CONTINUOUS TIME SYSTEM IDENTIFICATION: A SELECTED CRITICAL SURVEY

Part II - INPUT ERROR METHODS AND OPTIMAL PROJECTION EQUATIONS

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Abstract. The part I and the part II of the paper refer to a critical survey on significant results available in the literature for identification of systems, linear in the present part and nonlinear in the following one. The most important trends in identification approaches to linear systems are from the development of optimal projection equations, which are argued by the complexity of numerical calculations and of practical applications. The perturbed a quasilinear and on Neuro-Fuzzy trends in representing nonlinear systems, i.e., functional series expansions of Wiener and Volterra, Modeling Robustness and structured numerical estimators are included. The limitations and applicability of the methods are discussed throughout.

4. INPUT ERROR METHODS

It has been shown in [36, 44 - 48] that by adopting input error methods one can avoid the direct use of time derivatives of system input signals. However, in the input error derivation, few terms and their relative are to be cleared first.

4.1. Definitions and lemmas

Definition 1. The model that is in antiparallel with the system having the output and input of the system as its respective input and output is named as a model inverse of the system [36, p. 12].

According to the above definition, the system of dynamical equations and its equivalence in the state variable description for describing model inverse of the system are readily obtained [36, p. 12, 13].

Definition 2. A description of the model inverse in the state variable space with minimal number of parameters is called canonical [14], for which realization of model inverse is also minimal and corresponding to this minimal, the dimension of matrix $\boldsymbol{A}$ is its order [36, p. 13].

Definition 3. Parameters of the model for a system are determinable if those of its model inverse are known and vice verse [36, p. 18].

Definition 4. A model of well specified structure having known parameters is called an assumed model (AM) [36, p. 25].

Definition 5. Let the response $\boldsymbol{y}(t)$ of a high order model to an input $\boldsymbol{u}(t)$ be given. A low order model is said to be the reduced model of the given one if the low order model has the response $\boldsymbol{y}(t)$ to an input close to $\boldsymbol{u}(t)$ or has a response close to $\boldsymbol{y}(t)$ to the same $\boldsymbol{u}(t)$ [36, p. 18].

Following lemmas are restated, their proofs are available in [36, p. 14-18].

Lemma 1. *A realization of model inverse is minimal if and only if it is controllable and observable jointly.*

Lemma 2. *Let a jointly controllable/observable model and a model inverse for a system be given. Assume that for an augmented of the model and model inverse, there exists a nonnegative definite steady state covariance matrix of the appropriate dimension satisfying Lyapunov equation. Then augmented system is stabilizable if and only if the model inverse is asymptotically stable.*

Lemma 3. *If the model is a minimal realization of the system, then there exists also a minimal realization for its model inverse.*

4.2. Derivation of the input error

With the use of model inverse

Assume that an AM is in parallel with a system. As parameters and order of AM are different from those of the system, for ensuring the output of the model to be matched with that of the system, AM should have a requested input different from the system input signal. Discrepancy between AM and the system is reflected at the input side of the system in term in terms of difference of two input signals. This difference between the two inputs is referred to as an input error.

Assume that a linear, continuous time system having input vector $\boldsymbol{u}(t)$ and output vector $\boldsymbol{y}(t)$ is modeled by the use of eqn. (2.1). By the definition 1, for the model there exists a model inverse described by:

$$\sum_{i=0}^{n_I} \hat{\alpha}_{i,k}(t) \frac{d^i u_k(t)}{dt^i} = \sum_{j=1}^{q} \sum_{i=0}^{n_I} \hat{\beta}_{i,j_k}(t) \frac{d^i y_j(t)}{dt^i}, \quad \text{for } k = 1, ..., p, \tag{4.1}$$

where superscript "∧" on parameters means that the parameters are to be estimated.

If the order and parameters of the model inverse are known, then it follows an AM, from definitions 1 and 3. Considering the coefficients of zero derivatives of the requested inputs be 1, the requested signal at the k-th input of AM is obtained:

$$\hat{u}_k(t) = \sum_{j=1}^{q} \sum_{i=0}^{n_I} \beta_{i,j_k}(t) \frac{d^i \hat{y}_j(t)}{dt^i} - \sum_{i=1}^{n_I} \alpha_{i,k}(t) \frac{d^i \hat{u}_k(t)}{dt^i}, \quad \text{for } k = 1, ..., p, \tag{4.2}$$

where parameters are known values, $y_j(t)$ for $j = 1, ..., q$ are the response at the j-th output of the system.

The input error vector in this case is obtained by defining the error at the k-input first then writing for all k input:

$$\boldsymbol{e}_i(t) = \boldsymbol{u}(t) - \hat{\boldsymbol{u}}(t), \tag{4.3}$$

where $\boldsymbol{e}_i(t) = \left[e_{i_1}(t), ..., e_{i_p}(t) \right]^T$, $\boldsymbol{u}(t) = \left[u_1(t), ..., u_p(t) \right]^T$ and $\hat{u}(t) = \left[\hat{u}_1(t), ..., \hat{u}_p(t) \right]^T$.

Whenever the system is described in the state variable space, then in the same description is used for an AM inverse whose response is also considered to be the requested input to an associated AM. If system response is considered to be the input of the AM inverse, which has time invariant parameters, then the input error vector becomes [36, p. 28]:

$$\boldsymbol{e}_i(t) = \boldsymbol{u}(t) - \mathcal{L}^{-1} \left[\boldsymbol{c}_I (Is - \boldsymbol{A}_I)^{-1} \boldsymbol{B}_I \boldsymbol{Y}(s) \right], \tag{4.4}$$

where $\mathcal{L}^{-1}$ stands for inverse Laplace and $\boldsymbol{Y}(s)$ is the Laplace transform of $\boldsymbol{y}(t)$.

However, if an AM is used, then the input error can be seen to be a vector of signals actuating AM in addition to the system input signals and the input error can be defined by employing an convolution operator.

With the use of convolution operator

The output of a system and that of an AM described by integral convolution are matched [36, p. 29], giving rise to the input error in an expression:

$$\int_0^t \left[\hat{\boldsymbol{u}}(\tau) - \boldsymbol{u}(\tau) \right] d\tau = \int_0^t \boldsymbol{H}^+(t - \tau) \left[\hat{\boldsymbol{H}}(t - \tau) - \boldsymbol{H}(t - \tau) \right] \boldsymbol{u}(\tau) d\tau, \tag{4.5}$$

where $\boldsymbol{H}^+(t - \tau)$ is the pseudoinverse of impulse response matrix of a bounded input bounded output AM.

The expression related to the input error using convolution summation is:

$$\sum_{k=0}^{n} \left[\hat{\boldsymbol{u}}(n - k) - \boldsymbol{u}(n - k) \right] = \sum_{k=0}^{n} \boldsymbol{H}^+(k) \left[\hat{\boldsymbol{H}}(k) - \boldsymbol{H}(k) \right] \boldsymbol{u}(n - k), \quad n = 0, ..., N. \tag{4.6}$$

The exists also a methodfor determination of input error by employing the factorization theory

[58, p. 82]. In this method, a transformation on the basis of an assumed model is made for interchanging the roles of its terminals. The requested input to AM is obtainable. The transformation method has however been pointed out to have the effect same as that of the use of a model inverse.

4.3. On stochastic input error and optimization processes

Stochastic input error

Depending upon the structure of the model inverse, its transfer function for the stochastic part $G_n(s)$ has different forms and corresponding to which different criterion would be considered for the optimization purpose. The formula for determining stochastic input error for single input single output case is [36, 39]:

$$w^*(t) = \mathrm{LD}\big[\mathcal{L}^{-1}\big[(\mathrm{LD})^{-1}\big[G_n^*(s).\big(U^*(s) - U_M^*(s)\big)\big]\big]\big], \tag{4.7}$$

where superscript "*" and "−1" stand for measured values and inverse operation respectively, (s) is meant in Laplace domain, $\mathcal{L}$ and LD are denoted for Laplace and linear dynamical operator respectively.
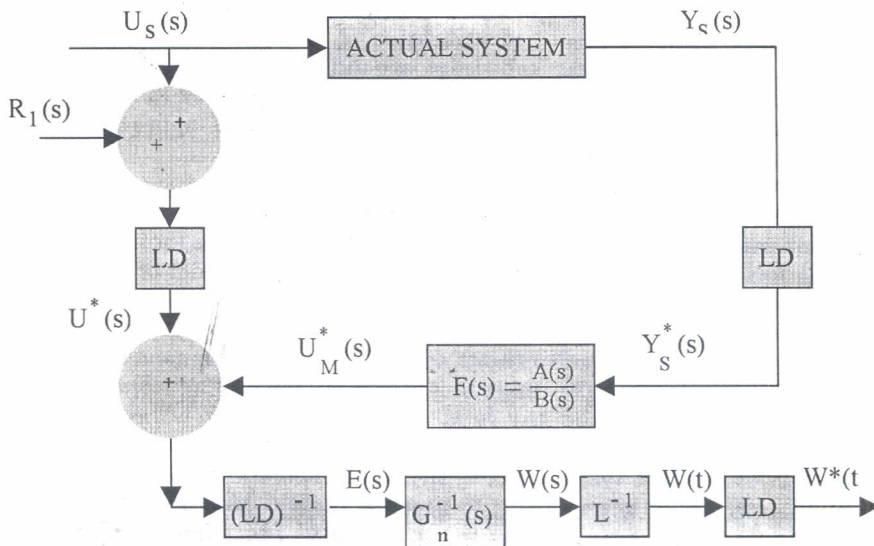

Fig. 2. Block diagram for determination of stochastic input error

Fig. 2 above and eqn. (4.7) imply that the deviation $\{U^*(s) - U_M^*(s)\}$ of the system input $U^*(s)$ from its deterministic behaviour $U_M^*(s)$ may be seen due to a white noise $w^*(t)$ operated upon a system consisting of LD operator and $G_n^*(s)$.

Optimization processes

An optimization approach to the identification for systems described in differential equations has been developed and through the development has been shown advantages of the input error methods over the other error methods [37, 39] with regards to parameter estimation and order reduction for models.

Different important aspects of system identification problem have also been considered for the case of input error method. Some can be cited for examples as a general treatment of noise measurements contaminated from different sources [39], equivalence between the effect of a transformation to get noise free data and that of IV technique [39, 41], recursive process of solution [39], an ill-conditioned problem for high noise sensitive data in estimation process [41] and a discrete mechanism for processing continuous time models [40], etc.

It should be addressed that with models described by differential equations there exists permanent two factors giving rise to troublesome in obtaining bias free estimate. Most influential one is of noise contaminated input/output measurements supplied by LD operator, the other is of initiating to high sensitive data including also rounding off number in estimation algorithm. Choice of a suitable estimation algorithm is not out of capability of escaping high sensitive data but LD.

5. OPTIMAL PROJECTION EQUATIONS (OPEQ)

5.1. On the input error

A few work has been so far reported in literature on estimating parameters for models in the state variable description on the basics of model adaptation using stability [42, 43]. However, the model adaptation method faces many difficulties with regards to the law for adaptation, which is rather arbitrary chosen from an error differential equations, in one side also to the applicability to complex systems on the other side. Further, model adaptation method does not enjoy the advantage of bias free estimates of IV method, satisfying a lot of adaptation rules meanwhile.

From the idea same as the way how normally human brain dealing with the case of incomplete (inexact or missed) information, methods based on fuzzy logic and neural network theory [17 - 19] have been developed for identifying systems incapable of getting complete information. Although, fuzzy methods are found yet to be successfully applicable to complex system in respect to a close approximation for high order systems and to an amount of calculation concern where the number of fuzzy (conditional) rules required to be set up is high. However, each of the soft computing components can be seen to be a numerical model free estimator and on which each method based has got various advantages with regards to identification of nonlinear dynamic or uncertainty or distributed systems etc., where the system knowledge are difficult incorporating all together in the form of mathematical expressions [17, 19] in one side and high complexity in realizing them in the other side. Both the mentioned, adaptation and soft computing based approaches can be further extended for accommodating OPEQ, undoubtedly.

The input error concept has been exploited for the identification of system modeled in state variable space for both known and unknown order cases. It has been observed that due to the existence of an optimal projection matrix (OPM), the first order necessary conditions for the optimality are able to be expressed in the form of OPEQ. In OPEQ, the parameters for model of full or reduced order are in terms of OPM components on satisfying a rank condition and different modified Lyapunov equations. Herewith, the problem of order determination for models has been also considered [36]. The system identification problem is found to switch over to that of development of suitable algorithm for finding controllability and observability anagramians or pseudoones [36] from OPEQ. It has been shown that the approach is applicable to any form of the system input signals in a hand, in the other hand, LD are not required for supplying measurements of time derivatives of signals from either side of the system. Thus, the system identification problem which so far has been considered be an experimental process, becomes applicable to practice reality. Moreover, the question of noise intercepting on LD hence does not arise, leading to a possibility of bias free estimates [36, 44 - 47].

Interest in most cases of system theory is on jointly controllable/observable part of the system. The rest two parts, i.e., uncontrollable and unobservable parts of the system can be made to be controllable/observable jointly, using standard linear quadratic Gaussial theory [52, 58] while looping the system. The identification for system in the closed loop is not exactly the same as that for controllable/observable jointly part of the system. However, the idea of considering a problem of order reduction to be the one of parameter estimation applicable to a misorder case has seen to be valid irrespective of the model description and loop-wise configuration. This enables one to cover on both loop-wise configuration of the present meaning for system identification with the development of OPEQ for parameter estimation of unknown order models and OPEQ for order reduction of system in a closed loop configuration [38, 48]. With respect to which, the preserve of existing optimal control strategy to the system and the guarantee of matching on the output side are the aims to be achieved. Two modes (open and closed mode) of treatments have been proposed [36, 38] and OPEQ for the order reduction of system in closed loop configuration are resulted different from two modes of treatment [38]. A higher complex, with respect to the development of computing algorithm, has been found with the closed loop mode of treatment.

In the context of uniqueness of the system identification, OPEQ are found to provide an additional constrained condition to the L_2 optimization problem. This additional condition is resulted from the effect of coupling among equations in the relevant OPEQ. Further, OPEQ are found to be well accommodated with as much as available constraint conditions.

There still exists, however, an important aspect of the system identification with regards to parsimony principle, i.e., hierarchical structure for models [14, 15]. However, it has been shown through analyzing optimum property [46] that for the case of state variable description both models

of full and lower order are optimized with respect to the actual system. This is due to the fact that in augmented system consisting of an AM, full order and reduced order model, there exist three mutual coupled each to other optimal projection matrices and owning their roles, the parsimony principle holds [36, 46].

However, it should address that a high complexity of mathematics has been involved in the development of OPEQ and for these equations a fairly complex algorithm would arise due to the coupling event. For the solvability of OPEQ, some more assumptions (very often, the internally balanced conditions are used) on the system are to be adopted for decoupling the equations. The complexity is resulted from the fact that the optimization has been performed with respect to the parameters and that with respect to the states has been obtained as a by-product. This complexity has been shown to be overcome by adopting the concept of state optimization to the system identification problem.

5.2. On state optimization processes

a. Basics of state optimization method

It has been observed that for an output function, the variables, i.e., the states and parameters, are inherently nonseparable between them. Then the questions have been arisen as follows.

For two models of order n and order m subjecting to the same excitation in the state variable description:

$$x'_n = A_n x_n + B_n u_n \tag{5.1}$$

$$y_n = C_n x_n \tag{5.2}$$

and

$$x'_m = A_m x_m + B_m u_n \tag{5.3}$$

$$y_m = C_m x_m \tag{5.4}$$

with u_n, y_n and y_m are p-, q- and q-dimensional vectors, A_n, B_n, C_n, A_m, B_m, and C_m are appropriately dimensioned, if it is possible to optimize (in some sense) a state vector with respect to other state vector, and if it is the case, then this optimization is sufficient in regard to their outputs. That is, weighted least squares output error will be minimized or not(?).

The above question has been cleared by a lemma being restated hereby:

Lemma 4. *Let the vector x_n of n independently specified states of a system be given. Assume that an AM is chosen, having the vector x_m of m independently specified states, $m < n$. Then, there exists a nonsimilarity transformation $T \in \mathbf{R}^{m \times n}$, $\rho(T) = m$, on x_n for obtaining x_m such that if the number of the system output, $q < m$, then $T^+ x_m$ leads to the minimum norm amongst least squares of output error.*

Proof details can be found in [36, 47]. However, it is worthwhile to address that with the conditions mentioned in the lemma the weighted least square criterion on the output error:

$$J_{\text{Out}} = \int_0^\infty (y_n - y_m)^T R (y_n - y_m) dt \tag{5.5}$$

is readily obtained from the *criterion for state optimization:*

$$J_{\text{Sopt}} = \int_0^\infty \left\| x_n - T^* x_m \right\|_R^2 dt. \tag{5.6}$$

In regard to the use of partial isometry concept for developing simpler form of the OPEQ, another lemma has been proposed, of which the detail proof is also available in [36, 47]:

Lemma 5. *Let the state vector x_n of the system be a transformed state vector of the AM as*

$$x_n = T^+ x_m , \quad T \in \mathbf{R}^{m \times n} , \quad \rho(T) = n < m. \tag{5.7}$$

Then T can be factorized as

$$T = EG = HE, \tag{5.8}$$

where $E = E(x_m x_n^T) \in \mathbf{R}^{m \times n}$ is a partial isometry, $G = E(x_n x_n^T) \in \mathbf{R}^{n \times n}$, $H = E(x_m x_m^T) \in \mathbf{R}^{m \times m}$ both are nonnegative definite matrices.

b. On the OPEQ

OPEQ have been developed for parameter estimation, reduced order (open loop thinking case [27, 37]) and for reduced order of system operating in a closed loop configuration (closed loop thinking case).

For the open loop thinking case, the OPEQ often consist of three equations, one for the parameters and the other two are of the standard like form Lyapunov equation. Simplicity in the form of OPEQ is resulted from the effect of factorizing the relevant nonsimilarity transformation in term of a partial isometry. As it has been shown [37, 48] that the effect of coupling amongst the equations in OPEQ is equivalent to that of an additional constrained condition to the L_2 criterion in the optimality problem. An other additional constrained condition has been obtained from the decoupling those coupled ones owing the factorization effect.

Few typical results for the open loop thinking problems

Theorem 1. *Let the measurements of a system of order n be available for the parameter estimation. Let a controllable and observable AM of order m, $m > n$, with known parameters be chosen. Then there exists an optimal, orthogonal projection matrix $\sigma = EE^T \in \mathbf{R}^{m \times n}$, $\rho(\sigma) = n$, and two nonnegative definite matrices $Q = HEW_cE^T$, $P = H^+EW_oE^T \in \mathbf{R}^{m \times n}$ both of rank n, such that the parameters of the controllable and observable part of the system are computable from:*

$$A_n = E^T H^+ A_m HE, \; B_n = E^T H^+ B_m, \; C_n = KC_m HE \tag{5.9}$$

which satisfy the following conditions:

$$\sigma(H^+ A_m Q + QA_m^T H^+ + H^+ B_m VB_m^T H^+)\sigma^T = 0 \tag{5.10}$$

$$\sigma^T(HA_m^T P + PA_m H + HC_m^T K^T RKC_m H)\sigma = 0 \tag{5.11}$$

where $E = E(x_m x_n^T) \in \mathbf{R}^{m \times n}$ is a partial isometry, $H = E(x_m x_m^T) \in \mathbf{R}^{m \times m}$ is a positive definite matrix, W_c and W_o are the controllability and observability gramians of the system and K is a similarity transformation for matching the output of the assumed model with that of the system.

Converse of theorem. *Let a controllable and observable assumed model of order m, $m > n$, be chosen. Assume that the parameters of the system are determinable with (5.9) on satisfying (5.10) and (5.11). Then σ, Q and P are optimal.*

The proofs are available in [36, 47]. It is however mentioned that two modified Lyapunov equations i.e., eqns. (5.10) and (5.11) have been successfully transferred into the standard like form owing the nature of σ.

Theorem 2. *For a given linear, time invariant system of the order n, there exists always an $r \times n$ partial isometry E and an $n \times n$ nonnegative definite matrix such that the optimal parameters of the reduced order model are given by:*

$$A_r = EHA_n H^+ E^T, \; B_r = EHB_n, \; C_r = C_n H^+ E^T. \tag{5.12}$$

Further, there exists an $n \times n$ optimal projector σ and two $n \times n$ nonnegative definite matrices Q and P such that if the optimal model is to be controllable and observable, the following condition are then to be satisfied:

$$\sigma[HA_n Q + QA_n^T H + HB_n V_1 B_n^T H] = 0, \tag{5.13}$$

$$[H^+ A_n^T P + PA_n H^+ + H^+ C_n^T R_2 C_n H^+]\sigma = 0, \tag{5.14}$$

where $V_1 = E(uu^T)$, R_2 is weighted matrix in the criterion for order reduction.

The converse theorem along with their proofs are available in [47]. Due to the nature of σ, which is no longer as obtained by adopting the input error method, two modified Lyapunov equations (5.13) and (5.14) have been transferred into the standard like form. However, an important advantage of the state optimization method is that the method permits one to preserve the physical significance of the modeled states into the reduced states. It is mentioned hereby that the results for robustness of modeling problem are also available in details in [36, 47].

With regard to the closed loop thinking case, it has been observed that there exist three transformation, two are nonsimilarity between the two state vectors, and between output and state vector the third one is similarity between the two output vectors. Like in the open loop thinking case, out of an equation for parameters, the OPEQ usually consist of two other equations. They are of the standard like form of Lyapunov equations, excepting the case where a compensation is required. In such cases, OPEQ may consist of three (like in a dynamical compensation) or four (like in an order reduction for controller) conditional equations. Amongst which two are Lyapunov like form equations having the responsibility for the reduction operation and the other one or two are of Riccati like form depending upon the requirement of compensation [48, 49].

Few typical results for the closed loop thinking problems

Theorem 3. *For a linear n-th order time invariant parameters system there exist full row rank matrices $K \in \mathbf{R}^{e \times n}$ and $L \in \mathbf{R}^{q_e \times q}$, $q_e \leq q$, such that optimal parameters of a state estimator of order e are given by:*

$$A_e = K(A_n - MC_n)K^+ , \quad B_e = K[B_n \,|\, M], \quad C_e = LC_nK^+ , \tag{5.15}$$

where M is a linear combination of the system outputs.

Maximum value which can be considered for the order of the reduced order state estimator to be controllable and observable is the irreducible order of system.

Further, there exists a partial isometry $E_e \in \mathbf{R}^{e \times n}$ and nonnegative definite $H_e \in \mathbf{R}^{n \times n}$, such that with optimal orthogonal projector $\sigma_e = E_e^T E_e \in \mathbf{R}^{n \times n}$, $\rho(\sigma_e) = e$, and two nonnegative definite matrices $Q_e^* = H_c^+ E_e^T Q_e E_e$, $P_e^* = H_e E_e^T P_e E_e \in \mathbf{R}^{n \times n}$, the following conditions are to be satisfied:

$$\sigma_e[H_e A_n Q_e^* + Q_e^* A_n^T H_e + H_e B_n V_1 B_n^T H_e] -$$
$$\sigma_e[H_e M(C_n Q_e^* - V_{12}^T B_n^T H_e) + (C_n Q_e^* - H_e B_n V_{12})M^T H_e + H_e M V_2 M H_e] = 0, \tag{5.16}$$

$$\sigma_e[H_e^+ A_n^T P_e^* + P_e^* A_n H_e^+] -$$
$$[\sigma_e H_e^+ C_n^T M^T P_e^* + P_e^* M C_n H_e^+ \sigma_e + \sigma_e H_e^+ C_n^T L^T R_4 L C_n H_e^+ \sigma_e] = 0. \tag{5.17}$$

Proofs of the theorem and its converse are available in [36, 48]. However, it may address that after some mathematical manipulations the closed loop thinking problem becomes an open loop one and the result obtained for order reduction of model can be adopted. Further, optimal reduced order state estimation has been turned to that of unregulated system. If some certain assumptions are made, the problem is turned out dealing with the order reduction for dynamic compensation. In such a case, the maximum value assignable to e is n. From the equation used for the implementation of full order state estimator (usually known as state observer), corresponding to the optimal parameters, it is seen that there exists an error, which is referred to the input side and is arisen due to nonsimilarity transformation.

Theorem 4. *For a linear n-th order time invariant parameters system there exists a partial isometry $E_c \in \mathbf{R}^{e \times n}$ and two nonnegative definite matrices Q_c, P_c such that the optimal parameters of a jointly controllable and observable controller of order e are given by:*

$$A_c = E_c H(A - BB^T \Pi - \Psi C^T C)H^{-1}E_c^T , \quad B_c = E_c H \Psi C^T , \quad C_c = -B^T \Pi H^{-1} E_c^T \tag{5.18}$$

in which two positive definite matrices Π and Ψ are unique solutions of Controlling Algebraic Riccati Equation CARE and Filtering Algebraic Riccati Equation FARE [32] respectively and H is related with states of full order LQG controller.

The following conditions are satisfied:

$$\sigma_e[H(BB^T \Pi + \Psi C^T C)Q_c - \frac{1}{2}H\Psi C^T V CH] \geq 0, \tag{5.19}$$

$$\sigma_e[H^{-1}(BB^T \Pi + \Psi C^T C)P_c - \frac{1}{2}H^{-1}\Pi BB^T \Pi H^{-1}] \geq 0, \tag{5.20}$$

where $Q_c = H^{-1}E_c^T Q E_c$, $P_c = H E_c^T P E_c$, $\sigma_e = E_c^T E_c$, and Q, P are the respective controllability and observability gramians of reduced order LQG controller.

It has been found that compensation of the system by a reduced controller is not generally guaranteed until some proper measures are taken. The normalized linear quasiGaussian (LQG) has been considered, forming an augmented system [49]. From the result of standard normalized LQG problem [58], (it said reduced order controller internal stabilizes system, the augmented system is to be stable) has given to a system of six equations deducing to four modified ones. Of which two modified Lyapunov equations have been resulted from the reduction and two modified Riccati equations have been found being responsible for the LQG. However, two of the four mentioned equations responsible for reduction have been found being the conditions (5.19) and (5.20) in the theorem. The other two (Riccati ones) are found decoupling readily due to the role of operational factorization.

Reduced order controller can be obtained by three steps. In the first one, LQG is used for obtaining an equivalent, open loop model. In the second step, model reduction is performed and in the last one, LQG is used for compensation. This implies that optimal performance of reduced controller can be obtained by steepwise design process.

6. CONCLUDING REMARKS

In this survey, it has concentrated specifically on the recently developed methods of identification for continuous time systems described in differential equations and in state variable space with regard to the parameter estimation and order reduction for models in both loopwise. Different aspects of the problem have also been dealt with such as the order determination for models, ill-conditioning, parsimony principle (hierarchical structure) for state variable descriptive models, etc. However, comments on the comparative performance of the various techniques are not satisfied and except in passing, other important aspects on special classes of nonlinear dynamic system of the problem will be discussed with in the coming part of the paper.

It is worthwhile to address that the OPEQ methods have got such important advantages that the problem of system identification has been found to switch from an experimental process over to that applicable to practice reality. Moreover, the OPEQ have got simpler forms by adopting the state optimization concepts.

State optimization concept can be employed to treating different optimum problems. Further, in the same direction, various researches can be carried out with respect to the case of an infinite dimensional system like distributed parameter one, nonlinear dynamic system modeled by series, etc., where partial or functional equations are required. Corresponding to which the concept of generalized Green function and its inverse would be adopted [57], which may gives rise to the concept of a polyoptimization in stead of state optimization concept. Under this direction, various researches in both theoretical and practical aspects can also be carried out for treating many different optimization problems happened to be in a nonfinite dimensional space.

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