lj}$ of the element of the square in the $l$ row and $j$ column:

$$x_{lj}=\sum_{j=1}^{j}S_{lj}=x_{lj-1}+S_{lj};$$
$$y_{lj}=\sum_{l=1}^{l}S_{lj}=y_{l-1j}+S_{lj} \qquad (7)$$

Where: $x_{lj}$- abscissa, $y_{lj}$- ordinate.

The resulting complex integers $z_{lj}= x_{lj}+iy_{lj}$ are Gaussian numbers that determine the coordinates of the elements of the antenna array. The coordinates $z_{lj}$ of the element are offset from the previous elements along the abscissa and ordinate by the value $S_{lj}$. At that the matrix $\|Z\|$ is obtained on the basis of the generating matrix $\|S\|$, whose elements are elements of the Latin square, determining the coordinates of the elements of the two-dimensional non-equidistant antenna array.

The spatial frequencies $x_m$, $y_p$ covering along the axes $x, y$ are determined by the difference in the coordinates of the array elements:

$$x_m = x_{ij} - x_{ik} \quad \textit{and} \quad y_p = y_{ij} - y_{ik} \qquad (8)$$

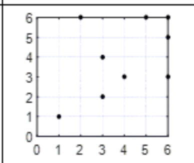
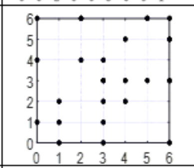
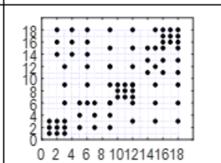
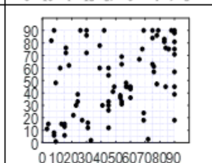
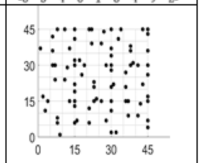
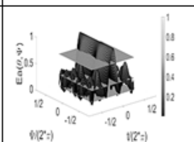
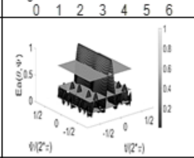
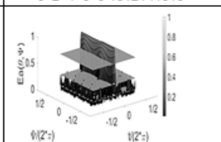
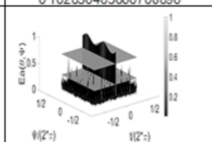
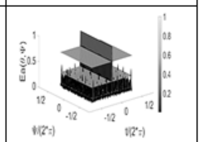
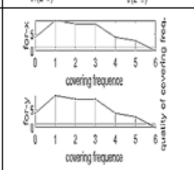
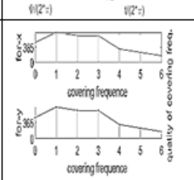
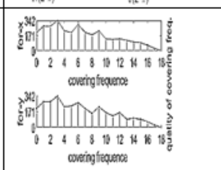
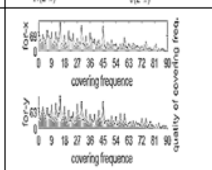
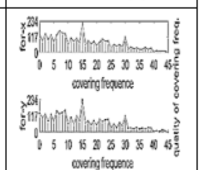
When developing antenna arrays, one tends to create such an array which with full desired coating of region of spatial frequencies has a minimum number of elements and acceptable values of sidelobe levels. One of the important tasks is to find a non-redundant configuration (NRC) with a given number of elements that provides a complete coverage of the central region of maximum size in the plane of spatial frequencies (*u*, *v*-plane).

Often the elements of “Latin” squares are the elements of an arithmetic progression, in a particular case, a series of natural numbers $i, j \in (1, n)$. If to each element of the generator matrix $S_{lj}$ we add a constant number $a$, i.e. $\hat{S}_{lj}=S_{lj}+a$, or multiply it by a constant $b$, i.e. $\hat{\hat{S}}_{lj}=bS_{lj}$, then the matrixes $\|\hat{S}\|$ and $\|\hat{\hat{S}}\|$ obtained in this way are also Latin squares. And the non-equidistant arrays obtained on their basis will be topologically equivalent, since adding a constant only leads to a shift of all lattice elements in space, and multiplication by a constant leads to compression/extension of spatial frequency regions.

## CHARACTERISTICS OF ANTENNA ARRAYS BASED ON LATIN SQUARES

Several antenna arrays (AA) were obtained using the main Latin square of the 3-th order. The results are shown in table 1. At that *S*1, *S*2,... *S*5 denote the different types of generating matrices and antenna arrays.

*Table 1.* Characteristics of antenna arrays based on Latin squares

| name | S1 | S2 | S3 | S4 | S5 |
|---|---|---|---|---|---|
| array | $\begin{bmatrix}1&2&3\\3&1&2\\2&3&1\end{bmatrix}$ | $\begin{bmatrix}1&0&0&2&0&0&3&0&0\\0&1&0&0&2&0&0&3&0\\0&0&1&0&0&2&0&0&3\\3&0&0&1&0&0&2&0&0\\0&3&0&0&1&0&0&2&0\\0&0&3&0&0&1&0&0&2\\2&0&0&3&0&0&1&0&0\\0&2&0&0&3&0&0&1&0\\0&0&2&0&0&3&0&0&1\end{bmatrix}$ | $\begin{bmatrix}1&1&1&2&2&2&3&3&3\\1&1&1&2&2&2&3&3&3\\1&1&1&2&2&2&3&3&3\\3&3&3&1&1&1&2&2&2\\3&3&3&1&1&1&2&2&2\\3&3&3&1&1&1&2&2&2\\2&2&2&3&3&3&1&1&1\\2&2&2&3&3&3&1&1&1\\2&2&2&3&3&3&1&1&1\end{bmatrix}$ | $\begin{bmatrix}8&1&6&16&2&12&24&3&18\\3&5&7&6&10&14&9&15&21\\4&9&2&8&18&4&12&27&6\\24&3&18&8&1&6&16&2&12\\9&15&21&3&5&7&6&10&14\\12&27&6&4&9&2&8&18&4\\16&2&12&24&3&18&8&1&6\\6&10&14&9&15&21&3&5&7\\8&18&4&12&27&6&4&9&2\end{bmatrix}$ | $\begin{bmatrix}8&1&6&6&7&2&2&9&4\\3&5&7&1&5&9&7&5&3\\4&9&2&8&3&4&6&1&8\\2&9&4&8&1&6&6&7&2\\7&5&3&3&5&7&1&5&9\\6&1&8&4&9&2&8&3&4\\6&7&2&2&9&4&8&1&6\\1&5&9&7&5&3&3&5&7\\8&3&4&6&1&8&4&9&2\end{bmatrix}$ |
| Distribution of AA |  |  |  |  |  |
| Directional pattern in the Cartesian system |  |  |  |  |  |
| Spatial frequency |  |  |  |  |  |
| $\Delta w_{0.707}$ (rad) | 0.9704 | 0.8213 | 0.3099 | 0.0609 | 0.1222 |
| $\Delta w_0$ (rad) | 2.9847 | 2.7826 | 1.6582 | 1.2878 | 1.3335 |
| $m$ | 0.3067 | 0.1900 | 0.1070 | 0.1093 | 0.1077 |
| N0 | 9 | 81 | 81 | 81 | 81 |
| M | 6*6 | 6*6 | 18*18 | 90*90 | 45*45 |
| S | 36 | 36 | 324 | 8100 | 2025 |
| $\alpha$ | 0.2500 | 2.2500 | 0.2500 | 0.0100 | 0.0400 |
| $\beta$ | 1.5000 | 13.5000 | 4.5000 | 0.900 | 1.8000 |

***Notes:*** $\Delta w_0$: width of main lobe; $\Delta w_{0.707}$: the effective width of the main lobe by the level of half power; m: average level of side lobes:

$$m = \sqrt{\sum_{\Delta\psi=-\pi}^{\pi} \left(\sum_{\Delta t=-\pi}^{-\Delta w/2} \sum_{n=1}^{N} \left|F(\Delta t, \Delta\psi)\right| + \sum_{\Delta t=\Delta w/2}^{\pi} \sum_{n=1}^{N} \left|F(\Delta t, \Delta\psi)\right|\right)^2} \Big/ N_{max} \tag{9}$$

Where the summation is carried out outside the main lobe of the radiation pattern along the $N_{max}$ pixels; $\alpha$: the fill factor for the antenna array:

$$\alpha = \frac{N_0}{v} \tag{10}$$

Where $N_0$ is the number of array elements, and $v$ is the number of nodes of an equidistant array in which they can be located; $\beta$: the redundancy factor for the antenna array:

$$\beta = \frac{N_0}{\sqrt{S}} \tag{11}$$

Where: $S$ is the area of the corresponding equidistant array. For rectangular fully filled equidistant arrays $S = v = n^2$, and $N_0 = n^2$. The size of the side of the array will be denoted by symbol $M$.

1. If AA is used as a set in the form of a unit matrix, for example $S2$, then AA is characterized by a large number of multiple covered frequencies (large redundancy). Comparison of AA $S1$ and $S2$ shows that the generating matrices are based on the "Latin" squares and the matrix of units does not increase the size of the AA, but improves its quality. This does not lead to a narrowing of the width of the main lobe, but it makes it possible to reduce the average level of the side lobes.

2. If we use a set in the form of a matrix of units, for example $S3$, then comparing the parameters of directional pattern of AA $S2$ and $S3$, it is clear that using a combination of a Latin square with a square of units can increase the AA size and narrow the main lobe width and reduce the level of side lobes opposite to the unit square.

Comparing the parameters of directional pattern of AA $S1$ and $S3$, it is clear that for a Latin square, with a nested n-th order of a unit square the size of the AA will be expanded by a factor of $n*n$ and the width of the main lobe will decrease by a factor of n. Similarly, for the average level of the side lobes, they decrease by a factor of n. Therefore, using nesting of n-th order of the unit square we can increase the AA size by $n$ times and reduce the effective main lobe width by a factor of n and also decrease the average level of the side lobes by a factor of n.

3. If we use the set in the form of a "magic" square multiplied by the coefficients from natural numbers (1, 2, 3), for example $S4$, then comparing the parameters of directional pattern for $S1$ and $S4$, we can conclude that for a Latin square, an n-th order of square nesting allows expanding the size of the aperture of AA in $M * M$ times and reducing the width of the main lobe by M times, while decreasing the average level of the side lobes by $n$ times. Therefore if nesting the unit square of n-th order, it is possible to increase the size of the AA by $M$ times, and decrease the width of the main lobe by a factor of $M$, and also reduce the average level of the side lobes by a factor of n, where M is the magic constant equal to $(n^2+1)*n/2$.. This makes it possible to reduce the duty cycle in $(M/n)^2$ times and the redundancy factor in $n^2/M$ times.

4. If we use a set as a "magic" square with a rotation of 90 degrees and multiplied by the coefficients of natural numbers (1, 2, 3), for example $S5$, then comparing the parameters of directional pattern for AA $S1$ and $S5$ shows that for a Latin square whose element is not number but rotation, the nesting of n-th order of magic square allows substantially increasing the area of AA in $(M/2n)*(M/2n)$ times, and reducing the width of the main lobe approximately by $(M/2n)$ times, simultaneously reducing the average level of the side lobes by $n$ times. Using such element as operation of rotation for a nested into Latin square of n-th order of a magic square allows decreasing AA area by $(M/2n)$ times, and reducing the main lobe width by $(M/2n)$ times, while reducing the average side lobe level in n times. Here M is the magic constant equal to $(n^2+1)*n/2$. At the same time, the filling coefficients decrease in $(M/2n^2)^2$ and the redundancy coefficient increases in $2n*n^2/M$ times. This design has the best characteristics and can be recommended for use in the construction of non-equidistant antenna arrays.

## CONCLUSION

There is the possibility of using the Latin squares for constructing two-dimensional non-equidistant antenna arrays. An algorithm for calculating the coordinates of sources of a non-equidistant antenna array using the values of elements of Latin squares is proposed. It is shown that in this case, it is possible to obtain, with small filling coefficients, an almost complete coverage of a grid of spatial frequencies in the region of the position of elements. Directional patterns are studied, the side lobe levels of non-equidistant antennas are obtained on the basis of Latin squares.

The possibility of synthesizing large AA on the basis of composite squares using the nesting to generate Latin squares is shown. The characteristics of the obtained arrays are studied. It is shown that using mutual rotations of individual layers entering in the synthesized array can significantly improve its characteristics.

The developed approaches to the synthesis of non-equidistant antenna arrays can be used in the design of antennas for radio telescopes and systems for monitoring seismic and thunderstorm activity by receiving lithospheric and atmospheric radiation.

**REFERENCES**

1. Shifrin, Y. S., 1970. Questions of the statistical theory of antennas. *Moscow: Sov. Radio*., 383 p.
2. Sodin, L. G., 1966. Statistical analysis of nonequidistant linear antenna arrays (Statistical parameters of linear antenna arrays after removal of emitters). *Radio Engineering and Electronic Physics*, **11**, 1715-1720.
3. Leeper, D. G., 1978. Thinned aperiodic antenna arrays with improved peak sidelobe level control. *U.S. Patent No. 4,071,848. Washington, DC: U.S. Patent and Trademark Office*.
4. Kopilovich, L. E., and Sodin, L. G., 2001. Linear Non-Equidistant Antenna Arrays. *In Multielement System Design in Astronomy and Radio Science* (pp. 77-96). *Springer, Dordrecht*.
5. Kopilovich, L. Ye., 2014. The non-redundant configurations of elements on square and hexagonal lattices of large dimensions. *Radiophysics and Electronics*, **5**(19), 80-84.
6. Kopilovich, L. E., and Sodin, L. G., 2001. Multielement System Design in Astronomy and Radio Science (Vol. 268). *Springer Science & Business Media*.
7. Lutsenko, V. I., 2015. Using the properties of magic squares for aperture synthesis. *The 8th International Conference on Acousto-optical and Radar Methods of Measurement and Information Processing 20, September 23, 2015, Suzdal, Russia Materials of the 8th International Scientific and Technical Conference. Russian National Technical University. A.S. Popova. Suzdal. Russia*. Pp. 215-219.
8. Lutsenko, V. I., Popov, I. V., Lutsenko, I. V., Yiyang, L., and Mazurenko, A. V., 2016. Nonequidistant two-dimensional antenna arrays are based on magic squares. In *Physics and Engineering of Microwaves, Millimeter and Submillimeter Waves (MSMW), 2016 9th International Kharkiv Symposium on* (pp. 1-4). IEEE.