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CONTINUOUS TIME SYSTEM IDENTIFICATION: A SELECTED CRITICAL SURVEY

Part I - GENERAL VIEW, SOME MODELS AND ESTIMATION METHODS

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Abstract. Part I and Part II of the paper refer to a critical survey on significant results available in the literature for identification of systems, linear in the present part and nonlinear in the following one. The most important trends in identification approaches to linear systems are from the development of optimal projection equations, which are argued by the complexity of numerical calculations and of practical applications. The perturbed a quasilinear and on Neuro-Fuzzy trends in representing nonlinear systems, i.e., functional series expansions of Wiener and Volterra, Modeling Robustness and structured numerical estimators are included. The limitations and applicability of the methods are discussed throughout.

1. A GENERAL VIEW

Most systems encountered in the really physical world are continuous and the development of automatic control owes a great deal to the concepts evolved originally in continuous time domain [1, p.3]. Further, all most control systems in practice are nonlinear to some extent. Although, it may be possible to present systems by perturbing over a quasilinear model for a restricted operating range, in general nonlinear dynamic processes can only be adequately characterized by a nonlinear model [2, 3]. However, whereas now through a huge number of reported works available in the literature, system identification techniques for linear systems are well established and have been widely applied, the identification of nonlinear systems has not received so far such attention or exposure. This may be attributed to the fact arising from inherent complexity happened to be in nonlinear dynamic systems, arising from the difficulty in obtaining acceptable identification processes to a large class of nonlinear dynamic systems also [4].

Prior to being into existence of input error concept [5], the identification of a system is known to be *an experimental process* of determining a mathematical model which is capable of describing the essential properties of the system from its input/output information [1, 15]. This process become necessary due to the fact that such a model plays the vital role in the analysis and design for ensuring an efficient operation and control of the system and obviously, the quality of control reaches to optimum only if the properties of the model are matched as close as possible to those of the actual system [1, 15].

Probably, it is known to most of us, there exists no model satisfying exactly all sets of input/output data of the system [1, 10]. This is because of the fact that the system identification problem mathematically belongs to a group of optimization ones [1, 15], of which solution is characterized by a model structure, a criterion of equivalence also. For a particular descriptive model, the equivalent criterion is the core giving rise to different various methodologies for estimation of the model parameters with respect to the choice of performance or cost function. Most cost functions in the identification theory evaluate the minimum deviation of the model from the system using one of the four error equations, namely the output error, equation error, prediction error and the input error equation [1, 5, 12]. That is, a suitable method of optimization is adopted on an error equation for estimating the model parameters [12]. It is clear that process of determining values for the model parameters plays an important role in governing the behavior of the system or in realizing the equivalent criterion [1, 14].

It is, however, found that for carrying out an estimation process successfully with respect to any one of the first three earlier mentioned error equations, the input signal of the system is to be *persistently exciting* [6] (the persistently exciting degree of the input signal is required to be twice system order [15]). As the input signal of system in most of the cases may be of any form and the order of the model for the system may be high, for meeting the said persistence requirement the system is to be subjected to a test signal which theoretically, is a white noise realizable as a pseudo random binary sequence signal [1, 15]. This requirement puts a restriction on real time estimation of model parameters, where the system under consideration is not permitted to be excited by any sort of external signals and the identifiability of the system becomes an uncertainty.

With respect to the identifiability aspect, various promissible approaches have been developed [16 - 18] for systems, to which there are uncountable difficulty in trying to describe by mathematical models. These approaches are on the human brain modeling basics, different from the above optimum methodology. Although, fuzzy methods are found yet to be successfylly applicable to complex high order systems with regard to a closer approximation and to an amount of calculation concern where the number of fuzzy rules is highly required to be set up. However, not only fuzzy methods but components of "soft computing" [18] based methods including neural network and evolutionary computing as the whole, each of them has got various advantages in identifying nonlinear dynamic or uncertainty system or the one where the system knowledge are difficult (in the sense of a fair complex realization) incorporating together all in the form of mathematical expressions [17 - 19]. Therefore, it would be better to discuss on adopting methods components of soft computing based to the identification of nonlinear dynamic systems.

In the case where the system is identifiable, a fair complex structure and high order model may be obtained from the theoretical consideration, the reduction of order for model in such a case becomes an evidently useful measure for a better understanding of and controlling the system [20].

Although several different approaches have been proposed in the literature for obtaining low order models for a given high order model of a system [20, 21], practically all of them belong to one of three main groups. First group of methods attempts to retain the important eigenvalues of the system and then obtains the remaining parameters of the low order model in such a manner that its response to certain inputs is a close approximation to that of the original system [22 - 25]. The second group of reduction methods is based on obtaining a model of a specified order such that its impulse or step response are matched with of the original system in an optimum manner [26 - 28] with no restriction on the location of eigenvalues. The third group of reduction methods is based on matching some other than the properties of the responses [29 - 31].

However, most methods developed so far require the knowledge of either transfer function or parameters of the high order model. This demands the process of parameter estimation for high order model to be performed before considering the model reduction. Consequently, the model reduction faces also the aforesaid restriction on the system excitation [21]. Further, due to an error arising out of the reduction, the response of the reduced model differs from that of the system for the same excitation. This difference makes the results established prior to the use of input error concept for the reduction problem unacceptable in the case where the system is in a closed loop configuration like that in a trajective control, output regulation or in state estimation problem [32 - 35], etc. The reason behind is due to the fact that in such cases a compensation of arisen error is required, leading to a change in the control strategy which may be no longer a linear control law [33, 36].

It has been pointed out that all the first three errors used in the parameter estimation and order reduction problems for linear, parametric, continuous models are referred on the output side of the system [36]. This restrains on considering the identification process to be either a vital stage in the overall task of modern control or linked with analysis of the system properties. The restraints to be removed for the sake of identification are summarized in three points as follows [36].

(i). Persistently exciting property imposed on the system input signal in a problem of parameter estimation.

(ii). Involvement of either parameters or impulse response (i.e., transfer function) of the high order model in a problem of order reduction.

(iii). Expense of existing optimal control to system for the attainment of a specified trajectory from the reduced order model.

It has been found also that for ignoring persistent excitation requirement in a problem of parameter estimation, the measurements of *time derivatives of the system input signal* should not be involved in the parameter estimation process [5]. In order to avoid the process of parameter estimation for high order model before considering order reduction, problem of order reduction can be considered as that of *parameter estimation applicable to a misorder case* [36, 37]. For ensuring the response of the reduced model to be as specified as that of the system, the *match of their output signals* becomes essential condition [36, 38].

From these arguments, the input error concept has been initiated firstly for problem of system parameter estimation then has been further exploited to problem of order reduction for models [36, 37] and to other related aspects of the problem of system identification [36, 39-41].

With regards to the parameter estimation for models described in the state variable space, in the literature there exists quite a few works reported on the basics of model adaptation using stability theory [42, 43]. Intensive works for identifying system described in this description have been reported in [44-50] not only with the use of input error concept but also the state optimization concept [47]. Solutions of various identification problems are expressed in the relevant forms of optimal projection equations (OPEQ), the term has been introduced firstly in [51] for order model reduction. By adopting OPEQ, the parameters for state variable descriptive models have been found to be estimated successfully with any form of the system input signal, without incurring linear operators (LD) on both sides of the system [36, 44]. System identification has been shown to *switch over to solution of OPEQ*, which got simpler forms by adopting the state optimization concept [36, 47]. The developing OPEQ has got some important significances from the view point of uniqueness in optimization process as the effect of different *additional constrained conditions* to the L_2 optimality criterion has been resulted from interpreting the inherent existence of coupling equations in one side, and in the other side different constraint conditions are able to be adopted alongwith [27, 47].

In this first article, it is made a selected review on the recent trends in the identification for linear systems in framework of interest to control and system area without an intention to bibliographic terms. Identification for nonlinear dynamic system and relevant matters are critical reviewed in the coming part of the paper. In this part, system and model are considered to be multi input multi output (MIMO) unless specified whereas. Bold and capital bold letters are used for notations of vectors and matrices respectively. Superscript "+" of a matrix stands for the notation of pseudoinverse of that matrix and $\rho(.)$, $E(.)$ and $R^{n\times m}$ are denoted for rank of a matrix, expectation and $(n \times m)$-real space.

2. LINEAR, CONTINUOUS TIME MODELS AND BASIC IDENTIFICATION PROCEDURES

1.1. Linear, continuous times models

The term "model" is used in general to mean a handy entity representing the actual system. Models are classified mainly in three groups and the first one is of conceptual or phenomenological. The second group is of physical or empirical and the last one is of mathematical or analytical.

It is concerned herewith the basic mathematical models for linear dynamic, continuous time systems, frequently used in control theory, characterizing by lumped parameters either in the ordinary differential equations or in some other equivalent finite parameter presentation, i.e., in the state space description, etc. For the brevity only deterministic models are mentioned for the purpose of this article, while stochastic models are obtained simply by adding the term corresponding to the random input although different model structures and related aspects are arisen from the stochastics. Further, the models considered are of the discrete type only due to the application of digital computer for identification and control purposes. The discretization of parametric models eliminates by approximation inherent the continuous time calculus from continuous time parametric models. Discretization of nonparametric models is mainly for computational convenience purpose, with less serious consequences with respect to parametric models.

Perhaps, the best known lumped parameter model for a linear continuous time system (p inputs and q outputs) is expressed in a system of q equations as:

$$\sum_{i=0}^{n} a_{i,j}(t)\frac{d^i y_j(t)}{dt^i} = \sum_{k=1}^{p}\sum_{i=0}^{n} b_{i,k_j}(t)\frac{d^i u_k(t)}{dt^i}, \quad \text{for } j = 1, \dots, q, \tag{2.1}$$

where n stands for the order of model, $u_k(t)$ and $y_j(t)$ are the respective excitation at the k^{th} input and the response at the j^{th} output of the system, $a_{ij}(t)$ and $b_{i,k}(t)$, for $n \geq 1$, $i = 0, \dots, n$, $k = 1, \dots, p$, are the parameters of the model. These parameters are usually referred to as process parameters [7, 14], which play the vital role in governing the behavior of the actual system.

The linear dynamic system can be equivalently described in a system of first order differential equations in the state variable space:

$$x_n(t) = A_n(t)x_n(t) + B_n(t)u_n(t), \tag{22}$$

$$y_n(t) = C_n(t)x_n(t) + D_n(t)x_n(t), \tag{2.3}$$

where equations (2.2) and (2.3) are referred to as the dynamical and output equations respectively [14, 36]. In these equations, $u_n(t)$, $x_n(t)$ and $y_n(t)$ are p-, n- and q-dimensional vectors of the inputs, state variables and outputs, and $A_n(t)$, $B_n(t)$, $C_n(t)$ and $D_n(T)$ are matrices of the dimensions of $n \times n$, $n \times p$, $q \times n$ and $p \times p$ respectively. A description of the model in this space with minimal number of parameters is called canonical [14]. In such a case, a realization of $\{A_n(t), B_n(t), C_n(t), D_n(T)\}$ is termed minimal and dimension of $A_n(t)$ is order of the minimal $A_n(t)$ [14, 36]. The intention to use in these equations subscript "n" is for indicating the irreducible order of the model to be referred to as a well specified order. Otherwise, the order of the system will not be well specified in either sense, reducible or unknown one [36].

From equations (2.2) and (2.3) above, other representation for the system is also obtained, referring to the observation space of the system. This description is in the form of a matrix consisting of multivariable transfer functions, which is known in the literature as polynomial matrix description. Important point in the polynomial matrix description lies on the usability of time derivative operator $s = d/dt$ and on the hypothetical "noise free output" of the system. However, a combined effect of input and output disturbances can be accounted in a model called "autoregressive moving average" (ARMA) [12, 13]. Further, with some certain assumption made on different coefficient matrices in ARMA model, an autoregressive moving average exogenous variable (ARMAX) model or dynamic adjustment [15] one is obtained.

2.2. Basis identification procedures

In fixing the course and goal of the identification process, it would be better to stress to the fact that the ultimate purpose for which the entire job is being planned. In other words, system identification is merely one of the phase of activity in an integrated effort and therefore should not be treated as an isolated task. In view of frequent mixing of "identification" and "parameter estimation" terms in the literature, it is necessary to clear the meaning of each term.

System identification that may be interpreted as the "inverse" of the system analysis problem, is characterized by a *class of signals*, *a class of models*, and by a *criterion of equivalence*. A particular situation, where some certain amount of a *priori knowledge* in the problem such as structure, time invariant, linear dynamic, etc. the system identification problem reduces to a simpler one, that of parameter estimation. Thus, the problem of parameter estimation may be seen as that of the experimental *determination of the values for parameters* governing the system behavior assuming that the system is modeled by parametric, known structure.

It is legitimate to say that observations and experiments are most favourable means for system understanding. In this connection, measuring data does not mean only measured input/output information but also that of *some auxiliary variables* such as noise [1, 12], state variables [13, 15], the controllability and observability gramians [45, 47], etc.

Fig. 1 below shows a general process of system identification. There exists various procedures for the system identification however. Basic principles behind this variety are concerned with the type of models first (either linear or nonlinear, parametric or nonparametric, distributed or lumped, deterministic or stochastic, etc.), then are related conceptual bases for parameter estimation (i.e., defined error equations) and in fact, are not very much different between methods for continuous models and those for discrete ones [11, 12].

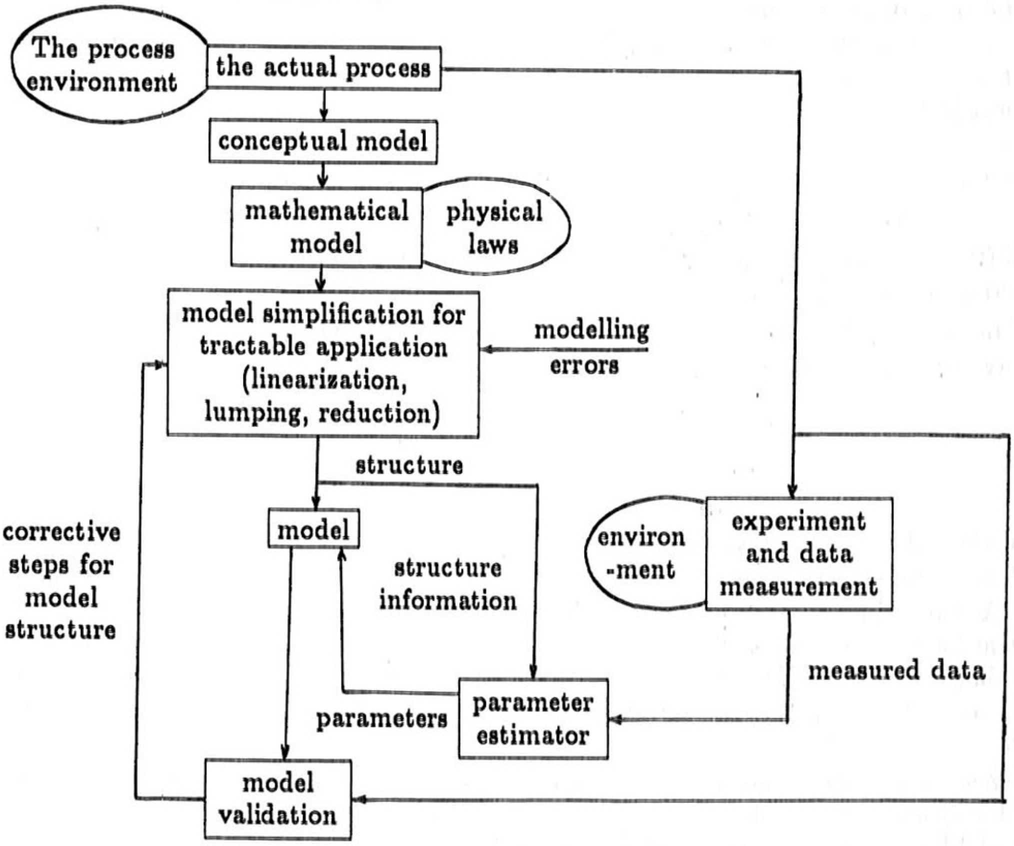

Fig. 1. General process of system identification

There are two basic methods of data processing for an optimality problem. These are the "off line" or batch processing approach and the "on line" or recursive approach. In the batch processing case, *the complete set data* as a whole is used for carrying out computation. In this case, the estimate updating unknown parameter vector is performed by reference to all of the data available over the observation interval. Contrast to this one, in the recursive approach the estimate of unknown parameter vector is basically *continuous updated while computing* serially through the data. The most common recursive algorithm is a gradient algorithm, where the update at any step (k say) of the recursive process is expressed as a function of the instantaneous gradient evaluated utilizing the estimate obtained from the previous recursive step, excepting the first one. An important aspect of data processing is the form of estimation algorithms in relation to the nature of models, i.e., with the use of a continuous time estimation procedure for a continuous time model (referred to as CC scheme) or a discrete time estimation procedure for a discrete time model (DD) or one of hybrid schemes, either CD or DC. The principles of identification and the state of the art with reference to parameter estimation for linear dynamic time invariant lumped continuous models are classified next.

3. CHOICE OF COST FUNCTION AND ESTIMATION METHODS

Most obvious approach to estimating parameter values in a mathematical model of a dynamic system is to minimize a scalar cost (or loss) function J. This cost function is usually formulated in terms of some norm in an error equation vector $e(t)$ which reflects the discrepancy between the model and the real system. The choice of $e(t)$ gives rise to particularities of each estimation methodology [12].

3.1. Choice of cost function

As regards the choice of cost function J for an optimization process, this depends to some extent on the nature of the problem. The most common cost is based on the integral of a weighted L_2 norm in $e(t)$, i.e.:

$$J = \int_{t_0}^{t_f} \{e^T(t) W e(t)\} dt = \int_{t_0}^{t_f} \|e(t)\|_W^2 dt, \tag{3.1}$$

where W is a positive definite weighted matrix of the appropriate dimension, and $(t_f - t_0)$ is the time interval over which the data are available.

In the simplest, scalar case the above integral is reduced to the integral of squared error. The discrete time equivalent of (3.1) cost function is:

$$J = \sum_{i=1}^{N} \|e_i(t)\|_W^2 dt, \tag{3.2}$$

where the subscript "i" indicates the value of the vector $e(t)$ at the i-th sampling instant, and N denotes the number of samples available over the observation interval from t_0 to t_f.

As the reduction of order for a model has been observed to deal with the parameter estimation problem for a misorder case [36, 37], for which cost function is no much different from the above one, excepting the case for reducing order of a system operating in closed loop configuration, where for some certain reasons, it is required to impose several constraint conditions on the optimality process [27, 28].

Also, constraint conditions are required to be exploited in a problem of estimating the parameters for either controllable or observable part or both jointly, i.e., parameters corresponding to the controllable/observable jointly part of the system [44, 45]. Further, different constraint conditions are to be used with an aim at the uniqueness for the solution of system identification problem [44]. With respect to the uniqueness question, OPEQ [44] are found to open an opportunity for imposing as many constraint conditions as possible, in one side and in other side, to provide effects of additional constrained conditions towarding to attainment of the absolute extremum. In such cases, above mentioned cost function, by some mathematical manipulations, becomes:

$$J \leq \operatorname{tr} RQ, \tag{3.3}$$

where "tr" stands for the trace of matrix, R and Q are appropriately dimensioned square matrices corresponding to the weight and data of the system respectively. In most cases, R is also related through matrix C with the system, to which Q is related through matrices A, B' and controllability grammian [59].

It is worthwhile to address herewith that constraint conditions frequently used nowadays in the control and system areas are those for a L_2 bound [27], a preassigned H_∞ bound [27,34], for internal balance [29,32], by principle of cost ranking [30], for the Lyapunov sense with regards to the stability, controllability, observability character of the system [52], of the robustness including the robust modeling [28], etc.

3.2. Estimation methods

As mentioned early, the problem of system identification becomes that of parameters estimation when some amount of a priori knowledge about the system is concentrated in a set of parameters [1,36]. The methods of parameter estimation can be generally seen to be adopted of two steps. First one arises out of avoiding a direct generation of time derivatives of the input/output signals of the system. In this step, the system of equations for parameter estimation is derived from the dynamical model of the system and in an ideal case, the number of equations is equal to that of south parameters. The second step is to deal with the methods of parameter estimation for models. It is obviously noted that the major difference in the treatment between the discrete time and continuous time models arises in the first step, while the second step is applicable to both the cases [1,36].

Most methods of parameter estimation for models are based on the principle of reference technique [10], where an error function is defined for reflecting the discrepancies between the model and the real system. There are available four error equations in the literature and various but related conceptual bases criteria imposed on the error equation for the estimation of model parameters. These are the output error, equation error, prediction error and the input error. The last one is defined on a different concept basics however [5,36], but the first three errors are found some how being defined similarly for some extent [12,36]. In order to distinguish those errors, it is convenient to consider a system described:

$$\sum_{i=0}^{n} a_{i,j}(t)\frac{d^i y_j(t)}{dt^i} = \sum_{k=1}^{p}\sum_{i=0}^{n} b_{i,k_j}(t)\frac{d^i u_k(t)}{dt^i}, \quad \text{for } j = 1,\dots,q. \tag{3.4}$$

By introducing operations:

$$f_j = \sum_{i=0}^{n} a_{i,j}(t)\frac{d^i}{dt^i} \quad \text{and} \quad g_{jk} = \sum_{i=0}^{n} d_{i,k_j}(t)\frac{d^i}{dt^i} \tag{3.5}$$

then one gets:

$$\mathcal{F}y(t) = \mathcal{S}u(t), \tag{3.6}$$

where $y(t) = [y_1(t),\dots,y_q(t)]^T$, $u(t) = [u_1(t),\dots,u_p(t)]^T$, $\mathcal{F}$ is of a diagonal matrix form having components f_j, for $j = 1,\dots,q$, $\mathcal{S}$ is also of a matrix form consisting of g_{jk}, for $j = 1,\dots,q$, $k = 1,\dots,q$. Actually, matrices $\mathcal{F}$ and $\mathcal{S}$ that consist of measurements of the operations on $y(t)$ and $u(t)$ respectively and that are required for the parameter estimation purpose.

When the system has time invariant parameters only, then the Laplace transformation can be used as the operation with $a_{i,j}d/dt^i = a_{i,j}s^i$ and $b_{i,k_j}d/dt^i = b_{i,k_j}s^i$. In such a case, (3.6) becomes:

$$\hat{H}(s)y(t) = \hat{K}(s)u(t), \tag{3.7}$$

where $\hat{H}(s) = \sum_{i=0}^{n} a_i s^i$ and $\hat{K}(s) = \sum_{i=0}^{n} b_i s^i$, in which a_i, b_i, for $i = 0,\dots,n$, are real coefficient matrices of appropriate dimension. Based on this equations, the first three error are readily defined.

Output error methods

Output or response error method is probably the most intuitively obvious approach to the problem of parameter estimation for models. The parameters in this case are chosen in such manner that the minimum of an instantaneous (for the purely deterministic case) or an integral (for the stochastic case) norm in the error between the model output $\frac{\hat{K}(s)}{\hat{H}(s)}u(t) = \hat{y}(t)$ and the actual output $y(t)$ of the system be achieved [36]. That is, error is defined as:

$$e_0(t) = y(t) - \hat{y}(t). \tag{3.8}$$

It is found that much of early research on deterministic output error methods are tied with self adaptive system design or model reference schemes [12] using a CC mechanization. To this CC mechanism for all output methods, there exists one difficulty in regard to the establishment of conditions for the convergence. It is also found that the output error methods have theoretical but less practical significance.

Equation error methods

Equation error is generated directly from the input/output dynamical (in the sense of time derivatives of the input/output) equation of the model as:

$$e_e(t) = \hat{H}(s)\hat{y}(t) - \hat{K}(s)u(t). \tag{3.9}$$

In the integral cost function case, the equation error approach clearly derives from an analogy with static regression analysis and linear least squares estimation. The equation error method has been shown to be closely related to the concept of differential approximation [53] and has also been termed as satisfaction error [10].

An alternative generalized equation error is often defined to avoid the obvious problems that arise from the differentiation of a possibly noisy signal. In this case, the input/output signals are passed through a set of state variable filters. This state filter set simultaneously filters the signals and provides the filtered time derivatives, which replace the exact but unobtainable derivatives in the definition of a modified error [54-56].

For equation error methods many CC and CD have been suggested, utilizing either off-line or on-line least squares algorithms with guarantee of convergence by an arbitrarily rapid rate [7, 12]. Simplest solution to the asymptotic bias problem associated with basic equation error for stochastic case is the instrumental variable (IV) method. Here, the least squares solution has to be modified so that one can include a vector of IV, which is highly correlated with the noise free output of the system but uncorrelated with the noise on the system measurements. By adopting an adaptive prefilter [13] on all measured signals, another IV scheme has been suggested (refined IV method [54]), which can be considered as an adaptive filter and state reconstructor for both continuous and discrete time stochastic systems.

Equation error methods demand some strong assumptions about the nature of noise and more sophisticated IV procedure for obtaining the bias free estimate. However, these methods are not found to work well by adopting equivalent DD for stochastical basics [13, 60].

Prediction error methods

Prediction error is defined as the error between the actual output $y(t)$ of the system and some best prediction of the output given the current estimate of the parameters characterizing the system and noise models. In other words, the best prediction of the output is conditional mean of the observed output given all current and past information on the system. That is, the error is defined as:

$$e_{pe}(t) = \frac{\hat{C}(s)}{\hat{D}(s)}\left[\hat{y}(t) - \frac{K(s)}{H(s)}u(t)\right], \tag{3.10}$$

where $\frac{\hat{C}(s)}{\hat{D}(s)}$ stands for the transfer function of the noise part of a general model. This prediction error within an equation context is obtainable by defining the error directly in a recognizable equation error like form as:

$$e_{pe}(t) = \frac{\hat{C}(s)}{\hat{D}(s)\hat{H}(s)}\left[\hat{H}(s)\hat{y}(t) - \hat{K}(s)u(t)\right]. \tag{3.11}$$

As mentioned in [12], there exists a considerable number of works reported on the merit, demerit of each of the above defined errors. Although all of these errors are centered around the output of the actual system, the prediction error is found to have more practical value, also to need however more complex process for optimization purpose, due to the consideration of noise part of the models.

Different choices of the noise part of a general model gives rise to different model structures [1], each structure needs to use a criterion. Among criteria, the least squares error criterion is used only in an ideal case, where the model has no noise part, i.e., $\frac{\hat{C}(s)}{\hat{D}(s)} = I$. For handling a realistic case, the generalized least squares error or generalized weighted least squares error [41, 57] criterion is used. In the latter case, with the use of transformations on the noise corrupted data, noise free data are obtainable [41]. These transformations can be seen to be equivalent to the introduction of an IV matrix for transfer function of noise part of the model to be unitary. This implies that generalized weighted least squares error criterion gives rise to the IV method [41] to be adopted. In the case of a random variable whose N measurements are available and statistically independent of one from another, the maximum likelihood criterion may be considered [61]. When a priori information on the probability distributions is to be included, as for example, in a recursive solution, where there is a requirement of linking a prior to a posterior probability statements, then the maximum likelihood criterion is to be imposed along with the Bayesian method [1, 12, 15].

However, an aspect which plays a key role in meeting a requirement [6] for a successful estimation, is the input signal design [1, 15]. The parameter estimation problem becomes an experimental job where the system under consideration has to be subjected to some test signals of specified characteristics. This is because of the fact that in the methods employing errors discussed so far, the measurements of time derivatives of the system input signals are directly used, demanding the input to be limited in a class of signals. In other words, the above error methods have been found to have a limitation in applying to the real time parameter estimation for models where an experiment on system like a bioone, a chemical reaction or an ecological process etc., can not be performed by injecting to the system any sorts of external signals.

If the parameter estimation process for high order models is not successfully performed due to the earlier mentioned reasons, it is not possible to tackle problem of order reduction for models by employing any of the methods which demands the knowledge of the model to be reduced before considering the reduction. The idea of considering problem of order reduction for models to be that of parameter estimation for models of unknown order enables one to avoid the involvement of high order models by the use of input/output signals of systems. That is, both the problems of parameter estimation and order reduction for models are viewed on the same angle from the point of system identification problem. However, the problem of order reduction for models should get rid off facing the limitation imposed on the system inputs too.

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