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# On Prediction and Filtering Problem of Long - Run Stationary Time Series

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## 1. Introduction.

Suppose by the irregularity of the reflectivity of the earth a seismic signal is not always stationary in usual sense, but only long-run stationary ( see [6] and [7] ). Then there arises a question :" Why is Wiener filter ,which as well known, is used in prediction and filtering of ergodic stationary time series, also applicable in processing seismic signals ? ", In this paper we try to give answer for this question.

## 2. Some remarks on Wiener filter.

For the sake of simplicity here we consider the following problem:

Let $x_n$ be a random process of second order and our task is to estimate $x_n$ from $x_{n-1}$. The criterion here is the least mean square (LMS), i.e. we find the value c that

$$E(x_n - cx_{n-1})^2 = \min_{d \in R} E(x_n - dx_{n-1})^2. \tag{1}$$

From (1) we get

$$c = \frac{\varphi_x(n, n-1)}{\varphi_x(n-1, n-1)}, \tag{2}$$

where

$$\varphi_x(m, n) = Ex_m x_n. \tag{3}$$

Then we estimate $x_n$ by

$$\tilde{x}_n = cx_{n-1}. \quad (4)$$

We call the coefficient $c$ the Wiener filter and $\tilde{x}_n$ the optimal estimate of $x_n$. Because (1) and (2) are equivalent, we can consider (2) as the definition of the Wiener filter. As we can see, in definition the Wiener filter requires only that the process $x_n$ is of second order, i.e. $Ex_n^2 < \infty$. However in practice we do not know the correlation function $\varphi(m,n)$, we can only estimate it from the observations $x_{n-1} - s$ in the case $x_n$ is an ergodic stationary process. Then

$$c = \frac{\varphi(1)}{\varphi_x(0)} \quad (5)$$

and we estimate it by

$$\hat{c} = \frac{r_x(1)}{r_x(0)}, \; r_x(s) = \frac{1}{N}\sum_{n=0}^{N-1} x_{n+s}x_n, \quad (6)$$

hence $x_n$ is estimated by

$$\hat{x}_n = \hat{c}x_{n-1}. \quad (7)$$

we call $\hat{x}_n$ the approximant of $\tilde{x}_n$. We probably keep in mind that:

a. $\hat{x}_n$ is a good approximation of $\tilde{x}_n$ only if the sample correlation $r_x(s)$ is a good estimation of $\varphi_x(s)$, i.e. the process $x_n$ is ergodic and stationary.
b. The optimal estimate $\tilde{x}_n$ is better than its approximant $\hat{x}_n$, especially in the case when the process $x_n$ is not ergodic and stationary.

By the following examples, we want to show that the above thought is not always true.

**Example 1.** Let $x_n$ satisfy

$$x_n + ax_{n-1} = u_n, \; n = 0, 1, 2, ..., \quad (8)$$

where $|a| < 1$ and

a. The variables $u_0, u_1, u_n, ...$ are independent with mean 0 and

$$E|u_n|^{2+\epsilon_0} < K < \infty \quad (9)$$

for some $K$ and $\epsilon_0 > 0$.

b.

$$lim \frac{1}{N}\sum_{n=0}^{N-1} Eu_n^2 = \sigma^2 > 0. \quad (10)$$

c.

$$1 + a_1 z + a_2 z^2 + \ldots + a_p z^p \neq 0, \ \forall\ z : |z| \leq 1, \tag{11}$$

i.e. $x_n$ is a long-run stationary autoregressive process of first order (see [6] and [7]).
We can see

$$E(x_n/x_{n-1}) = E(-ax_{n-1} + u_n/x_{n-1}) = -ax_{n-1} + Eu_n = -ax_{n-1}, \tag{12}$$

hence $\tilde{x}_n = -ax_{n-1}$ and $c = -a$. Using the main theorem in [6] or [7] we have

$$-a = \frac{g_x(1)}{g_x(0)} = \frac{\lim_{N\to\infty} r_x(1)}{\lim_{N\to\infty} r_x(0)} = c = \frac{\varphi_x(n, n-1)}{\varphi_x(n-1, n-1)} \tag{13}$$

Because the process $x_n$ defined above is not stationary, its correlation function is bivariate and unestimable by the sample correlation $r_x(s)$. However from (13) we can see that the optimal filter is estimable . More exactly speaking, in this case if we wrongly suppose that the process were ergodic and stationary and estimate $c$ by (6) then we get good estimate for $c$ because

$$\lim_{N\to\infty} \frac{r_x(1)}{r_x(0)} = \frac{\varphi_x(n, n-1)}{\varphi_x(n-1, n-1)} = c \tag{14}$$

although

$$\lim_{N\to\infty} r_x(1) \neq \varphi_x(n, n-1),$$

$$\lim_{N\to\infty} r_x(0) \neq \varphi(n-1, n-1).$$

In this case the correlation function is " a bad intermediatory device" in definition of the Wiener filter, because the formula (2) make us thinking that the Wiener filter were not estimable. If instead of (2) we define the Wiener filter by

$$c = \frac{g_x(1)}{g_x(0)} \tag{15}$$

then in this case the filter is the same as one defined by (2) and it seems estimable as it is.

**Example 2.** Let $x_n$ = cosnw , where $w$ is a uniformly distributed random variable on $[0, 2\Pi]$. In [8] we have shown that $x_n$ is not ergodic ( although it is stationary ). As well known $x_n$ is white noise, therefore $c = 0$ and the optimal estimate $\tilde{x}_n = 0$, the mean square error for $c$ is

$$E(x_n - \tilde{x}_n)^2 = 0.5. \tag{16}$$

Suppose that $x_n$ is ergodic and we can estimate $c$ by

$$\hat{c} = \frac{r_x(1)}{r_x(0)} = \frac{\sum_{n=1}^{N} cosnwcos(n+1)w}{\sum_{n=1}^{N} cos^2 nw} \tag{17}$$

$$= cosw + \frac{(cos(2N+1)w - cosw)cosw}{sin(2N+1)w + (2N-1)sinw}.$$

We can see

$$lim_{N\to\infty}\hat{c} = cosw = 0 = c, \quad a.s. \tag{18}$$

In this case $c$ is really not a good estimate of $c$. It is a natural thought that perhaps the approximant $\hat{x}_n$ is worse estimate than $\tilde{x}_n$. However we have

$$E(x_n - \hat{x}_n)^2 = E(x_n - \hat{c}x_{n-1})^2 \tag{19}$$

$$\simeq E(cosnw - coswcos(n-1)w)^2 = E(sin^2 w sin^2(n-1)w)$$

$$< E\, sin^2(n-1)w = 0.5.$$

Thus

$$E(x_n - \hat{x}_n)^2 < E(x_n - \tilde{x}_n)^2.$$

In this case the approximant gives better results than the optimal estimate. If we define our filter by (15) instead of (2) then we have the following advantages:

a. It is estimable from the observations $x_n - s$. ( As we have seen, the Wiener filter is not estimable ).

b. It gives better estimate for $x_n$ than the Wiener filter.

In the next section we propose a definition of such a filter, which in the above cases gives (15).

**3. Least time-mean square filter.**

*Definition 1.*

A process $x_n$, $n = 0, 1, 2, \ldots$ is called a long-run stationary process, if there exist the limits

$$\lim_{N\to\infty} \overline{m} = \lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N-1} x_n = A \quad a.s. \tag{20}$$

$$\lim_{N\to\infty} r(s) = \lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N-1} x_{n+s}x_n = g_x(s) \quad a.s.\ s = 0, 1, 2, \ldots, \tag{21}$$

We call $A_x$ the time mean and $g_x(s)$ the time correlation function of the process $x_n$.

*Definition 2.*

Let $x_n$ and $y_n$ be long-run stationary processes. We say that there exists the time crosscorrelation function between them if

$$\lim_{N\to\infty} r_{xy}(s) = \lim_{N\to\infty} \frac{1}{N} x_{n+s}y_n = g_{xy}(s) \quad a.s., s = 0, 1, 2, \ldots \tag{22}$$

If $g_{xy} = 0$ for $s = 0, 1, 2, \ldots$ then we say that $x_n$ and $y_n$ are time-uncorrelated.

*Definition 3.*

A long-run stationary process $u_n$ is called a long-run white noise, if $A_u = 0$ and

$$g_u(s) = \begin{cases} \sigma^2, & \text{for } s = 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\sigma$ may be a random variable (see [8] in more details for properties of long-run stationary time series).

Suppose $x_n$ and $y_n$ are long-run stationary processes and there exists the time-crosscorrelation function $g_{xy}(s)$. Our task is to estimate $x_n$ from the observations $y_n, y_{n-1}, \ldots, y_{n-q}$ where q is a non-negative integer.

For arbitrary vector-random variable $\beta = (\beta_0, \beta_1, \beta_2, \ldots, \beta_q)$ by using the long-run stationarity of the processes $x_n$ and $y_n$ it is easy to prove that there exists the limit

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} (x_n - \beta_0 y_n - \beta_1 y_{n-1} - \ldots - \beta_q y_{n-q}) = e^2(\beta) \quad a.s. \tag{23}$$

we call the quantity $e^2(\beta)$ the time-mean square error (TMSE) of the filter $\beta$. We will prove that there exists $\gamma$ for which

$$e^2(\gamma) = \min e^2(\beta). \tag{24}$$

*Definition 4.*

We call $\gamma$ the least time-mean square (LTMS) filter and we estimate $x_n$ by

$$\hat{x}_n = \gamma_0 y_n + \gamma_1 y_{n-1} + \ldots + \gamma_q y_{n-q}. \tag{25}$$

Thus, the LTMS filter is optimum for the TMSE ,i.e. its TMSE is minimum.

**Theorem 1.** *The LTMS filter satisfies the equations*

$$\begin{bmatrix} g_y(0) \; g_y(1) \ldots g_y(q) \\ g_y(1) \; g_y(0) \ldots g_y(q-1) \\ \\ \ldots\ldots \\ \\ g_y(q) \; g_y(q-1) \ldots g_y(0) \end{bmatrix} \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \\ \\ \\ \gamma_q \end{bmatrix} = \begin{bmatrix} g_{xy}(0) \\ g_{xy}(1) \\ \\ \\ \\ g_{xy}(q) \end{bmatrix} \tag{26}$$

*Proof.*

For fixed $N$ we define $\gamma(N) = (\gamma_0(N), \gamma_1(N), \ldots, \gamma_q(N))$ as follows

$$\frac{1}{N} \sum_{n=0}^{N-1} (x_n - \gamma_0(N) y_n - \gamma_1(N) y_{n-1} - \ldots - \gamma_q(N) y_{n-q})^2 \tag{27}$$

$$= min_{\beta}\frac{1}{N}\sum_{n=0}^{N-1}(x_n - \beta_0)y_n - \beta_1(N)y_{n-1} - \ldots - \beta_q(N)y_{n-q})^2,$$

where $y_n = 0$ for $n < 0$. Then we can see that $\gamma(N)$ satisfies the equation

$$\begin{bmatrix} r_y(0,0)\ r_y(1,0)\ldots r_y(q,0) \\ r_y(0,1)\ r_y(1,1)\ldots r_y(q,1) \\ \\ \ldots\ldots \\ \\ r_y(0,q)\ r_y(1,q)\ldots r_y(q,q) \end{bmatrix}\begin{bmatrix}\gamma_0(N)\\ \gamma_1(N)\\ \\ \\ \\ \gamma_q(N)\end{bmatrix} = \begin{bmatrix} r_{xy}(0)\\ r_{xy}(1)\\ \\ \\ \\ r_{xy}(q)\end{bmatrix} \tag{28}$$

where

$$r_y(l,s) = \frac{1}{N}\sum_{n=-s+1}^{N-s} y_{n+s-l}y_n,\ r_{xy} = \frac{1}{N}\sum_{n=-s+1}^{N-s} y_{n+s}y_n,\ s,l = 0,1,\ldots q.$$

As $x_n$ and $y_n$ are long-run stationary, we have

$$\lim r_y(l,s) = \lim r_y(l-s) = g_y(l-s),\ \lim r_{xy}(s) = g_{xy}(s).$$

After limiting (28) we get (26). Similarly by taking the limit of (27) we obtain (24). Thus the theorem is proved.

**Theorem 2.** *Let $x_n$ be a long-run stationary autoregressive process of order p, i.e. $x_n$ satisfies the equation*

$$x_n + a_1x_{n-1} + a_2x_{n-2} + \ldots + a_px_{n-p} = u_n,\ n = 0,1,2,\ldots \tag{29}$$

*and the conditions (9), (10), (11). Our task is to predict the value $x_n$ from $x_{n-1}, x_{n-2}, \ldots, x_{n-q}$, where $q >' p$.*

*Let $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_q)$ be the LTMS filter; $c = (c_1, c_2, \ldots, c_q)$ be the Wiener filter, i.e.*

$$E(x_n - c_1x_{n-1} - \ldots - c_qx_{n-q})^2 = \min_{d\in R^q} E(x_n - d_1x_{n-1} - \ldots - d_qx_{n-q})^2.$$

*Then $\gamma = c$. ( As we want that the index i of $\gamma_i$ denotes the time lag, we write $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_q)$ instead of $\gamma = (\gamma_0, \gamma_1, \ldots, \gamma_{q-1})$.*

*Proof.*

In [6] and [7] we have shown that the process $x_n$ is a long-run stationary and the coefficients $a_1, a_2, \ldots, a_p$ satisfy the following equations

$$\begin{bmatrix} g_x(0)\ g_x(1)\ \ldots\ g_x(p-1) \\ g_x(1)\ g_x(0)\ \ldots\ g_x(p-2) \\ \\ \ldots\ldots \\ \\ g_x(p-1)\ g_x(p-2)\ \ldots\ g_x(0) \end{bmatrix}\begin{bmatrix}a_1\\ a_2\\ \\ \\ \\ a_p\end{bmatrix} = \begin{bmatrix} g_x(0)\\ g_x(1)\\ \\ \\ \\ g_x(p-1)\end{bmatrix} \tag{30}$$

It follows from Theorem 1 that the LTMS filter $\gamma$ satisfies the equations

$$\begin{bmatrix} g_x(0) & g_x(1) & \dots & g_x(p-1) \\ g_x(1) & g_x(0) & \dots & g_x(p-2) \\ & & \dots\dots & \\ g_x(q-1) & g_x(q-2) & \dots & g_x(0) \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \\ \gamma_q \end{bmatrix} = \begin{bmatrix} g_x(0) \\ g_x(1) \\ \\ g_x(q-1) \end{bmatrix} \tag{31}$$

Hence

$$\gamma_i = \begin{cases} -a_i, & \text{for } i = 1, 2, \dots p \\ 0, & \text{for } i = p+1, p+2, \dots, q \end{cases} \tag{32}$$

and therefore

$$\hat{x} = -a_1 x_{n-1} - a_2 x_{n-2} - \dots - a_p x_{n-p}. \tag{33}$$

On the other hand

$$E(x_n / x_{n-1}, x_{n-2}, \dots, x_{n-q}) = E(-a_1 x_{n-1} - a_2 x_{n-2} - \dots - a_p x_{n-p} + u_n / x_{n-1}, \dots, x_{n-q})$$

$$-a_1 x_{n-1} - a_2 x_{n-2} - \dots - a_p x_{n-p} + E u_n = -a_1 x_{n-1} - a_2 x_{n-2} - \dots - d_p x_{n-p}.$$

This means that for $q \geq p$

$$E(x_n - a_1 x_{n-1} - \dots - a_q x_{n-q})^2 = \min_{d \in R^q} E(x_n - d_1 x_{n-1} - \dots - d_q x_{n-q})^2.$$

From which we get

$$c_i = \begin{cases} -a_i, & \text{for } i = 1, 2, \dots, p \\ 0, & \text{for } i = p+1, p+2, \dots, q. \end{cases} \tag{34}$$

The statement of the theorem follows from (32) and (34). Thus the theorem is proved.

**Theorem 3.** *Suppose a process $y_n$ has the form*

$$y_n = x_n + v_n, \tag{35}$$

*where $x_n$ is a process satisfying (29) and the conditions (9), (10), (11); $v_n$-s are independently distributed random variables with mean 0 and independent of $x_n$; furthermore*

$$E|v|^{2+\mu_0} < L < \infty, \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} E v_n^2 = \lambda^2 \tag{36}$$

*for some $\mu_0 > 0, L > 0, \lambda > 0$.*

*Let our task be the estimation of the value $x_n$, $n = 1, 2, \dots$ from $y_n, y_{n-1}, \dots, y_{n-q}$; where $q$ is some fixed positive integer. Then there exists the LTMS filter $\gamma = (\gamma_0, \gamma_1, \dots, \gamma_q)$, which is uniquely determined by the equations*

$$\begin{bmatrix} g_y(0) & g_y(1) & \dots & g_y(q) \\ g_y(1) & g_y(0) & \dots & g_y(q-1) \\ & & \dots\dots & \\ g_y(q) & g_y(q-1) & \dots & g_y(0) \end{bmatrix} \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \\ \gamma_q \end{bmatrix} = \begin{bmatrix} g_y(0) - \lambda^2 \\ g_y(1) \\ \\ g_y(q) \end{bmatrix} \tag{37}$$

*where the matrix on the left side is positive definite.*

*Proof.*

In [6] and [7] we have shown that $u_n$ and $v_n$ are long-run white noises , i.e.

$$g_u(s) = \begin{cases} \sigma^2, & \text{for } s = 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$g_v(s) = \begin{cases} \lambda^2, & \text{for } s = 0 \\ 0, & \text{otherwise.} \end{cases} \tag{38}$$

By (36) $Eu_n^2 < K_1$, $Ev_n^2 < L_1$ for some $K_1 > 0$, $L > 0$. For $k, s = 0, 1, 2, \dots$ let us consider

$$E\Big(\frac{1}{N}\sum_{n=0}^{N-1} u_{n-k}v_{n+s}\Big)^2 = \frac{1}{N}\sum_{n=0}^{N-1} Eu_{n-k}^2 Ev_{n+s}^2 \le \frac{K_1L_1}{N} \to 0.$$

Therefore by Tchebychef's inequality we get

$$P\lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N-1} u_{n-k}v_{n+s} = 0.$$

(Here Plim means the limit in Probability).

Note that the variables $z_n = u_{n-k}u_{n+s}$, $n = 0, 1, 2, \dots$ are independent, furthermore we can see

$$E|z_n|^{\mu_0} < M < \infty \; \textit{for some } 1 < \mu_0 \le 2$$

and hence

$$\sum_{n=1}^{\infty} \frac{E|z|^{\mu_0}}{n^{\mu_0}} < \sum_{n=1}^{\infty} \frac{1}{n^{\mu_0}} < \infty.$$

By the theorem 3.2 in [1], p.159 we have

$$\lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N-1} u_{n-k}v_{n+s} = 0. \; a.s. \tag{39}$$

By (11) we can take the reciprocal $B(z)$ of the $Z$-transform $A(z)$

$$B(z) = \frac{1}{1 + a_1 z + \dots + a_p z} = 1 + b_1 z + b_2 z^2 + .$$

and $x_n$ can be written in the form

$$x_n = \sum_{s=0}^{\infty} b_s u_{n-s}, \quad u_n = 0, \ \forall n < 0.$$

Then

$$g_{vx}(s) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} x_n v_{n+s} = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=0}^{\infty} b_k u_{n-k} v_{n+s}$$

$$= \sum_{k=0}^{\infty} \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} u_{n-k} v_{n+s}$$

Using this fact we can see there exists the time-crosscorrelation function $g_{xy}(s)$ and

$$g_{xy}(s) = g_y(s) - g_{vx}(s) - g_V(s) \tag{40}$$

$$= \begin{cases} g_y(0) - \lambda^2, & \text{for } s = 0 \\ g_y(s), & \text{for } s = 1, 2, \ldots \end{cases}$$

By theorem 1 there exists the LTMS filter $\gamma$ satisfying (26) , from which using (40) we get (37). Now we prove that the matrix on the left side of (37) is positive definite.

Let us denote

$$G_x = \begin{bmatrix} g_x(0) & g_x(1) & \ldots & g_x(p) \\ g_x(1) & g_x(0) & \ldots & g_x(p-1) \\ \vdots & \vdots & \ddots & \vdots \\ g_x(p-1) & g_x(p-2) & \ldots & g_x(0) \end{bmatrix}$$

$$G_y = \begin{bmatrix} g_y(0) & g_y(1) & \ldots & g_y(q) \\ g_y(1) & g_y(0) & \ldots & g_y(q-1) \\ \vdots & \vdots & \ddots & \vdots \\ g_y(q) & g_y(q-1) & \ldots & g_y(0) \end{bmatrix}$$

Then $G_y = G_x + \lambda^2 I$, where I is a unit matrix. In [6] we have shown that $G_x$ is positive definite. Let $d = (d_0, d_1, \ldots, d_q)$ be an arbitrary non-zero vector. Then

$$d'G_y d = d'G_x d + \lambda d'd > d'G_x d > 0$$

which means that $G_y$ is positive definite. Thus the theorem is proved.

**Remark:** In practice it is usually a case when we have to estimate the signal $x_n$ from the noisy signal $y_n$, which we can observe. It is reasonable to think that in many cases we can invesigate the strength of a noise before producing the sequence $y_n$. ( For example before we start an explosion to produce an elastic wave into the ground ). When there only the noise $v_n$ appears, we can estimate $\lambda^2$ by $r_v(0)$. Thus we can suppose that $\lambda$ is known. Then we can estimate $\gamma$ by

$$\begin{bmatrix} r_y(0)\ r_y(1)\ ...r_y(q) \\ r_y(1)\ r_y(0)\ ...r_y(q-1) \\ ...... \\ r_y(q)\ r_y(q-1)\ ...r_y(0) \end{bmatrix} \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \\ \gamma_q \end{bmatrix} = \begin{bmatrix} r_y(0)-\lambda^2 \\ r_y(1) \\ \\ r_y(q) \end{bmatrix} \quad (41)$$

then $lim \gamma = \gamma$, a.s. When the processes $x_n$, , $y_n$ are stationary , $u_n$ and $v_n$ are white noises with $Eu_n = \sigma$ and $Ev_n = \lambda$ it can be shown that the Wiener filter $c = (c_0, c_1, ..., c_q)$ satisfies the equations

$$\begin{bmatrix} \varphi_y(0)\ \varphi_y(1)\ ...\varphi_y(q) \\ \varphi_y(1)\ \varphi_y(0)\ ...\varphi_y(q-1) \\ ...... \\ \varphi_y(q)\ \varphi_y(q-1)\ ...\varphi_y(0) \end{bmatrix} \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \\ \gamma_q \end{bmatrix} = \begin{bmatrix} \varphi_y(0)-\lambda^2 \\ \varphi_y(1) \\ \\ \varphi_y(q) \end{bmatrix} \quad (42)$$

If furthermore the above processes are ergodic then we can estimate c by

$$\begin{bmatrix} r_y(0)\ r_y(1)\ ...r_y(q) \\ r_y(1)\ r_y(0)\ ...r_y(q-1) \\ ...... \\ r_y(q)\ r_y(q-1)\ ...r_y(0) \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ \\ c_q \end{bmatrix} = \begin{bmatrix} r_y(0)-\lambda^2 \\ r_y(1) \\ \\ r_y(q) \end{bmatrix} \quad (43)$$

which is formally the same as (42).

**Conclusion.**

As well known, the problem of recognition of ergodic stationary time series is very difficult in practice. It can be shown that for arbitrary time series $x_n$ ( not necessarily ergodic and stationary ) if there exist the limits

$$lim_{N\to\infty} r_x(s) = g_x(s), s = 0, 1, 2, ...,$$

then $g_x(s)$ is non-negative definite, i.e. it is a correlation function of some stationary process. Thus if $r_x(s)$ converges when N tends to $\infty$ then for N large enough it likes a correlation function of a stationary process, inspite the process $x_n$ is stationary or not. Therefore we can not recognize, the stationarity of the time series $x_n$ by its sample correlation function. By this reason in practical application sometimes we may wrongly suppose that our time series were ergodic and stationary ( but in fact they are not ) and apply, the Wiener filter . What happens then ?

a. If $x_n$ is a long-run stationary autoregressive process and our task is one-step prediction then the result is an approximation of the Wiener filter, inspite of the fact that the correlation function is unestimable.

b. If our time series are long-run stationary only then in many cases the result is not an approximation of Wiener filter, it is an approximation of the LTMS filter only. However this does not always cause much matter, because by the example 2 we can see sometimes the LTMS filter gives better result than the Wiener filter even by the least mean square criterion ( by definition the LTMS filter is optimum for the least time-mean square criterion ).

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**Abstract.**

On Prediction and Filtering Problem of Long - Run Stationary Time Series

*Suppose by the irregularity of the reflectivity of the earth a seismic signal is not always stationary in usual sense, but only long-run stationary (see [6] and [7] ). Then there arises a question: 'why is wiener filter, which is as well known is used in prediction and filtering of ergodic stationary time series, also applicable in processing seismic signals?'*

*In this paper we try to give answer to this question.*