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SOME INTUITIONISTIC LINGUISTIC AGGREGATION OPERATORS

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Tóm tắt. Gộp thông tin là một yêu cầu xảy ra hằng ngày. Gộp thông tin bằng từ là một dạng gộp khi các thông tin đầu vào được cho dưới dạng nhãn ngôn ngữ. Việc sử dụng các nhãn ngôn ngữ xuất phát từ bản chất của thông tin (thông tin là ngôn ngữ), cũng có thể từ thói quen dùng ngôn ngữ để đánh giá của chuyên gia. Trong bài báo này, chúng tôi lần đầu đưa ra khái niệm nhãn ngôn ngữ trực cảm. Khái niệm này sẽ có lợi khi gộp các thông tin cho dưới dạng một cặp nhãn ngôn ngữ, trong đó, nhãn đầu tiên diễn tả độ thuộc, nhãn còn lại diễn tả độ không thuộc như trong lý thuyết tập mờ trực cảm [1, 2]. Sau đó, vài toán tử tích hợp thông tin ngôn ngữ trực cảm được giới thiệu.

Từ khóa. Phép gộp ngôn ngữ, tập mờ trực cảm.

Abstract. Information aggregation is a usual task in human activity. Linguistic aggregation operators are used to aggregate information given in terms of linguistic labels. The use of linguistic labels has been posed due to the nature of the information or the habit of experts when they give assessments. In this paper, the notion of intuitionistic linguistic label is first introduced. This notion may be useful in situations when evaluations of experts are presented as two labels such that the first expresses the degree of membership, and the second expresses the degree of non-membership as in the intuitionistic fuzzy theory [1, 2]. Some intuitionistic linguistic aggregation operators are also proposed.

Keywords. Linguistic aggregation operator, intuitionistic fuzzy set.

1. INTRODUCTION

Group decision making (GDM) has played an important role in daily activities, such as economic, engineering, education, medical, etc. In GDM, one of the problems involves gathering many sources of information, giving the final result via aggregating process. Due to the nature of the information or the habit of experts when they give assessments, information could be given as linguistic labels. Many aggregation operators and linguistic aggregation procedures in GDM problems were presented (see [13] for an overview). In this paper, the novel notion of intuitionistic linguistic label, which inherits ideas of intuitionistic fuzzy set and linguistic label, is first introduced. Then, some linguistic aggregations are presented in intuitionistic linguistic environment.

In this section, a short overview of linguistic aggregation operators and intuitionistic fuzzy sets are presented.

1.1. Linguistic Aggregation Operators

In many problems, the information about quality, comforts, suitability, efficiency, etc., of objects may be given as linguistic labels [3, 6, 9]. For example, the comforts of a car can be evaluated using linguistic labels: poor, fair, good, etc. The set of linguistic labels can be constructed depending on the characteristic real word problems. However, it generally contains an odd number of linguistic labels (7 and 9 for example). The set of linguistic labels is theoretically given by $S = \{s_1, s_2, \ldots, s_n\}$, where the odd number n is the cardinality of S, s_i is a possible value of linguistic evaluation in some situations. The set S is equipped an order relation and a negation operator [9]:

$$s_i \geqslant s_j \Leftrightarrow i \geqslant j;$$

$$neg\,(s_i) = s_j \Leftrightarrow j + i = n + 1.$$

Linguistic aggregation operators are including [16]: linear order based linguistic aggregation operators, extension principle and symbols based linguistic aggregation operators, linguistic 2-tuple based linguistic aggregation operators, linguistic aggregation operators computing with words directly.

In this paper, the linear order based linguistic aggregation operators should be extended to intuitionistic case.

1.2. Linear order based linguistic aggregation operators

Let $\{a_1, a_2, \ldots, a_m\}$ be a collection of linguistic labels, $a_i \in S$, and $\{b_1, b_2, \ldots, b_m\}$ is a permutation of $\{a_1, a_2, \ldots, a_m\}$ yields $b_1 \geqslant b_2 \geqslant ... \geqslant b_m$. Yager et al. [18-20] introduced some simple linguistic aggregation operators:

linguistic max operator:$\max(a_1, a_2, \ldots, a_m) = b_1$;

linguistic min operator: $\min(a_1, a_2, \ldots, a_m) = b_m$;

and linguistic median operator: $med\,(a_1, a_2, \ldots, a_m) = \begin{cases} b_{\frac{m+1}{2}} & if\ m\ is\ odd, \\ b_{\frac{m}{2}} & if\ m\ is\ even. \end{cases}$

Using above operators, many other operators were developed for aggregating linguistic information: ordinal ordered weighted averaging operator (Yager [14]), linguistic weighted disjunction and linguistic weighted conjunction operators (Herrera and Herrera-Viedma [10]), hybrid aggregation operators (Xu [15]), etc.

As a similarity of weighted median in statistics, Yager [15, 16, 17] defined weighted median of linguistic labels:

Considering a collection of linguistic labels $\{a_1, a_2, \ldots, a_m\}$, each label a_i has corresponding weight: w_i, $w_i \in [0,1]$, $\sum_{i=1}^{m} w_i = 1$. Such collection is denoted by $\{(w_1, a_1), (w_2, a_2), \ldots, (w_m, a_m)\}$. Assume that $\{(u_1, b_1), (u_2, b_2), \ldots, (u_m, b_m)\}$ is the decreasingly ordered collection of $\{(w_1, a_1), (w_2, a_2), \ldots, (w_m, a_m)\}$, i.e., b_j is the j-th largest of a_i, and u_j is the weight of j-th largest of a_i. Let $T_i = \sum_{j=1}^{i} u_j$ be, the linguistic weighted median (LWM) operator was defined as:

$$LMW\,((w_1, a_1), (w_2, a_2), \ldots, (w_m, a_m)) = b_k,$$

where k is the value such that T_k first crosses 0.5. Yager [15] proved that LWM operator is idempotent, commutative, and monotonous.

1.3. Intuitionistic Fuzzy Set

The intuitionistic fuzzy set first launched by Atatanssov [1] is one of the significant extensions of Zadeh's fuzzy set [20]. An intuitionistic fuzzy set has two components: a membership function and a non-membership function, it is different from fuzzy set which characterized by only a membership function.

Definition 1.1 ([1]) An intuitionistic fuzzy set$A$ on a universe X is an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,|\, x \in X\}$, where $\mu_A(x) \in [0,1]$ is called the "degree of membership of x in A", $\nu_A(x) \in [0,1]$ is called the "degree of non-membership of x in A", and following condition is satisfied

$$\mu_A(x) + \nu_A(x) \leqslant 1, \quad \forall x \in X.$$

Some recent developments of the intuitionistic fuzzy set theory with applications could be found in [4, 5, 7, 10, 11].

2. INTUITIONISTIC LINGUISTIC LABELS

The intuitionistic linguistic label defined below can be seen as a linguistic aspect supplement of intuitionistic fuzzy set. It may be helpful when the information is expressed in terms of pair of labels (s_i, s_j), where s_i represents the degree of membership and s_j the degree of non-membership.

Example 2.1. We recall the intuitionistic approach of De and Biswas in medical diagnosis [9], the correspondences between the set of patients and the set of symptoms were be described via an intuitionistic fuzzy relation as in Table 1 (see [5] for intuitionistic fuzzy relation). It is reasonable and meaningful that we allow experts to use linguistic labels instead of numbers. Such situation raised the need of using linguistic in intuitionistic assessments. Using linguistic label set S containing $s_1 = $ *impossibly*, $s_2 = $ *very unlikely*, $s_3 = $ *less likely*, $s_4 = $ *likely*, $s_5 = $ *more likely*, $s_6 = $ *very likely*, and $s_7 = $ *certainly*, experts' assessments may be given in Table 1 (membership degree of Paul to the set of all patients who have a temperature is assigned to $s_7 = $ *certainly*, non-membership degree of Paul to the set of all patients who have a temperature is assigned to $s_1 = $ *impossibly*).

Q	Temperature	Headache	Stomach pain	Cough	Chest pain
Paul	$(0.8, 0.1)$	$(0.6, 0.1)$	$(0.2, 0.8)$	$(0.6, 0.1)$	$(0.1, 0.6)$
Jadu	$(0, 0.8)$	$(0.4, 0.4)$	$(0.6, 0.1)$	$(0.1, 0.7)$	$(0.1, 0.8)$
Kundu	$(0.8, 0.1)$	$(0.8, 0.1)$	$(0, 0.6)$	$(0.2, 0.7)$	$(0, 0.5)$
Rohit	$(0.6, 0.1)$	$(0.5, 0.4)$	$(0.3, 0.4)$	$(0.7, 0.2)$	$(0.3, 0.4)$

Table 1: Intuitionistic fuzzy relation between patients and symptoms [9]

Moreover, in intuitionistic fuzzy set theory, the membership degree and the non-membership degree of x in the set A ($\mu_A(x)$ and$\nu_A(x)$ respectively) must satisfy $\mu_A(x) + \nu_A(x) \leqslant 1$. This condition can be rewritten as $\mu_A(x) \leqslant neg(\nu_A(x))$, where $neg : [0,1] \to [0,1]$, $x \mapsto 1 - x$. So, we propose that for(s_i, s_j) the condition $s_j \leqslant neg(s_i) = s_{n+1-i}$ should be satisfied. Then, this implies $s_j \leqslant s_{n-i+1}$ or $i + j \leqslant n + 1$.

Q	Temperature	Headache	Stomach pain	Cough	Chest pain
Paul	(s_7, s_1)	(s_6, s_1)	(s_2, s_5)	(s_6, s_1)	(s_1, s_6)
Jadu	(s_1, s_7)	(s_4, s_4)	(s_6, s_1)	(s_1, s_6)	(s_1, s_7)
Kundu	(s_5, s_1)	(s_4, s_1)	(s_1, s_7)	(s_2, s_6)	(s_1, s_4)
Rohit	(s_5, s_1)	(s_5, s_3)	(s_3, s_4)	(s_6, s_1)	(s_2, s_3)

Table 2: Relation between Patients and Symptoms

Definition 2.1. An intuitionistic linguistic label is defined as a pair of linguistic labels $(s_i, s_j) \in S^2$, such results in $i + j \leqslant n + 1$, where $S = \{s_1, s_2, \ldots, s_n\}$ is the linguistic label set, s_i, $s_j \in S$ respectively define the degree of membership and the degree of non-membership of an object in a set.

The set of all intuitionistic linguistic labels is denoted by IS, i.e.

$$IS = \left\{ (s_i, s_j) \in S^2 \;\middle|\; i + j \leqslant n + 1 \right\}.$$

Example 2.2. If the linguistic label set S, which may be used in medical diagnoses, contains s_1 = *impossibly*, s_2 = *very unlikely*, s_3 = *less likely*, s_4 = *likely*, s_5 = *more likely*, s_6 = *very likely* and s_7 = *certainly*; then, the corresponding intuitionistic linguistic label set of IS is given below:

$$\begin{array}{lllllll}
(s_7, s_1) & & & & & & \\
(s_6, s_1) & (s_6, s_2) & & & & & \\
(s_5, s_1) & (s_5, s_2) & (s_5, s_3) & & & & \\
(s_4, s_1) & (s_4, s_2) & (s_4, s_3) & (s_4, s_4) & & & \\
(s_3, s_1) & (s_3, s_2) & (s_3, s_3) & (s_3, s_4) & (s_3, s_5) & & \\
(s_2, s_1) & (s_2, s_2) & (s_2, s_3) & (s_2, s_4) & (s_2, s_5) & (s_2, s_6) & \\
(s_1, s_1) & (s_1, s_2) & (s_1, s_3) & (s_1, s_4) & (s_1, s_5) & (s_1, s_6) & (s_1, s_7)
\end{array}$$

3. ORDER RELATIONS ON IS

In order to define the linear order based intuitionistic linguistic aggregation operators, it is necessary to define order relations on the IS set.

Let A, B be an intuitionistic fuzzy set on X, relation $A \supset B$ is defined as [1, 2]:

$$A \supset B \Leftrightarrow (\forall x \in X)\left(\mu_A(x) \geqslant \mu_A(x) \;\&\; \nu_A(x) \leqslant \nu_A(x)\right).$$

Order relation on two intuitionistic linguistic labels (μ_1, ν_1), (μ_2, ν_2) can be defined similarly to "$\supset$" relation of intuitionistic fuzzy sets:

$$(\mu_1, \nu_1) \geqslant (\mu_2, \nu_2) \Leftrightarrow \mu_1 \geqslant \mu_2 \text{ and } \nu_1 \leqslant \nu_2, \tag{1}$$

where (μ_1, ν_1), (μ_2, ν_2) are intuitionistic linguistic labels.

It is easily seen that there are intuitionistic linguistic labels which cannot be compared by this relation (for example (s_1, s_5) and (s_2, s_6)). However, when comparing two intuitionistic linguistic labels, first we can compare two membership degrees, then two non-membership degrees, vice versa. Then, we can define two order relations on IS as following definition.

Definition 3.1. For all of (μ_1, ν_1), (μ_2, ν_2) on IS, membership based order relation $\geqslant_M$ and non-membership based order relation $\geqslant_N$ are defined as the following:

$$(\mu_1, \nu_1) \geqslant_M (\mu_2, \nu_2) \Leftrightarrow \mu_1 > \mu_2 \text{ OR } (\mu_1 = \mu_2 \;\;\&\;\; \nu_1 \leqslant \nu_2)\,;$$

$$(\mu_1, \nu_1) \geqslant_N (\mu_2, \nu_2) \Leftrightarrow \nu_1 < \nu_2 \text{ OR } (\nu_1 = \nu_2 \;\;\&\;\; \mu_1 \geqslant \mu_2)\,.$$

Theorem 3.1. *$\geqslant_M$ and $\geqslant_N$ are total orders.*

Proof. Let's consider $\geqslant_M$. It is easily seen that $\geqslant_M$ is reflexive. Now we consider the anti-symmetry, transitivity and totality. Let (μ_1, ν_1), (μ_2, ν_2), (μ_3, ν_3) be arbitrary intuitionistic linguistic labels, we obtain:

$$\text{Anti-symmetry: } \begin{cases} (\mu_1, \nu_1) \geqslant_M (\mu_2, \nu_2) \\ (\mu_2, \nu_2) \geqslant_M (\mu_1, \nu_1) \end{cases} \Leftrightarrow \left[\begin{array}{l} \mu_1 > \mu_2 \\ \begin{cases} \mu_1 = \mu_2 \\ \nu_1 \leqslant \nu_2 \end{cases} \end{array} \right. \;\;\&\;\; \left[\begin{array}{l} \mu_2 > \mu_1 \\ \begin{cases} \mu_2 = \mu_1 \\ \nu_2 \leqslant \nu_1 \end{cases} \end{array} \right.$$

$$\Leftrightarrow \underbrace{\begin{cases} \mu_1 > \mu_2 \\ \mu_2 > \mu_1 \end{cases}}_{false} \text{ OR } \underbrace{\begin{cases} \mu_1 > \mu_1 \\ \mu_2 = \mu_1 \\ \nu_2 \leqslant \nu_1 \end{cases}}_{false} \text{ OR } \underbrace{\begin{cases} \mu_2 > \mu_1 \\ \mu_1 = \mu_2 \\ \nu_1 \leqslant \nu_2 \end{cases}}_{false} \text{ OR } \begin{cases} \mu_1 = \mu_2 \\ \nu_1 \leqslant \nu_2 \\ \nu_2 \leqslant \nu_1 \end{cases} \Leftrightarrow \begin{cases} \mu_1 = \mu_2 \\ \nu_1 = \nu_2 \end{cases}$$

$$\Leftrightarrow (\mu_1, \nu_1) = (\mu_2, \nu_2)\,.$$

$$\text{Transitivity : } \begin{cases} (\mu_1, \nu_1) \geqslant_M (\mu_2, \nu_2) \\ (\mu_2, \nu_2) \geqslant_M (\mu_3, \nu_3) \end{cases} \Leftrightarrow \left[\begin{array}{l} \mu_1 > \mu_2 \\ \begin{cases} \mu_1 = \mu_2 \\ \nu_1 \leqslant \nu_2 \end{cases} \end{array} \right. \;\;\&\;\; \left[\begin{array}{l} \mu_2 > \mu_3 \\ \begin{cases} \mu_2 = \mu_3 \\ \nu_2 \leqslant \nu_3 \end{cases} \end{array} \right.$$

$$\Leftrightarrow \begin{cases} \mu_1 > \mu_2 \\ \mu_2 > \mu_3 \end{cases} \text{ OR } \begin{cases} \mu_1 > \mu_2 \\ \mu_2 = \mu_3 \\ \nu_2 \leqslant \nu_3 \end{cases} \text{ OR } \begin{cases} \mu_2 > \mu_3 \\ \mu_1 = \mu_2 \\ \nu_1 \leqslant \nu_2 \end{cases} \text{ OR } \begin{cases} \mu_1 = \mu_2 \\ \nu_1 \leqslant \nu_2 \\ \mu_2 = \mu_3 \\ \nu_2 \leqslant \nu_3 \end{cases}$$

$$\Rightarrow \mu_1 > \mu_3 \text{ OR } \begin{cases} \mu_1 = \mu_3 \\ \nu_1 \leqslant \nu_3 \end{cases} \Leftrightarrow (\mu_1, \nu_1) \geqslant_M (\mu_3, \nu_3)\,.$$

Totality: If $\mu_1 > \mu_2$, then $(\mu_1, \nu_1) \geqslant_M (\mu_2, \nu_2)$. If $\mu_1 < \mu_2$, then $(\mu_2, \nu_2) \geqslant_M (\mu_1, \nu_1)$.
If $\mu_1 = \mu_2$, then there are following cases:
Case 1. If $\nu_1 \leqslant \nu_2$, then $(\mu_1, \nu_1) \geqslant_M (\mu_2, \nu_2)$.
Case 2. If $\nu_1 > \nu_2$, then $(\mu_2, \nu_2) \geqslant_M (\mu_1, \nu_1)$.
So, $\geqslant_M$ is a total order. Similarly, $\geqslant_N$ is also a total order.

In the following, the relationship between $\geqslant$, $\geqslant_M$ and $\geqslant_N$ is explored. For convenience, in each $A = (s_i, s_j) \in IS$, s_i and s_j are respectively denoted by μ_A, ν_A.

Theorem 3.2. *For all $A, B \in IS$, we obtain*

$$A \geqslant B \Leftrightarrow A \geqslant_M B \;\;\&\;\; B \geqslant_N A \quad ,$$

where $\geqslant$ is defined as (1).

Proof. If $A \geqslant B$, then

$$\begin{cases} \mu_A \geqslant \mu_B \\ \nu_A \leqslant \nu_B \end{cases} \Rightarrow \mu_A > \mu_B \text{ OR } \begin{cases} \mu_A = \mu_B \\ \nu_A \leqslant \nu_B \end{cases} \Rightarrow A \geqslant_M B;$$

$$\begin{cases} \nu_A \leqslant \nu_B \\ \mu_A \geqslant \mu_B \end{cases} \Rightarrow \nu_A < \nu_B \text{ OR } \begin{cases} \nu_A = \nu_B \\ \mu_A \geqslant \mu_B \end{cases} \Rightarrow A \geqslant_N B \quad .$$

We assume that $A \geqslant_M B$ and $B \geqslant_N A$. Then,

$$\left[\begin{array}{l} \mu_A > \mu_B \\ \begin{cases} \mu_A = \mu_B \\ \nu_A \leqslant \nu_B \end{cases} \end{array} \right. \quad \& \quad \left[\begin{array}{l} \nu_B > \nu_A \\ \begin{cases} \nu_A = \nu_B \\ \mu_A \geqslant \mu_B \end{cases} \end{array} \right.$$

$$\Leftrightarrow \begin{cases} \mu_A > \mu_B \\ \nu_B > \nu_A \end{cases} \text{OR} \begin{cases} \mu_A > \mu_B \\ \nu_A = \nu_B \end{cases} \text{OR} \begin{cases} \mu_A = \mu_B \\ \nu_B > \nu_A \end{cases} \text{OR} \begin{cases} \mu_A = \mu_B \\ \nu_A = \nu_B \end{cases}$$

$$\Leftrightarrow \begin{cases} \mu_A > \mu_B \\ \nu_B \geqslant \nu_A \end{cases} \text{OR} \begin{cases} \mu_A = \mu_B \\ \nu_B \geqslant \nu_A \end{cases} \Rightarrow A \geqslant B.$$

4. LINEAR ORDERING BASED INTUITIONISTIC LINGUISTIC AGGREGATION OPERATORS

4.1. Definitions

Definition 4.1. Let $\{A_1, A_2, \ldots, A_m\}$ be a collection of intuitionistic linguistic labels on IS, and $\{B_1, B_2, \ldots, B_m\}$ be a permutation of $\{A_1, A_2, \ldots, A_m\}$, such yields $B_1 \geqslant_M B_2 \geqslant_M \cdots \geqslant_M B_m$.

- Membership based intuitionistic linguistic max and min operators are determined as

  $\max_M (A_1, A_2, \ldots, A_m) = B_1$and $\min_M (A_1, A_2, \ldots, A_m) = B_m$.

- Membership based intuitionistic linguistic median operator is determined as

$$med_M (A_1, A_2, \ldots, A_m) = \begin{cases} B_{\frac{m+1}{2}} & if\ m\ is\ odd, \\ B_{\frac{m}{2}} & if\ m\ is\ even. \end{cases}$$

- Membership based intuitionistic linguistic weighted median: The collection of $\{(w_1, A_1),$ $(w_2, A_2), ..., (w_m, A_m)\}$ is considered, where A_i is an intuitionistic linguistic label, and w_i is its associated weight, $w_i \in [0, 1]$, $\sum_{i=1}^{m} w_i = 1$. We assume that $\{(u_1, B_1), (u_2, B_2),$ $..., (u_m, B_m)\}$ is the ordered collection of $\{(w_1, A_1), (w_2, A_2), ..., (w_m, A_m)\}$, where B_j is the j-th largest of the A_i , and u_j is the weight of the j-th largest of A_i. Let $T_i = \sum_{j=1}^{i} u_j$, membership based intuitionistic linguistic weighted median ($ILWM_M$) operator was defined as

$$ILWM_M ((w_1, A_1), (w_2, A_2), ..., (w_m, A_m)) = B_k,$$

  where k is the value such that T_k first crosses 0.5.

Similarly, notions of non-membership based intuitionistic linguistic max, min, median and weighted median ($\max_N$, $\min_N$, med_N, $ILWM_N$) are also given:

$\{B_1, B_2, \ldots, B_m\}$ is a permutation of $\{A_1, A_2, \ldots, A_m\}$, such result in $B_1 \geqslant_N B_2 \geqslant_N \ldots \geqslant_N B_m$.

$\max_N(A_1, A_2, \ldots, A_m) = B_1$ and $\min_N(A_1, A_2, \ldots, A_m) = B_m$.

$$med_N(A_1, A_2, \ldots, A_m) = \begin{cases} B_{\frac{m+1}{2}} & if\ m\ is\ odd, \\ B_{\frac{m}{2}} & if\ m\ is\ even. \end{cases}$$

We assume that u_j is the weight of the j-th largest of A_i. Let $T_i = \sum_{j=1}^{i} u_j$.

$$ILWM_N((w_1, A_1), (w_2, A_2), \ldots, (w_m, A_m)) = B_k,$$

where k is the value such that T_k first crosses 0.5.

Example 4.1. Considering $S = \{s_1, s_2, \ldots, s_9\}$, $A_1 = (s_1, s_6)$, $A_2 = (s_2, s_7)$, $A_3 = (s_5, s_4)$, $A_4 = (s_7, s_3)$ and $A_5 = (s_4, s_2)$. We obtain

- $_4 >_M A_3 >_M A_5 >_M A_2 >_M A_1$ and $A_5 >_N A_4 >_N A_3 >_N A_1 >_N A_2$.
- $\max_M(A_1, A_2, A_3, A_4, A_5) = A_4$ and $\min_M(A_1, A_2, A_3, A_4, A_5) = A_1$.
  $\max_N(A_1, A_2, A_3, A_4, A_5) = A_5$ and $\min_N(A_1, A_2, A_3, A_4, A_5) = A_2$.
- $med_M(A_1, A_2, A_3, A_4, A_5) = A_5$ and $med_N(A_1, A_2, A_3, A_4, A_5) = A_3$.
- Considering $w_1 = 0.15$, $w_2 = 0.34$, $w_3 = 0.25$, $w_4 = 0.12$, $w_5 = 0.14$.

  If order relation is $\geqslant_M$, we obtain

j	B_j	u_j	T_j
1	A_4	$w_4 = 0.12$	0.12
2	A_3	$w_3 = 0.25$	0.37
3	A_5	$w_5 = 0.14$	0.51
4	A_2	$w_2 = 0.34$	
5	A_1	$w_1 = 0.15$	

  So, $ILWM_M((w_1, A_1), (w_2, A_2), (w_3, A_3), (w_4, A_4), (w_5, A_5)) = A_3$.

  Similarly, if order relation is $\geqslant_N$, we obtain

$$ILWM_N((w_1, A_1), (w_2, A_2), (w_3, A_3), (w_4, A_4), (w_5, A_5)) = A_3.$$

Remark 4.1. The application of $\max_M$, $\min_M$, med_M, $ILWM_M$or $\max_N$, $\min_N$, med_N, $ILWM_N$ may obtain different results from each other. Subject to more due attention to membership or non-membership degree of assessment, the first or second group of operator is respectively proposed to use.

4.2. Some properties

The following theorem gives an efficient method for calculating with operators $\max_M$ and $\min_M$.

Theorem 4.1. *Let $\{A_1, A_2, \ldots, A_m\}$ be a collection of intuitionistic linguistic arguments on IS. The following properties yield:*

$$\max_M (A_1, A_2, \ldots, A_m) = \left(\max(\mu_{A_1}, \mu_{A_2}, \ldots, \mu_{A_m}), \min_{i:\mu_{A_i}=\max(\mu_{A_1},\mu_{A_2},\ldots,\mu_{A_m})} \{\nu_{A_i}\} \right),$$

$$\min_M (A_1, A_2, \ldots, A_m) = \left(\min(\mu_{A_1}, \mu_{A_2}, \ldots, \mu_{A_m}), \max_{i:\mu_{A_i}=\min(\mu_{A_1},\mu_{A_2},\ldots,\mu_{A_m})} \{\nu_{A_i}\} \right).$$

Proof. It is easily seen that

$$\left(\max(\mu_{A_1}, \mu_{A_2}, \ldots, \mu_{A_m}), \min_{i:\mu_{A_i}=\max(\mu_{A_1},\mu_{A_2},\ldots,\mu_{A_m})} \{\nu_{A_i}\} \right) \in \{A_1, A_2, \ldots, A_m\}.$$

For each $A_j \in \{A_1, A_2, \ldots, A_m\}$, there are two following cases:
If $\mu_{A_j} < \max(\mu_{A_1}, \mu_{A_2}, \ldots, \mu_{A_m})$, then

$$A_j <_M \left(\max(\mu_{A_1}, \mu_{A_2}, \ldots, \mu_{A_m}), \min_{i:\mu_{A_i}=\max(\mu_{A_1},\mu_{A_2},\ldots,\mu_{A_m})} \{\nu_{A_i}\} \right).$$

If $\mu_{A_j} = \max(\mu_{A_1}, \mu_{A_2}, \ldots, \mu_{A_m})$, then $\nu_{A_j} \geqslant \min_{i:\mu_{A_i}=\max(\mu_{A_1},\mu_{A_2},\ldots,\mu_{A_m})} \{\nu_{A_i}\}$. So,

$$A_j \leqslant \left(\max(\mu_{A_1}, \mu_{A_2}, \ldots, \mu_{A_m}), \min_{i:\mu_{A_i}=\max(\mu_{A_1},\mu_{A_2},\ldots,\mu_{A_m})} \{\nu_{A_i}\} \right).$$

Therefore,

$$\max_M (A_1, A_2, ..., A_m) = \left(\max(\mu_{A_1}, \mu_{A_2}, ..., \mu_{A_m}), \min_{i:\mu_{A_i}=\max(\mu_{A_1},\mu_{A_2},\ldots,\mu_{A_m})} \{\nu_{A_i}\} \right).$$

The rest of proof runs as before.

Theorem 4.2. *If $\langle \{A_1, A_2, \ldots, A_m\}, \geqslant \rangle$ has the smallest and largest elements, then*

$$\max_M (A_1, A_2, \ldots, A_m) = \max_N (A_1, A_2, \ldots, A_m) = \max(A_1, A_2, \ldots, A_m),$$

$$\min_M (A_1, A_2, \ldots, A_m) = \min_N (A_1, A_2, \ldots, A_m) = \min(A_1, A_2, \ldots, A_m).$$

Proof. We have $(\max(\mu_{A_1}, \mu_{A_2}, \ldots, \mu_{A_m}), \min(\nu_{A_1}, \nu_{A_2}, \ldots, \nu_{A_m})) = \max(A_1, A_2, \ldots, A_m)$

$$\leqslant_M \max_M (A_1, A_2, \ldots, A_m) = \left(\max(\mu_{A_1}, \mu_{A_2}, \ldots, \mu_{A_m}), \min_{i:\mu_{A_i}=\max(\mu_{A_1},\mu_{A_2},\ldots,\mu_{A_m})} \{\nu_{A_i}\}\right)$$

$$\Rightarrow \min(\nu_{A_1}, \nu_{A_2}, \ldots, \nu_{A_m}) \geqslant \min_{i:\mu_{A_i}=\max(\mu_{A_1},\mu_{A_2},\ldots,\mu_{A_m})} \{\nu_{A_i}\}.$$

On the other hand, $\min(\nu_{A_1}, \nu_{A_2}, \ldots, \nu_{A_m}) \leqslant \min_{i:\mu_{A_i}=\max(\mu_{A_1},\mu_{A_2},\ldots,\mu_{A_m})} \{\nu_{A_i}\}$. Then

$$\min(\nu_{A_1}, \nu_{A_2}, \ldots, \nu_{A_m}) = \min_{i:\mu_{A_i}=\max(\mu_{A_1},\mu_{A_2},\ldots,\mu_{A_m})} \{\nu_{A_i}\}$$

$$\Rightarrow \max_M (A_1, A_2, \ldots, A_m) = \max(A_1, A_2, \ldots, A_m).$$

The remains are similarly proved.

Theorem 4.3. *$ILWM_M$ is idempotent, compensative, commutative and monotonous, i.e.*

- $ILWM_M(A, A, \ldots, A) = A$ *for all* $A \in IS$.
- $\min_M(A_1, A_2, \ldots, A_m) \leqslant ILWM_M(A_1, A_2, \ldots, A_m) \leqslant \max_M(A_1, A_2, \ldots, A_m)$ *for all* $A_1, A_2, \ldots, A_m \in IS$.
- $ILWM_M((w_1, A_1), (w_2, A_2), \ldots, (w_m, A_m)) = ILWM_M\left(\left(w_{\sigma(1)}, A_{\sigma(1)}\right), \left(w_{\sigma(2)}, A_{\sigma(2)}\right), \ldots, \left(w_{\sigma(m)}, A_{\sigma(2)}\right)\right)$, *forall* $A_1, A_2, \ldots, A_m \in IS$, σ *is an arbitrary permutation on the set* $\{1, 2, \ldots, m\}$.
- $ILWM_M((w_1, A_1), (w_2, A_2), \ldots, (w_m, A_m)) \leqslant ILWM_M((w_1, C_1), (w_2, C_2), \ldots, (w_m, C_m))$ *if* $A_i \leqslant C_i$ *for all* $i = 1, 2, \ldots, m$.

Proof. (1), (2) are straightforward. And (3) is implied from the fact that the j-th largest of $\{A_1, A_2, \ldots, A_m\}$ is equal to that of $\left\{A_{\sigma(1)}, A_{\sigma(2)}, \ldots, A_{\sigma(m)}\right\}$. It is easily shown that the j-th largest of $\{A_1, A_2, \ldots, A_m\}$ is smaller or equal to that of $\{C_1, C_2, \ldots, C_m\}$; so, (4) is also proved.

Remark 4.2. $\max_N$, $\min_N$, $ILWM_M$ also have similar properties.

5. CONCLUSIONS

In this paper, the notion of intuitionistic linguistic label is first launched. Some intuitionistic linguistic aggregation operations are also introduced. Besides, some properties of these operators are considered. In future, some new operators should be proposed, and applications in group decision making problems should be presented.

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