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SOME COMMENTS ABOUT "AXIOMATISATION OF FUZZY MULTIVALUED DEPENDENCIES IN A FUZZY RELATIONAL DATA MODEL"

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Abstract. In "Axiomatisation of fuzzy multivalued dependencies in a fuzzy relational data model" [1], Bhattacharjee and Mazumdar have introduced an extension of classical multivalued dependencies for fuzzy relational data models. The authors also proposed a set of sound and complete inference rules to derive more dependencies from a given set of fuzzy multivalued dependencies. We are afraid an important result that was used by the authors to prove the soundness and completeness of the inference rules has been stated incorrectly (Lemma 3.1 [1]). In fact, there are some logically vicious and insufficient reasoning in the proof of the soundness in [1]. This paper aims at correction of the above result (Lemma 3.1), gives a proof of its soundness and by the way, proposes some opinions.

Tóm tắt. Trong bài báo "Axiomatisation of fuzzy multivalued dependencies in a fuzzy relational data model" [1], Bhattacharjee và Mazumdar đã đề xuất một mở rộng của phụ thuộc đa trị cổ điển cho mô hình cơ sở dữ liệu mờ. Các tác giả đã đưa ra một tập luật suy dẫn xác đáng và đầy đủ để có thể dẫn ra thêm các phụ thuộc từ một tập các phụ thuộc đa trị mờ đã được biết. Chúng tôi sợ rằng một kết quả quan trọng mà các tác giả bài báo dùng để chứng minh tính xác đáng và tính đầy đủ của các luật suy dẫn đã được phát biểu chưa chính xác (Bổ đề 3.1 [1]). Chứng minh tính xác đáng của [1] còn chưa đầy đủ và đôi chỗ dường như không chặt chẽ về logic. Trong bài báo này chúng tôi chính xác hóa lại kết quả nói trên và đề xuất một chứng minh cho tính xác đáng, đồng thời nêu một số ý kiến trao đổi thêm.

1. INTRODUCTION

Integrity constraints play a crucial role in logical database design theory. Various types of dependencies such as functional, multivalued, join dependencies, etc... have been studied in the classical relational database literature. These dependencies are used as guidelines for design of a relational schemas, which are conceptually meaningful and are able to avoid certain update anomalies. Inference rule is an important concept, related to data dependencies. A set of rules help the database designers to find other dependencies which are logical consequences of the given dependencies. It is very important that the inference-rules can only be useful if they form a sound and complete data dependencies. This means the generated dependency is valid in all instances in which the given set of inferences are also valid, and all valid dependencies can be generated when only these rules are used.

But the ordinary relation database model introduced by Codd [3] does not handle imprecise, inexact data well. Several of extensions have been brought to the relational model to capture the imprecise parts of the real world. A fuzzy relational data model is an extension of the classical relational model [5]. It is based on the mathematical framework of the fuzzy set theory invented by Zadeh [9]. Several authors have proposed extended dependencies in fuzzy relational data model. A definition of fuzzy multivalued dependencies (FMVDs) is proposed by Bhattacharjee and Mazumdar [1]. The authors have shown that FMVDs are more generalized than classical multivalued dependencies. A set of sound and complete inference rules, similar to Amstrong's axioms is also proposed to derive more dependencies from a given set of FMVDs. The inter-relationship between two-tuple subrelations and the relation, to which they belong, with reference to FMVDs was established. The proof of the inference rules given in [1] is based on this relationship.

This paper is organized as follows. To get an identical understanding of terminology, notations, basic definitions and concepts related to fuzzy relational data model are given, and a few definitions and results from the similarity relation of domain of elements [2, 5] are reviewed in section 2. Section 3 contains all of the main result of [1] in brief. In section 4, by giving out a counterexample, we suppose that Lemma 3.1 in [1] seem to be incorrect. A revised version of this lemma is proposed and proved. Through this correction, several consequential results, such as the completeness of inference axioms are still valid. Then the proof of the soundness of inference axioms is discussed. We can have the soundness directly from the definition of FMVD without the result of Lemma 3.1 in [1].

2. BACKGROUND

First, similarity relations are described as defined by Zadeh [10]. Then a characterization of similarity relation is provided. Finally, the basic concepts of fuzzy relational database model are reviewed.

Similarity relations are useful for describing how similar two elements from the same domain are.

Definition 2.1 [5]. A similarity relation $S_D(x, y)$, for given domain D, is a mapping of every pair of elements in the domain onto the unit interval $[0, 1]$ with the following properties, $x, y, z \in D$:

1. Reflexivvity $S_D(x, x) = 1$
2. Symmetry $S_D(x, y) = S_D(y, x)$
3. Transitivity $S_D(x, z) \geq \text{Max}\left(\text{Min}\left[S_D(x, y),\ S_D(y, z)\right]\right)$ (T1)

(or

3'. Transitivity $S_D(x, z) = \text{Max}\left(\left[S_D(x, y) * S_D(y, z)\right]\right)$ (T2)

where $*$ is arithmetic multiplication).

Theorem 2.1 [5]. *Let D be a set with a transitive similarity relation S_D. Suppose that D contains a certain value r, such that for the two values $y, z \in D$:*

$$S_D(r, y) \neq S_D(r, z).$$

Then the similarity relation is entirely determined, there is only one possible choice for $S_D(y, z)$

$$S_D(y, z) = \min\left(S_D(r, y),\ S_D(r, z)\right).$$

Definition 2.2. A fuzzy relation r on a relational schema $R = \{A_1, A_2, \ldots, A_n\}$ is fuzzy subset of the cartesian product of $\text{dom}(A_1) \times \text{dom}(A_2) \times \ldots \times \text{dom}(A_n)$ and is characterized by the n-variable membership function

$\mu_r :\quad \text{dom}(A_1) \times \text{dom}(A_2) \times \ldots \times \text{dom}(A_n) \rightarrow [0, 1]$,

where '$\times$' represents 'cross-product'.

Thus, a tuple t in r is characterized by a membership value $\mu_r(t)$, which represents the compatibility of component values of t in representing an entity in the instance r. To simplify the matter, it is assumed that $\mu_r(t) = 1$ for all the tuples in base relations.

In order to compare two elements of a given domain in fuzzy relations, a fuzzy measure, a relation EQ(UAL) is associated with each domain. Thus EQ can be a similarity relation of elements in a domain. Furthermore, the fuzzy equality measure EQ is extended to two tuples on a set of attributes X

$$\mu_{\text{EQ}}(t_1[X], t_2[X]) = \min\left(\mu_{\text{EQ}}(a_1^1, a_1^2), \mu_{\text{EQ}}(a_2^1, a_2^2), \ldots, \mu_{\text{EQ}}(a_k^1, a_k^2)\right),$$

where $X = A_1 A_2 \ldots A_k$.

3. FUZZY DEPENDENCIES AND SET OF INFERENCE RULES OF BHATTACHARJEE AND MAZUMDAR

Definition 3.1. A fuzzy functional dependency (FFDs) $X \rightsquigarrow Y$ in a fuzzy relation r is said to hold, if for every two-tuple subrelations of r, the pair of tuples t_1 and t_2, the inequality $\mu_{\rm EQ}(t_1[X], t_2[X]) \geq \alpha$ implies that,

$$\mu_{\rm EQ}(t_1[X], t_2[X]) \leq \mu_{\rm EQ}(t_1[Y], t_2[Y]),$$

where α is a threshold value for the similarity relation EQ.

Inference axioms:

FFD1	*Reflexivity*	If $Y \subseteq X$ then $X \rightsquigarrow Y$
FFD2	*Augmentation*	If $X \rightsquigarrow Y$ holds, then $XZ \rightsquigarrow YZ$ hold
FFD3	*Transitivity*	If $X \rightsquigarrow Y$ and $Y \rightsquigarrow Z$ hold, then $X \rightsquigarrow Z$ hold

The following inference axioms are infered from the above axioms:

FFD4	*Union*	If $X \rightsquigarrow Y$ and $X \rightsquigarrow Z$ hold, then $X \rightsquigarrow YZ$ hold
FFD5	*Decomposition*	If $X \rightsquigarrow YZ$ holds, then $X \rightsquigarrow Y$ and $X \rightsquigarrow Z$ hold
FFD6	*Pseudotransitivity*	If $X \rightsquigarrow Y$ and $YW \rightsquigarrow Z$ hold, then $XW \rightsquigarrow Z$ hold

The soundness and completeness of the above inference axioms are proved in [6].

In a relation r of a scheme R, for X-value x,

$$Y_r(x) = \{y \mid \text{for some tuple } t \in r, \text{ such that } t[X] = x, \ t[Y] = y\}.$$

Then, a relation r on the scheme R obeys the classical multivalued dependency (MDV) $m : X \twoheadrightarrow Y$ if for every XZ-value xz that appears in r we have

$$Y_r(xz) = Y_r(x), \text{ where } Z = R - XY. \qquad (*)$$

In other words, the MVD m is valid in r if the set of Y-values that appears in r with a given x appears with any combination of x, z.

The idea of extension of multivalued dependency on fuzzy relational database in [1] is the extension of the equal relation '=' to the relation α-equivalent. $Y_r(x)$ is considered as a set of Y-values, which appears in r with not only a given x, but also with $x's$, which are α-equivalent to x. $X_r(x)$ was used to denote a set of such $x's$.

$$X_r(x) = \{x' \mid \exists t \in r, \text{ such that } t[X] = x', \ \mu_{\rm EQ}(x, x') \geq \alpha\}.$$

$Y_r(x)$ is defined as follows:

$$Y_r(x) = \{y \mid \exists t \in r, \text{ such that } t[X] \in X_r(x), \ t[Y] = y\}.$$

It is clear that $Y_r(x)$ is independent of Z-values. The equal relation '=' in $(*)$ is extended to *α-equivalent of the two sets $Y_r(x)$, $Y_r(xz)$*. The relation *α-equivalent of two sets* means that for every y of one, there is existing y' of the other, such that $(\mu_{\rm EQ}(y, y') \geq \alpha$ and vice versa. We use $\cong$ for the relation α-equivalent of two sets.

Definition 3.2. A fuzzy multivalued dependency (FMVD) m on a scheme R, is a statement $m : X \rightsquigarrow\!\!\!\rightarrow Y$, where X, Y are subsets of R. Let $Z = R - XY$. A relation r on the scheme R obeys the FMVD $m : X \rightsquigarrow\!\!\!\rightarrow Y$ if for every XZ-value xz that appears in r we have $Y_r(x) \cong Y_r(xz)$.

In the two-tuple relation $s(t_1, t_2)$ on the scheme R, if $\mu_{\rm EQ}(t_1[X], t_2[X]) < \alpha$ then it can be easily concluded that s trivially satisfies the FMVD $X \rightsquigarrow\!\!\!\rightarrow Y$. Obviously, when s satisfies nontrivially the FMVD $X \rightsquigarrow\!\!\!\rightarrow Y$, we must have $\mu_{\rm EQ}(t_1[X], t_2[X]) \geq \alpha$. For this case we coud say that the FMVD $X \rightsquigarrow\!\!\!\rightarrow Y$ holds *actively* in s.

In order to simplify the notational complexity, a fuzzy truth assigment function Ψ for two tuples is defined:

$$\Psi_{r(t_1,t_2)}(X) = \mu_{\mathrm{EQ}}(t_1[X],\, t_2[X]).$$

The comprehensive definition of FMVD for two-tuple relation is provided by Lemma 3.1 in [1].

Lemma 3.1. *Let R be a relation scheme, and let X, Y and Z be a partition of R. Let $s = \{t_1,\, t_2\}$ be a two-tuple relation on R. Relation s actively satisfies the FMVD $X \sim\!\!\twoheadrightarrow Y$ if and only if*

(1) $\Psi_s(X) \geq \alpha$,

(2) $\Psi_s(X) \leq \max\big(\Psi_s(Y),\, \Psi_s(Z)\big)$.

The relationship between two-tuple subrelations with the parent relation during reference to fuzzy dependencies is presented. The soundness of the inference axioms is proved by using this result. Therefore, Lemma 3.1 plays an important role in [1].

A set of inference rules are proposed in [1]:

FMVD0	*Complementation*	If $X \sim\!\!\twoheadrightarrow Y$ holds, then $X \sim\!\!\twoheadrightarrow Z$ holds, where $Z = R - XY$
FMVD1	*Reflexitivity*	$X \sim\!\!\twoheadrightarrow X$ always holds
FMVD2	*Augmentation*	If $X \sim\!\!\twoheadrightarrow Y$ holds, then $XZ \sim\!\!\twoheadrightarrow Y$ holds
FMVD3	*Additivity or Union*	If $X \sim\!\!\twoheadrightarrow Y$ and $X \sim\!\!\twoheadrightarrow Z$ hold, then $X \sim\!\!\twoheadrightarrow YZ$ hold
FMVD4	*Projectivity or decomposition*	If $X \sim\!\!\twoheadrightarrow Y$ and $X \sim\!\!\twoheadrightarrow Z$ hold, then $X \sim\!\!\twoheadrightarrow Y \cap Z$ and $X \sim\!\!\twoheadrightarrow Y - Z$ also hold
FMVD5	*Transitivity*	If $X \sim\!\!\twoheadrightarrow Y$ and $Y \sim\!\!\twoheadrightarrow Z$ hold, then $X \sim\!\!\twoheadrightarrow (Z - Y)$ holds
FMVD6	*Pseudotransitivity*	If $X \sim\!\!\twoheadrightarrow Y$ and $YW \sim\!\!\twoheadrightarrow Z$ hold, then $XW \sim\!\!\twoheadrightarrow Z - YW$ holds

The proof of the soundness and completeness of the inference rules (FMVD0 - FMVD6) was also given in [1]. We are made to be very interested in this result, because it is a natural, meaningful one to be further developed. Therefore we would like to have some following comments and by the way propose a proof of the soundness.

4. THE SOUNDNESS OF INFERENCE RULES

4.1. Correction of Lemma 3.1

Lemma 3.1 gives a necessary and sufficient condition for a two-tuple relation that actively satisfies a FMVD. But in fact, it only holds in the direction '$\Rightarrow$'. Easily to propose a counterexample for '$\Leftarrow$' direction. So the lemma can be restated:

Let R be a relation scheme, and let X, Y and Z partition R. Let $s = \{t_1,\, t_2\}$ be a two-tuple relation on R. Relation s actively satisfies the FMVD $X \sim\!\!\twoheadrightarrow Y$ if and only if

(1) $\Psi_s(X) \geq \alpha$,

(2) $\alpha \leq \max\big(\Psi_s(Y),\, \Psi_s(Z)\big)$.

Proof. ($\Leftarrow$) If s satisfies (1) and (2), it will be showed that s actively satisfies the FMVD $X \sim\!\!\twoheadrightarrow Y$

	X	Y	Z
t_1	x_1	y_1	z_1
t_2	x_2	y_2	z_2

Since $\Psi_s(X) \geq \alpha$ from (1), we have $Y(x_1) = \{y_1,\, y_2\}$. There are two possible cases for $Y(x_1 z_1)$:

Possibility 1: $\mu_{\mathrm{EQ}}(z_1, z_2) \geq \alpha$, then $Y(x_1 z_1) = \{y_1, y_2\} \cong Y(x_1)$.

Possibility 2: $\mu_{\mathrm{EQ}}(z_1, z_2) < \alpha$, then from (2) we have $\mu_{\mathrm{EQ}}(y_1, y_2) \geq \alpha$, i.e.

$$Y(x_1z_1) = \{y_1\} \cong \{y_1, y_2\} = Y(x_1).$$

Thus s actively satisfies FMVD $X \rightsquigarrow\!\!\!\rightarrow Y$.

($\Rightarrow$) Suppose that s actively satisfies FMVD $X \rightsquigarrow\!\!\!\rightarrow Y$. Obviously we have (1). Consider $Y(x_1) = \{y_1, y_2\}$ and $y_1 \in Y(x_1z_1)$.

Case 1: If $Y(x_1z_1) = \{y_1\}$ then from the definition of FMVD, we can infer that $Y(x_1z_1) \cong Y(x_1)$, which implies $\mu_{\mathrm{EQ}}(y_1, y_2) \geq \alpha$.

Case 2: If $Y(x_1z_1) = \{y_1, y_2\}$ then from the meaning of $Y(x_1z_1)$, we must have $\mu_{\mathrm{EQ}}(z_1, z_2) \geq \alpha$.

Thus, $\max\left(\mu_{\mathrm{EQ}}(y_1, y_2), \mu_{\mathrm{EQ}}(z_1, z_2)\right) \geq \alpha$.

In other words, $\max\left(\Psi_s(Y), \Psi_s(Z)\right) \geq \alpha$.

α can not be replaced with $\Psi_s(X)$ in (2) as Lemma 3.1 because that make ($\Rightarrow$) not valid.

A counterexample:

s actively satisfies $X \rightsquigarrow\!\!\!\rightarrow Y$, but $\Psi_s(X) \geq \max\left(\Psi_s(Y), \Psi_s(Z)\right) \geq \alpha$

s		X	Y	Z
	t_1	x_1	y_1	z_1
	t_2	x_2	y_2	z_2

EQ			μ_{EQ}	$\alpha = 0.5$
	x_1	x_2	0.7	
	y_1	y_2	0.6	
	z_1	z_2	0.3	

4.2. Comment about the soundness of inference axioms

In [1], the soundness is proved (in Lemma 3.5) by using the result of Lemma 3.4. Lemma 3.4 is infered from Lemma 3.3 by contradiction. But during the proof of Lemma 3.3, Lemma 3.4 is used. Consider paragraph below:

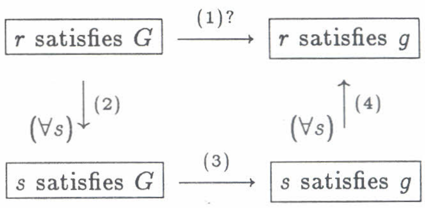
"Since r does not satisfy the FMVD $X \rightsquigarrow\!\!\!\rightarrow Y$, there exist two tuples $t_1 = \langle x_1, y_1, z_1\rangle$ and $t_2 = \langle x_2, y_2, z_2\rangle$, thus that $\Psi_{1,2}(X) \geq \alpha$ and $\max[\Psi_{1,2}(Y), \Psi_{1,2}(Z)] < \Psi_s(X)$" [1] (p. 347).

That means, if r does not satisfy the FMVD $X \rightsquigarrow\!\!\!\rightarrow Y$, there exists a two-tuple subrelation, which does not satisfy the FMVD. This statement is equivalent to Lemma 3.4: If every sub-relation of a relation r satisfies an FMVD then r satisfies that FMVD.

We suppose that it was a vicious reasoning. In fact, we can infer the result of Lemma 3.4 directly without Lemma 3.2 and Lemma 3.3.

In addition, we want to discuss more about Lemma 3.5 in [1], which states and proves that the set of FMVD axioms (FMVD0 - FMVD6) is sound. It is known that, a set **R** of inference rules is sound, if for every FMVD $g : X \rightsquigarrow\!\!\!\rightarrow Y$ which is deduced from a set of dependencies G, using **R** then g holds in any relation in which G holds.

In [1], in order to prove the soundness, it only was showed that for any s two-tuple subrelation of r, if s actively satisfies every FMVD, which is in G, then s also actively satisfies g. Now, let us consider the procedure of proof for FMVD5, which is presented by diagram below



We need (1). We have (4) from Lemma 3.4. In the proof of Lemma 3.5 in [1], (3) is showed. But we can not conclude (1) because (2) is not true in general case. For relation r, which satisfies G, there are two cases. The first case: all of two-tuple subrelations s of r satisfies G; the second case: there exists a two-tuple subrelation s which does not satisfy some FMVD g', belonging to G.

Thus, after adjustment, which is corresponding to new version of Lemma 3.1, the FMVD5 is proved only in the first case, where r satisfies G and every s (a two-tuple subrelation of r) also satisfy G. The second one is still open.

We would like to propose a proof for FMVD5 in general case, by using only the definition of FMVD.

FMVD5 *Transitivity* : If $X \rightsquigarrow\!\!\!\rightarrow Y$ and $Y \rightsquigarrow\!\!\!\rightarrow Z$ hold, then $X \rightsquigarrow\!\!\!\rightarrow (Z - Y)$ holds.

Proof. To prove this, we first prove that, if r satisfies $X \rightsquigarrow\!\!\!\rightarrow Y$ and $Y \rightsquigarrow\!\!\!\rightarrow Z$ then r satisfies $X \rightsquigarrow\!\!\!\rightarrow YZ$. From the meaning of $X \rightsquigarrow\!\!\!\rightarrow YZ$, we need to show $YZ(x) \cong YZ(xv)$, where $R = XYZV$.

Obviously, $YZ(xv) \subseteq YZ(x)$. Therefore, we only need to show

$$\forall\, y_0 z_0 \in YZ(x)\ \exists\, y'z' \in YZ(xv) : \mu_{\mathrm{EQ}}(y_0 z_0,\, y'z') \geq \alpha. \qquad (**)$$

- From $y_0 z_0 \in YZ(x)$, we have

  $\mu_{\mathrm{EQ}}(x_0,\, x) \geq \alpha$ (I.a)

- From (I.a) we have $(z_0 v_0) \in ZV(x)$. Since r satisfy $X \rightsquigarrow\!\!\!\rightarrow Y$ and by axiom complementation r must satisfy $X \rightsquigarrow\!\!\!\rightarrow ZV$, i.e. $ZV(x) \cong ZV(xy)$. Therefore, $\exists t_1 = \langle x_1 y_1 z_1 v_1 \rangle \in r : (z_1 v_1) \in ZV(xy)$ and $\mu_{\mathrm{EQ}}(z_1 v_1,\, z_0 v_0) \geq \alpha$.
  It mean that

  $\mu_{\mathrm{EQ}}(x,\, x_1) \geq \alpha$ (II.a)
  $\mu_{\mathrm{EQ}}(y,\, y_1) \geq \alpha$ (II.b)
  $\mu_{\mathrm{EQ}}(z_0,\, z_1) \geq \alpha$ (II.c)
  $\mu_{\mathrm{EQ}}(v_0,\, v_1) \geq \alpha$ (II.d)

- Obviously, we have $z_1 \in Z(y_1)$. Since r satisfy $Y \rightsquigarrow\!\!\!\rightarrow Z$, $\exists t_2 = \langle x_2 y_2 z_2 v_2 \rangle \in r : z_2 \in Z(xyv)$ and $\mu_{\mathrm{EQ}}(z_1,\, z_2) \geq \alpha$.
  It mean that

  $\mu_{\mathrm{EQ}}(x,\, x_2) \geq \alpha$ (III.a)
  $\mu_{\mathrm{EQ}}(y,\, y_2) \geq \alpha$ (III.b)
  $\mu_{\mathrm{EQ}}(z_1,\, z_2) \geq \alpha$ (III.c)
  $\mu_{\mathrm{EQ}}(v_1,\, v_2) \geq \alpha$ (III.d)

- From (I.a) and (III.a), by transitivity of similarity relation EQ, we have $\mu_{\mathrm{EQ}}(x_0,\, x_2) \geq \alpha$, which implies, $y_0 \in Y(x_2)$. Since r satisfy $X \rightsquigarrow\!\!\!\rightarrow Y$, $\exists t_3 = \langle x_3 y_3 z_3 v_3 \rangle \in r : y_3 \in Y(x_2 y_2 v_2)$ and $\mu_{\mathrm{EQ}}(y_0,\, y_3) \geq \alpha$.
  It mean that

  $\mu_{\mathrm{EQ}}(x_2,\, x_3) \geq \alpha$ (IV.a)
  $\mu_{\mathrm{EQ}}(y_0,\, y_3) \geq \alpha$ (IV.b)
  $\mu_{\mathrm{EQ}}(z_2,\, z_3) \geq \alpha$ (IV.c)
  $\mu_{\mathrm{EQ}}(v_2,\, v_3) \geq \alpha$ (IV.d)

  Consider $y_3 z_3$, we have

  $\mu_{\mathrm{EQ}}(x,\, x_3) \geq \alpha$, from (III.a) and (IV.a) and transitivity of EQ
  $\mu_{\mathrm{EQ}}(v,\, v_3) \geq \alpha$, from (III.d) and (IV.d) and transitivity of EQ

  which implies $y_3 z_3 \in YZ(xv)$.

  We have also

$\mu_{\mathrm{EQ}}(y_0, y_3) \geq \alpha$ (IV.b)

$\mu_{\mathrm{EQ}}(z_0, z_3) \geq \alpha$, from (II.c), (III.c), (IV.c) and transitivity of EQ

which implies $\mu_{\mathrm{EQ}}(y_0 z_0, y_3 z_3) \geq \alpha$.

Thus, the existing of $y'z'$ in $(**)$ is pointed (let $y'z' = y_3 z_3$), i.e. r satisfies $X \rightsquigarrow\!\!\rightarrow YZ$. Combining $X \rightsquigarrow\!\!\rightarrow Y$ and $X \rightsquigarrow\!\!\rightarrow YZ$ by FMVD4 we have $X \rightsquigarrow\!\!\rightarrow (Z - Y)$ (FMVD5).

Similarly, we can prove FMVD0, FMVD1, FMVD2, FMVD3 directly from the definition. As pointed out in [1], procedure of proofs for FMVD4 and FMVD6 are very similar to the classical case involving algebraic manipulation which bases on other proven axioms.

5. CONCLUSIONS

From the meaning of FMVD, which is given in [1], we have corrected a necessary and sufficient condition for a two-tuple subrelation that actively satisfies a FMVD. In the proof procedure for the soundness, we are afraid it is insufficient to prove on two-tuple subrelations. We suppose that, the soundness of these axioms for a class FMVD has been established by using definition of FMVD and the properties of similarity relation EQ.

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