ON THE METHOD OF STATE INVARIANCE

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Abstract. *This paper develops a lena-kumpati methodology for the design of adaptive control systems which guarantee the state invariance of the plant although there is invariance in its work condition.*

0. INTRODUCTION

The necessity of the model for the elasstical adaptive control which was introduced by Gutkin and by Craxovski [4, 5] and for the estimate adaptive control which was introduced by Williamson and by Lena-Kumpati [6, 8] implies that the state equation of the plant is not enough to solve a problem of the adaptive control and that in order to solve this problem it is necessary to introduce some new equations or some new conditions for an adaptable property of the given plant.

The real control plant in general is varying. Therefore there is always a different between the real state of the plant and its given resquisite state in its standard work condition chosen for the design.

The aim is, lossely speaking, to construct the adaptive control system which guarantees the state invariance of the plant and stabilizes its dynamic process although there is invariance in its work condition. And an expected new equation will be established analyzing these requirements.

I. A DYNAMIC ASSOCIATION EQUTION

Examine the main aims of the Lena-Kumpati adaptive control to obtain an expected new extensive state equation.

1.1. The condition for the state invariance

Consider the linear control plant described by differential equation

$$y^{(n)} + \sum_{i=1}^{n} a_i y^{(n-i)} = \sum_{j=0}^{r-1} b_j f^{(r-1-j)} + \sum_{k=0}^{s} c_k d^{(s-k)}, \tag{1.1}$$

where y denotes the coordinate of the plant, f denotes the control exister, d denotes the external disturbance exister, and the coefficiants a_i, b_j and c_k depend on the set Θ of the variant dynamic l-characteristics of the given plant.

Note [1, 2]

$$y^{(i-1)} = x_i, \quad i = 1, 2, ..., n,$$
$$f^{(j-1)} = u_j, \quad j = 1, 2, ..., r,$$

from (1.1) to obtain the state equation of the plant

$$\dot{x} = Ax + Bu + z, \tag{1.2}$$

where

$$\begin{cases} x = (x_1, x_2, ..., x_n)^T \\ u = (u_1, u_2, ..., u_r)^T \\ z = (0, 0, ..., z_n)^T \\ \dots\dots\dots\dots\dots\dots \\ z_n = \sum_{k=0}^{s} c_k d^{(s-k)} \end{cases} \tag{1.3}$$

$n \times n$-matrix A and $n \times r$ matrix B have the shape

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{bmatrix} \tag{1.4}$$

$$B = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ b_{r-1} & b_{r-2} & \dots & b_0 \end{bmatrix} \tag{1.5}$$

In the real condition the state of control plant is always different from its given requisite tate in its standard work condition chosen for the design.

In the standard condition

$$\Theta = \Theta_c = \text{ constant}$$

then

$$z = z_c(t),$$

therefore the requisite state is achieved

$$x = x_c(t);$$

the optimal control must be also achieved

$$u = u_{\rm opt} = u_c(t)$$

and the state equation (1.2) of the plant becomes

$$\dot{x}_c = A_c x_c + \mathcal{B}_c u_c + z_c, \tag{1.6}$$

where

$$A_c = A|_{\Theta_c} = \text{ const,; } \quad \mathcal{B}_c = \mathcal{B}_{\Theta_c} = \text{ const.}$$

In the real condition

$$\Theta \neq \Theta_c$$

therefore although before the control u enough time to change, i.e.

$$u = u_c$$

the state of plant and the disturbance are varied

$$x \neq x_c,$$
$$z \neq z_c.$$

These varied vectors can be always written

$$\begin{cases} \Theta = \Theta_c + \Delta\Theta \\ z = z_c + \Delta z \\ x = x_c + \Delta x \end{cases} \tag{1.7}$$

Let such available control r-vector φ can be formed so that with

$$u = u_c + \varphi. \tag{1.8}$$

The state of the plant becomes invariant in comparision with its requisite state although there is a variance in the work condition of the given plant.

The Lena-Kumpati state invariance o0f the plant

$$\dot{x} = Ax + Bu + z$$

means that [8]

$$\lim_{t\to\infty} = 0 \tag{1.9}$$

is derived, noting

$$\Delta x = p. \tag{1.10}$$

Use (1.7) and the fact (1.8) in (1.2), with (1.6), to obtain

$$\begin{aligned} \dot{x} - \dot{x}_c &= Ax - A_c x + A_c x - A_c x_c + Bu - B_c u + B_c u - B_c u_c + z - z_c \\ \dot{\Delta x} &= A_c \Delta x + B_c \varphi + \Delta A x + \Delta B u + \Delta z. \end{aligned}$$

or, with (1.10),

$$\begin{cases} \dot{p} = A_c p + B_c \varphi + z_s \\ z_s = \Delta A x + \Delta B u + \Delta z \end{cases} \tag{1.11}$$

The state equation (1.2) of the plant then can be also written as

$$\dot{x} = A_c x + B_c u + z_e.$$

Noting

$$z_e = \Delta A x + \Delta B u + z$$

and $\Delta A x + Bu$ is some parameter disturbance.

Next, only the case, when the plant is assumed to be observable and all the signals in the plant are uniformly bounded, will be examined, (when the plant signals cannot be assumed to be uniformly bounded neither approach works directly and special analysis is needed).

Choosing $n \times n$-matrix B arbitrarily from $(A\ B)$ with as small as possible m, provided

$$B_c^{\Theta_i} = 0$$

cannot occur for any i, $i = 1, 2, ..., l$, where

$$B_c^{\Theta} = \frac{\partial B}{\partial \Theta}|_{\Theta_c} = \text{ const},$$

and according to the special shape of the vector z (1.3-3) and of matrices A (1.4) and B (1.5), for the pair of given values $B_c^{(\Theta_k)} v$ and σ_k, such value $\Delta\Theta_{ek}$ can be defined that, with $B_c^{(\Theta_k)} v \neq 0$,

$$B_c^{(\Theta_k)} v \Delta_{ek} = (0, 0, ..., \sigma_k)^T, \ k = 1, 2, ..., l \tag{1.11a}$$

is always satisfied, where v is m-vector chosen appropriately to B from $(x,u)^T$;

$$\sigma_k = \Big(-\sum_{i=1}^{n} a_{ic}^{(\Theta_k)} x_{n+l-i} + \sum_{j=0}^{r-l} b_{jc}^{(\Theta_k)} u_{r-j} + z_{nc}^{(\Theta_k)} \Big) \Delta\Theta_k$$

$$a_{ic}^{(\Theta)} = \frac{\partial a_i}{\partial \Theta}|_{\Theta_c};\ b_{jc}^{(\Theta)} = \frac{\partial b_j}{\partial \Theta}|_{\Theta_c};\ z_{nc}^{(\Theta)} = \frac{\partial z_n}{\partial \Theta}|_{\Theta_c}.$$

Then it can be seen that

$$\begin{cases} \sum_{k=1}^{l} \sigma_k = z_{sn} \\ z_{ns} = -\sum_{i=1}^{n} \Delta a_i x_{n+l-i} + \sum_{j=0}^{r-l} \Delta b_j u_{r-j} + \Delta z_n \\ (0,0,...,z_{sn})^T = z_s. \end{cases} \tag{1.11 b}$$

According to (1.4) and (1.5) matrix $B_c^{(\Theta)}$ can be written as

$$B_c^{(\Theta)} = b\Phi_c^{(\Theta)T},$$

where b is n-vector, and

$$b = (0,0,...,1)^T,$$

$$\Phi_c^{(\Theta)} = \frac{\partial \Phi}{\partial \Theta}|_{\Theta_c} = const,$$

Φ is m-vector which has been defined from

$$\begin{bmatrix} (-a_n & -a_{n-1} & \dots & -a_1)^T \\ (b_{r-1} & b_{r-2} & \dots & b_0)^T \end{bmatrix}$$

choosing matrix B.

According to (1.4), (1.5) and (1.11a), (1.11b), substitute

$$\begin{cases} B_c\varphi = B_c^{(\Theta)} vw = b\Phi_c^{(\Theta)T} vw \\ z_s = b\Phi_c^{(\Theta)T} v\Delta\Theta_e \\ q = \Phi_c^{(\Theta)}(w + \Delta\Theta_e) \end{cases} \tag{1.12}$$

from (1.11) to obtain

$$\dot{p} = A_c p + B_c^{(\Theta)} vw + z_s \tag{1.13}$$

or, since $\Phi_c^{(\Theta)T} v = v^T \Phi_c^{(\Theta)}$,

$$\dot{p} = A_c p + bv^T q, \tag{1.13 *}$$

when r-vector φ has the shape as r-vector u, i.e.

$$\varphi = (\varphi_1, \varphi_2, ..., \varphi_r)^T$$

$$\varphi_i = \psi^{(i-1)},\ i = 1,2,...,r,$$

where ψ is some function of time which will be defined by (1.12-l) through l-vector w, i.e.

$$\sum_{i=0}^{r-1} b_{ic} s^{r-1-i} \psi = v^T \Phi_c^{(\Theta)} w$$

$$b_{ic} = b_i|_{\Theta_c}, \quad i = 1, 2, ..., r$$

$$\Phi_c^{(\Theta)} w = \sum_{i=1}^{l} \Phi_c^{(\Theta_i)} w_i,$$

$$\Theta = (\Theta_{e1}, \Theta_{e2}, ..., \Theta_{el})^T,$$

w is l-vector which is going to be defined next.

The equation (1.13), or (1.13 *), is called centralized state equation of the plant, and m-vector w can be assumed to be the control of this centralized process. This control vector must be defined according to the condition for state invariance.

Remark

In the case if

$$b_{r-lc}^{(\Theta_i)} \equiv 0$$

cannot occur for any i, $i = 1, 2, ..., l$, where

$$b_{r-lc}^{(\Theta)} = \frac{\partial b_{r-l}}{\partial \Theta}|_{\Theta_c}$$

then m-vector v must be

$$v = u_i = f, \tag{1.14}$$

i.e. v is control scalar, and therefore

$$m = l$$

when $n \times m$-matrix B is chosen as

$$B = (0, 0, ..., b_{r-l})^T$$

therefore

$$B_c^{\Theta)} = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ b_{r-lc}^{(\Theta)} \end{bmatrix} = b b_{r-lc}^{(\Theta)}$$

and the controlized state equation (1.13) and (1.13 *) become

$$\dot{p} = A_c p + b_{r-lc}^{(\Theta)} b f w + z_s, \tag{1.13a)}$$

and

$$\dot{p} = A_c p + b f q, \tag{1.13 *a)}$$

where

$$q = b_{r-lc}^{(\Theta)} (w + \Delta\Theta_e).$$

2. In the case if

$$B_c^{(\Theta_i)} \equiv 0$$

cannot occur for any i, $i = 1, 2, ..., l$ only when

$$B = (A \quad \mathcal{B})$$

therefore the chosen appropriately to it vector

$$v = \begin{bmatrix} x \\ u \end{bmatrix}$$

then the centralized state equation (1.13) and (1.13 *) of the plant have the shape

$$\dot{p} = A_c p + (A_c^{(\Theta)} x + \mathcal{B}_c^{(\Theta)} u) w + z_s \tag{1.13b}$$

and

$$\dot{p} = A_c p + b v^T q, \tag{1.13 *b}$$

where

$$q = \Phi_c^{(\Theta)} (w + \Delta\Theta_e)$$

$$\Phi = \begin{bmatrix} (-a_n & -a_{n-1} & \dots & -a_1)^T \\ ((b_{r-1} & b_{r-2} & \dots & b_0)^T \end{bmatrix}$$

$$\Phi_c^{(\Theta)} = \frac{\partial \Phi}{\partial \Theta}|_{\Theta_c}$$

and according to (1.11-2)

$$z_s = (A_c^{(\Theta)} x + \mathcal{B}_c^{(\Theta)} u) \Delta\Theta + \Delta z. \tag{1.11-2a}$$

Proposition 1, for the state invariance

If

$$\lim_{t \to \infty} q = 0$$

occurs, a state invariance of the plant with its centralized state equation

$$\dot{p} = A_c p + b v^T q$$

will be derived if and only if every real part of all of eigenvalues of matrix A_c is negative.

Actually, with

$$\lim_{t \to \infty} q = 0$$

i.e. with an enough great t_c

$$q = 0, \quad t \geq t_c$$

must be derived, then system

$$\dot{p} = A_c p + b v^T q$$

becomes a linear homogeneous dynamic system

$$\dot{p} = A_c p, \quad t \geq t_c$$

And it is well known that for this system

$$p = 0, \quad t \geq t_c$$

or

$$\lim_{t \to \infty} p = 0, \tag{1.9 *}$$

i.e. a state invariance is derived, if and only if every real part of all of eigenvalues of the matrix A_c is negative.

This proposition is same condition for the Lena-Kumpati adaptive property of the given plant.

The control w in the system

$$\dot{p} = A_c p + B_c^{(\Theta)} w + z_s$$

must be defined according to the condition when matrix A_c is Hurwitz and

$$\lim_{t \to \infty} q = 0.$$

The control w defined in this condition is called an adaptive algorithm. And the dynamic system with such algorithm w, then is said to be an adaptive control process.

The adaptive control process which guarantees so that

$$\lim_{t \to \infty} p = 0$$

occurs is called Lena-Kumpati adaptive process. The the control w of the dynamic centralized system

$$\dot{p} = A_c p + B_c^{(\Theta)} v w + z_s$$

of the plant is called Lena-Kumpati adaptive algorithm.

Remark

In the case when

$$\Delta z = 0,$$

from (1.12-2 a)

$$z_s = \left(A_c^{(\Theta)} x + B_c^{(\Theta)} u\right) \Delta\Theta$$

$$z_s = \left(A_c^{(\Theta)} B_c^{(\Theta)}\right) \begin{bmatrix} x \\ u \end{bmatrix} \Delta\Theta$$

$$z_s = B_c^{(\Theta)} v \Delta\Theta. \qquad (1.11-2b)$$

from (1.11 a) and (1.11 b)

$$z_s = B_c^{(\Theta)} v \Delta\Theta_e \qquad (1.11\ c)$$

If (1.11 c) is compared with (1.11-2 c) we can see that

$$\Delta\Theta_e = \Delta\Theta$$

therefore, from (1.12-3),

$$q = \phi_c^{(\Theta)}(W + \Delta\Theta).$$

Then the state invariance can occur if with the great enough t_c

$$q \approx 0, \quad t \geq t_c$$

i.e.

$$-W \approx \Delta\Theta, \quad t \geq t_c.$$

It means that in the case, when

$$B = (A,\ B),$$

$$v = \begin{bmatrix} x \\ u \end{bmatrix},$$

$$\Delta \equiv 0,$$

the state invariance will be derived if $-w$ is plant parameter error estimation vector.

I.2. A stability of the adaptive dynamic process

In this section conditions which have to be satisfied for the Lyapunov's stalbility theory to be successfully applied to adaptive controlers are examined.

Proposition 2, for a stability

If there exist some real positive definite $n \times n$ - and $m \times m$ - matrices P and Q such that the equation

$$\dot{q} = -Q^{-1} v b^T P p$$

holds at each time, then an adaptive dynamic process, with the centralized state equation

$$\dot{p} = A_c p + b v^T q$$

will be asymptotically stable about zero in an extended state space $\{q, p\}$ if and only if the matrix A_c is Hurmitz.

A control plant, with a control w,

$$\dot{p} = A_c p + B_c^{(\Theta)} v w + z_s$$

or a centralized dynamic process

$$\dot{p} = A_c p + b v^T q$$

in the condition

$$\dot{q} = -Q^{-1} v b^T P p \tag{1.14}$$

can be examined as a diametral dynamic process

$$\begin{cases} \dot{p} = A_c p + b v^T q \\ \dot{q} = -Q^{-1} v b^T P p \end{cases} \tag{1.15}$$

And this process [1, 2, 4, 5, 9] will be asymptotically stable about zero in an extended state space $\{p,\ q\}$ if its extensive Lyapunov energy assumed to be the Lyapunov function candidate $E \to 0$ as $t \to \infty$, where

$$E = p^T L p + q^T M q \tag{1.16}$$

and L and M are some positive definite $n \times n$ and $m \times m$ matrices.

The main problem consists of finding a monotonic norm.

A condition for asymptotic stability can be obtained [1, 2, 4, 5, 9] if

$$\dot{E} < 0 \tag{1.17}$$

is always satisfied.

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