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THE MINIMUM TOTAL HEATING LANDER

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Abstract. The article will research a lander flying into the atmosphere with flow velocity constraint, i.e. the total load by means of minimizing the total thermal energy at the end of the landing process. The lander's distance at the last moment depends on the variables selected from the total thermal energy minima. To deal with the problem, the Pontryagin maximum principle and scheme Dubovitskij Milutin will be applied. Boundary value problems are solved by the introduction and continuation of the perturbation parameters and solutions for the selected parameter. The results of simulations perform on Matlab.

Keywords. Maximum principle, control, the overload, total heat, minimum.

1. INTRODUCTION

Research is on the problem of choosing an angle to launch the flying object which is reducing velocity in atmospheric conditions under which the minimizing of total heat flow with the load limits of aircraft equipment is taken into account. The total heat output of the device is the integral form of the following:

$$Q = \int_0^T CV^3 \rho^{\frac{1}{2}} dt \tag{1}$$

Required to detemine a control $C_y(t)$, which minimizes $Q(T)$ (1) under the following restrictions:

$$n_\sigma = \sqrt{C_{(x)}^2 + C_y^2}\dot{q}\frac{S}{G} \leq N, \quad q = \frac{\rho V^2}{2}, \quad G = mg, \tag{2}$$

$$C_y^{\min} \leq C_y \leq C_y^{\max}, \quad C_x = C_{x0} + kC_y^2, \tag{3}$$

$$\rho = \rho_0 e^{-\beta H}, \quad g = g_0\frac{R^2}{(R+H)^2}, \quad \dot{V} - C_x q\frac{S}{m} - g\sin\theta \tag{4}$$

$$\dot{\theta} = C_y q\frac{S}{mV} + \left(\frac{V}{R+H} - \frac{g}{V}\right)\cos\theta, \dot{H} = V\sin\theta \tag{5}$$

$$\dot{L} = \frac{RV\cos\theta}{R+H} \tag{6}$$

where n_σ- full overload, q - speed pressure, ρ - atmospheric density, V- velocity of the vehicle, θ-path angle, $H-$ height, L - the remote, G- the weight of the machine, $m - -$ mass, g_0 - acceleration due to gravity on the surface of the planet, R - the radius of the planet, C_x- the drag coefficient, C_y-lift coefficient, S- characteristic area apparatus, C_{x0}, k, ρ_0, β, $C_y^{\min}$, $C_y^{\max}$, N - constants.

For the system (1) - (5) the initial conditions:

$$V(0) = V_0, \quad \theta(0) = \theta_0, \quad H(0) = H_0, \quad L(0) = L_0, \quad Q(0) = 0 \tag{7}$$

and conditions and limitations:

$$L(T) = a, \quad V(T) = V_1, \quad \theta(T) = \theta_1, \quad H(T) = H_1, \quad T - \text{not fixed.} \tag{8}$$

where a – parameter.

2. APPLICATION OF MAXIMUM PRINCIPLE IN THE REGULAR CASE

Let lander come from the initial state (7) in a washed-position (8) in an optimal way in the sense of minimizing the total amount of heat under the assumption of optimal trajectory regularity condition [1,2]. In the above problem, the regularity condition is equivalent to

$$\frac{\partial n_\sigma}{\partial C_y} \neq 0, n_\sigma = N \tag{9}$$

In this case, the maximum principle is as follows:

$$\Pi = P_\theta \dot{\theta} + P_H \dot{H} + P_V \dot{V} + P_L \dot{L} + P_Q \dot{Q}, \quad L_1 = \Pi = -\lambda(t)(n_\sigma - N) \tag{10}$$

$$\overset{\bullet}{P_\theta} = -\frac{\partial \Pi}{\partial \theta}, \quad \overset{\bullet}{P_V} = -\frac{\partial \Pi}{\partial V}, \quad \overset{\bullet}{P_H} = -\frac{\partial \Pi}{\partial H}, \quad \overset{\bullet}{P_L} = -\frac{\partial \Pi}{\partial L}, \quad \overset{\bullet}{P_Q} = -\frac{\partial \Pi}{\partial Q}. \tag{11}$$

Here $\lambda(t)$- the Lagrange multiplier, which is determined from the condition of Bliss [1,2].

$$\frac{\partial \Pi}{\partial C_y} - \lambda(t) \frac{\partial n_\sigma}{\partial C_y} = 0 \tag{12}$$

$\Pi-$Pontryagin function, L_1 - Lagrange function.

P_θ, P_V, P_H, P_L, P_Q - corresponding conjugate variables. For inequality constraints (2) satisfies the complementary slackness.

$$\lambda(t)(n_\sigma - N) = 0 \tag{13}$$

Since the system (1) - (6) is autonomous and there is no descent of restrictions, the Pontryagin function (10) is identically zero, i.e.

$$\Pi(P, x, u) \equiv 0, \quad u = C_y, \quad x = (\theta, V, H_y, L), \quad P = (P_\theta, P_V, P_H, P_L, P_q) \tag{14}$$

Conjugate variable $P_Q(t)$ normalized by the condition

$$P_Q(t) \equiv -1. \tag{15}$$

The initial conditions for the system (11) are unknown parameters of the problem. Condition $P_Q(t) \equiv -1$ and $\Pi(P, x, u) \equiv 0$ is essentially determined by three free parameters

$$P_\theta(0) = C_1, \quad P_V(0) = C_2, \quad P_L(0) = C_3 \tag{16}$$

since $P_H(0)$ is determined from the condition $\Pi(P, x, u) \equiv 0$.

In this case, the number of controlled functions at the end of the trajectory (8) coincides with the number of free parameters of the problem (1) - (8), (10), (11), because the time T is not fixed and is a free parameter.

According to the principle of maximum control program chosen from the condition:

$$\Pi \to \max_{C_y} \ \text{ while } \ Q(T) \to \min \tag{17}$$

The part Pontryagin function (10) can be written down, which clearly depends on the control $C_y\,(t)$.

$$\Pi_0 = P_\theta \frac{C_y \rho V S}{2m} - P_V \frac{C_x \rho V^2 S}{2m} \tag{18}$$

$C_y(t)$ can take control of not only limit values (3), but also an intermediate, which is determined from the condition

$$\frac{\partial \Pi_0}{\partial C_y} = 0, \quad C_y^* = \frac{P_\theta}{2kP_V V}, \quad C_y^{\min} < C_y^* < C_y^{\max} \tag{19}$$

Three values of the function Π_0 are calculated in (18)

$$\Pi_1 = \Pi_0\left(C_y^{\min}\right), \quad \Pi_2 = \Pi_0\left(C_y^{\max}\right), \quad \Pi_3 = \Pi_0\left(C_y^*\right)$$

and

$$\Pi_0^{\max} = \max\left\{\Pi_1,\, \Pi_2,\, \Pi_3\right\} \tag{20}$$

Equation (20) determines the nature of the optimal control problem of Pontryagin, i.e. provided that $n_\sigma \leq N$. Solution to the problem is greatly simplified if the right end of the trajectory is controlled by the condition

$$H\,(T) = H_1 \tag{21}$$

In this case, the solution to (1) - (8) is determined by the boundary conditions

$$\theta\,(T) = \theta_1, \quad V(T) = V_1, \quad L\,(T) = a \tag{22}$$

and depends on three arbitrary constants C_1, C_2 and C_3.

Thus, the initial problem is reduced to a three-parameter problem (1) - (8), (16), (11), (22), and the optimal control $C_y\,(t)$ is determined at each point t of the maximum principle (22).

3. RESTRICTION ON OVERLOAD

The task difficulty of determining the geometry of optimal trajectory is the identification of points coming off the disabled $n_\sigma = N$.

Note that the total overload (2) has two components n_x and n_y. The first is called a longitudinal overload, and the second - normal.

$$n_y = \frac{\rho V^2 S}{2mg_0} C_y, \quad n_x = \frac{\rho V^2 S}{2mg_0} C_x, \quad n_\sigma = \sqrt{n_x^2 + n_y^2}. \tag{23}$$

Instead of limiting (2), a new restriction is introduced

$$|n_y| + n_x \leq N_1\,, \qquad |n_y| + n_x - N_1 = \varphi\,(x, u) \leq 0 \tag{24}$$

With an appropriate choice N_1 of the inequality (24) is known to be satisfied constraint (2). This fact follows from

$$N_1 \geq [|n_y| + |n_x|] \geq \sqrt{n_x^2 + n_y^2} \tag{25}$$

equal sign occurs when $C_y = 0$.

Now, it is to compute the derivative of $\varphi(x, u)$ (24) following C_y

$$\frac{\partial \varphi}{\partial C_y} = \frac{\rho V^2 S}{2 m g_0} [sign C_y + 2k C_y] \tag{26}$$

In this case, the Lagrange multiplier $\lambda(t)$ for limiting $\varphi(x, u) \leq 0$ (24) is determined by the formula

$$\lambda(t) = \frac{2\left(\frac{P_\theta}{2} - k P_V C_y V\right) g_0}{V [sign C_y + 2k C_y]} \tag{27}$$

4. NECESSARY OPTIMALITY CONDITIONS IN THE IRREGULAR CASE

Now consider the case when the optimal trajectory contains an interval, when $n_\sigma = N$ and in this interval at some point $\frac{\partial n_\sigma}{\partial C_y} = 0$.

The set of points defined by the equations

$$\frac{\partial n_\sigma}{\partial C_y} = 0, \quad n_\sigma = N \tag{28}$$

following [1], it is to call irregular points. For the problem $\frac{\partial n_\sigma}{\partial C_y} = 0$ at $C_y = 0$. For the given problem the results of of A. I. Dubovitskij and A. A. Miliutin [1,2] can be used. According to Refs. [1,2] in the presence of irregular points, conjugate system of equations is

$$\begin{aligned}
\dot{P}_\theta &= -\frac{\partial}{\partial \theta} \\
\dot{P}_H &= -\frac{\partial}{\partial H} + \lambda(t)\frac{\partial n_\sigma}{\partial H} + \frac{d\mu}{dt}\frac{\partial n_\sigma}{\partial H}, \\
\dot{P}_V &= -\frac{\partial}{\partial V} + \lambda(t)\frac{\partial n_\sigma}{\partial V} + \frac{d\mu}{dt}\frac{\partial n_\sigma}{\partial V}, \\
\dot{P}_L &= 0, \\
\dot{P}_Q &= 0.
\end{aligned} \tag{29}$$

Here - $\lambda(t)$ Lagrange multiplier - a $\frac{d\mu}{dt}$generalized function. For these objects, complementary slackness condition is made

$$\lambda(t)(n_\sigma - N) = 0, C_y \frac{d\mu}{dt} = 0. \tag{30}$$

From (29) it follows that in the irregular point (28) and the conjugate variables will experience racing on the values of $\mu \frac{\partial n_\sigma}{\partial H}$ and $\mu \frac{\partial n_\sigma}{\partial V}$ when $\mu > 0$. This is the essential difference between the case of irregular regular, where the conjugate variables are continuous functions for mixed class constraints [1,2].

Besides the conditions (28) - (30) the optimal trajectory should be the conditions of integrability of the Lagrange multipliers and the normalization condition (non-triviality condition of the maximum principle).

5. REGULARIZATION DEGENERATE OF THE MAXIMUM PRINCIPLE

One of the possible ways of constructing a nondegenerate optimal trajectory is to change the structure of restriction (28). Limitation (29) has been used previously for sustainable iterative search for the optimal trajectory for small $C_y(t)$. This Lagrange multiplier is calculated by the formula (27). Changes in the structure of mixed constraints (23) do not impose additional requirements on the function $P_\theta(t)$ in an irregular point $(P_\theta(t_*) = 0)$. However, to continue the path through the point t_* is necessary to satisfy the condition $q^\bullet(t_*) = 0$. As a result, there are three conditions on an irregular optimal trajectory

$$\dot{q}(t_*) = 0, \qquad P_V(T) = 0, \qquad P_\theta(T) = 0. \tag{31}$$

that can be performed by selecting the jumps conjugate variables of the form (16) and arbitrary constants $P_V(0)$, $P_\theta(0)$.

With this approach, a non-degenerate maximum principle throughout the optimal trajectory can be gotten.

The presence of several irregular points also leads to degeneration of the maximum principle, however, complicates the search for the optimal trajectory.

Let us now consider another approach to the construction of a non-degenerate maximum principle. For this purpose, the construction of Pontryagin function (10) value $P_\theta \frac{C_y \rho V S}{2m}$ considers a small parameter at sufficiently small $C_y(t)$. Then the expression for the Lagrange multiplier $\lambda(t)$ takes the form:

$$\lambda(t) = -\frac{2kP_V}{1 + 2kC_x} g_0 \sqrt{C_x^2 + C_y^2} \tag{32}$$

In this case, the integrability conditions $\lambda(t)$ are performed automatically.

The result is a non-degenerate maximum principle with irregular points. In addition, the expression (32) allows for a steady iterative search for the optimal trajectory for small.

Another way of regularization of the degenerate maximum principle is mentioned. Suppose that the optimal trajectory condition $n_\sigma = N$ (2), then it yields

$$\frac{1}{2} \ln\left(C_x^2 + C_y^2\right) + \ln \frac{\rho V^2 S}{2mg_0} = \ln N \tag{33}$$

Now consider separately the members of (33), which are associated with the management of

$$\begin{aligned}
\frac{1}{2} \ln\left(C_x^2 + C_y^2\right) &= \ln\left[(C_x + C_y)^2 - 2C_y C_x\right] \\
&= \frac{1}{2} \ln (C_x + C_y)^2 \left[1 - \frac{2C_y C_x}{(C_y + C_x)^2}\right] \\
&= \ln (C_x + C_y) + \frac{1}{2} \ln \left[1 - \frac{2C_y C_x}{(C_y + C_x)^2}\right]
\end{aligned}$$

These expressions do not have singularities at $C_y = 0$. This means that in this case a Lagrange multiplier for the constraint (33) will be finite. Thus, the irregular point does not impose any restrictions on the conjugate variables $P_\theta(t)$. The result is a non-degenerate maximum principle.

6. EXAMPLE AND NUMERICAL RESULT

For more details of the problem, it is to solve the problem of finding the minimum maximum total heating of space shuttle [3, 5], constants and the boundary conditions are:

$C_y^{min} = -0.5$; $C_y^{max} = 0.6$; $\frac{S}{m} = 50000$; $\left[\text{km}^2\text{kg}^{-1}\right]$; $\rho_0 = 2.3769 \times 10^{-3}$

$R = 6371.2$ [km] ; $C_{x0} = 0.88$; $k = 0.5$; $g_0 = 9.8 \times 10^{-3}$ [kms^{-2}]

$C = 20$; $N = 4$; $\beta = 0.145$; $\theta\,(0) = -1.25$ [deg] ; $V(0) = 0.35$ [km] ;

$H(0) = 100$ [km]; $L(0) = 0$ [km]; $Q(0) = 0$;

The result received the following number:

Figure 1 illustrates the shuttle's altitude over time, it is seen that the height H decreases rapidly from100 km down to 40 km over a period [0, 200 s]. Figure 2 shows the velocity of the shuttle which also drops significantly during this period..

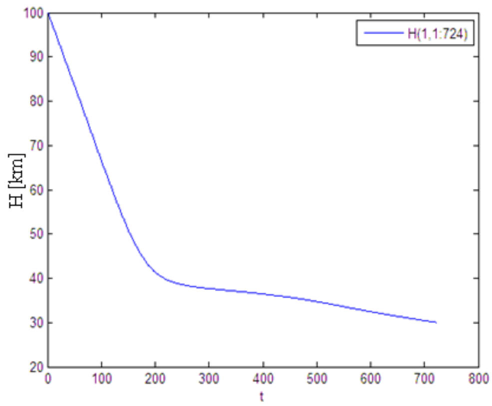

Figure 1: Height H [km]

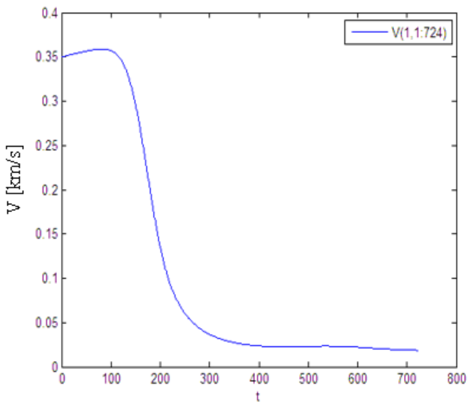

Figure 2: Velocity V[km/s]

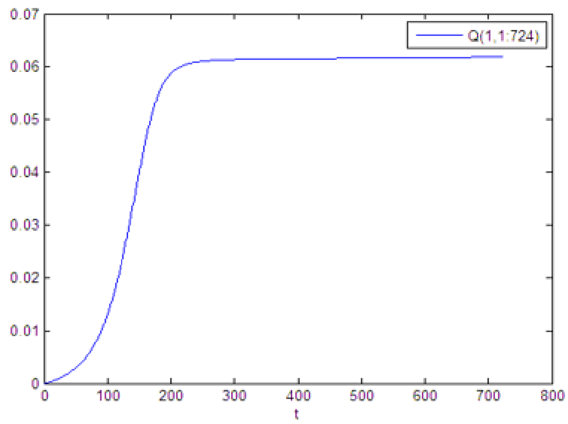

Figure 3: Total heat $Q(t)$

In Figure 3, in the interval [0, 200 s] the total amount of surface temperature increases and stabilizes the ship during the period close to landing [200-720s]. According to simulations, the heat at the surface of the vessel can be considered to have been minimized during landing.

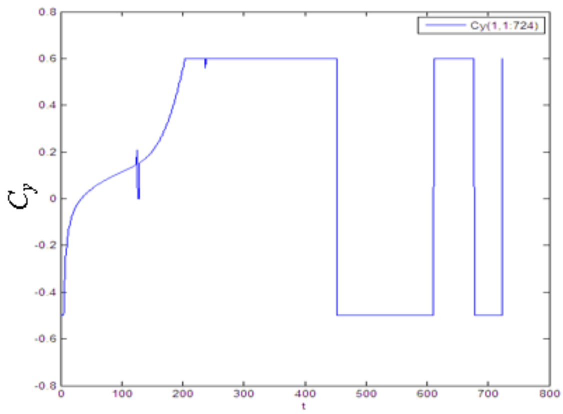

Figure 4: Lift coefficient $C_y(t)$

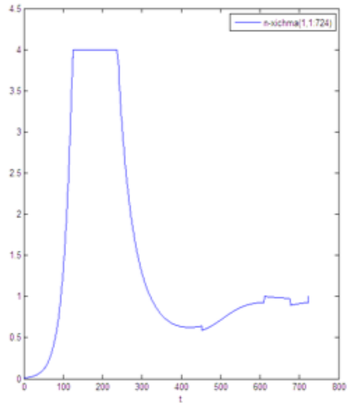

Figure 5: Full overload n_σ

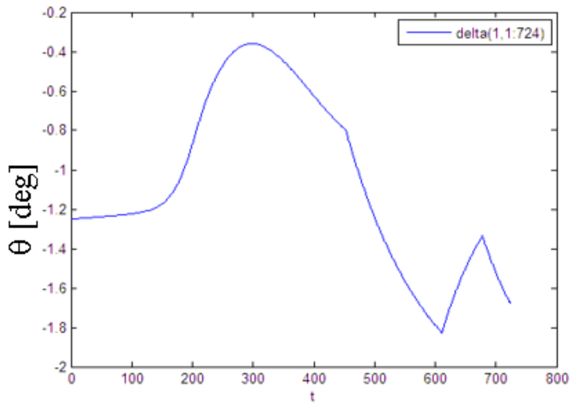

Figure 6: Path angle θ

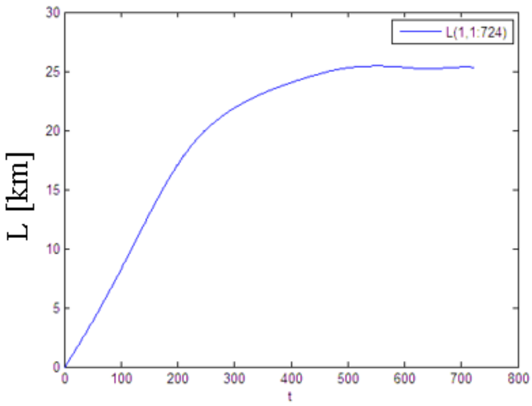

Figure 7: Remote L [km]

Figure 4 illustrates the state of the control variables changing over time and Figure 7 shows the remote of the space shuttle.

7. CONCLUSIONS

This paper solves the problem of minimizing the total heating (1) with the constraints (2) - (8) by using the Pontryagin maximum principle and Dubovistkij Miliutin system and the result is illustrated by numerical solution of software Matlab. Three parameter boundary value (22), the problem is solved for a fixed value a. Next, the desired value a can be chosen from the minimum of the minimum value of the functional (1). The boundary value problem is solved by the continuation of solutions to the parameter [4].

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