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SOME PROPERTIES OF THE POSITIVE BOOLEAN DEPENDENCIES IN THE DATABASE MODEL OF BLOCK FORM

TRAN MINH TUYEN[1], TRINH DINH THANG[2], AND NGUYEN XUAN HUY[3]

[1] *Vietnam Trade Union University; tuyentm@dhcd.edu.vn*
[2] *Hanoi Pedagogical University No. 2; thangsp2@yahoo.com*
[3] *Institute of Information Technology, Vietnam Academy of Science and Technology; nxhuy564@gmail.com*

Abstract. The report proposes the concept of positive boolean dependency in the database model of block form, proving equivalent theorem of three derived types, necessary and sufficient criteria of the derived type, the member problem... In addition, some properties related to this concept in the case of block r degenerated into relation are also expressed and demonstrated here.
Keywords. Positive boolean dependence, block, block scheme.

1. THE DATABASE MODEL OF BLOCK FORM

1.1. The block, slice of the block

Definition 1.1 ([1]) Let $R = (id; A_1, A_2, \ldots, A_n)$ be a finite set of elements, where id is non-empty finite index set, A_i $(i = 1 \ldots n)$ is the attribute. Each attribute A_i $(i = 1 \ldots n)$ there is a corresponding value domain dom (A_i). A block r on R, denoting $r(R)$ consists of a finite number of elements that each element is a family of mappings from the index set id to the value domain of the attributes A_i $(i = 1 \ldots n)$.

$$t \in r(R) \Leftrightarrow t = \{t^i : id \to \mathrm{dom}(A_i)\}_{i=1..n}.$$

The block denotes $r(R)$ or $r(id; A_1, A_2, \ldots, A_n)$, sometimes without fear of confusion it simply denotes r.

Definition 1.2 ([1]) Let $R = (id; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block over R. For each $x \in id$, $r(R_x)$ denotes a block with $R_x = (\{x\}; A_1, A_2, \ldots, A_n)$, such is:

$$t_x \in r(R_x) \Leftrightarrow t_x = \{t_x^i = t^i\Big|_x\}_{i=1..n},\ t \in r(R),\ t = \{t^i : id \to \mathrm{dom}(A_i)\}_{i=1...n},$$

where $t_x^i(x) = t^i(x)$, $i = 1 \ldots n$.

Then $r(R_x)$ is called a slice of block $r(R)$ at point x.

1.2. Functional dependencies

Here for simplicity, the following notation is used:
$x^{(i)} = (x; A_i)$; $id^{(i)} = \{x^{(i)} | x \in id\}$.
$x^{(i)}(x \in id,\ i = 1 \ldots n)$ is called an index attribute of the block.

Definition 1.3 ([1]) Let $R = (id; A_1, A_2, \dots, A_n)$, $r(R)$ is a block over R, $X, Y \subseteq \bigcup\limits_{i=1}^{n} id^{(i)}$, X→Y is a notation of functional dependency. A block r satisfies $X \to Y$ if for any $t_1, t_2 \in r$ such is $t_1(X) = t_2(X)$ then $t_1(Y) = t_2(Y)$.

Definition 1.4 ([3])

Let $R = (id; A_1, A_2, \dots, A_n)$, F is the set of functional dependencies over R. Then, the closure of F denoting F^+ is defined as follows:

$$F^+ = \{X \to Y | F \Rightarrow X \to Y\}.$$

If $X = \{x^{(m)}\} \subseteq id^{(m)}$, $Y = \{y^{(k)}\} \subseteq id^{(k)}$ then functional dependency $X \to Y$ denotes simply $x^{(m)} \to y^{(k)}$.

The block r satisfies $x^{(m)} \to y^{(k)}$ if for any t_1, $t_2 \in r$ such is $t_1(x^{(m)}) = t_2(x^{(m)})$ then $t_1(y^{(k)}) = t_2(y^{(k)})$, where:

$$\begin{aligned} t_1(x^{(m)}) =& t_1(x; A_m), t_2(x^{(m)}) = t_2(x; A_m), \\ t_1(y^{(k)}) =& t_1(y; A_k), t_2(y^{(k)}) = t_2(y; A_k). \end{aligned}$$

Let $R = (id; A_1, A_2, \dots, A_n)$, the subsets of functional dependency over R denote:

$$\begin{aligned} F_h =& \{X \to Y | X = \bigcup_{i \in A} x^{(i)}, Y = \bigcup_{j \in B} x^{(j)}, A, B \subseteq \{1, 2, \dots, n\} \text{ and } x \in id\}, \\ F_{hx} =& F_h\Big|_{\bigcup\limits_{i=1}^{n} x^{(i)}} = \left\{ X \to Y \in F_h | X, Y \subseteq \bigcup_{i=1}^{n} x^{(i)} \right\}. \end{aligned}$$

Let $R = (id; A_1, A_2, \dots, A_n)$, F is the set of functional dependencies over R. Then $\alpha = (R, F)$ is called a block scheme, if $F = \emptyset$ then the notation R is used.

$\alpha_x = (R_x, F_x)$ is called a slice scheme at point x, $F_x = F\Big|_{\bigcup\limits_{i=1}^{n} x^{(i)}}$, if $F_x = \emptyset$ then the notation R_x is used.

Definition 1.5 ([3]) Let block scheme $\alpha = (R, F_h)$, $R = (id; A_1, A_2, \dots, A_n)$, then F_h is called the complete set of functional dependencies over R if F_{hx} is the same for all $x \in id$.

A more specific way:

F_{hx} is the same for all $x \in id$, i.e. $\forall x, y \in id$: $M \to N \in F_{hx} \Leftrightarrow M' \to N' \in F_{hy}$ with M', N', respectively formed from M, N by replacing x by y.

1.3. Closure of the index sets attributes

Definition 1.6 ([4]) Let block scheme $\alpha = (R, F)$, $R = (id; A_1, A_2, \dots, A_n)$, F is a set of functional dependencies over R.

For each $X \subseteq \bigcup\limits_{i=1}^{n} id^{(i)}$, it is to define the closure of X for F denoting X^+ as follows:

$$X^+ = \{x^{(i)}, x \in id,\ i = 1 \dots n | X \to x^{(i)} \in F^+\}.$$

The set of all subsets of $\bigcup_{i=1}^{n} id^{(i)}]$ denotes a subset $(\bigcup_{i=1}^{n} id^{(i)})$.

Let $\Re$, $\Im \subseteq$ SubSet $(\bigcup_{i=1}^{n} id^{(i)})$ and $M, P \in$ SubSet $(\bigcup_{i=1}^{n} id^{(i)})$, then operations $\oplus$ on the SubSet$(\bigcup_{i=1}^{n} id^{(i)})$ is defined as follows:

$M \oplus P = MP$, $(MP = M \cup P)$,
$M \oplus \Re = \{MX | X \in \Re\}$,
$\Re \oplus \Im = \{XY | X \in \Re,\ Y \in \Im\}$.

1.4. Key of the block scheme $\alpha = (R, F)$

Definition 1.7 ([4]) Let block scheme $\alpha = (R, F)$, $R = (id; A_1, A_2, \ldots, A_n)$, F is a set of functional dependencies over R,$K \subseteq \bigcup_{i=1}^{n} id^{(i)}$. K called a key of block schema α if it satisfies two conditions:

a) $K \to x(i) \in F^+$, $\forall x \in id$, $i = 1 \ldots n$.
b) $\forall K' \subset K$ then K' has no properties a).

If K is a key and $K \subseteq K''$ then K'' called a super key of block scheme R for F.

2. BOOLEAN FORMULAS

2.1. Boolean formula

Definition 2.1 ([2]) Let $U = \{x_1, x_2, \ldots, x_n\}$ be a finite set of Boolean variables, B is Boolean value set, $B = \{0, 1\}$. Then the Boolean formulas, also known as logic formulas are constructed as follows:

(i) Each value 0/1 in B is a Boolean formula.

(ii) Each variable taking the value of U is a Boolean formula.

(iii) If a is a Boolean formula, then (a) is a Boolean formula.

(iv) If a and b are the Boolean formulas, then $a \wedge b$, $\neg a$ and $a \to b$ is a Boolean formula.

(v) Only the formula created by the rules from (i) - (iv) is Boolean formula.

$L(U)$ denotes the set of Boolean formulas built on a set of variables U.

Definition 2.2 ([2]) Each vector of the elements 0/1, $v = \{v_1, v_2, \ldots, v_n\}$ in the space $B^n = B \times B \times \ldots \times B$ is called a value assignment. Thus, with each Boolean formula $f \in L(U)$, it yields $f(v) = f(v_1, v_2, \ldots, v_n)$ as the value of the formula f for value assignment v.

In case of confusion, it is understood that the symbols X performances are for the following subjects:

- A set of the attributes in U.
- A set of the logical variables in U.
- A Boolean formula is a logical association of the variables in X.

On the other hand, if $X = \{B_1, B_2, \ldots, B_m\} \subseteq U$, it denotes:
$\wedge$X = B$_1\wedge$ B$_2\wedge$ ... $\wedge$ B$_m$ and called the opportunity form.
$\vee$X = B$_1\vee$ B$_2\vee$ ... $\vee$ B$_m$ and called the recruitment form.
The formula $f : Z \to V$ is called as:

- Derived formula if Z and V have the opportunity form, i.e. $f : \wedge Z \to \wedge V$.
- Strong derived formula if Z have the recruitment form and V have the opportunity form:

$$f : \vee Z \to \wedge V.$$

- Weak derived formula if Z have the opportunity form and V have the recruitment form:

$$f : \wedge Z \to \vee V.$$

- Duality derived formula if Z and V have the recruitment form:

$$f : \vee Z \to \vee V.$$

For every finite set of Boolean formula $F = \{f_1, f_2, \ldots, f_m\}$ in $L(U)$, F is seen as a formula $F = f_1 \wedge f_2 \wedge \ldots \wedge f_m$. Then it results in:

$$F(v) = f_1(v) \wedge f_2(v) \wedge \ldots \wedge f_m(v).$$

2.2. Value table and truth table

For each formula f on U, the value table of f, denoting V_f contains $n+1$ column, with n the first column contains the values of the variables in U, and the last column contains the value of f for each value assignment of the respective line. Thus, the value table contains 2^n line, n is the number of elements of U.

Definition 2.3 ([2]) Truth table of f, denoting T_f is the set of value assignment v such that $f(v)$ takes the value 1:

$$T_f = \{v \in B^n | f(v) = 1\}$$

Also, truth table T_F of a finite set of formulas F on U, is the intersection of the truth table of each member formulas in F.

$$T_F = \bigcap_{f \in F} T_f.$$

Yields: $v \in T_F$ if and only if $\forall f \in F : f(v) = 1$.

2.3. Logic derived

Definition 2.4 ([2]) Let f, g be two Boolean formulas, said formula g derives from formula f logic, and symbols $f \vDash g$ if $T_f \subseteq T_g$. It is to say that f is equivalent to g and notation $f \equiv g$ if $T_f = T_g$.

With F and G in $L(U)$, said G logic derives from f, denoting that $F \vDash G$ if $T_F \subseteq T_G$. Furthermore, it is to say F and G are equivalent, denoting $F \equiv G$ if $T_F = T_G$.

2.4. Positive Boolean formula

Definition 2.5 ([2]) The formula $f \in L(U)$ is called positive Boolean formulas if $f(e) = 1$ with $e = (1, 1, ..., 1)$, and the set of all positive Boolean formulas over U denoting $P(U)$.

3. RESEARCH RESULTS

3.1. Truth block of the block

Definition 3.1. Let $R = (id; A_1, A_2, \dots, A_n)$, $r(R)$ is a block on R, $u, v \in r$. $\alpha(u, v)$ is called as value assignment: $\alpha(u, v) = (\alpha_1(u.x^{(1)}, v.x^{(1)}), \alpha_2(u.x^{(2)}, v.x^{(2)}), \dots, \alpha_n(u.x^{(n)}, v.x^{(n)}))_{x \in id}$, in which:

$\alpha_i(u.x^{(i)}, v.x^{(i)}) = 1$ if $u.x^{(i)} = v.x^{(i)}$ and

$\alpha_i(u.x^{(i)}, v.x^{(i)}) = 0$ if otherwise, $i = 1 \dots n$, $x \in id$.

Then, with each block r, the truth block of r denotes T_r:

$$T_r = \{\alpha(u, v) | u, v \in r\}.$$

From the definition it can be seen that the truth block of r is a binary block.

In the case $id = \{x\}$, then the block r is degenerated into relation and the truth block of the block r becomes the truth table of the relations in the relational data model. In other words, the truth block of the block is expanded concept of the truth table of relations in the relational data model.

3.2. Positive Boolean dependencies on the block

Definition 3.2. Let $R = (id; A_1, A_2, \dots, A_n)$, $r(R)$ are a block on $R, U = \{x^{(1)}, x^{(2)}, \dots, x^{(n)} | x \in id\}$, each positive Boolean formula in $P(U)$ is called as a positive Boolean dependency.

It is to say the block r satisfies positive Boolean dependency f denoting $r(f)$ if $T_r \subseteq T_f$.

The block r satisfies the set of positive Boolean dependencies F denoting $r(F)$ if the block r satisfies any positive Boolean dependency f in F:

$$r(F) \Leftrightarrow \forall f \in F : r(f) \Leftrightarrow T_r \subseteq T_F.$$

Example:

Let $R = (\{1, 2\}; A_1, A_2, A_3, A_4)$, with the block r satisfies:

$t_1(1^{(1)}) = 1, t_1(1^{(2)}) = 1, t_1(1^{(3)}) = 0, t_1(1^{(4)}) = 0, t_1(2^{(1)}) = 1, t_1(2^{(2)}) = 1, t_1(2^{(3)}) = 1, t_1(2^{(4)}) = 1$

$t_2(1^{(1)}) = 1, t_2(1^{(2)}) = 0, t_2(1^{(3)}) = 1, t_2(1^{(4)}) = 1, t_2(2^{(1)}) = 1, t_2(2^{(2)}) = 1, t_2(2^{(3)}) = 0, t_2(2^{(4)}) = 1$

$t_3(1^{(1)}) = 1, t_3(1^{(2)}) = 1, t_3(1^{(3)}) = 1, t_3(1^{(4)}) = 0, t_3(2^{(1)}) = 1, t_3(2^{(2)}) = 0, t_3(2^{(3)}) = 1, t_3(2^{(4)}) = 0$

$t_4(1^{(1)}) = 0, t_4(1^{(2)}) = 0, t_4(1^{(3)}) = 0, t_4(1^{(4)}) = 1, t_4(2^{(1)}) = 0, t_4(2^{(2)}) = 0, t_4(2^{(3)}) = 0, t_4(2^{(4)}) = 1$

$t_5(1^{(1)}) = 0, t_5(1^{(2)}) = 0, t_5(1^{(3)}) = 0, t_5(1^{(4)}) = 0, t_5(2^{(1)}) = 0, t_5(2^{(2)}) = 0, t_5(2^{(3)}) = 0, t_5(2^{(4)}) = 1$

where 1: Saturday, 2: Sunday, A_1: Bread, A_2: Milk, A_3: Butter, A_4: Egg.

Then, it results in a positive Boolean dependency $(1^{(1)}2^{(1)}) \to ((1^{(2)}2^{(2)}) \vee (1^{(3)}2^{(3)}))$: this means that, in the Saturday and Sunday who buy bread also buy milk or butter.

Although it does not have: $(1^{(1)}2^{(1)}) \to (1^{(2)}2^{(2)}1^{(3)}2^{(3)})$

For the set of positive Boolean dependencies F and a positive Boolean dependency f:

- It is said that f derives from F according to block and denotes $F \vdash f$ if:

  ∀r: r (F) ⇒r (f).

- It is to say that f derives from F according to block having not more than 2 elements and symbols $F \vdash_2 f$ if $\forall r_2$: $r_2(F) \Rightarrow r_2(f)$.

It yields the following equivalence theorem:

Theorem 3.1. *For the set of positive Boolean dependencies F and a positive Boolean dependency f, $R = (id; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block on R. Then the following three statements are equivalent:*

i) $F \vDash f$ *(logic derived),*

(ii) $F \vdash f$ *(derived from the block),*

(iii) $F \vdash_2 f$ *(derived from the block has no more than 2 elements).*

Proof. (i) ⇒ (ii): By assumption it has $F \vDash f \Rightarrow T_F \subseteq T_f$ $^{(1)}$. Suppose r is any block satisfying F, then by definition: $T_r \subseteq T_F$ $^{(2)}$. From (1) and (2) it deduces that: $T_r \subseteq T_f$, so resulting in: $r(f)$.

(ii) ⇒ (iii): Obviously, because the derived from block having not more than 2 elements is a special case of the derived from the block.

(iii) ⇒ (i): Suppose $t = (x^{(1)}, x^{(2)}, \ldots, x^{(n)})_{x\in id}$, $t \in T_F$, it is needed to prove $t \in T_f$.

Indeed, if $t = e$, it has $t \in T_f$ because it is known that f is a positive Boolean formula. If $t \neq e$, block r consists of two element u and v as follows: $u = (y^{(1)}, y^{(2)}, \ldots, y^{(n)})_{y\in id}$, $v = (z^{(1)}, z^{(2)}, \ldots, z^{(n)})_{z\in id}$ such that $\alpha(u, v) = t$. Thus r is the block having 2 elements and $T_r = \{e, t\} \subseteq T_F$, where e is the element of the block that all values are equal to 1.

It follows that $r(F)$. Under the assumption: $r(F) \Rightarrow r(f)$, so $T_r \subseteq T_f$ $^{(1)}$.

From the inclusion (1) it deduces that $t \in T_f$.

In the case $id = \{x\}$, then the block r is degenerated into relation and the equivalent theorem above becomes the equivalent theorem of the relational data model. Specifically, it yields the following corollary:

Corollary 3.1. *For the set of positive Boolean dependencies F and a positive Boolean dependency f, $R = (id; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block on R. Then, if $id = \{x\}$ then the block r is degenerated into relation and the following three propositions are equivalent:*

i) $F \vDash f$ *(logic derived),*

(ii) $F \vdash f$ *(derived from the block),*

(iiii) $F \vdash_2 f$ *(derived from the block has no more than 2 elements).*

This is proven results in the relational data model.

For dependencies on the block r, block r has been defined satisfying the functional dependency $f : X \to Y$, denoting $r(f)$ if: $\forall u, v \in r$: $u.X = v.X \Rightarrow u.Y = v.Y$

When seeing dependencies as a particular case of the positive Boolean formulas, it has adopted the definition of the block r satisfies the functional dependency $f : X \to Y$ if $T_r \subseteq T_f$.

Necessary and sufficient theorem below confirms the equivalence of two definitions:

Theorem 3.2 (necessary and sufficient condition)
Let $R = (id; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block on R, a functional dependency $f : X \to Y$ with $X, Y \subseteq \bigcup_{i=1}^{n} id^{(i)}$. Then: $r(f) \Leftrightarrow T_r \subseteq T_f$.

Proof. Suppose r is a block, which satisfies the functional dependency $f : X \to Y$ and $t \in T_r$, it is needed to prove $t \in T_f$. Indeed, because it has $t \in T_r \Rightarrow$ in block existing 2 elements u and v such that $t = \alpha(u, v)$. If $t(X) = 1 \Rightarrow u.X = v.X$ $^{(1)}$.

On the other hand, it has $r(f)$, from $^{(1)} \Rightarrow u.Y = v.Y$. Therefore: $t(Y) = 1 \Rightarrow t \in T_f$.

Conversely, suppose $T_r \subseteq T_f$ with $f : X \to Y$, it is needed to demonstrate r to satisfy functional dependencies f, i.e. $r(f)$. Indeed, two arbitrary element u and v in r yield: $u.X = v.X$, we set $t = \alpha(u, v) \Rightarrow t \in T_r$, $t.X = 1$. Other hand, it results in: $T_r \subseteq T_f \Rightarrow t \in T_f$, more it yields $t.X = 1 \Rightarrow t.Y = 1$. So it obtains: $u.Y = v.Y$.

Thus, from $u.X = v.X \Rightarrow u.Y = v.Y \Rightarrow r$ satisfies the functional dependency f, i.e. $r(f)$.

Corollary 3.2. *Let $R = (id; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block on R, a functional dependency $f : X \to Y with X, Y \subseteq \bigcup_{i=1}^{n} id^{(i)}$. If $id = \{x\}$ then the block r is degenerated into relation, whereas in the relational data model, it results in: $r_x(f_x) \Leftrightarrow T_{r_x} \subseteq T_{f_x}$.*

In the case F is a set of functional dependencies on the block, then T_F is the intersection of members T_f with any f in F, so there are following results:

Theorem 3.3 (necessary and sufficient condition)
Let $R = (id; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block on R, set of functional dependencies $F = \{f : X \to Y | X,\ Y \subseteq \bigcup_{i=1}^{n} id^{(i)}\}$. Then: $r(F) \Leftrightarrow T_r \subseteq T_F$.

Corollary 3.3. *Let $R = (id; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block on R, set of functional dependencies $F = \{f : X \to Y | X,\ Y \subseteq \bigcup_{i=1}^{n} id^{(i)}\}$. If $id = \{x\}$ then the block r is degenerated into relation, whereas in the relational data model, it results in: $r(F) \Leftrightarrow T_r \subseteq T_F$.*

Theorem 3.4 (necessary and sufficient condition) *Let $R = (id; A_1, A_2, \ldots, A_n)$, $r(R)$ is a block on R, a functional dependency $f : X \to Y$ with $X, Y \subseteq \bigcup_{i=1}^{n} id^{(i)}$, $X_x = X \cap \bigcup_{i=1}^{n} x^{(i)}$, $Y_x = Y \cap \bigcup_{i=1}^{n} x^{(i)}$, $f(X_x) = Y_x$, $f \cap \bigcup_{i=1}^{n} x^{(i)} = f_x$, $\forall x \in id$.*

Then: $r(f) \Leftrightarrow r_x(f_x)$, $\forall x \in id$.

Proof. Suppose r is a block, which satisfies the functional dependency $f : X \to Y$, from the Theorem 3.2 yields:

$$r(f) \Leftrightarrow T_r \subseteq T_f \Leftrightarrow T_{r_x} \subseteq T_{f_x}, \forall x \in id \tag{1}$$

On the other hand, from the corollary 3.2 results in:

$$T_{r_x} \subseteq T_{f_x} \Leftrightarrow r_x(f_x), \forall x \in id \tag{2}$$

Thus, from (1) and (2) it deduces that: $r(f) \Leftrightarrow r_x(f_x)$, $\forall x \in id$.

From here onwards it is understood that the set F in block schema $\alpha = (R, F)$ is the set of positive Boolean dependencies in R.

Suppose $X \subseteq \bigcup\limits_{i=1}^{n} id^{(i)}$, $v \in B^{nxm}$, $(m = |id|)$ then, it results in:

$$\wedge X(v) = 1 \Leftrightarrow \forall x^{(i)} \in X : v.x^{(i)} = 1,$$
$$\vee X(v) = 1 \Leftrightarrow \exists x^{(i)} \in X : v.x^{(i)} = 1,$$

and

$$\wedge X(v) = 0 \Leftrightarrow \exists x^{(i)} \in X : v.x^{(i)} = 0,$$
$$\vee X(v) = 0 \Leftrightarrow \forall x^{(i)} \in X : v.x^{(i)} = 0.$$

Applying the above results the following theorem is proven:

Theorem 3.5. *Let R = (id; A_1, A_2,..., A_n), r (R) is a block on R, F is the set of positive Boolean dependencies in R; X, Y $\subseteq . \bigcup\limits_{i=1}^{n} id^{(i)}$. Then:*

i) $F| = \wedge X \to \wedge Y \Leftrightarrow \forall v \in T_F$: $((\exists x^{(i)} \in X : v.x^{(i)} = 0) \vee (\forall y^{(j)} \in Y : v.y^{(j)} = 1))$

ii) $F| = \wedge X \to \vee Y \Leftrightarrow \forall v \in T_F : ((\exists x^{(i)} \in X : v.x^{(i)} = 0) \vee (\exists y^{(j)} \in Y : v.y^{(j)} = 1))$

iii) $F| = \vee X \to \wedge Y \Leftrightarrow \forall v \in T_F : ((\forall x^{(i)} \in X : v.x^{(i)} = 0) \vee (\forall y^{(j)} \in Y : v.y^{(j)} = 1))$

iv) $F| = \vee X \to \vee Y \Leftrightarrow \forall v \in T_F : ((\forall x^{(i)} \in X : v.x^{(i)} = 0) \vee (\exists y^{(j)} \in Y : v.y^{(j)} = 1))$

Corollary 3.4. *Let $R = (id; A_1, A_2, ..., A_n)$, $r(R)$ is a block on R, F is the set of positive Boolean dependencies in R; $X, Y \subseteq \bigcup\limits_{i=1}^{n} id^{(i)}$. As such, if $id = \{x\}$ then the block r is degenerated into relation and it results in:*

i) $F| = \wedge X \to \wedge Y \Leftrightarrow \forall v \in T_F : ((\exists A \in X : v.A = 0) \vee (\forall B \in Y : v.B = 1))$

ii) $F| = \wedge X \to \vee Y \Leftrightarrow \forall v \in T_F : ((\exists A \in X : v.A = 0) \vee (\exists B \in Y : v.B = 1))$

iii) $F| = \vee X \to \wedge Y \Leftrightarrow \forall v \in T_F : ((\forall A) \in X : v.A = 0) \vee (\forall B \in Y : v.B = 1))$

iv) $F| = \vee X \to \vee Y \Leftrightarrow \forall v \in T_F : ((\forall A \in X : v.A = 0) \vee (\exists B \in Y : v.B = 1))$.

Definition 3.3. Let $R = (id; A_1, A_2, ..., A_n)$, $B = \{0, 1\}$. Then for all $v \in B^{n \times m}$ denoting:

$$Set(v) = \{x^{(i)} \in \bigcup_{i=1}^{n} id^{(i)} | v.x^{(i)} = 1\},$$

and with each block $T \subseteq B^{n \times m}$ it denotes:

$$Set(T) = \{Set(v) | v \in T\},$$

In addition to any $X \in SubSet(\bigcup\limits_{i=1}^{n} id^{(i)})$ it denotes: $Vec(X) = v$, here $v \in B^{nxm}$, $v.x^{(i)} = 1$ if $x^{(i)} \in X$ and $v.x^{(i)} = 0$ in the opposite case $(x \in id,\ i = 1 \ldots n)$.

From Theorem 3.5 the following theorem is deduced:

Theorem 3.6. *Let* $R = (id; A_1, A_2, ..., A_n)$, $r(R)$ *is a block,* F *is the set of positive Boolean dependencies on* R*;* $X, Y \subseteq \bigcup_{i=1}^{n} id^{(i)}$*. Then:*

i) $F| = \wedge X \to \wedge Y \Leftrightarrow \forall V \in Set(T_F) : (X \subseteq V \Rightarrow Y \subseteq V)$

ii) $F| = \wedge X \to \vee Y \Leftrightarrow \forall V \in Set(T_F) : (X \subseteq V \Rightarrow Y \cap V \neq \emptyset)$

iii) $F| = \vee X \to \wedge Y \Leftrightarrow \forall V \in Set(T_F) : (X \cap V \neq \emptyset \Rightarrow Y \subseteq V)$

iv) $F| = \vee X \to \vee Y \Leftrightarrow \forall V \in Set(T_F) : (X \cap V \neq \emptyset \Rightarrow Y \cap V \neq \emptyset)$

Corollary 3.5. *Let* $R = (id; A_1, A_2, \dots, A_n)$, $r(R)$ *is a block,* F *is the set of positive Boolean dependencies in* R*;* $X, Y \subseteq \bigcup_{i=1}^{n} id^{(i)}$*. As such, if* $id = \{x\}$ *then the block* r *is degenerated into relation and in the relational database model, we have:*

i) $F \vDash \wedge X \to \wedge Y \Leftrightarrow \forall V \in Set(T_F) : (X \subseteq V \Rightarrow Y \subseteq V)$

ii) $F \vDash \wedge X \to \vee Y \Leftrightarrow \forall V \in Set(T_F) : (X \subseteq V \Rightarrow Y \cap V \neq \emptyset)$

iii) $F \vDash \vee X \to \wedge Y \Leftrightarrow \forall V \in Set(T_F) : (X \cap V \neq \emptyset \Rightarrow Y \subseteq V)$

iv) $F \vDash \vee X \to \vee Y \Leftrightarrow \forall V \in Set(T_F) : (X \cap V \neq \emptyset \Rightarrow Y \cap V \neq \emptyset)$

3.3. The relationship between the type of functional dependencies and positive Boolean formulas

Let $R = (id; A_1, A_2, ..., A_n)$, is a block on R; $X, Y \subseteq \bigcup_{i=1}^{n} id^{(i)}$. Then we said:

- The block r satisfies strongly functional dependency $f_s : X \to_s Y$, denoted $r(f_s)$ if: $\forall u, v \in r$: $\exists x^{(i)} \in X$ such that $u.x^{(i)} = v.x^{(i)} \Rightarrow u.Y = v.Y$.

- The block r satisfies weak functional dependency $f_w : X \to_w Y$, denoted $r(f_w)$ if: $\forall u, v \in r$: $u.X = v.X \Rightarrow \exists y^{(j)} \in Y$ such that $u.y^{(j)} = v.y^{(j)}$.

- The block r satisfies the duality functional dependency $f_d : X \to_d Y$, denoted $r(f_d)$ if: $\forall u, v \in r : \exists x^{(i)} \in X$ such that $u.x^{(i)} = v.x^{(i)} \Rightarrow \exists y^{(j)} \in Y such that u.y^{(j)} = v.y^{(j)}$.

Theorem 3.7. *Let* $R = (id; A_1, A_2, \dots, A_n)$, $r(R)$ *is a block on* R*;* $X, Y \subseteq \bigcup_{i=1}^{n} id^{(i)}$, $f_s : X \to_s Y$ *is a strongly functional dependency,* $g_s : \vee X \to \wedge Y$ *is a strongly derived formula. Then:*

$$r(f_s) \text{ if and only if } r(g_s).$$

Proof. $\Rightarrow$) Suppose it has $r(f_s)$, it is needed to demonstrate $r(g_s)$. Indeed, if $t \in T_r \Rightarrow t = \alpha(u, v) with u, v \in r$, assuming $(\vee X)(t) = 1 \Rightarrow \exists x^{(i)} \in X$ such that $t.x^{(i)} = 1 \Rightarrow u.x^{(i)} = v.x^{(i)}$, and $r(f_s)$ should be inferred: $u.Y = v.Y \Rightarrow (\wedge Y)(t) = 1$. Thus, from $(\vee X)(t) = 1 \Rightarrow (\wedge Y)(t) = 1$, so it yields: $t \in Tg_s$. Therefore, from the assumption, $t \in T_r \Rightarrow (t \in Tg_s) \Rightarrow (T_r \subseteq Tg_s) \Rightarrow r(g_s)$.

$\Leftarrow$) Conversely, suppose it has $r(g_s)$, it is needed to demonstrate $r(f_s)$. Indeed, $\forall u, v \in r$: if $\exists x^{(i)} \in X$ such that $u.x^{(i)} = v.x^{(i)}$, set $t = \alpha(u, v) \Rightarrow (\vee X)(t) = 1$, from the assumption $r(g_s) \Rightarrow (\wedge Y)(t) = 1 \Rightarrow u.Y = v.Y$. Hence it deduces that $r(f_s)$.

Theorem 3.8. *Let* $R = (id; A_1, A_2, ..., A_n)$*,* $r(R)$ *is a block on R;* $X, Y \subseteq \bigcup_{i=1}^{n} id^{(i)}$*,* $f_w : X \rightarrow_w Y$ *is a weak functional dependency,* $g_w : \wedge X \rightarrow \vee Y$ *is a weak derived formula. Then:*

$$r(f_w) \text{ if and only if } r(g_w).$$

Proof. $\Rightarrow$) Suppose it has $r(f_w)$, it is needed to demonstrate $r(g_w)$. Indeed, if $t \in T_r \Rightarrow t = \alpha(u, v)$ with $u, v \in r$, assuming $(\wedge X)(t) = 1 \Rightarrow u.X = v.X$, $r(f_w)$ should be inferred: $\exists y^{(j)} \in Y : u.y^{(j)} = v.y^{(j)} \Rightarrow (\vee Y)(t) = 1$. Thus, from $(\wedge X)(t) = 1 \Rightarrow (\vee Y)(t) = 1$, so it yields: $t \in Tg_w$. Therefore from the assumption, $t \in T_r \Rightarrow (t \in Tg_w) \Rightarrow (T_r \subseteq Tg_w) \Rightarrow r(g_w)$.

$\Leftarrow$) Conversely, suppose it has $r(g_w)$, it is needed to demonstrate $r(f_w)$. Indeed, $\forall u, v \in r : if u.X = v.X$, set $t = \alpha(u, v) \Rightarrow (\wedge X)(t) = 1$, from the assumption $r(g_w) \Rightarrow (\vee Y)(t) = 1 \Rightarrow \exists y^{(j)} \in Y : u.y^{(j)} = v.y^{(j)}$. Hence it deduces that $r(f_w)$.

Theorem 3.9. *Let* $R = (id; A_1, A_2, \ldots, A_n), r(R)$ *is a block on R;* $X, Y \subseteq \bigcup_{i=1}^{n} id^{(i)}$*,* $f_d : X \rightarrow_d Y$ *is a dual functional dependency,* $g_d : \vee X \rightarrow_d \vee Y$ *is a duality derived formula. Then:*

$$r(f_d) \textit{ if and only if } r(g_d).$$

Proof. $\Rightarrow$) Suppose it has $r(f_d)$, it is needed to demonstrate $r(g_d)$. Indeed, if $t \in T_r \Rightarrow t = \alpha(u, v) with u, v \in r$, assuming $(\vee X)(t) = 1 \Rightarrow \exists x^{(i)} \in X : t.x^{(i)} = 1 \Rightarrow u.x^{(i)} = v.x^{(i)}$, from $r(f_d)$, should be inferred: $\exists y^{(j)} \in Y : u.y^{(j)} = v.y^{(j)} \Rightarrow (\vee Y)(t) = 1$. Thus, from $(\vee X)(t) = 1 \Rightarrow (\vee Y)(t) = 1$, so it yields: $t \in Tg_d$. Therefore from the assumption, $t \in T_r \Rightarrow (t \in Tg_d) \Rightarrow (T_r \subseteq Tg_d) \Rightarrow r(g_d)$.

$\Leftarrow$) Conversely, suppose it has $r(g_d)$, it is needed to demonstrate $r(f_d)$. Indeed, $\forall u, v \in r : set t = \alpha(u, v), if \exists x^{(i)} \in X : u.x^{(i)} = v.x^{(i)} \Rightarrow (\vee X)(t) = 1$, from the assumption $r(g_d) \Rightarrow (\vee Y)(t) = 1 \Rightarrow \exists y^{(j)} \in Y : u.y^{(j)} = v.y^{(j)}$. Hence it deduces that $r(f_d)$.

4. CONCLUSION

The above results are from the concept of positive Boolean dependencies on the block; especially the necessary and sufficient conditions show clearly that the structure of this kind of logic dependencies in the design theory of the database model of block form. In case the index set $id = \{x\}$, block degenerate into a relation, then these results coincided with the results of many authors has been given to the relation in relational database model. On the basis of these results it is possible to study the relationship between types of different logic depending on the block scheme. . . some other results can be reviewed in the particular case of the set of functional dependencies F as a set of functional dependencies F_h, set of dependencies $F_{hx} \ldots$, contributing to the more complete designed theory of the database model of block form.

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