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HOW MUCH INFORMATION OF CONCURRENCY CAN BE GOT FROM FIRING SEQUENCES IN PETRI NETS

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Abstract. It is well known from [2], [3] that in general processes in Petri nets are not recoverable from firing sequences. However, firing sequences in Petri nets say something about concurrency. The paper presents a way to define concurrency from firing sequences of nets. It turns out that the information of concurrency in a firing sequences characterizes all its processes.

1. INTRODUCTION

In concerning the concurrent and distributed systems, the way in which the temporal/causal ordering events is described is a problem being under discussion. In the interleaving approach, the fact that a set of events may occur in parallel is described by saying that they may occur in any order. Models based on true concurrency use instead partial orderings to explicitly describe the temporal/causal relations among events [4, 6, 8, 9]. In [2, 3], a comparison between the two approaches has been treated. These authors proved that in P/T nets processes (corresponding to the latter) are not recoverable from firing sequences (corresponding) to the former), while in C/E systems they are. This means that in general in P/T nets true concurrency cannot be obtained from firing sequences. As firing sequences play an important role in studying the behaviours of P/T nets, and as a part of true concurrency is carried in them, it is worth studying the ways to decide what we can say about concurrency from firing sequence of P/T nets. By following the approach of Mazurkiewicz [7]. Best [2] and Degano [3] to the behaviours of concurrent systems and developing some results in [5], the paper presents a way to study concurrency from firing sequences. We show that in order to obtain information of true concurrency from firing sequences, only the statistical structures of nets comes into play. We also give a necessary and sufficient condition to a net for which processes are recoverable from firing sequences.

2. LIKE-DEPENDENCY

We follow Mazurkiewicz/s approach to the behaviours of C/E systems [1], [7] in studying firing sequences of P/T nets.

Our starting point is the notion of so-called like - dependency. Intuitively speaking, when there may be causal dependencies among occurrences of two actions, we consider them to be in like - dependence. Formally like - dependency is defined below.

Let A be a finite set whose members are referred to as actions. Let A^* (A^ω respectively) denote the set of all finite (infinite) sequences (or words) over A, $A^\infty := A^* \cup A^\omega$. The empty sequence is denoted by ε.

For $\omega \in A^\infty$ and $a \in A$, $\#_a w$ will denote the number of the occurrences of a in w and $\mathcal{O}(w)$ denotes the set $\{(a,i) | \#_a w > 0 \wedge 0 < i < \#_a w + 1\}$.

Definition 1. A like-dependency on A is a reflexive binary relation on A.

Since in general an action can depend on other action while the latter is independent of the former, a like-dependency is not required to be symmetrical.

Let D be a like-dependency on A. Each $w \in A^*$, w may represent a computation. Then the partial ordering of causal dependency relation $\leq_w$ is defined as follows.

Definition 2. The partial ordering generated by w over D is $(\mathcal{O}(w), \leq_w)$, where $\leq_w$ is the reflexive and transitive closure F_w^* of the relation F_w defined by $(a,i)\ F_w (b,j)$ iff the j-th occurrence of b precedes the i-th occurrence of a in w and $(a,b) \in D$.

Example 1. Let $A = \{a, b\}$, $D = \{(a,a),\ (b,b),\ (b,a)\}$, $w = (ab)^\omega = abab... \in A^\omega$. Then $(\mathcal{O}(w), \leq_w)$ is represented by the following graph (the transitive arcs are omitted).

$$
\begin{tikzcd}
\boxed{(a,1)} \arrow[d] & \\
\boxed{(a,2)} \arrow[d] \arrow[r, dash] & \boxed{(b,1)} \arrow[d] \\
\vdots & \vdots \\
\boxed{(a,n)} \arrow[d] \arrow[r, dash] & \boxed{(b,n-1)} \arrow[d] \\
\vdots & \boxed{(b,n)} \arrow[d] \\
& \vdots
\end{tikzcd}
$$

Now, we introduce a partial order among members of A^∞.

Definition 3. For $w,\ w' \in A^{\infty}$, $w \sqsubseteq w'$ iff $\mathcal{O}(w) = \mathcal{O}(w')$ and $\leq_w \subseteq \leq_{w'}$ ($w \sqsubseteq w'$ iff the partial ordering generated by w over D is coarser than the one generated by w').

We consider the behaviour of a computation system as a pair of

- A like-dependency, which approximately represents dependency in the system.
- subset C of A^{∞}, which represents possible computation of the system (the interleaving behaviours of the system).

Then, for each $w \in C$, $(\mathcal{O}(w), \leq_w)$ represents uncertainty the causal dependencies among occurrences of actions in w, some causal dependencies of which are introduced by going to extremes.

Now, we consider what the relation $\subseteq$ means.

In the sequel, let *name* : $A \times \{1,\ 2, \dots\} \longrightarrow A$ be defined as

$$\text{name}\,((a, i)) = a \text{ for all integers } i > 0,$$

and let pref : $A^{\infty} \longrightarrow 2^{A^*}$ be a mapping which returns all prefixes of its argument. The mapping *name* is extended to a homomorphism from $(A \times \{1,\ 2, \dots\})^{\infty}$ to A^{∞} in the obvious way. Furthermore, for $w,\ w' \in A^*$, we write $w \xrightarrow[D]{*} w'$ iff there is a derivation from w to w' in the rewriting system (A, P) with $P = \{ab \rightarrow ba | (a, b) \notin D\}$.

Theorem 1.

(i) *Let $w,\ w' \in A^*$, $w \sqsubseteq w'$ if and only if $w \xrightarrow{*} w'$.*

(ii) *Let $w,\ w' \in A^{\omega}$, $w \sqsubseteq w'$ if and only if* $(\mathcal{O}(w) = \mathcal{O}(w')$
$\wedge (\forall v \in \text{pref}(w')\ \exists u \in \text{pref}(w)\ \exists x \in A^* : (u \xrightarrow{*} vx))$.

Proof.

(i) Only the 'only if' part is not obvious and can be shown by induction on the length $|w|$ of w, and we leave it to the readers.

(ii) ($\Leftarrow$): Let $e_1,\ e_2 \in \mathcal{O}(w) = \mathcal{O}(w')$ and $e_1 \leq_w e_2$. There must be v in pref(w') such that $e_1, e_2 \in \mathcal{O}(v)$. Let u and x be such that $u \in \text{pref}(w)$ and $u \xrightarrow{*} vx$. From (i) it follows $e_1, e_2 \in \mathcal{O}(u)$. By the definition of $\leq_w$ we have $e_1 \leq_u e_2$, and thus $e_1 \leq_v e_2$ by (i). Hence, $e_1 \leq_{w'} e_2$ by the definition of $\leq_w$.

($\Rightarrow$): Let $v \in \text{pref}(w')$. Then $\mathcal{O}(v) \subseteq \mathcal{O}(w)$. Let $u \in \text{pref}(w)$ such that $\mathcal{O}(u) \supseteq \mathcal{O}(v)$. It follows that $\leq_u = (\leq_w) \cap (\mathcal{O}(u) \times \mathcal{O}(u)) \subseteq (\leq_{w'}) \cap (\mathcal{O}(u) \times \mathcal{O}(u))$. Let α be a topology sorting of $(\mathcal{O}(u) \setminus \mathcal{O}(v))$ by $\leq_{w'}$ and $x = \text{name}(\alpha)$. It can be seen from the definition of $\leq_{w'}$ that $(\leq_{w'} \cap (\mathcal{O}(u) \times \mathcal{O}(u)) \sqsubseteq \leq_{vx}$. Hence, $\leq_u \sqsubseteq \leq_{vx}$. By (1) we get $u \xrightarrow{*} vx$. $\square$

Theorem 1 says that for w, $w' \in A^{\infty}$, $w \sqsubseteq w'$ if and if they have the same set of action occurrences and w' is derived from w by applying a (finite or infinite) number of rewriting rules $ab \to ba$ with $(a,b) \notin D$.

Since independent events can occur in any order, it follows from Theorem 1 that if $w \in C$, for each w' such that $w \sqsubseteq w'$, $w' \in C$ as well.

Corollary 1. *Let $\equiv$ be defined as $w \equiv w'$ iff $\leq_w = \leq_{w'}$ then $w \equiv w'$ if and only if $w \xrightarrow[D_s]{*} w'$, where D_s is the symmetrical closure of D.*

3. INFORMATION OF TRUE CONCURRENCY IN FIRING SEQUENCES OF P/T NETS

In this section we investigate how much information of true concurrency can be got from firing sequences of P/T nets. We shall compare the partial orderings among events introduced by processes in P/T nets to the partial orderings generated by firing sequences with respect to the natural like-dependencies defined by the structure of nets.

A nets is a triple $\langle S, T; F \rangle$, where

- $S \cap T = \infty$;
- $F \subseteq (S \times T) \cup (T \times S)$.

Let, as usual, $t^{\bullet} = \{s \in S | tFs\}$, ${}^{\bullet}t = \{s \in S | sFt\}$ for a net $\langle S, T; F \rangle$.

An occurrence net is a net $= \langle S, T; F \rangle$ such that

- The transitive closure of F, defined by F^{+}, is irreflexive;
- $\forall s \in S$, $|{}^{\bullet}s| \leq 1 \wedge |s^{\bullet}| \leq 1$.

Furthermore,

- $S \cup T$ is considered as ordered by $<$, defined as F^{+};
- The slices of K are maximal subsets of S which do not contain elements related by $<$.

A marked place/transition net (P/T net) is a quintiple $N = \langle S, T; F, W, M \rangle$, where

- $\langle S, T; F \rangle$ is a net, with S and T finite;
- $W : F \to N$ assigns a positive weight to each arc;
- $M : S \to N$ is the initial marking of N.

Given a P/T net $N = \langle S, T; F, W, M \rangle$, a firing sequence of N is $\{M_0 t_0 M_1 t_1 M_2 ...\}$, where for $i = 1, 2, ...$

- M_i are markings of S and $M_0 = M, t_i \in T$;

- $M_i[t_i > M_{i+1}$, where $M[t > M'$ implies that $\forall s \in S$, $M(s) \geq W(s,t)$ and $M'(s) = M(s) - W(s,t) + W(t,s)$.

We shall call the sequences obtained from firing sequences by dropping the markings also firing sequences without fear of confusions.

Given a P/T net $N = \langle S, T, F; W, M\rangle$ and an occurrence net $K = \langle S', T'; F'\rangle$, a P/T process of N is a function

$$p : K \to N$$

such that

- $p(S') \subseteq S$, $p(T') \subseteq T$;
- $(S' \cup T', <')$ is finitely preceded. Let ${}^\circ K$ be the set of its minima;
- $\forall s \in S$, $M(s) = \{p^{-1}(s) \cap {}^\circ K|$;
- $\forall t' \in T'$. $\forall s \in S$
  (i) $W(s, p(t')) = |p^{-1}(s) \cap {}^\bullet t'|$,
  (ii) $W(p(t'), s) = |p^{-1}(s) \cap t'^\bullet|$.

Definition 4. The labeled partial ordering generated by a process $p : K \to N$ of a P/T net N (denoted as above) is $(T', p|_{T'}, \leq_p)$, where $\leq_p$ is $F'^*|_{T' \times T'}$.

From the results in [2], [3] it follows:

For a P/T net N, α is a firing sequence of N if and only if there exists a process $p : K \to N$ of N such that $\alpha = p(\beta)$, where β is a topology sorting of T' by $\leq_p$, p is extended to a homomorphism on sequences in obvious way.

It can be seen easily that if $p : K \to N$ is a process of N with $K = \langle S', T'; F'\rangle$, $S' \cup T'$ is countable. Furthermore, since isomorphic processes are not distinguished, in the sequel T' is usually considered as a subset of $T \times \{1, 2, \ldots\}$ satisfying:

(i) $t' = (a, n) \in T'$ implies $(a, i) \in T'$ for $0 < i \leq n$,

(ii) (a, n), $(a, n') \in T'$ and $n < n'$ implies $(a, n') \not\leq_p (a, n)$,

(iii) $p(t) = \text{name}(t)$ for all $t \in T'$.

As in [4] processes are considered to be equivalent iff the partial order of event occurrences agrees in them. We give the following definition.

Definition 5. Let K and N be denoted as above. $p : K \to N$ is a process of N. $(T', \leq_p')$ is called a concurrency characteristic of p (characteristic of p for short).

As in [2], let us denote for a firing sequence α and for a process $p : K \to N$ of N with $K = (S', T'; F')$:

$\text{Lin}(p) := \{\alpha | \alpha = \text{name}(\beta)$ with β being a topology sorting of T' by $\leq_p'\}$.

$$\text{Proc}(\alpha) := \{p | \alpha \in \text{Lin}(p)\}.$$

From the resul' in [2], [3], we have

Theorem 2. *Let N be a P/T net, α is a firing sequence of N if and only if there exists a process $p : K \to N$ of N with $K = \langle S', O(\alpha), F' \rangle$ such that $\alpha = name(\beta)$ with β being a topology sorting of $O(\alpha)$ by $\leq_p$ (p is said to correspond to α).*

This is the first result on the relationship between firing sequences and processes. Now we give some anothers.

Definition 6. Let $N = \langle S, T; F, W, M \rangle$ be a P/T net.

$D = \{(t,t') | t,t' \in T \wedge (t^\bullet \cap {}^\bullet t' \neq \emptyset \vee t = t')\}$ is called like-dependency generated by N.

Let, in the sequel, N be a P/T net, D its like dependency, α a firing sequence of N with $(O(\alpha), \leq_\alpha)$ being its partial ordering on D, and let $\leq$ be defined in as Definition 3 w.r.t D.

Theorem 3. *Let $K = (S', O(\alpha), F')$ and $p : K \to N$ be a process of N corresponding to α, $\leq'_p = F'^*$. Then $(O(\alpha), \leq_p)$ is coarser than $(O(\alpha), \leq_\alpha)$.*

Proof. It is sufficient to prove:

$$\forall (a,i),\ (b,j) \in O(\alpha) : (a,i)\, F'^2 (b,j) \Rightarrow (a,i) \leq_\alpha (b,j)\,.$$

we have

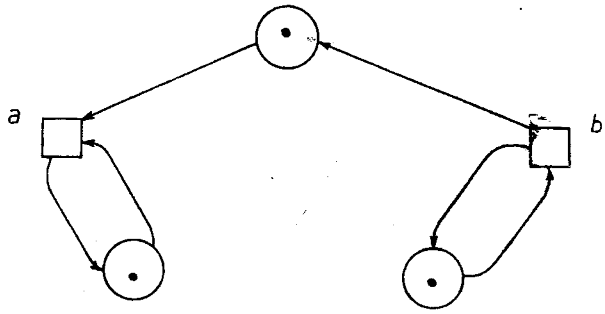
$$\begin{aligned}(a,i)F'^2(b,j) &\Rightarrow \exists s \in S' : (a,i)F'sF'(b,j) \\ &\Rightarrow a^\bullet \cap {}^\bullet b \supseteq \{p(s)\} \Rightarrow (a,b) \in D\,.\end{aligned}$$

Since $\alpha \in \mathrm{Lin}(p)$, the i-th occurrence of a precedes the j-th occurrence of b in α. By Definition 2 $(a,b) \in D$ implies $(a,i) \leq_\alpha (b,j)$. □

Theorem 3 says that if (a,i) and (b,j) are not related by $\leq_\alpha$, neither are they by any process corresponding to α. That means we can get some information of concurrency from firing sequences of the net by its like-dependency.

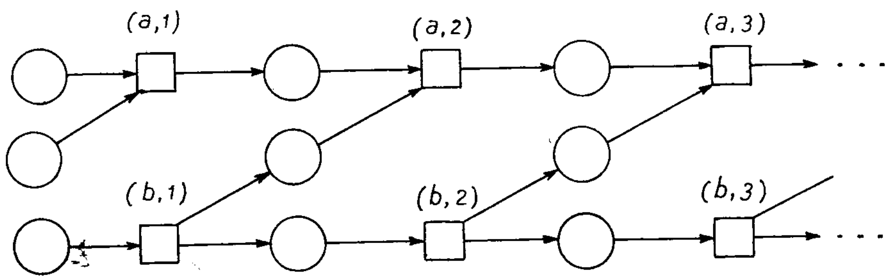
Example 2.

Let N =



Then, $D = \{(a,a),\ (b,b),\ (b,a)\}$.

$\alpha = (ab)^\omega$ is a firing sequence of N, and $(O(\alpha), \leq_\alpha)$ is the same as in Example 1. A process corresponding to α is the following



In this case, Proc(α) contains one process, and $(\mathcal{O}(\alpha), \leq_\alpha)$ is its characteristic.

Theorem 4. *If the parallel occurrence of the same transitions is impossible in N, (this means that if $p : (S', T'; F') \to N$ is a process of N, and if $(t,i), (t,j) \in T'$ with $i < j$, $(t,i) \leq_p (t,j)$, then* $\leq_\alpha = \bigcup_{p \in \mathrm{Proc}(\alpha)} \leq_{p'}$.

Proof. From Theorem 3 we have

$$\bigcup_{p \in \mathrm{Proc}(\alpha)} \leq_{p'} \subseteq \leq_\alpha .$$

Now we have to show the inverse inclusion. It is sufficient to prove that $\forall (a,i), (b,j) \in \mathcal{O}(\alpha) : (a,i) \leq_\alpha (b,j) \Rightarrow \exists p \in \mathrm{Proc}(\alpha) : (a,i) \leq_p (b,j)$.

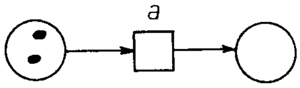
Let p be a process of N corresponding to α. If $(a,i) \leq_p (b,j)$, or $a = b$, the theorem has been proved. Suppose that $(a,i) \not\leq_p (b,j)$ and $a \neq b$. Since $(a,b) \in D \Rightarrow \exists s \in a^\bullet \cap {}^\bullet b$. Let $s_1 \in (a,i)^\bullet$ and $s_2 \in {}^\bullet(b,j)$ such that $p(s_1) = p(s_2) = s$. Of course, $s_1 \neq s_2$. Now we construct a occurrence net $K' = (S', \mathcal{O}(\alpha), F'')$, where

$$F'' = F' \setminus \{(s_2, (b,j))\} \cup \{(s_1, (b,j))\},$$
$$p' = p .$$

It can be seen that $F'')^+$ is acyclic and $p' : K' \to N$ is a process in Proc(α) as well. Furthermore, $(a,i) \leq_{p'} (b,j)$. □

The theorem will not be true in general without the assumption that the parallel occurrence of the same transitions is impossible. Let us consider the following example.

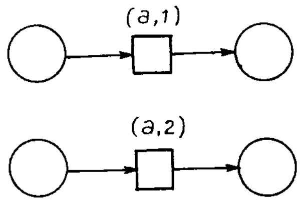
Example 3. Let

$N =$ 

Then, aa is a firing sequence of N with $(\mathcal{O}(aa), <_{aa})$ is represented by

while

Proc(aa) =



and its characteristics is $(\{(a,1),\ (a,2)\},\emptyset)$.

Theorem 4 says, in the case when the parallel occurrence of the same transitions is impossible in N, that the information of concurrency in each firing sequency is maximal amount derived from all processes corresponding to the firing sequence.

Theorem 5. *If α is a firing sequence of N and $\alpha \sqsubseteq \beta$ then β is a firing sequence of N also. Moreover* $\text{Proc}(\alpha) \subseteq \text{Proc}(\beta)$.

Proof. It follows from Theorem 1 that if is a firing sequence, then $\text{Proc}(\alpha) \not\subseteq \emptyset$. Let $p \in \text{Proc}(\alpha)$, from Theorem 3 we have $\leq_p \subseteq \leq_\alpha \subseteq \leq_\beta$, which implies $\beta \in \text{Lin}(p)$. Hence, β is a firing sequence of N, and every process in $\text{Proc}(\alpha)$ is a process in $\text{Proc}(\beta)$, too. $\square$

In the sequel, we assume that N be such net in which the parallel occurrence of the same transitions in impossible. We have the following corollaries.

Corollary 2. *For firing sequences α, β of N,* $\text{Proc}(\alpha) = \text{Proc}(\beta)$ *if and only if $\alpha \equiv \beta$, where $\equiv$ is defined as in Corollary 1.*

Proof. The 'only if' part follows from Theorem 4, and the 'if' part follows from Theorem 5. $\square$

Corollary 3. *For a firing sequence α of N, all processes in* $\text{Proc}(\alpha)$ *are equivalent (by $\equiv$ defined in [2]) if and only if $(\mathcal{O}(\alpha), \leq_\alpha)$ is their characteristic.*

Corollary 3 shows that the firing sequence approach and the process approach to the behaviour of P/T nets coincide only for a restricted class of Petri nets concluding 1-safe nets.

REFERENCES

1. Albersberg I. J., Rozenberg G., *Theory of trances*, Theoret. Comput. Sci., **60** (1988), 1-82.
2. Best E., *Concurrent behaviour: sequences, processes and axioms*, in S. D. Brookes et al., eds., Seminar on Concurrency, Lecture Notes in computer Science 197, Springer, Berlin, 1985, 221-245.
3. Degano P., Montanari U., *Concurrent histories: A basis for observing distributed systems*, Jour. of Comput. and Sys. Sci., **34** (1987), 422-461.
4. Goltz U. and Reisig W., *The non-sequential behavior of Petri nets*, Information and control, **57** (1983), 125-147.

5. Hung D. V. and Knuth E., *Semi-computations and Petri nets*, Theoret. Comput. Sci., **64** (1989), 67 - 81.

6. Loogen R. and Goltz U., *A non-interleaving semantic model for nondeterministic concurrent processes*, RWTH Aachen Fachgruppe Informatic, Nr. 87-15.

7. Mazrukiewicz A., *Concurrent program schemes and their interpretations*, DAIMI Report PB-78, Aarhus University, 1977.

8. Nielsen M., Plotkin G., and Winskel G., *Petri nets, event structures and domains*, Theoret. Comput. Sci., **13** (1983), 85 - 108.

9. Starker P. H., *Processes in Petri nets*, Informationsverarbeitung und Kybernet. EIK, **17** (1981), 389 - 416.

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