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A COMMON H_∞-OPTIMAL CONTROLLER FOR A COLLECTION OF MIMO PLANTS

VU NGOC PHAN

Abstract. The purpose of the present paper is to find a common controller for a collection of MIMO plants. The plant models are described in the frequency domain because of their advantages for technological controller designing. Control structure of two degrees of freedom is chosen. The main results are given by Theorem 3.1 and Theorem 3.2 in Section 3.

1. INTRODUCTION

The problem of designing a common controller for a collection of plants belongs to the concept of the robust control theory. This problem has been already studied by many researchers (Youla 1974, Vidyasagar 1985, Olbrot 1987, Toker 1996, Sakin and Petersen 1997). The motivation for this issue could be derived from the fact that a non-linear system can be approximately modeled by a finite set of linear models. Moreover, it is often required due to cost reduction of commercial controllers that a controller may be used to plants of different kind which could have different mathematical models.

To solve the problem above the first question to be answered is whether there exists such a controller for a given collection of plants. Recently, Sakin and Petersen focused their attention on systems described by state equations [2]. In the addressed paper the plant description in the frequency domain will be used because of its facilities for design of technological process controllers.

2. PROBLEM FORMULATION

The following notations are used throughout this paper.

K	number of plants of given collection
$\Re^n$	n-dimensional complex space
$\Re^{m\times n}$	$m\times n$–dimensional complex space
s	complex frequency ($s=\sigma+j\bar{\omega}$)
$D(s)$, $N(s)$, $P(s)$, $Q(s)$	polynomial matrices
$G(s)$, $G_i(s)$, $C(s)$, $R(s)$	transfer function matrices

(The independent variable s of polynomial or transfer function matrices will be skipped if there is no need to avoid any confussion)

I	identity matrix
$\|\ \|_\infty$	H_∞–norm

Now, consider a collection of plants described by

$$y_i(s)=G_i(s)\,u_i(s)\quad \forall i\in K, \tag{2.1}$$

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where $u_i \in \Re^{ni}$ is the input, $y_i \in \Re^{mi}$ is the output and $G_i \in \Re^{mi \times ni}$ is the transfer function matrix of the i^{th} plant. A control law is chosen as below:

$$u_i(s) = -C(s)\, y_i(s) + R(s)\, r_i(s). \tag{2.2}$$

Here, the feed-back $C(s)$ is designed for stabilising the plants, $R(s)$ is a output regulator needed for achieving the control performance and $r_i \in \Re^{mi}$ is the reference input. It is well known that $C(s)$ and $R(s)$ can be designed separately. Thus, the first step of the design problem here is to find a stabiliser $C(s)$ that can stabilise all the plants of the collection. Then, $R(s)$ will be found by solving an H_α-optimal problem. It is to make a note of that there may not always exist a meaningful common stabiliser for all plants. The stability of the closed loop systems requires that the transfer function matrices $[I + G_i(s)\, C(s)]^{-1}\, G_i(s)$ for all $i \in K$ must not have any pole in the right-half plane. We denote

$$[I + G_i(s)\, C(s)]^{-1}\, G_i(s) = S_i(s), \tag{2.3}$$

so $S_i(s)$ are stable matrices. To solve Equation (2.3) we can go the way below. Put

$$S(s) = \begin{bmatrix} S_1(s) & 0 & 0 & \cdots & 0 \\ 0 & S_2(s) & 0 & \cdots & 0 \\ 0 & 0 & S_3(s) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & 0 & 0 & S_k(s) \end{bmatrix} \tag{2.4}$$

$$G(s) = \begin{bmatrix} G_1(s) & 0 & 0 & \cdots & 0 \\ 0 & G_2(s) & 0 & \cdots & 0 \\ 0 & 0 & G_3(s) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & 0 & 0 & G_k(s) \end{bmatrix} \tag{2.5}$$

then Equation (2.3) becomes

$$[I + G(s)\, C(s)]^{-1}\, G(s) = S(s). \tag{2.6}$$

By using the co-prime factorizations to $G(s)$ and $C(s)$, Formula (2.6) gets the form

$$Q[DQ + NP]^{-1} N = S. \tag{2.7}$$

Formula (2.7) may be solved for given $G(s)$ and $S(s)$, but $C(s)$ can have a very high order and is then regardless to the practical use. Control engineering practitioners prefer a low-order controller to a high-order one [1, 2]. In the following section the necessary and sufficient conditions of a low-order common controller for a plants collection will be given based on the well known small gain theorem [3]. It is to note that the order of $C(s)$ must, of course, be not lower than that necessary for the worst case. That means, the order of $C(s)$ must be as high as the highest order of all plants. This requirement is valid also for $R(s)$ being needed to get the tracking performance. Furthermore, the dimension of $C(s)$ and $R(s)$ must be adequate with the dimension of each plant. However, this question will not be answered in the addressed paper.

3. MAIN RESULT

3.1. Stabilisation

For stabilisation design it can be assumed that all plants are excited by the same input signal $u(s)$. Because only the input-output relation of the plants is known, the BIBO stability is taken in account. Suppose

$$y_i(s) = G_i(s)\,u(s)\,. \tag{3.1}$$

Because the plants of the collection are different, the output responses to the same input signal are in generality also different. Denote $\Delta(s)$ the largest output difference of two plants, also

$$\eta(s) = \max_{i,j}\{y_i(s) - y_j(s)\}, \quad i,\,j \in K. \tag{3.2}$$

It follows from (3.1) and (3.2) that

$$\eta(s) = \max_{i,j\in K}\{G_i(s) - G_j(s)\}\,u(s) = \Delta(s)\,u(s). \tag{3.3}$$

Without losing the generality it can be assumed that

$$G_i(s) = G_j(s) + \Delta(s). \tag{3.4}$$

Theorem 3.1. *Assume that $C(s)$ stabilises one of the plants of a given collection. $C(s)$ stabilises all plants of the collection if and only if*

$$\frac{\|\eta(j\omega)\|_\infty}{\|u(j\omega)\|_\infty} \le 1 \quad \forall\omega. \tag{3.5}$$

Proof. According to equations (3.3) and (3.4), $\Delta(s)$ presents an additive uncertainty. For a $C(s)$ stabilising $G_i(s)$ all closed loop systems are stable iff $\|\Delta(j\omega)\|_\infty \le 1\ \forall\omega$ [3].

From (3.4) it follows

$$\|\eta(i\omega)\|_\infty \le \|\Delta(j\omega)\|_\infty \cdot \|u(j\omega)\|_\infty \tag{3.6}$$

and (3.5) is derived directly by (3.6). □

3.2. Tracking performance

In this section $S_i(s)$ designates a stable plant or a stabilised one. It is to be assumed, that the structure of $R(s)$ is priori chosen and only its parameters are to be determined. By using a parameterisation $R(s)$ can be interpreted as $R(p,\,s)$ where $p \in \Omega^\mu$ is the parameter vector determining $R(s)$. The parameter space Ω^μ is characterised here by a lower and upper bound, also

$$\Omega^\mu = \{p\,|\,p_0 \le p \le p_u\}. \tag{3.7}$$

For each plant of the collection the tracking error is determined by

$$e_i(s) = [I + S_i(s)\,R(p,\,s)]^{-1}\,r_i(s). \tag{3.8}$$

In order to assign the parameter values of $R(p)$, all errors $\|e_i\|_\infty$ are to be minimized together. It is well known that

$$\|e_i\|_\infty \leq \|I + S_i\, R(p)\|_\infty \cdot \|r_i\|_\infty. \tag{3.9}$$

Instead of minimising $\|e_i\|$ the right hand side of (3.9) will be taken in consideration. Here it is dealing with a so called polyoptimization problem. Moreover, make a note that only r_i with a power spectral bound is meaningful [4]. Therefore, $\|r_i\|$ $\forall i \in K$ has no effect to the minimum. Now, it is very easy to prove the theorem below.

Theorem 3.2. *Suppose that all plants of the collection are stable or stabilisable. Then, a low-order controller for tracking performance can be found by solving the polyoptimization problem following:*

$$\min_{p \in \Omega^\mu} \|I + S_i\, R(p)\|_\infty \quad \forall i \in K. \tag{3.10}$$

4. CONCLUSION

It is always a large necessity of finding a common controller for a collection of plants. Theorem 3.1 gives the necessary and sufficient existence condition of a common stabiliser. Theorem 3.2 shows how a common output regulator can be received for the tracking performance. One problem having been remained in this paper is how to find the adequate dimension of $C(s)$ and $R(s)$ for all MIMO plants. Obviously, the results of this paper are valid for SISO systems.

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Tóm tắt. Thiết kế bộ điều khiển chung cho một họ các đối tượng MIMO là một trong những vấn đề quan trọng của điều khiển bền vững và đã được nhiều tác giả nghiên cứu. Bài này tập trung giải quyết vấn đề cho các đối tượng MIMO được mô hình hóa trong miền tần số. Việc thiết kế được tiến hành trên cơ sở bộ điều khiển hai bậc tự do. Sau phần mở đầu và phần đặt bài toán, những kết quả chính được phát biểu qua Định lý 3.1 và Định lý 3.2 của phần 3.

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Institute of Information Technology
National Centre for Sciences and Technology of Vietnam