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# Extracting Invariants Based on Coordinate Transformations

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## 1. The statistic invariants

Let $f(x,y) \geq 0$ be a real bounded function with the support on finite region $R$ we define $(p+q)$th-order moments by

$$m_{pq} = \int\int_{-\infty}^{\infty} f(x,y)x^p y^q dxdy, \; p,q = 0,1,2,... \quad ($$

Note that, setting $f(x,y) = 1$ the equation (1) gives the moments of region $R$ that could represe a shape. Thus, the results presented here would be applicable to arbitrary objects as well as th shape.

Theorem: *The finite set moments $\{m_{pq},\ p,q = 0,1,2,..\}$ uniquement determine $f(x,y)$, and v versa[4].*

The center moment $\mu_{pq}$ is defined as follows:

$$\mu_{pq} = \int\int_{-infty}^{\infty} f(x,y)(x-\overline{x})^p(y-\overline{y}^q)dxdy, \; p,q = 0,1,2,... \quad ($$

where $\overline{x} = \frac{m_{10}}{m_{00}}$, $\overline{y} = \frac{m_{01}}{\mu_{00}}$ and the standard moment $\eta_{pq}$ for scaling transformation is defined follows:

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}}, \; \gamma = 1/(2(p+q)) + 1.$$

7 invariants were indicated by Hu [1] for above transformation

$$\Phi_1 = \mu_{20} + \mu_{02}$$
$$\Phi_2 = (\mu_{20} - \mu_{02})^2 + \mu_{11}^2$$
$$\Phi_3 = (\mu_{30} - 3\mu_{12})^2 + (\mu_{03} - 3\mu_{21})^2$$
$$\Phi_4 = (\mu_{30} + \mu_{12})^2 + (\mu_{03} + \mu_{21})^2$$
$$\Phi_5 = (\mu_{30} - 3\mu_{12})(\mu_{30} + \mu_{12})((\mu_{03} + \mu_{12})^2 - 3(\mu_{21} + \mu_{03})^2) + (3\mu_{21} - \mu_{03})(\mu_{03} + \mu_{21})((\mu_{03} + \mu_{12})^2 - 3(\mu_{12} + \mu_{30})^2)$$
$$\Phi_6 = (\mu_{20} - \mu_{02})[(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2] + 4\mu_{11}(\mu_{30} + \mu_{12})(\mu_{21} + \mu_{03})$$
$$\Phi_7 = (3\mu_{21} - \mu_{03})(\mu_{30} + \mu_{12})((\mu_{30} + \mu_{12})^2 - 3(\mu_{21} + \mu_{12})^2) - (3\mu_{12} - \mu_{30})(\mu_{03} + \mu_{21})((\mu_{03} + \mu_{21})^2 - 3(\mu_{21} + \mu_{03})^2).$$

In the paper we use polar coordinate transformations for proving their correctness.

Let us denote

$$x - \overline{x} = r\cos\phi,$$
$$y - \overline{y} = r\sin phi$$

We can obtain the centre moments in polar coordinate as follows:

$$\mu_{pq} = \int_0^{2\pi}\int_0^{\infty} f(r,\phi)r^{p+q+1}\cos^p\varphi\sin^q\phi dr d\phi.$$

Let $g_\theta(\phi)$ is the function

$$g_\theta(\phi) = \int_0^{\infty} r^\theta f(r,\phi)dr \tag{2b}$$

then we obtain the expression

$$\mu_{pq} = \int_0^{2\pi} g_\theta p + q + 1(\phi)dr \tag{3}$$

It is not difficult to prove that $g_\theta(\alpha)$ is circulating with period $2\pi$.

If the range $R$ is rotated around point $(x, y)$ with angle $\alpha$, then $m_{pq}$ will be defined by:

$$\mu_{pq}(\alpha) = \int_0^{2\pi} g_\theta p + q + 1(\phi + \alpha)dr \tag{4}$$

Obviously, we have the results:

$$\Phi_1 = \mu_{20}^2(\alpha) + \mu_{02}^2(\alpha) \tag{5}$$
$$= |\int_0^{2\pi} g_\theta^3(\alpha + \phi)e^{i\phi}d\phi|^2 = |e^{-i\alpha}\int_0^{2\pi} g_\theta^3(\phi)e^{i\phi}d\phi|^2 = |\int_0^{2\pi} g_\theta^3(\phi)e^{i\phi}d\phi|^2$$
$$\Phi_2 = (\mu^{20}(\alpha) - \mu_{02}(\alpha))^2 + \mu_{11}^2 \tag{6}$$
$$= |\int_0^{2\pi} g_\theta^3(\alpha + \phi)e^{2i\phi}d\phi|^2 = |e^{-2i\alpha}\int_0^{2\pi} g_\theta^3(\phi)e^{2i\phi}d\phi|^2 = |\int_0^{2\pi} g_\theta^3(\phi)e^{2i\phi}d\phi|^2$$
$$\Phi_3 = (\mu_{30}(\alpha) - 3\mu_{12}(\alpha))^2 + (\mu_{03}(\alpha) - 3\mu_{21}(\alpha))^2$$
$$= |\int_0^{2\pi} g_\theta^3(\alpha + \phi)e^{3i\phi}d\phi|^2 = |e^{-3i\alpha}\int_0^{2\pi} g_\theta^3(\phi)e^{3i\phi}d\phi|^2 = |\int_0^{2\pi} g_\theta^3(\phi)e^{3i\phi}d\phi|^2$$

To prove the reality of the remaint invariants we can use the Euler formula as follows:

$$\cos\phi = \frac{(e^{i\phi} + e^{-i\phi})}{2}; \quad \sin\phi = \frac{(e^{i\phi} - e^{-i\phi})}{2}; \tag{7}$$

It is easy to obtain the invariant moments $\Phi_4$, $\Phi_5$, $\Phi_6$, $\Phi_7$, .

From the equations (5), (6), (7), (4) we see that

$$vert\int_0^{2\pi} g_\theta(\phi)e^{it\phi}dr \tag{8}$$

is invariant for the translation and rotation, where $\theta$ and $t$ are the arbitrary parameters. When $\theta = 3$, $t = 1$ and $\theta = 3$, $t = 2$, we obtain the moments $\Phi_1$, $\Phi_2$, respectively.

## 2. Extracting fhe invariants based on the contour

If $f(r,\phi) = 1$, from equations (2) ,(8) we can rewrite (8) as follows:

$$\int_0^{2\pi}\int_0^{r_{max}(\phi)} r^\theta e^{it\phi}d\phi \tag{9}$$

where $r_{max}(\phi)$ is a distance from the central point of the range $R$ to contour with the polar angle $\phi$. When $\theta = 2$, from equation (9), the expression

$$|\int_0^{2\pi} \frac{1}{r_{max}(\phi)} e^{it\phi}d\phi| \tag{10}$$

is invariant for rotation. It is a characstic function of $\frac{1}{r_{max}(\phi)}$. When $t$ takes the values 0,1,2,... we have the Fourier effects on the expansion of function $\frac{1}{r_{max}(\phi)}$

If the range $R$ is a discrete set of contour points $(x_1, y_1)$, $(x_2, y_2)$... and the¡N¿central point lies on the coordinate¡N¿of origin. We can caculate $r(\phi)$ - the distance from center point to the contour with the angle $\phi$ as follows:

$$lorcrr(\phi) = l_i\cos\phi + r_i\sin\phi. \tag{11}$$

where

$$l_i = \frac{x_{i+1} - x_i}{(y_{i+1} - y_i)x_i - (x_{i+1} - x_i)y_i}; \quad r_i = \frac{x_{i+1} - x_i}{(y_{i+1} - y_i)x_i - (x_{i+1} - x_i)y_i} \&(12)$$

and $\phi_i \le \phi \le \phi_{i+1}$: $\phi_i = \arctan\frac{y_i}{x_i}$; $i = 0, 1, ...$

The effects $a_k$ and $b_k$ are calculated by expressions:

$$a_k = \frac{1}{2\pi}\sum_{i=1}^{n} l_i[\frac{\sin(k+1)\phi}{2(k+1)} + \frac{sin(k-1)\phi}{2(k-1)}]|_{\phi=\phi_i}^{\phi=\phi_i+1} + \tag{13}$$

$$r_i[\frac{\sin(k-1)\phi}{2(k-1)} - \frac{\sin(k+1)\phi}{2(k+1)}]|_{\phi=\phi_i}^{\phi=\phi_{i+1}}$$

$$b_k = \frac{1}{2\pi}\{\sum_{i=1}^{n} l_i[\frac{\cos(k-1)\phi}{2(k+1)} - \frac{\cos(k+1)\phi}{2(k+1)}]|_{\phi=\phi_i}^{\phi=\phi_i+1} \tag{14}$$

$$r_i[\frac{\sin(k-1)\phi}{2(k-1)} - \frac{\sin(k+1)\phi}{2(k+1)}]|_{\phi=\phi_i}^{\phi=\phi'+1}\}$$

$$a_0 = \sum_{i=1}^{n}(l_i\sin\phi - r_i\cos\phi)|_{\phi=\phi_i}^{\phi=\phi_i+1}. \tag{15}$$

herefore, the expression $z_k = \sqrt{a_k^2 + b_k^2}$ is invariant for the rotation and translation. Further ore, the ratio $\frac{z_k}{a_0}$ is invariant for the scale transformation.

## The SCC model (SCC- Star Contour Code)

This model is based on a very simple fact that the convexity is unchangeable under projection:

1. Angles that are greater than 180 are always greater 180
2. Angles that are less than 180 are always less 180
3. Angles that are equal to 180 are always equal to 180

The SCC is used for every vertex. The codes are assigned according to the follows:
Each point in the set is considered as central point and all others around it are connected by aight lines.

1. Convert vertex point is assigned '0'
2. Concave vertex point is assigned '1'
3. The point on line (vertex angle equal to 180) is assigned '2'.
4. Lines are drawn in certain direction, say clockwise. If serval points lies on the straight e with the central point, will be draw from far side to near side of the central point.
5. Points located on the border of the set may only have partial contour. In this case, the o terminal point will be assigned '3'.

Matching by use of SCC Let set 1 and set 2 be two n-point set (this limit is actually unnecessary t it makes computation simple). There are k points in set 1 difference from point s in set 2 and ners are same each other. Two sets suffer different perspective transformations. An algorithm the matching by SCC is given as follows.

Give names to the point in two set, say A, B, C,... and a, b, c,...computer chain code for every int in set 1' and set 2. List the names every of contour point with them to gether.
Estimate the value of k or k/n, the degree of distortion, from ratio of signal to noise:
Matching is progressed in two level: compare every chain code of set 1 with that of set 2 in ircular manner. Record the number of codes in the greatest common part. Set up threshold When the number of common codes is greater than T, the matching of these two points will acceptable. The second level : for each acepted matched pair of points, compare every pair of ir contour points in the common part in the same way as in the first level. For acceptable pair contour points judged by the same T, add 1 to the corresponding element of matrix $M$.
Select L pairs of points that have the L greatest values of element in $M$. We will see that L is und 2k. For every pair of these $L$ pairs, use any three couples of its contour point in common t to set up the restoring transformations and use them to convert set 2 into space of set 1. Now re are new L pictures of set 2.
Compare each one of the L pictures of set 2 with set 1 to see how many couples of points are ncident. If the number of couples of points is near $n - k$, the matching of set 1 with set 2 will acceptable.

## 4. Conclusion

This paper has presented the correctness of invariant moments and given a method for computing invariant moments from coordinate transformation with an invariant extracting based on the contour points, this is a simple calculation and can be used for industrial objects identification. Some evalutions of statistical matching, fourier expansion based matching and graphical matching are given in the last section of the paper. This paper is fulfilled by the aiding of the basic researching program in the nature science field.

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## Abstract

Extracting Invariants Based on Coordinate Transformations

*This paper deals with some methods extracting invariants for the translation, rotation and scaling and finding invariants which are depended on statistic invariants. It collates them with the Fourier transform. For discrete point set we can use the polar coordinate to construct SCC (Star Contour Code) and give match algorithm for SCC.*