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REVIEW PAPER

AN OVERVIEW AND THE TIME-OPTIMAL CRUISING TRAJECTORY PLANNING

SOMLÓ JÁNOS

Obuda University, Budapest Hungary; somlo@uni-obuda.hu

Abstract. In the practical application of robots, the part processing time has a key role. The part processing time is an idea borrowed from manufacturing technology. Industrial robots usually are made to cover a very wide field of applications. So, their abilities, for example, in providing high speeds are outstanding. In most of the applications the very high speed applications are not used. The reasons are: technological (physical), organizational, etc., even psychological. Nevertheless, it is reasonable to know the robot's abilities. In this paper, a method which intends to provide the motion in every point of the path with possible maximum velocity is described. In fact, the path is divided to transient and cruising parts and the maximum velocities are required only for the latter. The given motion is called "Time-optimal cruising motion". Using the parametric method of motion planning, the equations for determining the motions are given. Not only the translation motions of tool-center points, but also the orientation motions of tools are discussed. The time-optimal cruising motion planning is also possible for free paths (PTP motions). A general approach to this problem is proposed too.

Keywords. Robot motion planning, path planning, trajectory planning, parametric method, path length, time-optimal, cruising motions, translation of tool-center points, orientation changes of tools, PTP motions, free paths

1. INTRODUCTION

Robot motions may be described by the Lagrange's equation

$$\mathbf{H}(\mathbf{q})\,\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \tau \tag{1}$$

where $\mathbf{H}(\mathbf{q})$ is the inertia matrix of the robot, the quantity $\mathbf{q}$ is the vector of joint displacements: $\mathbf{q} = (q_1, q_2, ..., q_n)^T$

The components of the joint displacement of the joint coordinates, $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}, t)$ is the nonlinear term containing Centrifugal, Coriolis, gravitational forces, frictions and also the external forces affecting the robot joints (including the forces (moments) acting at the end-effectors, too), τ is the vector of joint torques. The components of τ torques (force, torque restricted by the torque characteristics of the driving motors. The vector of joint accelerations is: $\ddot{\mathbf{q}} = (\ddot{q}_1, \ddot{q}_2, ..., \ddot{q}_n)^T$

The $\dot{\mathbf{q}} = (\dot{q}_1, \dot{q}_2 \ldots \dot{q}_n)^T$ components of the joint velocity vector are constrained by the possible maximum number of rotations (in time units) of the motors. As it is well known, the maximums of torques (forces) are the decisive factors to determine optimal (dynamical) processes. As it will be clear from what is detailed below, the constraints of joint velocities determine the time-optimal cruising motions.

Formulating an optimization problem (for example: to move the robot end-effector center-point from one point to another in the space in minimum time), it can be solved by using the mathematical theory of optimal processes: the Pontriagin's maximum principle, or Dynamic programming of R. Bellman, or by other methods.

It is back to the Lagrange's equation. In the extended form it is

$$u_i = \sum_{j=1}^{n} I_{ij}\ddot{q}_j + \sum_{j=1}^{n}\sum_{k=1}^{n} C_{ijk}\dot{q}_j\dot{q}_k + \sum_{j=1}^{n} R_{ij}\dot{q}_j + g_i \tag{2}$$

$i = 1, 2, \ldots, n;\ u_{i\,\min} \leqslant u_i \leqslant u_{i\,\max}$

In (2):

- I_{ij} are the components of the inertia matrix,
- C_{ijk} are coefficients for the Coriolis and Centrifugal forces. (These terms are (usually) also nonlinear functions of joint displacements)
- R_{ij} is the viscous damping coefficient
- $g_i$is the gravitational term,
- $u_i$is the force or torque given by the actuator of the i-th joint.

In (2) the external forces are not indicated (when needed, they can be included). It is not indicated in the above equations either that the components of joint velocity vectors are constrained, too.

Solving the optimal control problem, one may have the solution in the form

$$\mathbf{u} = \tau = \mathbf{u_{opt}}(\dot{\mathbf{q}}, \mathbf{q}, t). \tag{3}$$

It can be realized in the computed torque manner. But in control practice it is desirable to solve the synthesis problem and generate the control signals depending on the error signals. The error signals are: $\varepsilon_i = q_{id} - q_i i$=1,2,......$n$. The q_{id} signals are the desired values (functions) of the joint coordinates. Their derivatives are: $\dot{\varepsilon}_i, \ddot{\varepsilon}_i$ etc.

Looking at Equation (2) (having in mind that the coefficients are also highly nonlinear) it can be imagined that to solve optimization task is not an easy task. But if the nonlinear effects can be neglected, in principle, for individual robot arms, the well-known optimal "bang-bang" control principles could be applied. As far as we know, it is not very frequently applied in robotics. The reason is: **a robot is not an artillery gun, or a spacecraft, or any similar**. The effort to solve the optimization problems is too high comparing the benefits.

Following this introduction, in the second part of the paper the motion planning problems will be specified and analyzed. Also a state-of-the-art summarization will be provided. The third part outlines the basic results concerning time-optimal cruising motions. In this part the basics of parametric method of motion planning are given, too. In part 4, time-optimal PTP motion is analyzed and solution method presented. In part 5, realization aspects are outlined. In part 6 trajectory tailoring methods will be outlined. In part 7 some conclusions will be given.

2. ROBOT MOTION PLANNING

Now, let us return to the rather exact formulation of the robot motion planning problems. The following tasks should be solved:

- **Path planning**
- **Trajectory planning**
- **Trajectory tracking**

2.1. Paths planning

Given a robot and its environment, the task is to plan a path which results in a transition of the end-effector center point:

a) from one position to another position;

b) through a series of positions;

c) along a continuous path.

During these actions, it may also be required that the orientation of the grippers, or working tools attached to the end-effector should have the given values. Sometimes, the path planning can be approached as a pure geometry problem, but in many cases, the path, trajectory planning and tracking problem are deeply interconnected. In cases when these levels can be considered separately, the optimization problems, with geometric criteria, can be formulated for path planning. For example, the goal may be to get the shortest path to walk over a series of points, or avoid obstacles, or avoid obstacles by volumetric bodies, etc. The powerful apparatus of computational geometry can be used to great extent to solve these problems.

2.2. Trajectory planning

Given a path to be followed by the working point (end-effector center-point) of a robot, and the corresponding orientations of tools attached to it. The dynamic characteristics of robot joints are known, including the constraints on torques, forces available at the actuators. The limit values of the joint speeds, the limit values of speeds in Cartesian coordinate system are also given. Possibly, the same is given for accelerations. Complex knowledge is available about the technological process characteristics (requirements, forces, etc.). The most general and practical requirement is to find the motion, giving minimum time for performing the task. Another goal may be to find the motion requiring minimum energy.

2.3. Trajectory tracking

The task of the trajectory tracking, as it was mentioned above, is to plan the control action that guarantees the realization of the desired trajectories with the necessary accuracies.

2.4. Optimal trajectory planning. State-of-the-art

In the introduction, the minimum time motions are reviewed. Bellow, more details is given.

The time optimal control problems can be classified into three categories:

1. Motion on constrained path between two endpoints;
2. Motion in free workspace between two endpoints;
3. Motion in a free workspace containing obstacles.

Concerning the robot motion in free workspace, a number of results are available. In Geering, Guzzella, Hepner, Onder (1986) [3], it has been shown that the time optimal controls of motion in free workspace, are regularly that of switching nature. The maximum torques (forces) are switched for accelerating and decelerating in an appropriate manner. A huge number of papers were dealing with different aspects of the above problem. An overview can be found in S. K. Singh (1991). In Singh's paper a general numerical method to the solution of similar optimization tasks was proposed, too. Discretion and the use of nonlinear programming method form the essence of this approach.

In many of the application problems the motion is constrained to a given path. Examples include arc welding, milling, grinding, painting, debarring using robots.

Several researchers have addressed the problem of this constrained motion of robots. Recently, it has turned out that the parametric description of the robot motion is one of the most promising ways of the investigation of constrained motion. The most detailed outline of this method can be found in K. G. Shin, N.D.McKay (1991) [7].

When using the parametric method, the differential equations characterizing the motion of the joints of an n-degree of freedom robot can be transformed to a form where instead of n joint coordinates $(q_1, q_2, \ldots, q_n)$ only the one path parameter $(\lambda(t))$ is present. The n non-linear, coupled (second order) differential equation of joint motion is transformed to a second order nonlinear differential equation formulated for one parameter. Shin, McKay, and others, based on the parametric description, proposed an approach to the solution of the time-optimal control of constrained motion of robots. Shin and McKay also used the parametric description method to the determination of other than the time-optimal motion. An example is the solution of optimal control problem using minimum energy criterion.

When using the method of parametric description, usually, the parameter is the length $(\lambda(t))$ along the path. In the present paper this approach will be used to a high extent, with the goal of investigating cruising motion rather than investigating dynamics.

In Paragraph 3 of the paper, some introduction to this method is given. For those who are unfamiliar with the parametric method, they should be asked (among other literature) to read the third Paragraph.

J. Podurajev and J. Somlo (1993) [8] used a parameter the time derivative of which is proportional to the square root of the entire kinetic energy of the robot mechanism. Using this parameter, the equation of motion becomes extremely simple. This approach made possible to develop optimal robot control, according to energy criterion in a straightforward manner.

Dynamic optimization problem may be solved using the parametric description of the dynamics of robots (see: Somló, Lantos, P.T. Cat [2]). Then Equation (2) may be transformed to

$$u_i = M_i \dot{\mu} + N_i \mu^2 + R_i \mu + S_i, \qquad i = 1, 2, ..., n \tag{4}$$

where $\mu = \frac{d\lambda}{dt} = \dot{\lambda}, \quad \dot{\mu} = \frac{d\mu}{dt} = \ddot{\lambda}$

Taking into account Equation (2) results in

$$\begin{aligned} M_i &= M_i(\lambda) = \sum_{j=1}^{n} I_{ij} \frac{df_j}{d\lambda} \\ N_i &= N_i(\lambda) = \sum_{j=1}^{n} I_{ij} \frac{d^2 f_j}{d\lambda^2} + \sum_{j=1}^{n} \sum_{k=1}^{n} C_{ijk} \frac{df_i}{d\lambda} \frac{df_k}{\lambda} \\ R_i &= R_i(\lambda) = \sum_{j=1}^{n} R_{ij} \frac{df_j}{d\lambda} \\ S_i &= S_i(\lambda) = g_i \end{aligned} \tag{5}$$

The above relations are obtained taking into attention that

$$\dot{q}_j = \frac{df_j}{d\lambda} \frac{d\lambda}{dt} = \frac{df_j}{d\lambda} \dot{\lambda} = \frac{df_j}{d\lambda} \mu$$

and $\ddot{q}_j = \frac{d^2 f_j}{d\lambda^2} \mu^2 + \frac{df_j}{d\lambda} \dot{\mu}$

Having the system differential Equation in the form (2) or (5) the processes in the system may be investigated. The constraints of the quantities have decisive effects on the performances. The optimal systems theories devote special attention to the limit values of the torques (forces) which determine the best values of performance characteristics (motion times, energy consumptions, etc.). **Our opinion is that in robotics the constraints of joint velocities have a role much more important than theirs when considered. Furthermore, these effects may be fully investigated during the more complicated optimization approaches. So, the investigation and use of velocity constrained motions have a bright future in robot motion planning**.

The above demonstrated parametric equation of dynamics of robot motion was used in Somló, Podurajev (1993), [11] (see also: Somló, Lantos, P.T. Cat (1997), [2]) for the determination of time-optimal motions. It was also shown that the motion with minimum energy consumption may also be solved.

Recently, the parametric equation of robot motion dynamics was applied to develop effective approaches to the determination of optimal robot motions. By Verscheure, Demeulenaere, Swevers, Schutter, Diehl (2008) [16] a complex optimization criterion was proposed. This contains components representing the time, energy, etc. aspects. Detailed investigation of the state-of-the-art was also given in the above paper (38 items for reference). Freely available computer program is provided by the authors (Verscheure "Time-optimal trajectory planning…" [Online]…) [17], too.

In the paper, the **time-optimal cruising trajectory planning** problem is introduced and its basic relations presented. Shortly, the cruising motion is described when a robot end-effector performs some application tasks and during that moves with velocity slowly changing absolute value. These changes are so slow that the times of transient motions from one state to another may be neglected. That is the motion times, which may be estimated by the sum of times of motions of small constant velocity sections. It seems that the above regime is very close to the working processes for the most of the industrial robots. Of course, the validity should be investigated. For that the theoretical apparatus (as it was mentioned) exists. Practical experience and measurements may also be clear whether the planning assumption is valid or not.

Later, in this paper, it is shown that *a cruising motion is time-optimal when at least one of the joint velocity values is at its limit value.*

In Figure 1, transient and cruising motions are shown together. Of course, on the cruising part shown in the Figure sections may exist which can be considered as transients. Taking into account that the above ideas (namely: cruising and transient) are provisional the exploitation characteristics have the importance.

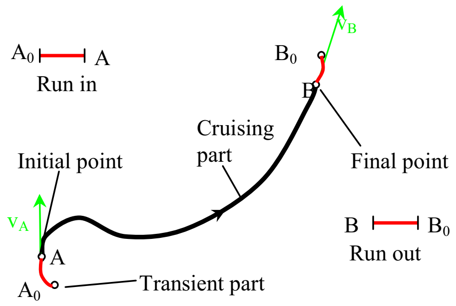

Figure 1: The cruising and transient parts of a path

In J. Somló and J. Poduraiev (1993) [9], as it was mentioned, a method is presented where, using the parametric method, it is solved that both acceleration and deceleration of the robot motion are at their limit values in the transient motions (see also Somló J., Lantos B., P.T. Cat (1997) [2]).

The proposals for trajectory planning in technical literature are based on rather simple approaches. Below, the method proposed in K.S. Fu, R.C. Gonzalez, C.S.G. Lee (1987) [10] is discussed. (Similar approach is outlined in L. Sciavicco and B. Siciliano [11].)

In this approach, the coordinates of a series of points in Cartesian coordinate system are given. The corresponding joint coordinates values are determined by inverse transformation. If the joint positions, speeds and possibly accelerations (deceleration) are known in the given points (and, also, the desired time of motion from point to point), the paths for joints satisfying the given conditions can be determined using proper-order splines.

For example, if in two points the $q_i(t_i), \dot{q}_i(t_i), q_i(t_{i+1}), \dot{q}_i(t_{i+1})$ joint coordinates and speed values are given, a third-order spline

$$q_i(t) = a_{0i} + a_{1i}t + a_{2i}t^2 + a_{3i}t^3$$

may be used for path determination of the motion. Because

$$\dot{q}_i(t) = a_{1i} + 2a_{2i}t + 3a_{3i}t^2;$$

the a_{0i} , a_{1i}, a_{2i}, a_{3i} parameter values can be determined from the 4 equations obtained at $t = t_i$ and $t = t_{i+1}$. When the accelerations are also specified, fifth-order spline with six adjustable parameters can be used. This method may also be used if N points are specified along the paths (see [13])..

These approaches are rather simple, but need some justification. Sometimes it may turn out that the trapezoidal speed profile is the adequately solution. This can be the case, for example, when the technological process constrains the speed. But, it can turn out only after the motion features are analyzed in detail. Indeed, in general, the speeds, the time of motion between the points, the acceleration, deceleration relations are unknown. In fact (see later) these quantities (among other factors) depend on the configuration of the paths. So, the proper order of investigations is to try to determine, in a systematic way, the above quantities, and then goes on with the solution of planning problems. The time-optimal cruising trajectory planning method outlined below solves these problems.

3. TIME-OPTIMAL CRUISING TRAJECTORY PLANNING

3.1. Motion on a given path

First, the case is analyzed when the path to move on is given. Assumed that the acceleration, deceleration abilities of the robot are so high, and that the transient motion part (see Figure 1) may be neglected. This condition, usually, is valid for most of the industrial robots and applications. In the next paragraph of the paper, one of the simplest cases to demonstrate the basic ideas is under discussion.

3.1.1. Time-optimal cruising motion planning for polar manipulator

In Figure 2, a 3-degrees of freedom cylindrical robot are shown. In Figure 3 the rotational and horizontal translation degrees are given. This mechanism is named as polar manipulator.

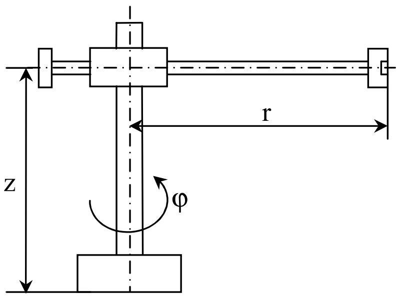

Figure 2: Cylindrical robot

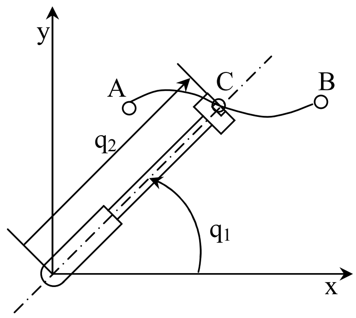

Figure 3: Polar manipulator

For time-optimal cruising trajectory planning, a general method in [2, 11] is proposed and its basic idea outlined below. The authors want to move the end-effector working point (point C) from point A to point B along the path indicated in the Figure.

The equations of the direct geometry are:

$$\begin{aligned} x =& q_2 \cdot \cos q_1, \\ y =& q_2 \cdot \sin q_1 \end{aligned} \tag{6}$$

To realize the motion, the equations of the inverse geometry are needful. These are:

$$\begin{aligned} q_1 =& arc\ \tan \frac{y}{x} \\ q_2 =& \sqrt{x^2 + y^2} \end{aligned} \tag{7}$$

At any given point, the velocity is directed tangentially to the path. The absolute value of the velocity is determined by the components given by the joints. Namely, the rotational joint results in a component

$$|v|_1 = q_2 \dot{q}_1 \tag{8}$$

The translation joint motion results in a component

$$|v|_2 = \dot{q}_2 \tag{9}$$

The absolute value of the velocity is

$$|v| = \sqrt{|v|_1^2 + |v|_2^2} \tag{10}$$

Now, let us try to determine the possible maximum absolute value of velocity. Let $\dot{q}_{1\,\max}$and $\dot{q}_{2\,\max}$be the maximum value of joint velocities. Clearly, in order to increase the absolute value of the velocity, the joint velocities should increase. Let us consider the case demonstrated in Figure 4.

Increasing $\dot{q}_2$ to his maximum value $\dot{q}_{2\,\max}$

$$|v|_{2\,\max} = \sqrt{(q_2\dot{q}_1)^2 + (\dot{q}_{2\,\max})^2} \tag{11}$$

The components of the velocity vector are interconnected because the motion should be directed along the tangent to the path. So, to get $|v|_{2\,\max}$ is very easy as demonstrated on Figure 4.

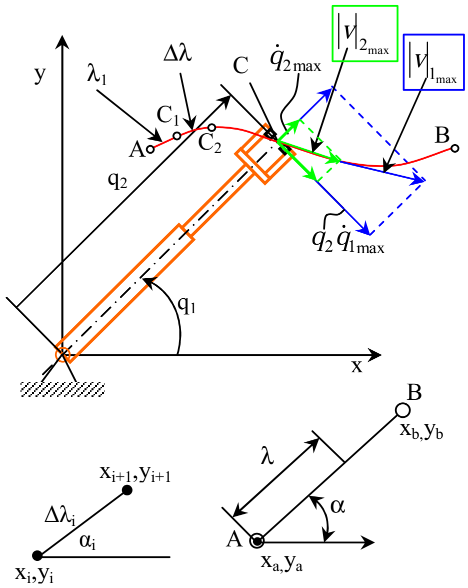

Figure 4: Time-optimal motion

(If analytical form is required, the following relation can be used to determine the required quantities

$$\frac{|v|_1}{|v|_2} = |\tan(\alpha_c - q_1)| \tag{12}$$

Here α_c is the angle of the path relating to axis x (see, α on Figure 4). It will be shown below that this step is unnecessary to perform.)

Increasing the absolute value of velocity more the maximum of other component may be reached

$$|v|_{1\,\max} = q_2 q_{1\,\max} \tag{13}$$

It is clear that this value may not be realized because at that the other component would exceed its limit value.

So, the optimal velocity value is

$$|v|_{opt} = Min\,[|v|_{1\,\max}\,; |v|_{2\,\max}] \tag{14}$$

In the given case this is $|v|_{2\,\max}$.

Let us now try to get an analytical expression for the realization of the above.

By the derivation of (6) we get

$$\dot{x} = -q_2\,[\sin(q_1)]\,\dot{q}_1 + [\cos(q_1)]\,\dot{q}_2 \text{ and } \dot{y} = q_2\,[\cos(q_1)]\,\dot{q}_1 + [\sin(q_1)]\,\dot{q}_2 \tag{15}$$

Solving (15) for $\dot{q}_1$ and $\dot{q}_2$ yields

$$\dot{q}_1 = \frac{x\dot{x} + y\dot{y}}{\sqrt{x^2 + y^2}} \text{ and } \dot{q}_2 = \frac{x\dot{y} - y\dot{x}}{x^2 + y^2} \tag{16}$$

Let in a point (for example, in C) world coordinates of which are x_i and y_i the angle of the tangent of the path with axis x be α_i (earlier we used for the same α_c). Then

$$\dot{x} = |v|\cos\alpha_i \text{ and } \dot{y} = |v|\sin\alpha_i \tag{17}$$

Substituting the quantities into (16) results in

$$\dot{q}_1 = S_1(x_i, y_i, \alpha_i)\,|v| \;\text{ and } \dot{q}_2 = S_2(x_i, y_i, \alpha_i)\,|v| \tag{18}$$

where S_1 and S_2 are the quantities obtained by the substitutions. So

$$\dot{q}_1 = S_1(x_i, y_i, \alpha_i)\,|v| \;\text{ and } \dot{q}_2 = S_2(x_i, y_i, \alpha_i)\,|v| \tag{19}$$

And for the maximum values we get

$$\dot{q}_{1\,\max} = S_1(...)\,|v|_{1\,\max} \text{ and } \dot{q}_{2\,\max} = S_2(...)\,|v|_{2\,\max} \tag{20}$$

Accordingly,

$$|v|_{1\,\max} = \frac{\dot{q}_{1\,\max}}{S_1(....)} \text{ and } |v|_{2\,\max} = \frac{\dot{q}_{2\,\max}}{S_2(....)} \tag{21}$$

For the determination of the optimal velocity value Equation (14) is valid.

According to the above, in every point of the paths the time-optimal cruising velocity can be determined. This velocity depends on the geometry of the paths (world coordinates of the points and direction of the tangents to the paths) and on the maximum possible values of the joint velocities.

We name ***dominant*** the joint the maximum velocity of which determines the optimum. The dominancy may change in some points. In these points both joints result in the same maximum velocity (for the given example in these points $|v|_{1\,\max}$ and $|v|_{2\,\max}$ are equal).

In fact, Equations (15) and (16) are the rows of the Jacobian matrices. So, all the above can be interpreted from the point of view of using Jacobian matrices. This has been done in Somló, Lantos, P.T.Cat [2]. In this case, it is to use

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{x})\dot{\mathbf{q}}, \quad \mathbf{x} = (x, y)^T, \quad \dot{\mathbf{q}} = \mathbf{J^{-1}}(\mathbf{q})\dot{\mathbf{x}}, \text{ and } \mathbf{q} = (q_1, q_2)^T \tag{22}$$

where $\mathbf{J}$ and $\mathbf{J}^{-1}$ are the Jacobian and inverse Jacobian matrices respectively.

3.1.2. Parametric method of trajectory planning

As mentioned above, it is proposed to solve the time-optimal cruising trajectory planning for the whole path, but how doing that is not outlined in a systematic way. In the followings, this task is trying to be settled.

In the introduction of the paper the differential equations of the robot motions are described using a parametric approach. The following introduces this approach.

Shin and McKay in [7, 8] proposed to use a parametric approach for robot planning problem. They proposed as a parameter the path length. It is clear that at any form of description of any path the world coordinates may easily be expressed as functions of path lengths. Indeed, the distance of any two points in a plane or in 3D space may be easily computed. Representing the world coordinates of a robot as a function of the determined path lengths, the parametric description problem is solved. The inverse transformations connect the joint coordinate values with the world coordinates values. So, if the last ones are expressed by the parameter, it leads straight to the opportunity to express also the joint coordinates as functions of the parameter. This is the essence of the parametric approach. In fact, the parameter is an independent variable for the planning.

Just back to the above analyzed example, let the world coordinates in a point of the path be x_i and y_i Let in the next (nearest) point of the path the coordinates be x_{i+1} and y_{i+1}. The distance between the two points is

$$\Delta\lambda_{i+1} = \sqrt{(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2}; \tag{23}$$

$$\alpha_i = \arctan\frac{y_{i+1}-y_i}{x_{i+1}-x_i} \tag{24}$$

For the point with index $(i-1)$ we have

$$\Delta\lambda_i = \sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2} \tag{25}$$

Let us assign to every point which we consider a serial number. This numbers are: i=1,2, ...N, and are also used as indices of the points. The last index value is N. The length of the path until point with index k is indicated asλ_k. Clearly λ_1=0. For others

$$\lambda_k = \sum_{i=2}^{k}\Delta\lambda_i \tag{26}$$

In the last point of the path $k = N$ and $\lambda_N$is the overall length of the path for which we will simply use λ. In any point of the path the λ_k value can be computed. λ_k belongs to the x_k and y_k world coordinates values. By inverse transformation the q_{1k} and q_{2k} values may be computed. In such a way the following functions may be determined:

$$q_1 = f_1(\lambda)\ \mathbf{and} q_2 = f_2(\lambda) \tag{27}$$

Of course, in this way the functions determined on discrete points are obtained. This is very much suitable for robot control tasks where, usually, similar representations are used. But, if necessary, sometimes, analytical relations may also be developed. The above example, and motion along a straight line are under consideration (from A to B; Figure 4).

In this case $x = x_A + \lambda\cdot\cos(\alpha)$ and $y = y_A + \lambda\cdot\sin(\alpha)$; $arc\tan\frac{y_A+\lambda\cos\alpha}{x_A+\lambda\sin\alpha}$; and $q_2 = f_2(\lambda) = \sqrt{(x_A+\lambda\cos\alpha)^2+(y_A+\lambda\sin\alpha)^2}$;

Assuming $\lambda = k \cdot \Delta\lambda$ $(k = 1, 2, \ldots, N)$, and returning to the discrete representation It is remarked that in this way (if necessary) sub-division of interpolation sections of paths is possible.

Now, the discrete velocity values are analyzed in the selected points of the path.

$$\frac{\Delta q_1}{\Delta t_k} = \frac{\Delta f_1}{\Delta t_k} = \frac{f_1(\lambda_k + \Delta\lambda_k) - f_1(\lambda_k)}{\Delta\lambda_k}\frac{\Delta\lambda_k}{\Delta t_k} \textbf{ and } \frac{\Delta q_2}{\Delta t_k} = \frac{\Delta f_2}{\Delta t_k} = \frac{f_2(\lambda_k + \Delta\lambda_k) - f_2(\lambda_k)}{\Delta\lambda_k}\frac{\Delta\lambda_k}{\Delta t_k} \tag{28}$$

It can be recognized that the velocity values may not exceed their limit values. So,

$$\dot{q}_{j\max} = \left(\frac{\Delta f_j}{\Delta t}\right)_{\max}; \quad (j = 1, 2) \tag{29}$$

$$\frac{\Delta f_j}{\Delta\lambda} = \frac{f_j(\lambda + \Delta\lambda) - f_j(\lambda)}{\Delta\lambda}; \quad (j = 1, 2) \tag{30}$$

$$|v|_{j\max} = \left(\frac{\Delta\lambda}{\Delta t}\right)_{\max}; \quad (j = 1, 2) \tag{31}$$

Based on all above

$$|v|_{j\max} = \frac{\dot{q}_{j\max}}{\frac{\Delta f_j}{\Delta\lambda}}; \quad (j = 1, 2) \tag{32}$$

Then,

$$|v|_{opt} = Min\left\{|v|_{j\max}\right\}; \quad (j = 1, 2) \tag{33}$$

It is easy to recognize that if for a 3D problem, the use the parametric representation results in then

$$q_i = f_i(\lambda); \quad (i = 1, 2, 3) \tag{34}$$

Using the discrete representations of the functions the velocity values of the joint coordinates may be obtained as

$$\frac{\Delta q_i}{\Delta t_i} = \frac{\Delta q_i}{\Delta\lambda}\frac{\Delta\lambda}{\Delta t_i}\ldots; \quad (i = 1, 2, 3) \tag{35}$$

$$\left(\frac{\Delta\lambda}{\Delta t}\right)_{i\max} = |v|_{i\max} = \frac{\dot{q}_{i\max}}{\frac{\Delta q_i}{\Delta\lambda}}; \quad (i = 1, 2, 3) \tag{36}$$

$$|v|_{opt} = Min\left\{|v|_{i\max}\right\}; \quad (i = 1, 2, 3) \tag{37}$$

Returning to the above, by determining the $|v| = \dot{\lambda}(\lambda)$ function for the whole path, in fact, the trajectory planning problem is solved. A graphic of a curve obtained for some planning tasks is demonstrated in Figure 6. This curve in itself is very much meaningful. At any λvalue the velocity along the path may not be higher than the corresponding $\dot{\lambda}$on the curve. On the contrary, any value below the curve may be applied. If a constant velocity along the path, is required for the whole path it may not be higher than$\dot{\lambda}_{\min}$. In some cases, different constant velocities along the path sections may be required. These may be constructed using the curve.

So, having the limit velocity curve different other permissible velocity curves may be generated. This generation is named as **"velocity characteristics tailoring".** The velocity tailoring problem will be mentioned later.

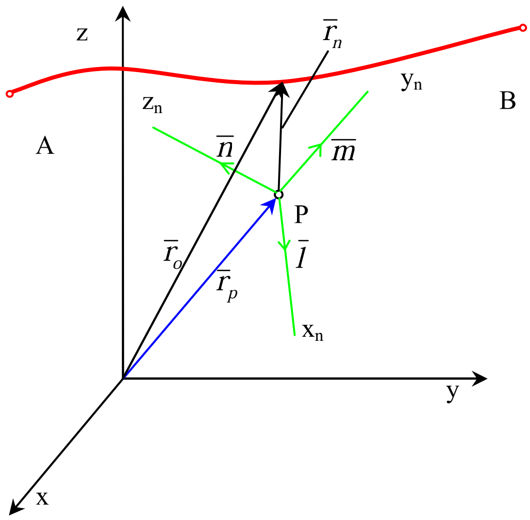

Figure 5: Changes of positions and orientations

One of the most important facts is that using the planning result, the path length motion time relation can be established. Indeed,

$$\frac{d\lambda}{dt} = |v(\lambda)| \tag{38}$$

So

$$t(\lambda) = \int_0^\lambda \frac{d\lambda}{|v(\lambda)|} \tag{39}$$

Of course, in practical robotics the (39) "integration" is performed by summation. In the use of the Equation (39) the time values belonging to any λcan be determined and so the

$$t(\lambda) = \int_0^\lambda \frac{d\lambda}{|v(\lambda)|} \tag{40}$$

$$q_i = g_i(t)....(i = 1, 2, 3, ...) \tag{41}$$

input signals for the drives may be determined which realize time-optimal cruising motion.

3.1.3. The changes of position and orientation together

In the previous paragraph only the translational motion of the end-effector central point is considered. It is well known that together with the translation motions, depending on the path, on the robot construction and parameters, the orientation also changes. Requirements are formulated, very frequently, not only for translation, but also for orientation motion. The proposed method used in this case remains unchanged.

The synchronized translation and orientation changes can be interpreted as shown in Figure 7.

The tool-center point motion is characterized by vector **RP**. The tool-frame x_n, y_n, z_n is determined by its orthogonal unit vectors **l**, **m**, **n**. As it is well known there are only 3 independent values

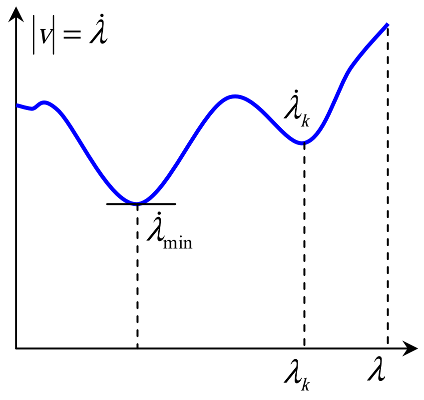

Figure 6: A $|v| = \dot{\lambda}(\lambda)$ function

among the components of these unit vectors. So, if

$$\begin{aligned} \mathbf{l} &= (l_x,\ l_y,\ l_z)^T \\ \mathbf{m} &= (m_x,\ m_y,\ m_z)^T \\ \mathbf{n} &= (n_x,\ n_y,\ n_z)^T \end{aligned} \tag{42}$$

Then, selecting any 3 independent components the x_4, x_5, x_6 orientation (world) coordinates can be introduced. Consequently, the $x = x_1$, $y = x_2$, $z = x_3$ are also introduced. Solving the path planning problem, which now can be interpreted in spaces x_1, x_2, x_3 and x_4, x_5, x_6, the trajectory planning problem can be solved. For that the parametric method is used.

The parametric method of planning yields:

$$q_4 = f_4(\lambda), \quad q_5 = f_5(\lambda), \text{ and } q_6 = f_6(\lambda) \tag{43}$$

It is remarked that, usually, in the equations of the inverse geometry, for q_4, q_5, q_6 the values of q_1, q_2, q_3 are involved. It does not give any difficulty when using the parametric method.

All the above is a clear indication for the fact that the time-optimal trajectory planning method and equations stay valid. That is Equations (36) and (37) results time-optimal cruising motion with the only change

$$i = 1,\ 2,\ 3,\ 4,\ 5,\ 6. \tag{44}$$

It is remarked that when 5D application problems are solved by the robots the orientation space becomes a plane. And that while for $i = 1, 2, 3$ the λ value is the real path length, for $i = 4, 5, 6$ the λ parameter value does not have any real physical meaning.

The velocity characteristics tailoring

On Figure 6 a velocity characteristic is shown. That is a real existing constraint for velocities in the given point of the path. Velocity "higher" the curve may not be realized. In fact, even the time-optimal control will result in curves (slightly) bellow this curve because of the transient processes. But finite many curves bellow the border may be applied. Examples are:

Reserves are implied to the limit velocities of joints (for example:

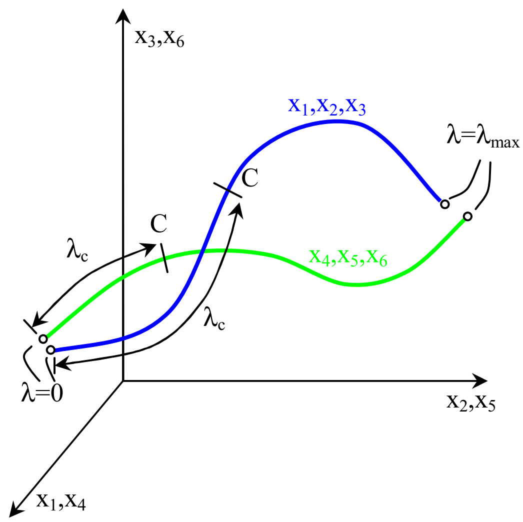

Figure 7: Translation and orientation paths

- decrease with the same or different percent). The benefit of this is straightforward. For example: decrease of velocity errors, more torque at technological applications, etc.
- The λ axis may be divided into a number of sections. Every section may have a constant velocity value below the limit curve. These constant velocities are applied in the given sections. It may be estimated which is the motion time increasing at any variants. Estimating the results reasonable solutions may be obtained. The less the number of sections is, the easier the program will be realized, etc.
- The time-optimal cruising trajectory planning method may be developed to take into attention rather complicated aspects. One field with most importance is to take into attention the process planning aspects. The use of manufacturing process planning (optimization) approaches in robotics is proposed in Somló, Lantos, P. T. Cat [2]. This may lead to add new borders to velocity limit curves at tailoring.
- Effects of joint drive characteristics may be estimated. There might be the cases when because drive design aspects exploitation shortcomings are present.
- Because the path planner mistakes trajectory planning problems may arise, time-optimal cruising trajectory planning helps to recognize these problems. For example: What is a sharp turn? may be formulated.
- Many other problems may be formulated and handled.

4. TIME-OPTIMAL MOTION FROM POINT TO POINT

The point-to-point (PTP) motion is simpler to analyze and realize than that for motion along a path. Nevertheless, it is very important to understand and use time-optimal motions. When PTP motion is used, proper step functions are input to the actuators of the joints, and the realized motions of the joints are step responses. The motions in Cartesian coordinate systems are determined by the equations of direct geometry of the robot. If the transient processes are short and the joint motions realize maximum velocities, the motion is close to the same time-optimal cruising one but with undetermined before geometry. Now, let us analyze the question what are the opportunities of realizing the point-to-point motion when the motion on different paths may be realized, too.

As an example, consider again the rotating and translating joint of a cylindrical robot (Fig. 3), let us suppose that the transient motion is very fast. So, the motion features are determined basically by the cruising.

Let the task be to get from point A to B of Fig. 3. Then the difference of coordinates is:

$$\Delta q_1 = q_{1B} - q_{1A}; \Delta q_2 = q_{2B} - q_{2A} \tag{45}$$

Let the velocity limits be

$$\dot{q}_{1\max} \textbf{ and } \dot{q}_{2\max} \tag{46}$$

They are constant values, independent of the states. Let

$$t_{1\min} = \frac{\Delta q_1}{\dot{q}_{1\max}} \text{ and } t_{2\min} = \frac{\Delta q_2}{\dot{q}_{2\max}} \tag{47}$$

It is clear that only the bigger of the two values in (47) may be realized. This will give the absolute minimum of the motion time.

In formal terms

$$t_{\min} = Max(t_{i\min}); \quad i = 1, 2 \tag{48}$$

The joint which determines this minimum time is named *dominant.* At minimum time motion the dominant joint will be at its maximum possible velocity in every moment. The other joint's velocity value may vary in the domain

$$\dot{q}_{j\min} \leqslant \dot{q}_i \leqslant \dot{q}_{j\max}; \quad j = 1, 2 \tag{49}$$

Now, let the dominant joint index be i and the non-dominant joint index j. Let us move the dominant joint with the velocity $\dot{q}_{i\max}$. It is clear that any velocity for the other joint in the $\dot{q}_{j\min}, \dot{q}_{j\max}$ domain (see (49)) may be applied. If moving from point A with $\dot{q}_{j\min}, \dot{q}_{j\max}$and $\dot{q}_{j\max}$ the borders of minimum time paths in the x,y plane may be determined. Using the equations of kinematics, it is very easy to get the boundary curves. It is equally easy to get the boundaries for the motion from point B using the method of backward time. (see Fig. 5). If there exists a common, contiguous area inside the borders containing A and B, then every realizable trajectory of this area is a minimum time one. Any trajectory of the domain is realizable if $\dot{q}_{i\max}$ and the (49) constraint $(j = 2)$ is satisfied.

Let us determine the border curves for the above example:

a.) Case when the rotation motion is dominant.

That is: $t_{1\min} > t_{2\min}$. The speed of a rotating joint should always be at its limit value. $\dot{q}_1 = \dot{q}_{1\max}$. For the motion of the translating joint one has the following limit values: For the motion from point A.

$$q_{2A} - \frac{q_1 - q_{1A}}{\dot{q}_{1\max}} |q_{2\min}| \leqslant q_2 \leqslant q_{2A} + \frac{q_1 - q_{1A}}{\dot{q}_{1\max}} \dot{q}_{2\max} \tag{50}$$

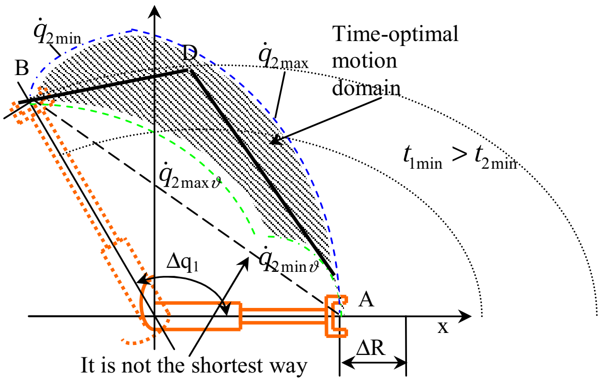

Figure 8: Time-optimal PTP motion

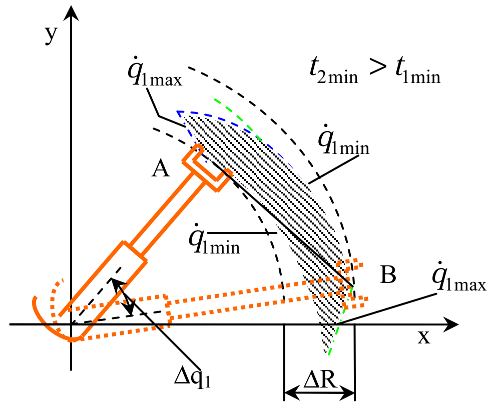

Figure 9: Time-optimal PTP motion

For the motion from (to) point B

$$q_{2B} - \frac{q_{1B} - q_1}{\dot{q}_{1\max}} \dot{q}_{2\max} \leqslant q_2 \leqslant q_{2B} + \frac{q_{1B} - q_1}{\dot{q}_{1\max}} |\dot{q}_{2\min}| \tag{51}$$

b.) Case when the translation motion is dominant.

That is $t_{2\min} > t_{1\min}$. The velocity of the translation motion should always be at its limit value$\dot{q}_2 = \dot{q}_{2\max}$. For the motion of the rotating joint one has the following limit values:

For the motion from point A:

$$q_{1A} - \frac{q_2 - q_{2A}}{\dot{q}_{2\max}} |q_{1\min}| \leqslant q_1 \leqslant q_{1A} + \frac{q_2 - q_{2A}}{\dot{q}_{2\max}} \dot{q}_{1\max} \tag{52}$$

For the motion from (to) point B:

$$q_{1B} - \frac{q_{2B} - q_2}{\dot{q}_{2\max}} \dot{q}_{1\max} \leqslant q_1 \leqslant q_{1B} + \frac{q_{2B} - q_2}{\dot{q}_{2\max}} |\dot{q}_{1\min}| \tag{53}$$

The realizable trajectories lie inside the border curves demonstrated in Figures 8 and 9. All the trajectories for which in every point the following relation is fulfilled:

$$\dot{q}_{i\min} \leqslant \dot{q}_i \leqslant \dot{q}_{i\max} \quad i = 1,\ 2,$$

and has the given initial and final points are realizable trajectories.

The border curves are also realizable trajectories. There is always a special realizable trajectory which results in the termination of the motions for the different joints at the same time. In the case of the given example, it simply means that the quicker joint velocity should slow down to have $t_2 = t_{1\min}$ (or $t_1 = t_{2\min}$). This implies the corresponding trajectory inside the border curves.

It is an interesting question how to choose from the set of realizable trajectories. All these trajectories realize the minimum of the time of the motion. It is possible to find trajectories which have some features more favorable than the others. For example: it can be a criterion to find the minimum time trajectory realizable by the minimum of energy.

It is also an interesting question what is the geometry of the minimum time trajectories for different robots and different tasks. The behavior of the robots from this point of view, including the investigation of the time of motions on different arcs, was analyzed by H. Doghiem (1993 a, 1993 b) [14, 15].

4.1. Case when the shortest way for a robot between two points is not the straight line

Looking at Figure 8 it can be recognized that if moving from A to B along the straight line it will take more time than moving on any time-optimal cruising trajectory. So, if the one which takes the least time is regarded as the shortest way, it will not be the straight line. To handle the case is very easy. First, the minimum time using Relation (48) is defined. Then the time-optimal cruising trajectory planning for the straight line is determined, which gives also the motion time, resulting in a clear picture about the quantitative characteristics.

4.2. Optimal PTP motion in space

What outlined in the example can be easily generalized for any degree of freedom. The dominant joint determination is exactly the same as in the 2D case. Of course, when ND (N=3, 4, 5, 6) problems are considered in Equations (45) $\div$ (49), the quantities with the proper indexation should be used. Furthermore, the individual representation of quantities of translational motion of the tool-center point and orientation motions is necessary (as it was introduced for the investigation of time-optimal motions on a given path). Assuming that the dominant joint moves with the limit speed, the other joints have some freedom of motion, then its limits can be determined similarly as in the 2D example above. So, the dominant joint with the maximum velocity is moved. Moving the other joints with $\pm$ limit velocities (from point A and with backward time from point B) results in sub-spaces which form time optimal domains. Any realizable trajectory of these sub-spaces is a time optimal trajectory. The optimal motion paths domains now become a 3D sub-space for translation and orientation motions. It is not as easy as in the 2D case to analyze the processes but it is fully possible. Even, graphical representations may help to find the most suitable paths.

4.3. Optimal motions in free space with obstacles

An excellent opportunity to solve the motion planning problem in the presence of obstacles is given by the approach outlined above. In this case, it is highly proposed to use graphical representations of the time-optimal motion domains. If realizable paths exist in between the optimal border and the obstacles borders, all these are time-optimal, that is given the same minimum time. The final choice may be made by considering a number of others than the time aspects.

5. REALIZATION OF TIME-OPTIMAL MOTIONS

Let us deal at first with the motions in a given path. As it is clear from the outlined the determination of the time-optimal trajectories is extremely simple. The equations which are necessary, for most of the industrial robots are known or can be easily derived. The maximums of joint velocities are included in the user handbooks of the robots. If in doubt, these values can easily be measured.

The optimal trajectory planning computations may be easily realized off-line. Then, using the data, the final choice of the velocities is possible with the guarantee of part processing time. These may prove much more favorable than the ones chosen without the knowledge of the consequences of the planning decisions

One may never know when the big problems occur. So, if a robot control is equipped with an option which realizes time-optimal motion, it is a significant step forward. To demonstrate the opportunity of realizing such a control system, an open system architecture robot control device was developed and experimented by Sokolov, Somlo, Lukanyin as reported in publications [16, 17].

It is remarkable that the time-optimal motion planning gives upper bounds for the velocities. The final choice of the velocity should take into account other constraints, too (see: Somlo, Lantos, P. T. Cat [2]). The other constraints may be grouped into: technological constraints, safety constraints, psychological constraints, etc.

Let us return to the question of free paths. For simplicity, let us consider the 2D case, but supposed that the device is able to realize not only PTP motions. Let us try to determine the path of the time-optimal motion. First (trivial) choice may be the straight line between A and B. If it is not time-optimal (see: Figure 8), we can try to choose two straight line sections (ADB in Figure 8). If these are realizable, they might construct the path. Application of the third and higher order splines for motion planning was reviewed in paragraph 2.1. Splines may also be candidates for realizing the time-optimal trajectories. For satisfying the time-optimality condition they should fully lie inside the optimal border, and the trajectories should be realizable.

Estimating these functions gives a thorough picture about velocity characteristics. Velocities may be properly decreased, manipulated. Paths may be relocated in the workspace. Paths may be modified (for example, smoothed to get smaller velocity differences), etc.

Trajectory tracking should provide the realization of the inputs given by trajectory planning. The problem is divided into two parts. The first part is the transient part realization, while the second comprises the realization of the motion on the time-optimal cruising part. As it is mentioned, to analyze the motion on the transient part and find suitable (optimal) control laws are not easy tasks and hard to realize. Nevertheless, this field is full of nice results.

Below, only the cruising part is dealt with. As, for every point of the paths we have the joint coordinates, and correspondingly, all the derivatives of them, we can determine, (using the Lagrange equation (1), (2) (or others)) the torques (forces) necessary for the given motion. If all the torques (forces) are in the available regions (below the limit values) there is a good chance that the robot control will provide processes close to the required. This is especially true if sophisticated robot control methods are used (see: Asada, Slotine [1], Somlo, Lantos, P. T. Cat [2], Fu, Gonzalez, Lee [12], Sciavicco, Siciliano [13] and many others).

Concerning trajectory tracking, the control science provides effective methods for the solutions. For example: Computed Torque, Model Reference Adaptive , Sliding Mode Control may be used (see, for example: [1, 2, 4–6, 12, 13].

6. SOME RESEACH TOPICS

Time-optimal and time-optimal cruising motions should be compared. Performance measures estimated.

Velocity profiles tailoring aspects should be developed. These may take into account process planning and as well control aspects. (Process planning and control aspects include not only the usual manufacturing process planning optimization but also a number of robot specific requirements (safe gripping, extended human safety, etc.)).

New, not technical features may arouse at the determination of robot velocities. Examples are: psychological (too big velocities horrify), scheduling, etc.

Time-optimal cruising trajectory planning seems very much suitable approach to investigate the above and many other interesting topics of robot motion planning.

7. CONCLUSIONS

The paper gives a general and simple approach to realize method for time-optimal cruising trajectory planning for industrial robots based on the parametric method. All the parameters needed for the application (for example, joint velocity limit values) are easily available. The basic relations reflecting the essence of the approach are given by Equations (34) $\div$ (37). The obtained velocity limit diagrams give the opportunity for tailoring. The usually too big limit velocities are tailored to suitable smaller values. Determining the $\dot{\lambda}(\lambda)$ function and from that the $t = t(\lambda)$ relations, the joint drives inputs may readily be determined and consequently the time-optimal cruising motion may be realized. A slightly different but in the spirit close method can be used for free paths.

The reduced velocity values make possible to decrease velocity errors, to improve the chance for good dynamical processes, to improve safety, etc.

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[18] *FANUC M-430iA robot manual.*

APPENDIX

An example for motion planning

A FANUC M-430iA robot is considered as an application example, when the TCP of the robot moves from point
$x = 100$; $y = -95$; $z = 1295$ to $x = 595$; $y = -95$; $z = 59$ along a straight line (all the data are in [mm]).

The limit velocities of the joints are: J1=300; J2=320; J3=320 (all are $[\circ/sec]$).

The geometrical construction and the arm lengths are taken from the robot manual (see [18]).

Time-optimal cruising trajectory planning

The curve of the time-optimal cruising velocity profile is given by V1. If the limit velocity values are decreased in the same proportion (changed to: kJ1, kJ2, kJ3 ; k=0,1; 0,2; 0,3 ;0,5) the limit velocity values will proportionally decrease (V0,1; V0,2; V0,3; V0,5)

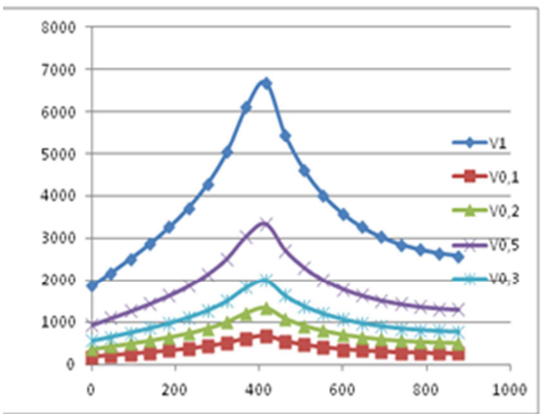

Figure A1: The velocity limit curve for FANUC M430iA example

(On the vertical axis of the Figure A1 the velocities along the path in $[mm/sec]$, on the horizontal axis the path length in [mm] is given.)

It can be recognized that the velocities along the path at original joint velocity limit values are very high. (The application of these velocities is impossible because the contour velocities of the robot are restricted by the “mechanisms building facts” and these restrictions are “built in” the robot control software.)

The application of the planning results with k=0,3 (V0,3) seems suitable. The motion times using this profile is very good and significant reserve for external forces is provided.

The computed motion time at original velocity limits is very small. The motion time is 0,281276 [sec]. The motion time at proportional decrease of limit velocities is proportionally more. For example 2,81276 [sec] at $k = 0,\, 1$.

Supposed that $k = 0,\, 3$ is applied, but instead of obtaining limit velocities the profile on Figure A2 is applied. The motion time then will be $T = 1,24439$ [sec]. If in some part of the path technological operation is performed at that part of the path the suitable “small” value of velocity should be used. In other parts (running in, stopping, running back to the initial point) the velocities according to the curve in Figure A2 can be used.

Centrifugal force constrained motion example

Now, a case is investigated when because the big mass in the gripper small limit velocity of the 1-st joint is allowed.

Let $\dot{q}_{1\,\max} = 0,2\left[\frac{\text{rad}}{\text{sec}}\right]$

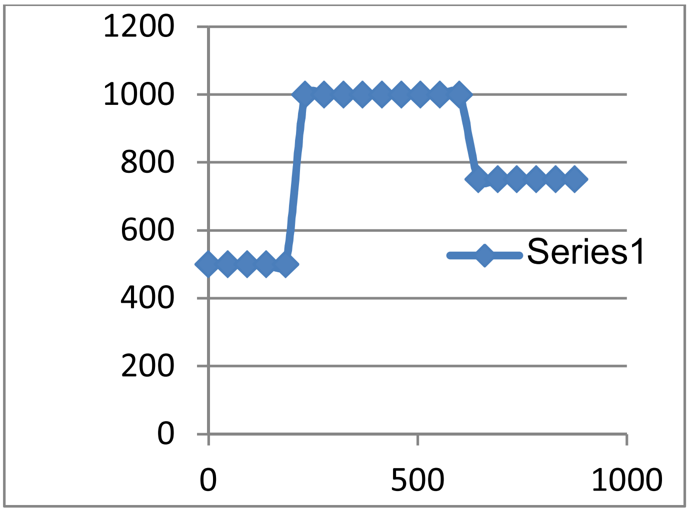

Figure A2: The tailored velocity diagram

For the 2 other joints $\dot{q}_{2\,\max} = 0,6\left[\frac{\text{rad}}{\text{sec}}\right]$ and $\dot{q}_{3\,\max} = 0,6\left[\frac{\text{rad}}{\text{sec}}\right]$ is applied.

The path is an “upper” half-circle with radius 100 [mm] in a horizontal plane with z=250 [mm]. The center point of the circle is x=250 [mm] and y=250 [mm].

The velocity limit curve is given in Figure A3

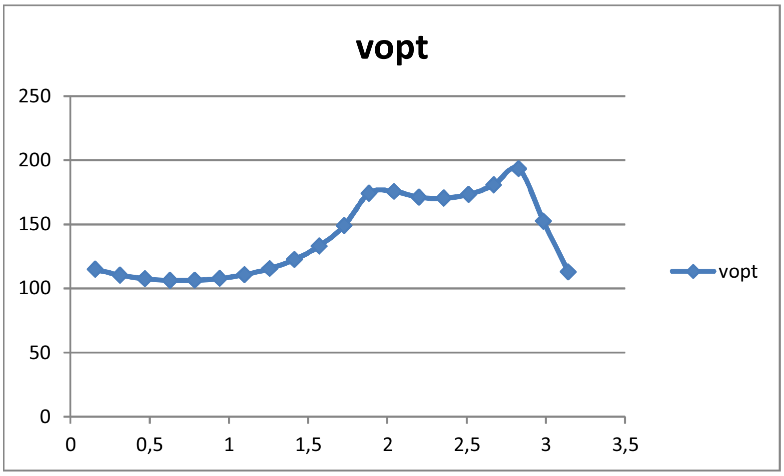

Figure A3: Centrifugal force constrained case

(In the horizontal axis the central angle in radians, in the vertical axis the velocity in $[mm/sec]$ are given.)

It can be recognized that to use 100 [mm/sec] velocity all along the path is a good solution of the planning problem. At that the motion time is 3.14 [sec] which seems a good value.

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