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Reasoning in knowledge bases with extrenal and internal uncertainties

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I. Introduction

In the recent years, in A.I. a great deal of attention has been devoted to formalisms dealing with various aspects of reasoning with uncertain information, and a number of theories and methods for handling uncertainty of knowledge has been proposed, notably, the probability theory, the Dempster-Shafer theory, and the possibility theory. Among these theories, the propobality one is surely the most developed by its long history and elaborated foundations.

In this paper the uncertainty of a sentence will be given by an interval of possible values for its truth probability. Two types of knowledge with uncertainty will be investigated: external uncertainty and internal uncertainty The former is given in the form $< S, I >$, in which S is a sentence, and $I =$ [a,b] is a closed subinterval of the unit interval [0,1]; it means that the truth probability of the whole sentence S lies in the interval I. I is then called the interval of truth probabities of S. In the later case, intervals of truth probabilities are given to subsentences of the given sentence S. For example,

$$< S_1, I_1 > \wedge < S_2, I_2 > \wedge \cdots \wedge < S_n, I_n > \rightarrow < S_{n+1}, I_{n+1} >$$

is a knowledge with internal uncertainty.

Let $\mathcal{B}$ be a knowledge base with these types of uncertainty and S be a any sentence. A semantics, which underlies a method of deducing the interval of truth probabilities of S from $\mathcal{B}$, will be given.

The paper is structured as follows: In Section 2 we shall review the probabilistic logic by N.J. Nilsson, and its extention to probabilistic logic with interval values. In Section 3, we shall discuss a method of reasoning from a set of knowledges with internal uncertainty. Section 4 is devoted to a method of reasoning from a knowledge base containing both external and internal uncertainties. Some illustrative examples will be given in Section 5.

2. External Uncertainty

We shall use methods of the interval-valued probabilistic logic for reasoning in knowledge bases with external uncertainty. This logic is based on the semantics given by N.J. Nilsson [4], and is presented in details, e.g., in [6].

Let us give a knowledge base

$$\mathcal{B} = \{< S_i, I_i > | i = 1, ..., L\},$$

and let S be a target sentence. We put $\sum = \{S_1, ...S_L, S\}$, and suppose that $W_1, ..., W_k$ are *all* $\sum$ *- classes of possible worlds.* Every class W_j is characterized by a consistent vector $(u_{1j}, ..., u_{Lj}, u_j)$ of truth values of sentences $S_1, ...S_L, S$.

Given a probability distribution $(p_1, ...p_K)$ over the classes $W_1, ..., W_K$, the truth probability of sentence S_i is defined to be the sum of probabilities of classes of worlds in which S_i is true, i.e., $\pi(S_i) = u_{i1}p_1 + \cdots + u_{iK}p_K$. From this semantics it follows that, given the knowledge base $\mathcal{B}$, the derived interval-value $[\alpha, \beta]$ for the truth probability of the sentence S is defined by

$$\alpha = \min \pi(S), \quad \beta = \max \pi(S),$$

where

$$\pi(S_i) = u_1p_1 + \cdots + u_Kp_K,$$

subject to the constraints

$$\begin{cases} \pi_i = u_{i1}p_1 + \cdots + u_{iK}p_K \in I_i \ (i = 1, \ldots, L) \\ \sum_{j=1}^{K} p_j = 1, \ p_j \geq 0 (j = 1 \ldots, K). \end{cases}$$

We denote the interval $[\alpha, \beta]$ by $F(S, \mathcal{B})$, and write $\mathcal{B} \vdash < S, F(S, \mathcal{B}) >$.

Now, let us give a set of sentence Γ. We define $\mathcal{I}$ to be the set of all mappings from Γ into $C[0,1]$ - the set of closed subintervals of $[0,1]$. A such mapping I assigns to each sentence $P \in \Gamma$ an interval $I(P) \in C[0,1]$.

The given knowledge base $\mathcal{B}$ defines an operator $R_{\mathcal{B}}$ from $\mathcal{I}$ into $\mathcal{I}$ as follows: For every $I \in \mathcal{I}$, we establish a new knowledge base

$$\mathcal{B}' = \mathcal{B} \cup \{< P, I(P) > | P \in \Gamma\}.$$

and then we take for every $P \in \Gamma$ the interval $I'(P) = F(P, \mathcal{B}')$. The mapping I' is $R_{\mathcal{B}}(I)$ defined to be the image of I by the operator $R_{\mathcal{B}}$: $R_{\mathcal{B}}(I) = I'$.

It is easy to see that if $R_{\mathcal{B}}(I) = I'$ then $R_{\mathcal{B}}(I') = I'$; therefore

$$R^n_{\mathcal{B}}(I) \;=\; R_{\mathcal{B}}(I) \quad \text{for any } n \geq 1. \qquad (*)$$

The calculation of the operator $R_{\mathcal{B}}$ is reduced to the solution of linear programming problems which has to face up a very large computational complexity whenever the sizes of $\mathcal{B}$ and Γ are large. Some attempts to reducing the size of linear programming problems have been investigated, e.g., a method of reduction is given in [7] for the cases when the core $\{S_1, \ldots, S_L\}$ of $\mathcal{B}$ forms a logic program.

Instead of the method presented above we can use methods of approximate reasoning, e.g., by means of deductions based on inference rules (see [2]), however, in this case the property (*) may not be satisfied.

3. Internal Uncertainty

In this section a method of reasoning on knowledges with internal uncertainty will be discussed. We limit ourself to consider knowledges given by rules of the form:

$$< S_1, I_1 > \wedge \cdots \wedge < S_n, I_n > \rightarrow < S, I >$$

Let us given a knowledge base $\mathcal{B} = \{J_j | j = 1, \ldots, M\}$, where J_j is the rule:

$$J_j \;=\; < A_{j1}, I_{j1} > \wedge \cdots \wedge < A_{jm_j}, I_{jm_j} > \rightarrow < A_{c_j}, I_{c_j} > .$$

Let Γ be a set of sentences containing all sentences occuring in rules $J_j (j = 1 \ldots M)$ of $\mathcal{B}$. As above we denote by $\mathcal{I}$ the set of mappnings I from Γ to $C[0,1]$.

For any $I_1, I_2 \in \mathcal{I}$, we say that $I_1 \leq I_2$ iff $I_1(P) \subseteq I_2(P)$ for every $P \in \Gamma$.

We say that the rule J_j *is satisfied* by the mappning $I \in \mathcal{I}$, iff $I(A_{jk}) \subseteq I_{j_k}$ for every $k = 1, \ldots, m_j$. Note that if $m_j = 0$ then J_j is satisfied by any mapping I.

Now we define an operator $t_{\mathcal{B}}$ from I into $\mathcal{I}$, which transforms any $I \in \mathcal{I}$ into a mapping $t_{\mathcal{B}}(I)$ such that for every $P \in \Gamma$:

$$t_{\mathcal{B}}(I)(P) \;=\; I(P) \cap (\bigcap_{j \in E} I_{c_j}),$$

where $E = \{j | A_{c_j} = P$ and J_j is satisfied by $I\}$. Here. for the sal assume that $\bigcap_{j \in E} I_{c_j} = [0,1]$ whenever $E = \emptyset$.

We have the following proposition:

Proposition 1. *For any mapping $I \in \mathcal{I}$ there exists always a natural number n such that $t_{\mathcal{B}}^{n+1}(I) = t_{\mathcal{B}}^{n}(I)$. In other orlds, the process of iteration of the operator $t_{\mathcal{B}}$ on any given $I \in \mathcal{I}$ always halts after a finite number of steps.*

Proof. Suppose that $E_0, E_1, E_2, \ldots$ are the set of indexes of rules which are satisfied satisfied by $I, t_{\mathcal{B}}(I), t_{\mathcal{B}}^2(I), \ldots$, respectively.

Let $h_i = |E_i|$ $(i = 1,2,...)$ be the number of elements of the set E_i $\{h_i/i = 1,2,...\}$ is then a sequence of integers such that

$$0 \leq h_0 \leq h_1, \ldots \leq h_n \leq \ldots \leq M.$$

Consider two cases

(i) There exists a number n such that $h_{n-1} = M$, i.e., J_j is satisfied by the mapping $t_{\mathcal{B}}^{n-1}(I)$ for every $J = 1, \ldots, M$. In this case we have $t_{\mathcal{B}}^{n}(I) = t_{\mathcal{B}}^{n+1}(I)$.

(ii) There exists a number n such that $h_{n-1} = h_n < M$. In this case we have $E_{n-1} = E_n$, and it is easy to see that

$$t_{\mathcal{B}}^{n}(I) = t_{\mathcal{B}}^{n+1}(I).$$

The proposition has been proved.

From this proposition we can define an operator $T_{\mathcal{B}}$ as follows: *For any $I \in \mathcal{I}$, $T_{\mathcal{B}}(I) = t_{\mathcal{B}}^{n}(I)$, where n is the least number such that $t_{\mathcal{B}}^{n}(I) = t_{\mathcal{B}}^{n+1}(I)$.*

Let S be a sentence. We denote by Γ the set consisting of S and of all sentences occuring in rules J_j of $\mathcal{B}$. Let I be the mapping which assigns the interval [0,1] for every sentence in Γ. Then $T_{\mathcal{B}}(I)(S)$ can be considered as the interval-value for the truth probability of the sentence S derived from the knowledge base $\mathcal{B}$.

4. A Method of Reasoning

We consider now the knowledge bases consisting both types of knowledges with external and internal uncertainty. Let $\mathcal{B}$ be such a knowledge base, we can write $\mathcal{B} = \mathcal{B}^E \cup \mathcal{B}^I$, where $\mathcal{B}^E$ consists of knowledges with external uncertainty. Suppose that

$$\mathcal{B}^E = \{< S_i, I_i > \mid i = 1, \ldots, L\},$$

$$\mathcal{B}^I = \{J_j \mid j = 1, \ldots, M\},$$

where

$$J_j = < A_{j1}, I_{j1} > \wedge \cdots \wedge < A_{jm_j}, I_{jm_j} > \rightarrow < A_{c_j}, I_{c_j} > .$$

Let S be any (target) sentence. Our problem is to deduce from the knowledge base $\mathcal{B}$ the interval value for the truth probability of the sentence S.

For this purpose we put Γ to be the set consisting of the sentence S and all distinct sentences occuring in $\mathcal{B}^E$ and $\mathcal{B}^I$. Denote by $\mathcal{I}$ the set of mappings from Γ to $C[0,1]$. Let I_0 be the mapping defined by

$$I_0(P) = \begin{cases} I_i, & \text{if } P = S_i \text{ for some } i = 1, \ldots, L \\ [0,1], & \text{otherwise.} \end{cases}$$

I_0 is called *the initial assignment* (of interval-value to sentences in Γ).

Now we define a sequence of assignments I_n $(n = 0, 1, \ldots,)$ initiated by I_0 and given recursively as follows:

$$I_n = \begin{cases} \mathcal{R}(I_{n-1}), & \text{if } n \text{ is odd} \\ \mathcal{T}(I_{n-1}), & \text{if } n \text{ is positive even.} \end{cases}$$

Here $\mathcal{R}$ stands for $R_{\mathcal{B}^E}$, and $\mathcal{T}$ stands for $T_{\mathcal{B}^I}$.

Proposition 2. *Let $\mathcal{B}$ be a knowledge base, and S be any given sentence. There exists a natural number n such that $I_{n+2} = I_{n+1} = I_n$.*

Proof. From the definition of the sequence $\{I_n\}$ $(i = 0, 1, \ldots)$ we can write

$$I_0 \overset{\mathcal{R}}{\longmapsto} I_1 \overset{\mathcal{T}}{\longmapsto} I_2 \overset{\mathcal{R}}{\longmapsto} \ldots \overset{\mathcal{R}}{\longmapsto} I_{n-2} \overset{\mathcal{T}}{\longmapsto} I_{n-1} \overset{\mathcal{R}}{\longmapsto} I_n \overset{\mathcal{T}}{\longmapsto} I_{n+1} \overset{\mathcal{R}}{\longmapsto} \ldots$$

Let h_i be a number of rules J_i satisfied by $I_i (i = 1, 3, 5, \ldots)$. Then, $\{h_i\}$ is a sequence of integers such that

$$0 \le h_1 \le h_3 \le \cdots \le h_{n-2} \le h_n \le \cdots \le M.$$

Consider two cases

(i) There exists a number n such that $h_{n-2} = M$, i.e, J_j is satisfied by the mapping I_{n-2}, for every $j = 1, \ldots, M$. Then, any rule $J_j (j = 1, \ldots, M)$ is also satisfied by the mappings

$$\begin{aligned} I_{n-1} &= \mathcal{T}(I_{n-1}) \\ I_n &= \mathcal{R}(I_n) \end{aligned}$$

Thus

$$\begin{aligned} I_{n+1} &= \mathcal{T}(I_n) = I_n \\ I_{n+2} &= \mathcal{R}(I_{n+1}) = \mathcal{R}(I_n) = I_n \end{aligned}$$

or $I_n = I_{n+1} = I_{n+2}$.

(ii) There exists a number n such that $h_n = h_{n-2} < M$. Then the sets of rules satisfied by I_n and by I_{n-2} are the same. Therefore,

$$\begin{aligned} I_{n+1} &= \mathcal{T}(I_n) = I_n \\ I_{n+2} &= \mathcal{R}(I_{n+1}) = I_{n+1} \end{aligned}$$

and we have again $I_n = I_{n+1} = I_{n+2}$.

This completes our proof.

Let n be *the least number* having the property sated in the proposition 2. We denote this I_n by I^*, and call it to be *the resulting assignment* deduced from $\mathcal{B}$ to sentences in Γ.

The interval $I^*(S)$ is defined to be *the interval value for the truth probability of the sentence S derived from the knowledge base $\mathcal{B}$.* We write also:

$$\mathcal{B} \vdash < S, I^*(S) > .$$

5. Examples

This section presents two examples illustrating the method of reasoning in a knowledge base consisting of both types of knowledge with internal and external uncertainty.

Example 1.

Given a knowledge base $\mathcal{B} = \mathcal{B}^E \cup \mathcal{B}^I$ where $\mathcal{B}^E$ is the set of sentences

$$B \rightarrow A : [1,1]$$
$$A \rightarrow C : [1,1]$$
$$B : [.2,.6]$$
$$C : [.6,.7]$$

and $\mathcal{B}^I$ is the set of rules

$$J_1 = C : [.5,.7] \longrightarrow B : [.3,.5]$$
$$J_2 = B : [.2,.5] \wedge C : [.5,.7] \longrightarrow A : [.2,.5]$$

Calculate the interval of truth probabilities of the sentence A.

Step 1. Applying the operator $\mathcal{R}$, we get

$$A : [.2,.7]$$
$$B : [.2,.6]$$
$$C : [.6,.7].$$

Step 2. Both rules J_1 and J_2 are satisfied, so applying the operator $\mathcal{T}$ we obtain

$$A : [.2,.5]$$
$$B : [.3,.5]$$
$$C : [.6,.7].$$

Step 3. Iterate $\mathcal{R}$, where $B : [.2,.6]$ is now replaced by B: $[.3,.5]$, we get

$$A : [.3,.5].$$

As both J_1 and J_2 are satisfied after *step 1*, it is not necessary to repeat $\mathcal{T}$ after *step 3* and we get the finall result $A : [.3,.5]$.

Note that $\mathcal{B}^E$ is a type-A problem (see [2]); hence, we can apply the type-A rules to computing the intervals for A, B, C instead of solving linear programming problems.

Example 2. This example is more complex than the above; it illustrates the iteration of the operator $\mathcal{T}$. Suppose that we wish to derive the interval of truth probabilities of the sentence $A \wedge D$ from the knowledge base $\mathcal{B} = \mathcal{B}^E \cup \mathcal{B}^I$ where $\mathcal{B}^E$ is the set defined as follows

$$B \rightarrow A : [.9,1]$$
$$D \rightarrow B : [.8,.9]$$
$$A \rightarrow C : [.6,.8]$$
$$D : [.8,1]$$
$$C : [.2,.4]$$

and $\mathcal{B}^I$ is the set of the rules $J_j (j = 1,2,3)$:

$$J_1 = C : [.1,.5] \longrightarrow B \wedge D : [.7,1]$$
$$J_2 = C : [.2,.3] \wedge (B \wedge D) : [.7,.9] \longrightarrow A \wedge D : [.4,.7]$$
$$J_3 = A \wedge D : [.6,.9] \longrightarrow C : [.2,.3].$$

Step 1 Applying $\mathcal{R}$, we have

$$A : [.2,.8]$$
$$B \wedge D : [.6,.9]$$
$$A \wedge D : [.5,.8]$$
$$C : [.2,.4]$$
$$D : [.8,1].$$

Step 2. Since only J_1 is satisfied, we get

$$B \wedge D : \ [.7,.9]$$

and the interval values of $A, A \wedge D, C, D$ are not varied.
Step 3. Repeating $\mathcal{R}$, only the interval value of $A \wedge D$ is varied

$$A \wedge D : \ [.6,.8]$$

Step 4. Repeating $\mathcal{T}$, now J_3 and J_2 are satisfied, so we have

$$C : [.2,.3]$$
$$A \wedge D : [.6,.7].$$

Step 5. $\mathcal{B}^E$ is now changed into $\mathcal{B}'$ which consists of:

$$B \rightarrow A : [.9,1]$$
$$D \rightarrow B : [.8,.9]$$
$$A \rightarrow C : [.6,.8]$$
$$D : [.8,1]$$
$$C : [.2,.3].$$

$\mathcal{R}$ is repeated and we have

$$A \wedge D : [.6..7].$$

By virtue of that all rules in $\mathcal{B}^I$ are satisfied in *step 4*, the interval value for the truth probability of $A \wedge D$ is [.6..7].

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Abstract

The paper presents a method of logical reasoning in knowledge bases with uncertainty; such a knowledge base is given by a set of "knowledges" of two following forms:

(1) $< S, I >$ *where* S *is a sentence, and* $I \subseteq [0,1]$ *is an interval of the possible values for truth probability of* S.

(2) $< S_1, I_1 > \wedge < S_2, I_2 > \wedge \cdots \wedge < S_n, I_n > \rightarrow < S, I >$, *where* $S_1, \ldots, S_n, S$ *are sentences, and* $I_1, \ldots, I_n, I$ *are the corresponding intervals of their truth probabilities.*

Let $\mathcal{B}$ *be a such knowledge base, and* S *be a goal sentence. The interval of truth probabilities of* S *derived from* $\mathcal{B}$ *can be found by the proposed method.*