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# STABILITY OF THE PRINCIPLES OF MINIMAL SPECIFICITY AND MAXIMAL BUOANCY

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**Abstract.** The aim of this paper is to investigate an use of the principles of minimal Specificity and maximal Buoancy (MB) proposed by R. R. Yager and to introduce some conditions of a set of possibility distributions and weights which guarantee uniqueness of possibility distribution selected by the use of these principles from a given set of distributions.

**Keywords.** Principle; minimal Specificity; Maximal Buoancy; mS-stable; MB-stable.

## 1. INTRODUCTION

In the theory of approximate reasoning (quantitative and qualitative) possibility distributions play an important and central role (see [2, 3, 14]). There exist many situations in which we need a determination of the approximate possibility distribution from the use of other possibility distributions. The most often used techniques for handling these problems are the principles of minimal Specificity (mS) for quantitative possibility distributions (qpd) and of maximal Buoancy (MB) for qualitative possibility distributions (Qpd) (see [14]).

The concept of specificity of qpds was originally introduced by R. R. Yager [9-13], D. Dubois and H. Prade [2-4], and A. J. Ramer [5-6],.... The principle of mS is used at least for two classes of following problems (see [14]):

1) In the first class, it must select a possibility value independently for each $x$ in a set **B** (in general, **B** is a set of all atoms of a Boolean algebra or is a set of possible wolds induced from the set of sentences in propositional language) if these values are given individual bounds on the elements of **B**.

In this case, it is simple to select the highest possibility value for each $x$ in **B**, then we will get a least specificity distribution from the set of all possible possibility distributions on **B**.

2) In the second class, let there be given a set of $m$ possibility distributions

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$\{\pi_1, \pi_2, ..., \pi_m\}$, we must pick one of these distributions satisfying some conditions.

Here the use of the principle of mS is to select from these distributions a possibility distribution that minimizes the chosen specificity measure.

Similarly, let have a quasi-ordering on the finite set $\mathbf{B}$, then there are many weak orderings that complete this quasi ordering. A proposed problem is to select a unique weak ordering (Qpd), that is the completion of this quasi ordering.

The use of the principle of MB for weak orderings is in the same spirit as the use of the principle of mS for qpds, this is to select a weak ordering that maximizes the chosen Buoancy measure.

We know that under these principles, a selected quantitative or qualitative possibility distribution depends on weights of the Specificity or Buoancy measure, respectively. In more detail, a possibility distribution was selected by the use of principle of mS (or MB) with chosen weights, can not be selected by the use of this principle with other weights. In other words, a possibility distribution selected by using one of these principles depends on individual opinions.

Two following problems have arisen:

1) *Which conditions must m given possibility distributions satisfy such that there exists a possibility distribution that is always selected from these distributions by the use of the principle of mS (or MB).* (1)

2) *For m given possibility distributions, which additional conditions must weights of Specificity measure (Buoancy measure) satisfy such that there exists a possibility distribution that is always selected from these distributions by the use of the principle of mS (or MB) with any weights satisfying these conditions?* (2)

The aim of this paper is to propose a use of these principles, this is to use simutalneously both the principles of mS and of MB for selecting one from given quantitative possibility distributions, and to give a part of answer of the questions above.

This paper is structured as follows: after introducing some background concepts in Section 2, in Section 3 we will explain why we propose a use simutalneously of the principles of mS and of MB for to select one from $m$ given quantitative possibility distributions. In sections 4, 5 we will introduce concepts of MB-stable, T-MB-stable, mS-stable, T-mS-stable of a set of possibility distributions and point out some conditions for these stability sorts.

## 2. PRELIMINARIES

Assume $A$ is a Boolean algebra with maximal and minimal elements $I$, $\Theta$. Non minimal element $x^*$ in $A$ is an *atom* (see [14]) of Boolean algebra $A$ if and only if for every $y$ in $A$ either $x^* \wedge y = x^*$ or $x^* \wedge y = \Theta$.

Let $\mathbf{B} = \{b_1, b_2, \ldots, b_n\}$ be the set of all atoms of $\mathbf{A}$, then for any element $x$ in $\mathbf{A}$, there exists an unique subset $\mathbf{B}_x$ or $\mathbf{B}$ such that $x = \vee y$, $y \in \mathbf{B}_x$ (see [7]).

A *quantitative possibility measure* (qpm) is a mapping $\Pi$ from $\mathbf{A}$ into $[0, 1]$ such that

1) $\Pi(\Theta) = 0$;
2) $\Pi(I) = 1$;
3) $\Pi(a, b) = \max(\Pi(a), \Pi(b))$ for all $a$, $b$ of $\mathbf{A}$.

A *quantitative possibility distribution* (qpd) $\pi$ is mapping from the set of all atoms of $\mathbf{A}$ into $[0,1]$, i.e., $\pi : B \to [0,1]$, $\pi$ is called normalized possibility distribution if there exists at least one element $x^*$ in $B$ such that $\pi(x^*) = 1$.

A *qualitative possibility measure on* $\mathbf{A}$ (Qpm) is an ordering relation $\mathbf{S}$ (where $a\mathbf{S}b$ means a is at least as possible as $b$) satisfying the following conditions:

1) $x\mathbf{S}y$ or $y\mathbf{S}x$ for all $x$, $y$ of $\mathbf{A}$;
2) if $x\mathbf{S}y$ and $y\mathbf{S}z$ then $x\mathbf{S}z$ for all $x$, $y$, $z$ of $\mathbf{A}$;
3) $I\mathbf{S}\Theta$ and $\Theta\overline{\mathbf{S}}I$;
4) $I\mathbf{S}a$ for all $a$ of $\mathbf{A}$;
5) if $b\mathbf{S}c$ then $(b \vee a)\mathbf{S}(c \vee a)$ for all $a$ of $\mathbf{A}$.

An ordering relation $\mathbf{S}$ satisfying $x\mathbf{S}x$ for every $x$ in $\mathbf{A}$ and the condition 2) above is called a *quasi ordering*. $\mathbf{S}$ is called a *weak ordering* if it satisfies the conditions 1), 2). A weak ordering on the set of all atoms of finite Boolean algebra is called a *qualitative possibility distribution* (Qpd).

In [14], R.R. Yager showed that if $\Pi$ is a qpm on $\mathbf{A}$ then $\pi = \Pi/\mathbf{B}$ is a normalized qpd, and conversely if $\pi$ is a normalized qpd, then there exists uniquely a qpm $\Pi$ on $\mathbf{A}$ such that $\Pi/\mathbf{B} = \pi$.

Similarly, if $\mathbf{S}$ is a Qpm then $s = \mathbf{S}/\mathbf{B}$ is a Qpd, and conversely if $s$ is a Qpd then there exists uniquely a relationship $\mathbf{S}$ satisfying the conditions 1), 2), 4), 5) in the definition of Qpm, and if $\mathbf{S}$ is added the condition 3) then $\mathbf{S}$ is a Qpm on $\mathbf{A}$ such that $\mathbf{S}/\mathbf{B} = s$ (see [14]).

In brief, it can say that both possibility measures (Qpm and qpm) are uniquely determined by their possibility distributions on the set of atoms.

For any two possibility distributions $\pi_1$, $\pi_2$, $\pi_1$ is more specific than $\pi_2$ if and only if $\pi_1(x) \leq \pi_2(x)$ for every $x$ in $\mathbf{B}$.

R.R. Yager proposed a class of linear measures of specific (see [14]), each of them is a function defining on the set of all possibility distributions and has the form

$S_p(\pi) = \pi_1 - \sum_{i \geq 2} v_i \pi_i$, here $\pi = \{\pi_i,\ i = 1, \ldots, n;\ \pi_1 \geq \pi_2 \geq \cdots \geq \pi_n\}$ is a quantitative possibility distribution and $\{v_i,\ i = 1, \dots, n\}$ are called weights of this measure if

1) $0 \leq v_i$, for $i = 2, \dots, n$;
2) $v_1 = 1$;
3) $v_2 \neq 0$;
4) $\sum_{i \geq 2} v_i \leq 1$;
5) $v_i \geq v_j$, if $i < j$.

Under this definition, for any weights of specificity measure, we get $S_p(\pi_1) \geq S_p(\pi_2)$ if $\pi_1 \leq \pi_2$, and in particular if $\pi$ is a normalized possibility distribution then $S_p(\pi) = 1 - \sum_{i \geq 2} v_i \pi_i$.

Let $S$ be a weak ordering, define $S(x, y) = \begin{cases} 1, & \text{if } xSy, \\ 0, & \text{otherwise} \end{cases}$ for every $x$, $y$ in **B** and $H(x_i) = \sum_{x \in B} S(x_i, x)$, $g(x) = H(x_i)/\max_{y \in B} H(y)$.

A *Buoancy measure Buo* (see [14]) is a mapping from the set of all qualitative possibility distributions into $[0,1]$ defined by $\text{Buo}(S) = \sum_{i \geq 1} a_i w_i$, where $a_1, a_2, \dots, a_n$ are $g(x)$ ($x \in \mathbf{B}$) ordered with $a_1 \geq a_2 \geq \cdots \geq a_n$ and $\{w_i,\ i = 1, \dots, n\}$ are called weigths of this measure if

1) $w_i \geq 0$ for $i = 1, \dots, n\}$;
2) $\sum_{i \geq 1} w_i = 1$;
3) $w_i \geq w_j$, if $i < j$.

## 3. USE OF THE PRINCIPLES OF mS and MB

From the principles of mS and MB, we get

1) The principle of mS says that if $\pi_1$, $\pi_2$ are two qpds and $\pi_1$ is more specific than $\pi_2$ then $\pi_2$ is selected.

In the same spirit, the principle of MB says that when $s_1$, $s_2$ are two weak orderings (Qpds), if $s_2$ is more relaxing than $s_1$ (i.e., $s_2 \supset s_1$) then $s_2$ is selected.

2) In the cases, when it can't be able to compare possibility distributions each other, these principles give us a formally framework for selecting one from these possibility distributions.

On other hand, we know that if $\pi$ is a qpd then there exists a weak ordering $S_\pi$ induced from $\pi$ by $xS_\pi y$ iff $\pi(x) \geq \pi(y)$, then $S_\pi$ is called a natural ordering associated with $\pi$ and the notation $xS_\pi y$ also means that $x$ is at least as possible as $y$.

In the possibility theory, a information says that the possibility degree of element $x$ is higher than one of element $y$, is more important than concretely given values about the possibility degree of these elements.

We can see that when $\pi_1$, $\pi_2$ are two qpds and $S_1$, $S_2$ are orderings associated respectively with them, there is not any relationship between $\pi_1(x) \leq \pi_2(x)$ for every $x$ in $\mathbf{B}$ and $S_1 \subseteq S_2$. In fact, there exists a case $\pi_1(x) \leq \pi_2(x)$ for every $x$, but $S_1 \supset S_2$.

For example, let $\mathbf{B} = \{x_1, x_2, x_3, x_4\}$,

$\pi_1 = \{\pi_1(x_1) = 1,\ \pi_1(x_2) = 0.5,\ \pi_1(x_3) = 0.5,\ \pi_1(x_4) = 0.5\}$,

$\pi_2 = \{\pi_2(x_1) = 1,\ \pi_2(x_2) = 0.9,\ \pi_2(x_3) = 0.8,\ \pi_2(x_4) = 0.7\}$.

Clearly that $\pi_1(x) \leq \pi_2(x)$ for every $x$ in $\mathcal{B}$ and $S_1 \supset S_2$, so $\pi_2$ is selected by the use of the principle of mS, and $S_1$ is selected by the principle of MB. Now a question arises: *which distribution we ought to select from $\pi_1$, $\pi_2$?*

By points of view of important information above and of most basic idea of the principles of MB and of mS (select a relaxing ordering or a least specificity possibility distribution), in this case selecting the possibility distribution $\pi_1$ seems more pertinent.

Therefore it must continue a discussion to select a qpd from the possibility distributions $\pi_1, \pi_2, \ldots, \pi_m$ by only the use of the principle of mS.

This example also suggests to us a use simutalneously of the principles of mS and MB for this problem.

Let $\pi_1, \pi_2, \ldots, \pi_m$ be qpds on the finite set $\mathbf{B}$ and $s_1, s_2, \ldots, s_k$ be Qpd's associated with these distributions, respectively. For selecting a qpd from the distributions $\pi_1, \pi_2, \ldots, \pi_m$, we can make as follows:

1) Select a Qpd $s_0$ by the use of the principle of MB for the weak orderings $s_1, s_2, \ldots, s_k$.

2) Select a qpd $\pi^*$ from the possibility distributions $\pi_1, \pi_2, \ldots, \pi_m$ by the use of the principle of mS for the distributions having the same natural ordering $s_0$.

A part of answer to the problems (1), (2) above will be executed on each of these processes.

## 4. BUOANCY MEASURE AND PRINCIPLE OF BUOANCY

Let $S_\pi$ be a weak ordering on $\mathbf{B}$, and $S$ be a relation defined on $\mathbf{B}$ by $xSy$ if and only if $yS_\pi x$ and $xS_\pi y$. Then $S$ is an equivalent relation on $\mathbf{B}$.

Define $\mathbf{B}/s = \{B_1, B_2, \ldots, B_m\}$ such that for every $x \in B_i$, $y \in B_j$, and if $i < j$ then $xS_\pi y$ but $y\overline{S}_\pi x$ (see [14]), then we say that $B_i > B_j$ and $S_\pi$ is called an $m$-classes ordering, the ordered collection $\{B_1, B_2, \ldots, B_m\}$ is called collection of equivalent classes associated with $S$.

In [14], R. R. Yager investigated the Buoancy measure and the principle of MB on the total orderings ($m = n$, $n$ is number of elements of $\mathbf{B}$) and on two-classes orderings ($m = 2$). We now intend to investigate the Buoancy measure and the use of the principle of MB on the m-classes orderings, here $2 < m < n$.

**Definition 4.1.** Let $S_1$, $S_2$ be two weak orderings in the finite set $\mathbf{B}$, and $\{E_1', E_2', \ldots, E_k'\}$, $\{E_1, E_2, \ldots, E_k\}$ are their collections of equivalent classes, respectively. We say $S_1$ *is more specific than* $S_2$ ($S_1 \angle S_2$) if and only if for $i = 1$ to $\max(k, k')$ we have
$\bigcup_{j\leq i} E_j' \subseteq \bigcup_{j\leq i} E_j$ (when $\max(k, k') > k$ then for $j = k+1$ to $k'$ we use $E_j = \emptyset$).

We see that there are not any correspondence between the specificity concept of two qpds and of two weak orderings (Qpds) associated with them, respectively, namely $\pi_1(x) \leq \pi_2(x)$ for every $x$ of $\mathbf{B}$ does not imply $S_1 \angle S_2$ and conversely.

On other hand, the relation $\angle$ is developed from the concept of specificity ordering proposed in [1], and it is deferent to the relation $\subseteq$ between the weak orderings.

However there exists a relationship between the relations $\subseteq$ and $\angle$. Following proposition will show this relationship.

**Proposition 4.2.** *Let $S_1$, $S_2$ be two weak orderings on $\mathbf{B}$, and $\{E_1', E_2', \ldots, E_k'\}$, $\{E_1, E_2, \ldots, E_k\}$ be the collections of equivalent classes associated with them, respectively.*

*If $S_1 \subseteq S_2$ then $S_1 \angle S_2$.*

*Proof.* From $S_1 \subseteq S_2$, we have

$$E_1' = \{x \in \mathbf{B} | x\tilde{S}_1 y, \ \forall y \in \mathbf{B}\} \subseteq \{x \in B | xS_2 y, \ \forall y \in \mathbf{B}\} = E_1 \, .$$

Assuming $\bigcup_{i\leq m} E_i' \subseteq \bigcup_{i\leq m} E_i$ to hold for $m < \max(k, k')$, we will prove it for $m+1$. For every $x \in \bigcup_{i\leq m+1} E_i' = \bigcup_{i\leq m} E_i' \cup E_{m+1}'$, under induction hypothesis $x \in \bigcup_{i\leq m} E_i'$ implies $x \in \bigcup_{i\leq m} E_i$, as $x \in E_{m+1}'$ implies $xS_1y$ for every $y \in \bigcup_{i\geq m+1} E_i'$. Since $\bigcup_{i\geq m+1} E_i \subseteq \bigcup_{i\geq m+1} E_i'$, we have $xS_2y$ for every $y \in \bigcup_{i\geq m+1} E_i$. This follows that $x \in \bigcup_{i\geq m+1} E_i$ and hence $S_1 \angle S_2$. □

**Proposition 4.3.** *$S_1 \subseteq S_2$ if and only if for every $i = 1, \ldots, k$ there exist the positive integers $1 \leq j_{i-1}$, $j_i \leq k'$ such that $E_i = \bigcup_{j_{i-1}\leq q\leq j_i} E_q'$, ($1 = j_0 \leq j_1, \leq \cdots \leq j_k = k'$).*

*Proof.* We first prove that when $S_1 \subseteq S_2$ and for any pair $(i, j)$, if $E'_j \cap E_i \neq \emptyset$, then $E'_j \subseteq E_i$.

Suppose $x \in E'_j \cap E_i$, then for every $y \in E'_j$, we have $xS_1y$ and $yS_1x$. From $S_1 \subseteq S_2$, it follows that $xS_2y$ and $yS_2x$, i.e., $E'_j \subseteq E_i$.

Set $j_{i-1} = \min\{q | E'_q \subseteq E_i\}$, $j_i = \max\{q | E'_q \subseteq E_i\}$. It follows $E_i \supseteq \bigcup_{j_{i-1} \leq q \leq j_i} E'_q$. If $E_i \neq \bigcup_{j_{i-1} \leq q \leq j_i} E'_q$ then there exists element $x_0 \in E_i$ but $x_0 \notin \bigcup_{j_{i-1} \leq q \leq j_i} E'_q$. Therefore $x_0 \in E'_q$ for $q < j_i$ or $x_0 \in E'_q$ for $q > j_i$. This contradict the definition of numbers $j_{i-1}$, $j_i$ and the proof above.

**Corollary 4.4.** *$S_1$, $S_2$ are k-classes ordering of* $\mathbf{B}$ *then $S_1 \subseteq S_2$ iff $S_1 = S_2$.*

The proof is given directly by the Proposition 4.3. □

Let $S_1$, $S_2$ be weak orderings on the finite set $B$. We know that if $S_1 \subseteq S_2$ then $\mathrm{Buo}(S_1) \leq \mathrm{Buo}(S_2)$ with any weights $\{w_i,\ i = 1, \ldots, n\}$, and in the case $S_1$, $S_2$ are two-classes orderings, if $S_1$ is more specific than $S_2$ then $\mathrm{Buo}(S_1) \leq \mathrm{Buo}(S_2)$ only with weights satisfying some conditions, for example $m.w_{m+1} \geq \sum_{i \geq m+2} w_i$, $m = 1, \ldots, k$ (see [14]).

This result is suggestion for following propositions.

**Proposition 4.5.** *Let $S_1$, $S_2$ be k-classes orderings, $S_1$ is more specific than $S_2$, if*
$$\mathrm{card}(E'_i \cap E_i) \geq \mathrm{card}(E'_{i+1} \cap E_{i+1}) \text{ for every } i = 1, \ldots, k-1, \tag{3}$$
*then $\mathrm{Buo}(S_1) \leq \mathrm{Buo}(S_2)$ with any weights $\{w_i,\ i = 1, \ldots, n\}$.*

*In the case $n = 2$, the condition (3) is also "only if".*

*Proof.* For $k \geq 2$ the condition of the proposition become $\mathrm{card}(E'_1) \geq \mathrm{card}(E_2)$. Define $n_1 = \mathrm{card}(E'_1)$, $r_1 = \mathrm{card}(E_1)$.
Assume $\mathrm{Buo}(S_1) \leq \mathrm{Buo}(S_2)$ for any weights $\{w_i,\ i = 1, \ldots, n\}$, so if we choose $\{w_i\}$ by $w_1 = w_2 = \cdots = w_n = \frac{1}{n}$ then $\mathrm{Buo}(S_2) - \mathrm{Buo}(S_1) = \frac{1}{n^2}(r_1 - n_1)[n_1 - (n - r_1)] \geq 0$, hence $n_1 \geq n - r_1$, i.e., $\mathrm{card}(E'_1) \geq \mathrm{card}(E_2)$.

For $k \geq 2$, define $n_i = \sum_{j \leq i} \mathrm{card}(E'_j)$. Since $\bigcup_{j \leq i} E'_j \subseteq \bigcup_{j \leq i} E_j$, we have $n_i \leq r_i$, for $i = 1, \ldots, k$.

In other side, since $S_2$ is the $k$-classes ordering we get $E_k \neq \emptyset$. From the $E_k \cap E'_k = E_k$ and under the hypothesis above, we have $E_i \cap E'_i \neq \emptyset$ for $i = 1, \ldots, k$. Therefore $0 = r_0 = n_0 \leq n_1 \leq r_1 \leq n_2 \leq r_2 \leq \cdots \leq n_k = r_k = n$.

From $\mathrm{card}(E'_i \cap E_i) \geq \mathrm{card}(E'_{i+1} \cap E_{i+1})$ we get $n_{i+1} - r_i \geq n_{i+2} - r_{i+1}$ for every $i = 0, \ldots, k-1$.

Then from $\mathrm{Buo}(S_1) = \sum_{i=1}^{n_1} w_i + (1 - \frac{n_1}{n}) \sum_{i=1+n_1}^{n_2} w_i + \cdots + (1 - \frac{n_{k-1}}{n}) \sum_{i=1+n_{k-1}}^{n} w_i$, and $\text{Buo}(S_2) = \sum_{i=1}^{r_1} w_i + (1 - \frac{r_1}{n}) \sum_{i=1+r_1}^{r_2} w_i + \cdots + (1 - \frac{r_{k-1}}{n}) \sum_{i=1+r_{k-1}}^{n} w_i$, we have

$$\begin{aligned}
\text{Buo}(S_2) - \text{Buo}(S_1) &= \sum_{i=1+n_1}^{r_1} w_i + \tfrac{1}{n}(n_1 - r_1) \sum_{i=1+r_1}^{n_2} w_i + \tfrac{1}{n}(n_2 - r_1) \sum_{i=1+n_2}^{r_2} w_i \\
&+ \tfrac{1}{n}(n_2 - r_2) \sum_{i=1+r_2}^{n_3} w_i + \cdots + \tfrac{1}{n}(n_{k-1} - r_{k-2}) \sum_{i=1+n_{k-1}}^{r_{k-1}} w_i + \tfrac{1}{n}(n_{k-1} - r_{k-1}) \sum_{i=1+r_{k-1}}^{n} w_i \\
&\geq \tfrac{1}{n}\{[n_1 w_{r_1} (r_1 - n_1) - (r_1 - n_1)(n_2 - r_1) w_{r_1+1}] \\
&\quad + [(r_2 - n_2)(n_2 - r_1) w_{r_2} - (r_2 - n_2)(n_3 - r_2) w_{r_2+1}] + \cdots \\
&\quad + [(r_{k-1} - n_{k-1})(n_{k-1} - r_{k-2}) w_{r_{k-1}} - (r_{k-1} - n_{k-1})(n_k - r_{k-1}) w_{r_{k-1}+1}]\} \\
&\geq 0.
\end{aligned}$$

Therefore $\text{Buo}(S_2) \geq \text{Buo}(S_1)$. □

**Proposition 4.6.** *Let $S_1$, $S_2$ be k-classes orderings, $S_1$ is more specific than $S_2$. If $E_i \cap E'_i \neq \emptyset$ for every $i = 1, \ldots, k$ then $\text{Buo}(S_1) \leq \text{Buo}(S_2)$ with any weights $\{w_i,\ i = 1, \ldots, n\}$ of Buoancy measure satisfying*

$$w_m \geq \sum_{i \geq m+1} w_i \text{ for } m = 1, \ldots, n-1. \qquad (\text{cw}_1)$$

*Proof.* As $S_1$ is more specific than $S_2$ and $E_i \cap E'_i \neq \emptyset$ for every $i = 1, \ldots, k$ we have $n_1 \leq r_1 < n_2 \leq r_2 < \cdots < n_k = r_k = n$.

Analytically as the Proposition 4.5, we also have

$$\begin{aligned}
\text{Buo}(S_2) - \text{Buo}(S_1) &= \sum_{i=1+n_1}^{r_1} w_i + \tfrac{1}{n}(n_1 - r_1) \sum_{i=1+r_1}^{n_2} w_i + \tfrac{1}{n}(n_2 - r_1) \sum_{i=1+n_2}^{r_2} w_i + \\
&+ \tfrac{1}{n}(n_2 - r_2) \sum_{i=1+r_2}^{n_3} w_i + \cdots + \tfrac{1}{n}(n_{k-1} - r_{k-2}) \sum_{i=1+n_{k-1}}^{r_{k-1}} w_i + \tfrac{1}{n}(n_{k-1} - r_{k-1}) \sum_{i=1+r_{k-1}}^{n} w_i \\
&\geq \left[\tfrac{n_1}{n} \sum_{j=1+n_1}^{r_1} \sum_{i=1+r_1}^{n_2} w_i + \tfrac{1}{n}(n_1 - r_1) \sum_{i=1+r_1}^{n_2} w_i\right] + \cdots \\
&\quad + \left[\tfrac{1}{n}(n_{k-1} - r_{k-2}) \sum_{j=1+n_{k-1}}^{r_{k-1}} \sum_{i=1+r_{k-1}}^{n} w_i + \tfrac{1}{n}(n_{k-1} - r_{k-1}) \sum_{i=1+r_{k-1}}^{n} w_i\right] \\
&\geq \tfrac{1}{n}(r_1 - n_1)(n_1 - 1) \sum_{i=1+r_1}^{n_2} w_i + \cdots + \tfrac{1}{n}(r_{k-1} - n_{k-1})(n_{k-1} - r_{k-2} - 1) \sum_{i=1+r_{k-1}}^{n} \\
&\geq 0. \quad \square
\end{aligned}$$

From the two propositions 4.5, 4.6 we have immediately following corollaries.

**Corollary 4.7.** *Let $S_1$, $S_2$ be k-classes orderings having the collections of equivalent classes $\{E'_i,\ i = 1, \ldots, k\}$, $\{E_i,\ i = 1, \ldots, k\}$, respectively, and $\{n_i, r_i\ i = 1, \ldots, k\}$ are defined as in the Proposition 4.5. If $n_1 \leq r_1 < n_2 \leq r_2 < \cdots < n_k = r_k = n$ then $\text{Buo}(S_1) \leq \text{Buo}(S_2)$ with any weights $\{w_i,\ i = 1, \ldots, n\}$ satisfying the condition $(\text{cw}_1)$.*

*If* $\{n_i\}$, $\{r_i\}$ *satisfy a additional condition*

$$n_{i+1} - r_i \geq n_{i+2} - r_{i+1} \text{ for } i = 0, \dots, k-1 \tag{4}$$

*then* $\mathrm{Buo}(S_1) \leq \mathrm{Buo}(S_2)$ *with any weights of Buoancy measure.*

**Corollary 4.8.** *Let* $S_1$ *be a k-classes ordering and* $S_2$ *be a m-classes ordering having the collections of equivalent classes* $\{E_i', \ i = 1, \dots, k\}$, $\{E_i, \ i = 1, \dots, m\}$, *respectively* $(m \leq k)$.

*If for every* $i = 1, \dots, m-1$ *exists* $i_0$, $1 \leq i_0 \leq k$ *such that*

$$\sum_{j=1}^{i} \mathrm{card}(E_j) < \sum_{j=1}^{i_0} \mathrm{card}(E_j') \leq \sum_{j=1}^{i+1} \mathrm{card}(E_j). \tag{5}$$

*then* $\mathrm{Buo}(S_1) \leq \mathrm{Buo}(S_2)$ *with any weights satisfying* $(\mathrm{cw}_1)$.

*Proof.*

Set $A_{i+1} = \{q | \sum_{j=1}^{i} \mathrm{card}(E_j) < q \leq \sum_{j=1}^{i+1} \mathrm{card}(E_j), \ q = \sum_{j=1}^{k} \mathrm{card}(E_j'), \ k \text{ depend } i\}$ for $i = 0, \dots, m-1$. Under the hypothesis, we have $A_{i+1} \neq \emptyset$ for $i = 0, \dots, m-1$.

Set $n_i = \max\{\underset{q \in A_{i+1}}{q}\}$ for $i = 0, \dots, m-1$, and $r_i = \sum_{j=1}^{i} \mathrm{card}(E_j)$, $i = 1, \dots, m$.

We will denote by $S^*$ a $m$-classes ordering having the collection of equivalent classes

$$\{F_i | F_i = \bigcup E_h', \ n_{i-1} < \sum_{j=1}^{h} \mathrm{card}(E_j') \leq n_i\}$$

where $n_0 = 0$ and $i = 1, \dots, m$.

Applying Proposition 4.6, we have $\mathrm{Buo}(S^*) \leq \mathrm{Buo}(S_2)$ with weights satisfying $(\mathrm{cw}_1)$, and under Proposition 4.3 we obtain $S_1 \subseteq S^*$, hence $\mathrm{Buo}(S_1) \leq \mathrm{Buo}(S^*)$ with any weights of the Buoancy measure. Therefore $\mathrm{Buo}(S_1) \leq \mathrm{Buo}(S_2)$ with any weights above defined. □

*Remark.* If $\{n_i\}$, $\{r_i\}$ (for $i = 1, \dots, m$ defined as in Corollary 4.8 satisfy the condition (4) then it is also obvious that $\mathrm{Buo}(S_1) \leq \mathrm{Buo}(S_2)$ for any weights of the Buoancy measure.

**Definition 4.9.** A set of quantitative possibility distributions (or qualitative) is *mS-stable* (or *MB-stable*) if and only if there exists at least one of these distributions such that it is always selected by the use of the principle of mS (or MB) with any weights of the Specificity (or Buoancy) measure.

This set is *T-mS-stable* (*T-MB-stable*) if any only if it is mS-stable (or MB-stable) with any weights satisfying the condition $T$.

From what has already been presented, we have

**Theorem 4.10.** *Let $\Phi$ be a finite set of weak orderings on a finite set* $\mathbf{B}$.

*If any two its possibility distributions satisfy the condition (5) then this set of is quasi $(cw_1)$-MB-stable.*

*If any two its possibility distributions satisfy (4), (5) then this set is MB-stable.*

*Remark.* A finite set of weak orderings is always MB-stable, if it has got the property that for any two its weak orderings, if both are not total orderings then there will exist the relation $\subseteq$ between them.

Indeed we can see that in this case any two possibility distributions of this set satisfy the conditions (4), (5). The proof is easy and omitted here.

## 5. SPECIFICITY MEASURE AND PRINCIPLE OF mS

In this section, we will investigate the use of the principle of mS only on a set of qpds having a same natural ordering.

From now on, we assume that the quantitative possibility distributions are normalized.

Let $S$ be weak ordering associated with the qpd $\pi$ on a finite set $\mathbf{B}$. Define $g^0(x) = g(x)\,\pi(x)$, where $g(x_i) = H(x_i)/\max H(y)$, $H(x_i) = \sum\limits_{i\geq 1} S(x_i, x)$ for $x \in \mathbf{B}$.

**Definition 5.1.** A *measure* $R$ is a mapping from the set of all possibility distributions into $[0,1]$ defined by $R(\pi) = \sum\limits_{i\geq 1} a_i\, w_i$, here $a_1, \ldots, a_n$ are $g^0(x)$ $(x \in \mathbf{B})$ ordered with $a_1 \geq a_2 \geq \cdots \geq a_n$ and $\{w_i,\ i = 1, \ldots, n\}$ are weights defined as ones of Buoancy measure (see [14]).

Formally, this measure is similar to Buoancy measure, but they are different each other by that the measure $R$ coincides both the value of the distribution $\pi$ and the weak ordering induced from $\pi$. This is illustrated by the expression $g^0(x) = g(x)\,\pi(x)$.

For $g^0(x)$ we have clearly following properties:

1. $0 \leq g^0(x) \leq 1$ for every $x \in \mathbf{B}$, this means that $0 \leq R(\pi) \leq 1$.
2. There exists $x^*$ in $\mathbf{B}$ such that $g^0(x^*) = 1$.
3. $g^0(x) \leq \pi(x)$ for every $x$ in $\mathbf{B}$.

4. $g^0(x) \leq g^0(y)$ iff $\pi(x) \leq \pi(y)$.

In the following, we can see that this measure will create a relationship between two measures above. First, we have a following lemma.

**Lemma 5.2.** *Assume that*

*1)* $\pi_1 = \{\pi_1^1 \geq \pi_1^2 \geq \cdots \geq \pi_1^n\}$ *and* $\pi_2 = \{\pi_2^1 \geq \pi_2^2 \geq \cdots \geq \pi_2^n\}$ *are two possibility distributions satisfying the conditions as:*

*If there exists* $i_0$ *satisfying* $\pi_1^{i_0} < \pi_2^{i_0}$ *then there exists* $r < i_0$ *such that* $\pi_1^r \geq \pi_2^r$. (6)

*2)* $\nu_i$, $i = 1, \dots, n\}$ *are weights of specificity measure satisfying the condition* $(\mathrm{cw}_1)$.

*3)* $k_i$, $i = 1, \dots, n\}$ *is a non creasing sequence of non negative reals.* (7)

*If* $\sum_{i \geq 1} \nu_i \pi_1^i \geq \sum_{i \geq 1} \nu_i \pi_2^i$ *then* $\sum_{i \geq 1} \nu_i k_i \pi_1^i \geq \sum_{i \geq 1} \nu_i k_i \pi_2^i$. (8)

*Proof.* We will prove this lemma by induction on the number of elements in **B**.

It is obvious that (8) holds for $n = 1$.

For $n = 2$, since $\nu_1 \pi_1^1 + \nu_2 \pi_1^2 \geq \nu_1 \pi_2^1 + \nu_2 \pi_2^2$ and if $\pi_1^i \geq \pi_2^i$ for $i = 1, 2$ then the inequality (8) is obvious.

Conversely, if $\pi_1^2 < \pi_2^2$ then under the hypothesis $\pi_1^1 \geq \pi_2^1$.

$\nu_1 \pi_1^1 + \nu_2 \pi_1^2 \geq \nu_1 \pi_2^1 + \nu_2 \pi_2^2$ implies $\left(\nu_1 + \nu_2 \frac{\pi_1^2}{\pi_1^1}\right) \pi_1^1 \geq \left(\nu_1 + \nu_2 \frac{\pi_2^2}{\pi_2^1}\right) \pi_2^1$.

Define $a = \frac{\pi_1^2}{\pi_1^1}$, $b = \frac{\pi_2^2}{\pi_2^1}$ and $q = \frac{k_1 \nu_1 + k_2 \nu_2 a}{1 + \nu_2 a}$, here $k_1 \geq k_2 \geq 0$, then we have $a < b$, and $q\,(\nu_1 + \nu_2\, a)\, \pi_1^1 \geq q\,(\nu_1 + \nu_2\, b)\, \pi_2^1$ i.e. $k_1 \nu_1 \pi_1^1 + k_2 \nu_2 \pi_1^2 \geq q\,(\nu_1 + \nu_2\, b)\, \pi_2^1$.

In other side $q\,(\nu_1 + \nu_2\, b)\, \pi_2^1 \geq k_1 \nu_1 \pi_2^1 + k_2 \nu_2 \pi_2^2$ iff $(k_1 - k_2)\, \nu_1 \nu_2\, (b - a) \geq 0$ and this is obvious.

Assume (8) to hold for $n = k$, we will prove if for $n = k + 1$.

We need only prove the case where there exists a positive integer $i_0$ ($i_0 > 2$) such that $\pi_1^{i_0} < \pi_2^{i_0}$. Then under the hypothesis there will exists $r < i_0$ such that $\pi_1^r \geq \pi_2^r > 0$. We can consider that $r = i_0 - 1$.

From $\sum_{i \geq 1} \nu_i \pi_1^i \geq \sum_{i \geq 1} \nu_i \pi_2^i$ we have

$$\sum_{i \leq i_0 - 2} \nu_i \pi_1^i + \left(\nu_{i_0 - 1} + \nu_{i_0} \frac{\pi_1^{i_0}}{\pi_1^{i_0 - 1}}\right) \pi_1^{i_0 - 1} + \sum_{i \geq i_0 + 1} \nu_i \pi_1^i \geq$$

$$\geq \sum_{i \leq i_0 - 2} \nu_i \pi_2^i + \left(\nu_{i_0 - 1} + \nu_{i_0} \frac{\pi_2^{i_0}}{\pi_2^{i_0 - 1}}\right) \pi_2^{i_0 - 1} + \sum_{i \geq i_0 + 1} \nu_i \pi_2^i. \tag{9}$$

Assume $\{k_i,\ i = 1, \dots, n\}$ satisfy the condition (7).

Set
$$a = \frac{\pi_1^{i_0}}{\pi_1^{i_0-1}},\ b = \frac{\pi_2^{i_0}}{\pi_2^{i_0-1}}\ q = \frac{k_{i_0-1}\,\nu_{i_0-1} + k_{i_0}\,\nu_{i_0}\,a}{\nu_{i_0-1} + \nu_{i_0}\,a}.$$

It follows that $k_{i_0-1} \geq q \geq k_{i_0+1}$, $a < b$.

Let $\pi_j^* = \{\pi_j^{*1} \geq \pi_j^{*2} \geq \cdots \geq \pi_j^{*k}\}$ for $j = 1,\ 2$, $\{\nu_i^*,\ i = 1,\dots,k\}$ and $\{k_i^*,\ i = 1,\dots,k\}$ are defined by

$$\pi_j^{*i} = \begin{cases} \pi_j^i, & \text{if } i \leq i_0 - 1, \\ \pi_j^{i+1}, & \text{if } i_0 \leq i \leq k. \end{cases} \quad \text{for } j = 1,\ 2$$

$$k_i^* = \begin{cases} k_i, & \text{if } i \leq i_0 - 2, \\ q, & \text{if } i = i_0 - 1, \\ k_{i_0-1}, & \text{if } i_0 \leq i \leq k. \end{cases} \quad \nu_i^* = \begin{cases} \nu_i, & \text{if } i \leq i_0 - 2, \\ \nu_{i_0-1} + \nu_{i_0}\,\frac{\pi_1^{i_0}}{\pi_1^{i_0-1}}, & \text{if } i = i_0 - 1, \\ \nu_{i-1}, & \text{if } i_0 \leq i \leq k. \end{cases}$$

respectively.

Then we can rewrite (9) as $\sum\limits_{1\leq i\leq k} \nu_i^*\,\pi_1^{*i} \geq \sum\limits_{1\leq i\leq k} \nu_i^*\,\pi_2^{*i}$. We can see that $\{\pi_1^{*i},\ i = 1,\dots,k\}$, $\{\pi_2^{*i},\ i = 1,\dots,k\}$ and $\{\nu_i^*,\ i = 1,\dots,k\}$, $\{k_i^*,\ i = 1,\dots,k\}$ satisfy the conditions (6), ($\mathrm{cw}_1$), (7), respectively.

Applying the hypothesis of induction we have

$$\sum_{1\leq i\leq k} \nu_i^*\,k_i^*\,\pi_1^{*i} \geq \sum_{1\leq i\leq k} \nu_i^*\,k_i^*\,\pi_2^{*i}, \text{ i.e.,}$$

$$\sum_{i\leq i_0-2} \nu_i\,k_i\,\pi_1^i + q\,\Big(\nu_{i_0-1} + \nu_{i_0}\,\frac{\pi_1^{i_0}}{\pi_1^{i_0-1}}\Big)\,\pi_1^{i_0-1} + \sum_{i\geq i_0+1} \nu_i\,k_i\,\pi_1^i$$
$$\geq \sum_{i\leq i_0-2} \nu_i\,k_i\,\pi_2^i + q\,\Big(\nu_{i_0-1} + \nu_{i_0}\,\frac{\pi_2^{i_0}}{\pi_2^{i_0-1}}\Big)\,\pi_2^{i_0-1} + \sum_{i\geq i_0+1} \nu_i\,k_i\,\pi_2^i \tag{10}$$

Similarly as in the case $n = 2$,

$$q\,\Big(\nu_{i_0-1} + \nu_{i_0}\,\frac{\pi_2^{i_0}}{\pi_2^{i_0-1}}\Big)\,\pi_2^{i_0-1} \geq \big(k_{i_0-1}\,\nu_{i_0-1}\,\pi_2^{i_0-1} + k_{i_0}\,\nu_{i_0}\,\pi_2^{i_0}\big) \text{ iff}$$

$(k_{i_0-1} - k_{i_0})\,\nu_{i_0-1}\,\nu_{i_0}\,(b - a) \geq 0$. Therefore the lemma is proved. $\square$

**Proposition 5.3.** *Let $\pi_1$, $\pi_2$ be two possibility distributions having a same natural ordering.*

*1. If $R(\pi_1) \leq R(\pi_2)$ with any weights $\{w_i\}$ of Buoancy measure, then $Sp(\pi_1) \geq Sp(\pi_2)$ with the weights* $\{\nu_1 = 1,\ \nu_i = (w_i\, H(x_i) \,/\, \sum_{k\geq 1} H(x_k)),\ i \geq 2\}$ (cw$_2$)

$$\text{or } \{\nu_1 = 1,\ \nu_i = (w_i\, H(x_i) \,/\, \sum_{k\geq 1} w_k\, H(x_k)),\ i \geq 2\} \qquad (\text{cw}_3)$$

*2. Conversely if $Sp(\pi_1) \geq Sp(\pi_2)$ for any weights of Specificity measure $\{\nu_i,\ i = 1, ..., n\}$ satisfying the condition* (cw$_1$)*, then $R(\pi_1) \leq R(\pi_2)$ with the weights $\{w_i = v_i/\Sigma v_k,\ i = 1, ..., n\}$.*

*Proof.*

1. $R(\pi_1) \leq R(\pi_2)$ with any weights $\{w_i\}$ implies $\sum_{i\geq 1} w_i\, H(x_i)\, \pi_1(x_i) \leq \sum_{i\geq 1} w_i\, H(x_i)\, \pi_2(x_i)$. It is easy to check that $\mathrm{Sp}(\pi_1) \geq \mathrm{Sp}(\pi_2)$ with the weights $\{v_i,\ i = 1, ..., n\}$ defined as in (cw$_2$) or (cw$_3$).

We see if $\{w_i,\ i = 1, ..., n\}$ satisfies (cw$_1$) then $\{v_i,\ i = 1, ..., n\}$ is also.

2. Conversely from $\sum_{i\geq 1} w_i\, \pi_1(x_i) \leq \sum_{i\geq 1} w_i\, \pi_2(x_i)$ and by Lemma 5.2, it can imply that $\sum_{i\geq 1} w_i\, H(x_i)\, \pi_1(x_i) \leq \sum_{i\geq 1} w_i\, H(x_i)\, \pi_2(x_i)$ i.e. $R(\pi_1) \leq R(\pi_2)$ with weights $\{w_i = v_i/\Sigma v_k,\ i = 1, ..., n\}$ and it is clear that $\{w_i,\ i = 1, ..., n\}$ also satisfies the condition (cw$_1$). □

If selecting a possibility distribution that maximizes the chosen measure $R$ is considered a *use of principle of maximal of measure R* (MR) then from the proposition 5.3, we have

**Corollary 5.4.** *Assume possibility distributions $\pi_1, ..., \pi_m$ have a same natural ordering. Then the use of the principle of mS with weights $\{\nu_i,\ i = 1, ..., n\}$ on the set of these distribution is equivalent the use of the principle of MR with any weights $\{w_i\}$, here $\{w_i\}$ and $\{v_i\}$ satisfy the additional condition* (cw$_1$) *and depend on each other as in the proposition 5.3.*

**Proposition 5.5.** *Let $S_1$, $S_2$ be two weak orderings associated with the possibility distributions $\pi_1$, $\pi_2$, respectively.*

*1. Proposition 5.3 also always holds when $S_1$, $S_2$ are total orderings.*

*2. Part (2) of Proposition 5.3 holds when $S_1$ is a total ordering, $S_2$ is a weak ordering.*

*3. Part (2) of Proposition 5.3 also holds when $S_1$, $S_2$ are any two weak orderings such that $S_1 \subseteq S_2$.*

*Proof.*

1. The proof is based on one of the proposition 5.3 and following remarks:

If $S_1$, $S_2$ are total orderings, we can consider that $\pi_1(x_1) > \pi_1(x_2) > \cdots > \pi_1(x_n)$, here $x_i \in \mathbf{B}$, $\pi_2(x_{k_1}) > \pi_2(x_{k_2}) > \cdots > \pi_2(x_{k_n})$, and $(k_1, k_2, \ldots, k_n)$ is a permutation of $(1, 2, \ldots, n)$.

Then

$$\mathrm{Sp}(\pi_1) = 1 - \sum_{i \geq 2} v_i\, \pi_1(x_i), \quad \mathrm{Sp}(\pi_2) = 1 - \sum_{i \geq 2} v_i\, \pi_2(x_{k_i})$$

$$R(\pi_1) = \sum_{i \geq 2} \frac{n-i+1}{n}\, w_i\, \pi_1(x_i)$$

$$R(\pi_2) = \sum_{i \geq 2} \frac{n-i+1}{n}\, w_i\, \pi_2(x_{k_i}),$$

here $\{v_i\}$, $\{w_i\}$ are weights of Specificity and Buoancy measures, respectively.

2. If $S_1$ is a total orderings, from $\mathrm{Sp}(\pi_1) \geq \mathrm{Sp}(\pi_2)$ with any weights of Specificity measure $\{v_i,\ i = 1, \ldots, n\}$ satisfying $(\mathrm{cw}_1)$ and according to Lemma 5.2 we have

$$\sum_{i \geq 2} \frac{n-i+1}{n}\, v_i\, \pi_1(x_i) \leq \sum_{i \geq 2} \frac{n-i+1}{n}\, v_i\, \pi_2(x_{k_i}),$$

where $(k_1, k_2, \ldots, k_n)$ is a permutation of $(1, 2, \ldots, n)$ and $\pi_2(x_{k_1}) > \pi_2(x_{k_2}) > \cdots > \pi_2(x_{k_n})$.

From $g(x_i) \geq \frac{n-i+1}{n}$, here $g(x_i)$ is defined by $\pi_2$ as in the Buoancy measure it may be concluded that the part 2 of this proposition holds.

3. Assume $S_1$, $S_2$ are any two weak and both not total orderings, but $S_1 \subseteq S_2$.

If $\pi_1(x_1) \geq \pi_1(x_2) \geq \cdots \geq \pi_1(x_n)$ then $\pi_2(x_1) \geq \pi_2(x_2) \geq \cdots \geq \pi_2(x_n)$ and $H_1(x_i) \leq H_2(x_i)$ for every $x_i \in \mathbf{B}$, here $H_k(x_i) = \Sigma S_k(x_i, x)$, $k = 1,\ 2$.

Since $\mathrm{Sp}(\pi_1) \geq \mathrm{Sp}(\pi_2)$ and under Lemma 4.2 we obtain

$$\sum_{i \geq 1} v_i\, H_1(x_i)\, \pi_1(x_i) \leq \sum_{i \geq 1} v_i\, H_1(x_i)\, \pi_2(x_i)$$

In other side

$$\sum_{i \geq 1} v_i\, H_1(x_i)\, \pi_2(x_i) \leq \sum_{i \geq 1} v_i\, H_2(x_i)\, \pi_2(x_i)$$

thus the part 3 of the proposition is proved. □

Propositions 5.3, 5.5 say that when the possibility distributions have a same natural ordering, or all orderings associated with them are total, if a possibility distribution is selected by the use of the principle of mS with and weights satisfying $(\mathrm{cw}_1)$, then it is also selected by the use of the principle of MR with the dependent weights and conversely.

In the cases, when the all orderings associated with these possibility distributions are placed in two groups, first group consists of total ordering, second group - non total weak orderings, but the relation $\subseteq$ become linear ordering on this group, if a possibility distribution is selected by the use of the principle of mS with any weights satisfying $(\mathrm{cw}_1)$, then it is also selected by the use of the principle of MR with the dependent weights, conversely in general it is not true.

We now return the requested problem above, i.e., analysis the process of the use of the principle of mS on possibility distribution having a same natural ordering.

Let $\pi_1, \ldots, \pi_m$ be possibility distributions satisfying this constraint, we will denote by $S$ the ordering associated with them.

Assume $\pi_1(x_1) \geq \pi_1(x_2) \geq \cdots \geq \pi_1(x_n)$ then $\pi_k(x_1) \geq \pi_k(x_2) \geq \cdots \geq \pi_k(x_n)$ for $k = 2, \ldots, m$.

**Theorem 5.6.** *Finite set of possibility distribution having a same natural ordering is $(cw_2)$-mS-stable and $(cw_3)$-mS-stable.*

*Proof.* Assume $\pi$ is a possibility distribution selected from $\pi_1, \ldots, \pi_m$ by the use of principle of mS with weights $\left\{v_1^* = 1,\ v_i^* = \dfrac{H(x_i)}{2^{i-1}\,\Sigma H(x_k)},\ i \geq 2\right\}$, we will point out that $\pi$ is also selected by the use of principle of mS with any weights defined as in $(\mathrm{cw}_2)$, or $(\mathrm{cw}_3)$.

Under the hypothesis of $\pi$, we have $\sum\limits_{i \geq 1} v_i^*\, \pi_k(x_i) \geq \sum\limits_{i \geq 1} v_i^*\, \pi(x_i)$ for every $k = 1, \ldots, m$.

Define $k_i = 2^{i-1} w_i$ for $i = 1, \ldots, n$, here $\{w_i,\ i = 1, \ldots, n\}$ is any weights of Buoancy measure. We can see that any two of the distributions $\pi_1, \ldots, \pi_m$ satisfy (6) (because they are normalized), $\{v_i^*,\ i = 1, \ldots, n\}$ and $\{k_i,\ i = 1, \ldots, n\}$ satisfy the conditions $(\mathrm{cw}_1)$ (7), respectively. Therefore, under Lemma 5.2. we obtain

$$\sum_{i \geq 1} v_i^*\, k_i\, \pi(x_i) \geq \sum_{i \geq 1} v_i^*\, k_i\, \pi_k(x_i), \quad \text{i.e.}$$

$$R(\pi) \geq R(\pi_k) \quad \text{for } k = 1, \ldots, n\,. \tag{11}$$

From (11), applying the proposition 5.3, we have $\mathrm{Sp}(\pi) \leq \mathrm{Sp}(\pi_k)$ for every $k = 1, \ldots, n$ with the weights of Specificity measure $\{v_i\}$ defined as in $(\mathrm{cw}_2)$ or $(\mathrm{cw}_3)$. Therefore, the proof of the theory is complete. □

*Remark.* If $\pi_1, ..., \pi_m$ are possibility distributions having a same natural ordering $S$, then it is also natural ordering associated with all possibility distributions in the format $\pi = \sum_{i=1}^{m} a_i\, \pi_i$, where $a_i \geq 0$ and $\sum_{i=1}^{m} a_i = 1$, and $\mathrm{Sp}(\pi) = \sum_{i=1}^{m} a_i\, \mathrm{Sp}(\pi_i)$.

Set $\mathbf{C}(\pi_1, ..., \pi_m) = \{\pi = \sum_{i=1}^{m} a_i\, \pi_i,\ a_i \geq 0,\ \sum_{i=1}^{m} a_i = 1\}$ then if $\pi_{i_0}$ is selected from the possibility distributions $\pi_1, ..., \pi_m$ by the use of the principle of mS with weights $\{v_i\}$ then $\pi_{i_0}$ is also selected by the use of this principle on $\mathbf{C}(\pi_1, ..., \pi_m)$. This is obvious, because

$$\mathrm{Sp}(\pi_{i_0}) = \sum_{i=1}^{m} a_i\, \mathrm{Sp}(\pi_{i_0}) \leq \sum_{i=1}^{m} a_i\, \mathrm{Sp}(\pi_i) = \mathrm{Sp}(\pi)$$

for every $\pi \in \mathbf{C}(\pi_1, ..., \pi_m)$.

**Corollary 5.7.** $\mathbf{C}(\pi_1, ..., \pi_m)$ *is* $(cm_2)$*-mS-stable and* $(cw_3)$*-mS-stable.*

## 6. CONCLUSION

In [8] we pointed out that there exists the relationship between the principles of mS in possibility theory and of maximal entropy (ME) in probability theory. Namely we showed some conditions of finite sets of possibility distributions such that the uses of the principles of mS and of ME to select one from these distributions are equivalent.

Since important role of possibility distributions, it is necessary to continue a discussion on the use of the principles of mS and MB.

In this paper, we proposed the concepts of mS-stable (MB-stable)... and showed some conditions of possibility distributions and weights of the Specificity (Buoancy) measure for these stability sorts.

Use of the principles of mS and of MB on infinite set of possibility distributions is one of our future research topics, there we will use the relationship between the principles of mS and of ME for defining a probability distribution that satisfies the given probability knowledge base such that at the distribution, the Entropy measure receives approximately maximum value, and then the concepts of mS-stable, (MB-stable)... of set of possibility distributions are useful.

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