Tạp chí Tin học và Điều khiển học, T. 14, S. 3 (1998) (42–44)

STRUCTURE OF MULTIVALUED DEPENDENCIES IN RELATION SCHEME

HO THUAN[(1)], LE VAN BAO[(2)], HO CAM HA[(3)]

Abstract. As pointed out in [1], Multivalued dependencies (MVD) depend on the context in which they are defined and thus are very hard to visualize. In this paper, continuing the study in [1], we give some results which may provide some more insight regarding the existence of possible MVDs in the relation schemes provided all their FDs are known in advance.

Throughout this paper, we assume that the reader is familiar with the basic concepts of the relational database [2, 3].

1. MULTIVALUED DEPENDENCIES

Let U be a set attributes, X and Y be disjoint subsets of U. We say that in the relation R over the set of attributes U, there is a multivalued dependency of the set Y on the set X, written $X \rightarrow\rightarrow Y$, if the following holds. Let $Z = U \setminus (X \cup Y)$, a tuple r in R can be viewed as the concatenation of its projection on X, Y and Z which we denote by $r = r[X]\, r[Y]\, r[Z]$. Let r_1 and r_2 be two tuples with the same X-component $r_1 = r_1[X]\, r_1[Y]\, r_1[Z]$ and $r_2 = r_1[X]\, r_2[Y]\, r_2[Z]$ then the interchanged tuples $r_1[X]\, r_1[Y]\, r_2[Z]$ and $r_1[X]\, r_2[Y]\, r_1[Z]$ are also in R. In simple words when $r[X]$ is given, the Y-values that appear with it in r are independent of the values of any other attributes. The definition can be generalized to nondisjoint sets X and Y. The multivalued dependency $X \rightarrow\rightarrow Y$ holds in R if and only if $X \rightarrow\rightarrow Y \setminus X$ holds in R.

We mention here some of the rules that we shall use later:

1. Augmentation axiom:
   if $X \rightarrow\rightarrow Y$ and $V \subseteq W$, then $WX \rightarrow\rightarrow VY$.
2. Coalescence axiom:
   if $X \rightarrow\rightarrow Y$ and $V \rightarrow W$ where $W \subseteq Y$ and $Y \cap V = \emptyset$ then $X \rightarrow W$.
3. Mixed pseudotransitivity axiom:
   if $X \rightarrow\rightarrow Y$ and $XY \rightarrow W$ then $X \rightarrow W \setminus Y$.

Let R be a relation scheme, an MVD $X \rightarrow\rightarrow Y$ is said to be nontrivial if $Y \neq \emptyset$, $Y \not\subseteq X$ and $XY \neq U$. Since $X \rightarrow\rightarrow Y$ is valid iff $X \rightarrow\rightarrow Y \setminus X$ is valid, we shall always assume that left and right sides of an MVD are disjoint.

Let $Z = U \setminus XY$, since $X \rightarrow\rightarrow Y$ holds iff $X \rightarrow\rightarrow Z$ holds, we often write the MVD $X \rightarrow\rightarrow Y$ as $X \rightarrow\rightarrow Y|Z$. If the FD $X \rightarrow Y$ or the FD $X \rightarrow Z$ is valid, then the MVD $X \rightarrow\rightarrow Y|Z$ also holds in R. In this case, we call the MVD $X \rightarrow\rightarrow Y|Z$, an MVD counterpart of an FD. We call an MVD $X \rightarrow\rightarrow Y|Z$ pure if it is nontrivial and it is not an MVD counterpart of an FD in R.

In [1] is has been shown that:

(a) Let $X \to\to Y$ be a nontrivial MVD in R and K any key for R, then either $X \to Y$ holds in R or $K \cap Y \neq \emptyset$.

(b) Let $X \to\to Y|Z$ be a nontrivial MVD in a relation scheme R. Let K be a key for K such that $Y \setminus K \neq \emptyset$. If $Y_1 = Y \setminus K$ and $Y_2 = Y \cap K$ then the FD $XY_2 \to Y_1$ holds in R.
If $X \to\to Y|Z$ be a pure MVD in a relation scheme R. Let K be a key for R such that $Y \setminus K \neq \emptyset$. If $Y_1 = Y \setminus K$ and $Y_2 = Y \cap K$ then the FD $XY_2 \to Y_1$ hodls in R, but the FD $XY_2 \to Z$ does not hold in R.

2. RESULTS

Theorem 1. *Let $K_1, K_2, \dots K_m$ be the keys for the relation scheme.*
If $Y \cap \left(\bigcap_{i=1}^{m} K_i\right) = \emptyset$ and $X \to\to Y \setminus K_i$ for $i = 1, 2, \dots m$ then $X \to\to Y$ is not a pure MVD.

Proof. By the hypothesis of the theorem it is obviously for all $i = 1, 2, \dots m$: $X \to\to Y \setminus K_i$ and $K_i \cap (Y \setminus K_i) = \emptyset$. Since K_i is a key for the relation scheme we have $K_i \to (Y \setminus K_i)$. By the coalescence axiom $X \to Y \setminus K_i$ holds in R. From this we have $X \to \bigcup_{i=1}^{m} (Y \setminus K_i)$.

On the other hand $\bigcup_{i=1}^{m} (Y \setminus K_i) = Y \setminus \left(\bigcap_{i=1}^{m} K_i\right) = Y$. Hence $X \to Y$. The proof is complete.

Remark: Theorem 1 in [1] is a special case of the above theorem with $m = 1$.

Corollary 1. *Let $X \to\to Y$ be a pure MVD in R and let $K_1, K_2, \dots K_m$ denote the keys for the relation scheme. If $X \to\to Y \setminus K_i$ for $i = 1, 2, \dots m$ then $Y \cap \left(\bigcap_{i=1}^{m} K_i\right) \neq \emptyset$.*

Theorem 2. *Let $X \to\to Y$ be a nontrivial MVD in a relation scheme R and $K_1, K_2, \dots K_m$ denote the keys of R such that $Y \setminus K_i \neq \emptyset$, $\forall i = 1, 2, \dots n$.*
If $Y \cap \left(\bigcap_{i=1}^{m} K_i\right) = \emptyset$ and $X \to\to Y \cap K_i$ for $i = 1, 2, \dots m$ then $X \to\to Y$ is not a pure MVD.

Proof. Let $Y_i^1 = Y \setminus K_i$, $Y_i^2 = Y \cap K_i$ and $Z = U \setminus XY$.
From (b) we have $XY_i^2 \to Y_i^1$ holds in R. Using $X \to\to Y_i^2$ and $XY_i^2 \to Y_i^1$ we have $X \to Y_i^1 \setminus Y_i^2$ i.e. $X \to Y \setminus K_i$. From this $X \to \bigcup_{i=1}^{m} (Y \setminus K_i)$. Consequently $X \to \left(Y \setminus \bigcap_{i=1}^{m} K_i\right)$ thus $X \to Y$. The proof is complete.

Corollary 2. *Let $X \to\to Y$ be a pure MVD in R, and let $K_1, K_2, \dots K_m$ denote the keys of the relation scheme. If $X \to\to (Y \setminus \cap K_i)$ for $i = 1, 2, \dots m$ then $Y \cap \left(\bigcap_{i=1}^{m} K_i\right) \neq \emptyset$.*

Theorem 3. *Let $X \to\to Y$ be a nontrivial MVD in R, and $K_1, K_2, \dots K_m$ denote the keys of the relation scheme, $Y \setminus K_i \neq \emptyset$ for $i = 1, 2, \dots m$.*
Then $X(Y \cap K_i) \to X(Y \cap K_j)$ for $i \neq j$.

Proof. We have $(Y \cap K_j) \setminus (Y \cap K_i) \subseteq Y \setminus (Y \cap K_i)$, thus $(Y \cap K_j) \setminus (Y \cap K_i) \subseteq Y \setminus K_i$.
Taking into account of (b), we get $X(Y \cap K_i) \to Y \setminus K_i$.
Consequently $X(Y \cap K_i) \to X(Y \cap K_j) \setminus X(Y \cap K_i)$.
On the other hand $X(Y \cap K_i) \to X(Y \cap K_j) \cap X(Y \cap K_i)$ and

$X(Y \cap K_j) = \{X(Y \cap K_j) \cap X(Y \cap K_i)\} \cup \{X(Y \cap K_j) \setminus X(Y \cap K_i)\}$.
From this we have $X(Y \cap K_i) \to X(Y \cap K_j)$. The proof is complete.

Theorem 4. *Let $X \to\to Y$ be a nontrivial MVD in R, $Y \cap K \neq \emptyset$ and K be any key for R then $X(Y \cap K) \to Y$.*

Proof. We have $X(Y \cap K) \to Y \cap K$. Taking into account of (b) we get $X(Y \cap K) \to Y \setminus K$. Consequently $X(Y \cap K) \to Y$.

Lemma 1. *Let $X \to\to Y|Z$ be a pure MVD in R. Then for any key K: $K \setminus Y \neq \emptyset$, $K \setminus Z \neq \emptyset$.*

Proof. Assume the contrary that $K \setminus Y = \emptyset$, i.e., $K \subseteq Y$. Then from $X \to\to Y$ and $XY \to Z$ we have $X \to Z$; which contradicts the fact that $X \to\to Y|Z$ is a pure MVD. Thus $K \setminus Y \neq \emptyset$. Similarly, we can prove $K \setminus Z \neq \emptyset$.

Theorem 5. *Let $X \to\to Y|Z$ be a pure MVD in R, and K be any key of R, then $|K| \geq 3$.*

Proof. Since $X \to\to Y|Z$ be a pure MVD in R, we have $X \to\to Y$ and $X \to\to Z$. Hence by (a) $K \cap Y \neq \emptyset$ and $K \cap Z \neq \emptyset$. Moreover, by Lemma 1: $K \setminus Y \neq \emptyset$ and $K \setminus Z \neq \emptyset$. Combining with $Y \cap Z = \emptyset$ we have $|K| \geq 3$.

Theorem 6. *Let $X \to\to Y$ be a nontrivial MVD. Then $X \to Y$ iff there exists a key K such that $X \to Y \cap K$.*

Proof. Let $X \to Y$. Then, of course, $X \to Y \cap K$ where K is any key for the relation scheme. Conversely, let K be a key such that $X \to Y \cap K$ (i).
From $X \to Y \cap K$, we have $X \to\to Y \cap K$. Combining with $X(Y \cap K) \to Y \setminus K$ (b), and using the mixed pseudotransitivity axiom, we have $X \to Y \setminus K$ (ii).
From (i) and (ii) we obtain $X \to Y$.

REFERENCES

[1] S. Jajoda, Recognizing multivalued dependencies in Relation Scheme, *Computer Journal* **29** (5) (1986).

[2] Ho Thuan and Le Van Bao, Some results about keys of relational schemes, *Acta Cybernetica*, Tom 7, Fasc 1, Szeged (1985).

[3] J. D. Ullman, *Principles of Database system*, 2nd ed., Computer Science Press, Rockville, Md, 1982.

Tóm tắt. Như đã chỉ rõ trong [1], các phụ thuộc đa trị (MVD) phụ thuộc vào ngữ cảnh trong đó chúng được xác định và do đó rất khó phát hiện. Trong bài báo này, tiếp tục hướng nghiên cứu trong [1], chúng tôi đưa ra một số kết quả làm sáng tỏ sự tồn tại của các MVD trong các lược đồ quan hệ với tất cả các phụ thuộc hàm (FD) đã biết trước.

Received: January 30, 1998

(1) Institute of Information Technology - NCST of Vietnam

(2) Open University of Hanoi

(3) Pedagogical Institute of Hanoi