

A KIND OF SELF-TUNING PID CONTROLLER

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Summary. A Self-tuning PID controller (STC) using a linguistic description of the control strategy and based on the theory of fuzzy set is presented in this paper.

I. INTRODUCTION

Despite the advent at many sophisticated control theories and techniques, the majority industrial process nowadays are still regulated PID controller. The PID controller and especially the self-tuning PID controller are in full play of control systems. For this reason we can introduce some kinds of them. Firstly, the self-tuning PID is designed based on Ziegler-Nichols princip (1942) and secondly, on other princip, such as

- STC by ultimate Sensitivity method
- STC by limit cycle method
- STC by least square method
- STC by pattern recognite method
- STC by Neuron Net method

In this work the STC by expert method is presented. There are two types of STC using expert system

1. To correct PID parameters according to the response ware patterns
2. To correct PID parameters using the fuzzy control

II. PROBLEM

The main ideas in this work are concerned with closed loops response (see Fig 1). The strategy to tune the parameters of the PID controller is based on I, II, III, IV stage of closed loops response. At the stages I and III the output of the system approaches to the setpoint. To accelerate the system response at these stages are needed. On the contrary at the stages II, IV the output derivates away from the setpoint, the system response must be slowdown.

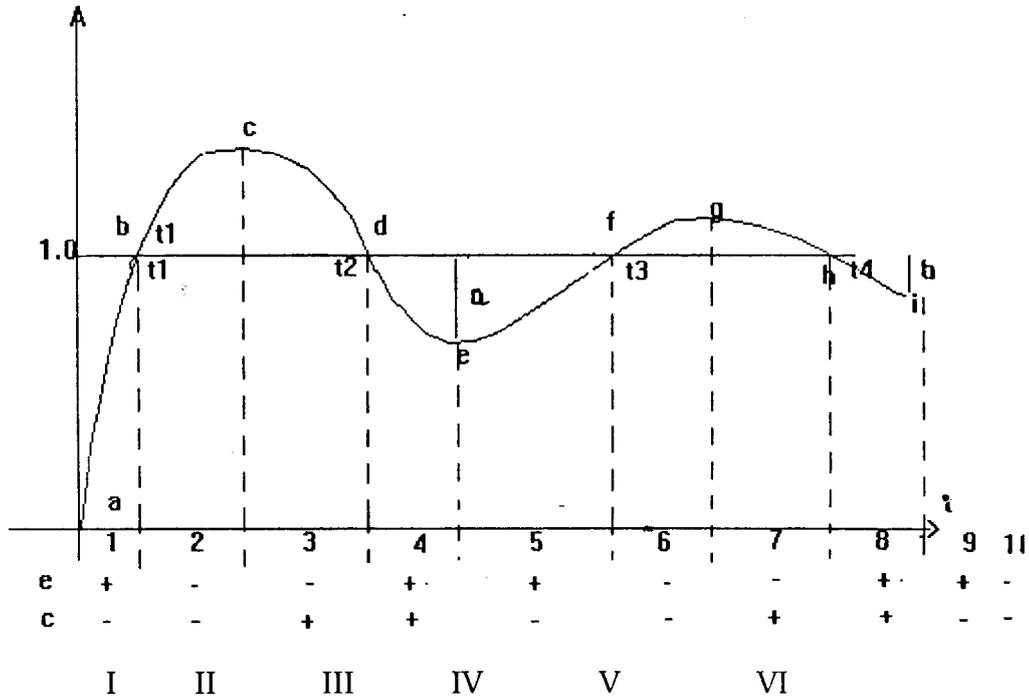


Fig 1. Typical closed loops response of control systems

III. THE SELF-TUNING PID CONTROLLER

Basic structure

In this paper we assume that the process to be controlled has single input $u(t)$ and single output $y(t)$. The STC PID controller must be to bring the process output $y(t)$ to prescribed setpoint $r(t)$. Fig 2 is the block scheme of STC PID controller. The closed loops error $e(t) = r(t) - y(t)$ and the control $u(t)$ has the following standard form in discrete time k

$$u(k) = K\{e(k) + \frac{T}{T_i} \sum e(k) + \frac{T_d}{T}[e(k) - e(k-1)]\} \quad (1)$$

or in continuous time t

$$u(k) = K_p(e(t) + K_d(de(t)/dt) + K_i \int e(t)dt$$

where

$u(k)$ control at the time k

$e(k)$ error at the time k

T_i The integral time

T_d derivative time

K propotional gain

T sampling periode

The K , T_d , T_i or (K_p, K_i, K_d) are the constants of the controller which are to be adjusted on_line.

On_line adaptation of the fuzzy control rules

At the Fig 1. the envelope of the peak values of the positive or negative values can be written

$$y_h = 1 \pm \frac{e^{-\xi \omega t}}{\sqrt{1 - \xi^2}} \quad (2)$$

where

ω undamped natural frequency,

ξ equivalent damping ratio,

$\omega = \omega_n \sqrt{1 - \xi^2}$ damped frequency.

The approximately of the sinusoid envelope with (Fig 1)

$$\omega(t_3 - t_1) = \omega(t_4 - t_2) = 2\pi. \quad (3)$$

Then it can to see the ratio of the two positive (negative) peaks of closed_loops response

$$\frac{b}{a} = e^{-2\pi\xi\sqrt{1-\xi^2}} = e^{-2\pi\Delta} \quad (4)$$

where

$$\Delta = \frac{\xi}{\sqrt{1 - \xi^2}}. \quad (5)$$

From Fig 1 the damped period of oscilation P is given by

$$P = (t_3 - t_1). \quad (6)$$

Then the ultimate period P_u is found

$$P_u = \frac{P}{\sqrt{1 + \Delta^2}} \quad (7)$$

Because

$$\sqrt{1 + \Delta^2} = \sqrt{1 + \frac{\xi^2}{1 - \xi^2}} = \frac{1}{\sqrt{1 - \xi^2}}.$$

According to the Ziegler-Nichols the proportional gain is

$$K_p = 0.5K_u. \quad (8)$$

On the other hand The ultimate gain K_u is a function of K_p and Δ . From (8) we can expansion for ratio K_u/K_p

$$K_u/K_p = \alpha_0 + \alpha_1\Delta + \alpha_2\Delta^2 + \dots \quad (9)$$

If $K_p = K_u$, then $\Delta = 0$ and $\alpha_0 = 1$.

The initial value of K_p can be written in the form *

$$K_{p0} = K_u/(1 + \alpha_1\Delta^*). \quad (10)$$

From (9) the values of α_1 can be calculated. By virtue of Ziegler - Nichols formula (1942) with $b : a = 1 : 4$ we can to get Δ^* from (8). From equation $K_p = K_u/(1 + \alpha_1\Delta^*)$, with $b : a = 1 : 4$ we get $\Delta^* = 0.2207$. Then

$$\alpha_1 = 4.5310. \quad (11)$$

From (9), (10), (11) we can get

$$K_{p0} = [1 + 4.5310/(1 + 4.5310\Delta^*)]K_p. \quad (12)$$

With $\Delta^* = 0.3$

$$K_p(k+1) = (0.423 + 1.784\Delta)K_p(k). \quad (13)$$

From (5), when $\Delta^* = 0.5$, i.e. $b : a = 1 : 22$ then

$$K_p(k+1) = (0.3 + 1.4\Delta)K_p(k). \quad (14)$$

From (1) we can get the parameters of PID by following Ziegler-Nichols formula

$$K_i = \frac{2\sqrt{1 + \Delta^2}}{p} \quad (15)$$

$$K_d = \frac{P}{8\sqrt{1 + \Delta^2}}. \quad (16)$$

We can get from Fig 1 with $(t_3 - t_1) = 2\pi$, when $0 << t_2 - t_1$ we have

$$d(t_3 + \tau) = \exp(-2\pi\Delta)d(t_1 + \tau).$$

It is easy to see

$$\exp(-2\pi\Delta) \int_{t_1}^{t_2} e(t)dt = \int_{t_3}^{t_4} e(t)dt. \quad (17)$$

Equation (17) points out that Δ is function of $e(k)$, then we can write

$$\Delta = F(e(k)). \quad (18)$$

From equation (18) can calculate Δ . According to the Fig 1. at the stage I, III, when the $y(t)$ approaches to the set_point $r(t)$, the Δ of (14), (15), (16) will be increased, then increasing K_p , K_i , and decreasing K_d . Similarly, when $y(t)$ diverges up/down from set_point $r(t)$ at the stages II, IV, the Δ of (14), (15), (16) will be decreased, then decreasing K_p , K_i and increasing K_d consequently. The divergence of the output $y(t)$ will be slowed down. The parameter adjustment of PID controller is proposed by

$$\Delta(k) = \Delta(k-1) - e(k-1)\{e(k-1) - e(k-2)\}/T\beta, \quad (20)$$

where:

T sampling period

$\Delta(\cdot)$ the above description

$e(\cdot)$ the above description

$[e(k-1) - e(k-2)]/T$ rate of change in errors of last time

$e(k-1)c(k-1)$ the sign of the stages I, II, III, IV

β parameter of the fuzzy inference

The calculation of fuzzy parameter

The calculation of the β is based on

1. The basic method [1]
2. A mathematical model and algorithm of fuzzy controller [3].

The basic of this calculation is described on the hand:

$$\text{If } e \text{ is } E_i \text{ and } c \text{ is } C_j, \text{ then } \beta \text{ is } \beta_{ij} \quad (20)$$

where I, J denote the index sets, E_i, C_j and β_{ij} denote linguistic values (fuzzy sets) of E, C, β , respectively, with the membership functions

$$\mu E_i(x) : X \rightarrow [0, 1]; \mu C_j(y) : Y \rightarrow [0, 1]; \mu(z) : Z \rightarrow [0, 1]. \quad (21)$$

Each rule (20) may be written as a fuzzy implication

$$E_i \rightarrow C_j \rightarrow \beta_{ij}, \text{ where } i \in I, j \in J.$$

Formula (20) gives a fuzzy relation R on the space $X \times Y = Z$ with the membership function

$$\mu R_{ij}(x, y, z) = \min(E_i(x), \mu C_j(y), \beta_{ij}(z)) \quad (22)$$

and

$$R = \max_{ij} R_{ij}. \quad (23)$$

Therefore every fuzzy value x and y are the observation on E', C' , respectively, then fuzzy control β is calculated according to the compositional rule of inference

$$\beta' = (E' \times C') \circ R. \quad (24)$$

Formula (24) has a form

$$\mu \beta'(z) = \max_{x,y} (\min(E'(x), C'(y), R(x, y, z))). \quad (25)$$

In the other hand, from 2 based on [3] and 1. (A mathematical model and algorithm of fuzzy controller) we can write that the output of fuzzy inference is described by

$$F(E', C')(z) = \max_{xy} (\min(E'(x), C'(y), R(x, y, z))), \quad (26)$$

where

E, C , are the normal classes of convex fuzzy set

$F(E', C')$ the output of fuzzy inference

E', C' are convex fuzzy set on the observation

The existence of fuzzy relation R is proved in [3]. We can see the fuzzy set of E and C below

$$E = \{B, NM, NS, ZO, PS, PM, PB\}$$

$$C = \{NB, NM, NS, ZO, PS, PM, PB\}$$

$$\beta = \{NB, NM, NS, ZO, PS, PM, PB\}.$$

Fig 3 The block scheme of Simulation of STC PID

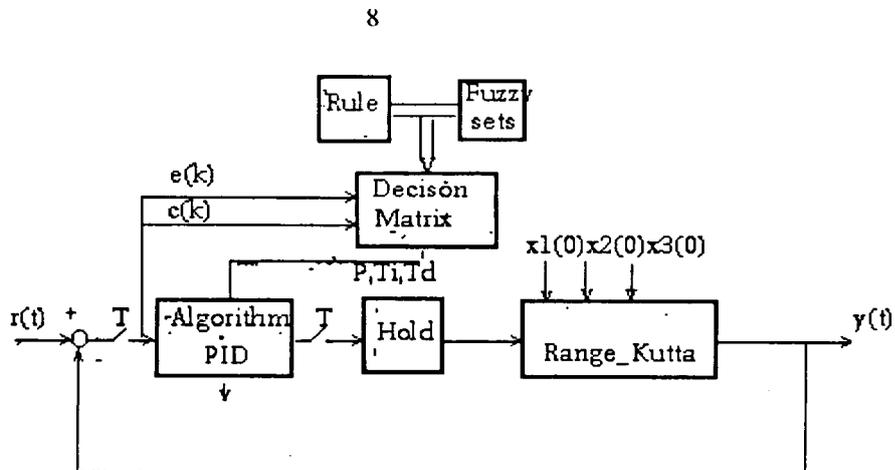


Fig.3 Simulation of STC-PID on PC by language C

IV. CONCLUSION

This paper proposes STC - PID. The model of STC - PID controller is based on two concepts: the inference of STC -PID is described by Generalized Modus ponens method and the mathematical model of fuzzy controller. The simulation is proved and the output of control system is approached to the setpoint, despite the parameters of controlled system are varied in wide range.

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