Journal of Computer Science and Cybernetics, V.29, N.2 (2013), 105–118

A SIMPLE WALKING CONTROL METHOD FOR BIPED ROBOT WITH STABLE GAIT

TRAN DINH HUY, NGUYEN THANH PHUONG, HO DAC LOC, NGO CAO CUONG

Ho Chi Minh City University of Technology, Vietnam;
Email: phuongkorea2005@yahoo.com

Tóm tắt. Bài báo giới thiệu một phương pháp đơn giản để điều khiển cho một rôbôt 2 chân 10 bậc tự do với một dáng đi ổn định và giống người sử dụng một cấu hình phần cứng đơn giản. Rôbôt 2 chân được mô hình như một con lắc ngược 3 chiều. Dáng đi của rôbôt được tạo bởi hệ thống điều khiển bám điểm mômen không (ZMP) của rôbôt 2 chân theo quỹ đạo là đường zigzac theo lòng bàn chân của rôbôt. Một bộ điều khiển tối ưu được thiết kế để điều cho hệ thống điều khiển bám điểm ZMP. Một quỹ đạo của khối tâm của rôbôt trong vùng ổn định được tạo ra khi ZMP của rôbôt bám theo quỹ đạo theo phương x và y luôn luôn nằm trong lòng bàn chân của rôbôt. Dựa vào quỹ đạo ổn định của khối tâm, bước đi của rôbôt được đề xuất bằng cách giải bài toán động học ngược và được tích hợp trên phần cứng dùng PIC18F4431 và DSPIC30F6014. Phương pháp đề xuất được kiểm chứng thông qua mô phỏng và thực nghiệm.

Từ khóa. Bộ điều khiển bám tối ưu, Hệ thống điều khiển bám ZMP, robot 2 chân.

Abstract. This paper proposes a simple walking control method for a 10 degree of freedom (DOF) biped robot with stable and human-like walking using simple hardware configuration. The biped robot is modeled as a 3D inverted pendulum. A walking pattern is generated based on ZMP tracking control systems, which are constructed to track the ZMP of the biped robot to zigzag ZMP reference trajectory decided by the footprint of the biped robot. An optimal tracking controller is designed to control the ZMP tracking control system. When the ZMP of the biped robot is controlled to track the x and y, ZMP reference trajectories always locates the ZMP of the biped robot inside stable region known as area of the footprint, a trajectory of the COM is generated as a stable walking pattern of the biped robot. Based on the stable walking pattern of the biped robot, a stable walking control method of the biped robot is proposed by using the inverse kinematics. The stable walking control method of the biped robot is implemented by simple hardware using PIC18F4431 and DSPIC30F6014. The simulation and experimental results show the effectiveness of the proposed control method.

Key words. Optimal tracking controller, ZMP tracking control system, biped robot.

1. INTRODUCTION

Research on humanoid robots and biped robots locomotion is currently one of the most exciting topics in the field of robotics and there exist many ongoing projects [1, 5, 13, 14, 15]. Although some of those works have already demonstrated very reliable dynamic biped walking [6, 11], it is still important to understand the theoretical background of the biped robot. The biped robot performs its locomotion relatively to the ground while it is keeping its balance and not falling down. Since there is no base link fixed on the ground or the base, the gait planning and control of the biped robot is very important but difficult. Numerous approaches have been proposed so far. The common method of these numerous approaches is to restrict zero moment point (ZMP) within a stable region to protect the biped robot from falling down [2].

In the recent years, a great amount of scientific and engineering research has been devoted to the development of legged robots in order to attain gait patterns more or less similar to human beings. Towards this objective, many scientific papers have been published on different aspects of the problem. Sunil, Agrawal and Abbas [3] proposed motion control of a novel planar biped with nearly linear dynamics. They introduced a biped robot but the model was nearly linear. The motion control for trajectory following used nonlinear control method. Park [4] proposed impedance control for biped robot locomotion so that both legs of the biped robot were controlled by the impedance control, where the desired impedance at the hip and the swing foot was specified. Huang and Yoshihiko [5] introduced sensory reflex control for humanoid walking so that the walking control consisted of a feed-forward dynamic pattern and a feedback sensory reflex. In those papers, the moving of the body of the robot was assumed to be only on the sagittal plane. The biped robot was controlled based on the dynamic model. The ZMP of the biped robot was measured by sensors so that the structure of the biped robot was complex and the biped robot required a high speed controller hardware system.

This paper presents a stable walking control of a biped robot by using the inverse kinematics with simple hardware configuration based on the walking pattern which is generated by ZMP tracking control systems. The robot's body can move on the sagittal and the lateral planes. Furthermore, the walking pattern is generated based on the ZMP of the biped robot so that the stability of the biped robot during walking or running is guaranteed without the sensor system to measure the ZMP of the biped robot. In addition, the simple inverse kinematics using the solid geometry is used to obtain angles of each joints of the biped robot based on the stable walking pattern. The biped robot is modeled as a 3D inverted pendulum [1]. The ZMP tracking control system is constructed based on the ZMP equations to generate a trajectory of COM. A continuous time optimal tracking controller is also designed to control the ZMP tracking control system. From the trajectory of the COM, the inverse kinematics of the biped robot is solved by the solid geometry method to obtain angles of each joint of the biped robot. It is used to control walking of the biped robot.

2. MATHEMATICAL MODEL OF THE BIPED ROBOT

A new biped robot developed in this paper has 10 DOF as shown in Fig. 1.

The biped robot consists of five links that are one torso, two links in each leg those are upper link and lower link, and two feet. The two legs of the biped robot are connected with torso via two DOF rotating joints which are called hip joints. Hip joints can rotate the legs in the angles θ_5 for right leg and θ_7 for left leg on sagittal plane, and in the angles θ_4 for right leg and θ_6 for left leg on in frontal plane. The upper links are connected with lower links via one DOF rotating joints those are called knee joints which can rotate on sagittal plane. The lower links of legs are connected with feet via two DOF of ankle joints. The ankle joints can rotate the feet in angle θ_1 (for right leg) and θ_{10} (for left leg) on the sigattal plane, and in angle θ_2 for left leg and θ_9 for right leg on the in frontal plane. The rotating joints are considered to be friction-free and each one is driven by one DC motor.

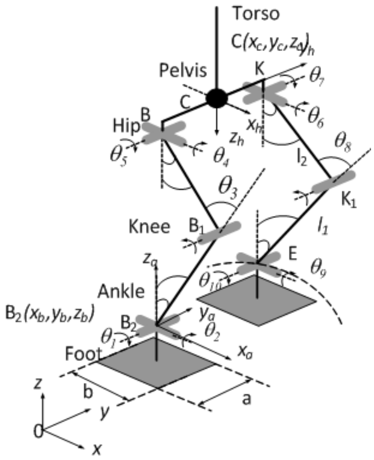


Fig. 1. Configuration of 10 DOF biped robot

2.1. Kinematics model of biped robot

It is assumed that the soles of robot do not slip. In the world coordinate system Σ_w which the origin is set on the ground, the coordinate of the center of the pelvis link and the ankle of swing leg can be expressed as follows

$$x_c = x_b + l_1 \sin\theta_1 - l_2 \sin(\theta_3 - \theta_1); \tag{1}$$

$$y_c = y_b + l_1 \sin\theta_2 + l_2 \cos(\theta_3 - \theta_1)\sin\theta_2 + \frac{l_3}{2}\cos(\theta_2 + \theta_4); \tag{2}$$

$$z_c = z_b + l_1 \cos\theta_1 \cos\theta_2 + l_2 \cos(\theta_3 - \theta_1)\cos\theta_2 - \frac{l_3}{2}\sin(\theta_2 + \theta_4). \tag{3}$$

In choosing Cartesian coordinate Σ_a which the origin is taken on the ankle, position of the center of the pelvis link is expressed as follows

$$x_{ca} = l_1 \sin\theta_1 - l_2 \sin(\theta_3 - \theta_1); \tag{4}$$

$$y_{ca} = l_1 \sin\theta_2 + l_2 \cos(\theta_3 - \theta_1)\sin\theta_2 + \frac{l_3}{2}\cos(\theta_2 + \theta_4); \tag{5}$$

$$z_{ca} = l_1 \cos\theta_1 \cos\theta_2 + l_2 \cos(\theta_3 - \theta_1)\cos\theta_2 - \frac{l_3}{2}\sin(\theta_2 + \theta_4); \tag{6}$$

where, $x_{ca},\ y_{ca},\ z_{ca}$ are the position of the center of the pelvis link in Σ_a.

Similarly, position of the ankle joint of swing leg is expressed in the coordinate system Σ_h which the origin is defined on the center of pelvis link as

$$x_{eh} = l_2 \sin\theta_7 - l_1 \sin(\theta_8 - \theta_7); \tag{7}$$

$$y_{eh} = \frac{l_3}{2} + l_2 \sin\theta_6 - l_1 \cos(\theta_8 - \theta_7)\sin\theta_6; \tag{8}$$

$$z_{eh} = l_2 \cos\theta_6 \cos\theta_7 + l_1 \cos(\theta_8 - \theta_7)\cos\theta_6. \tag{9}$$

It is assumed that the center of mass of each link is concentrated on the tip of the link and the initial position is located at the origin of the Σ_w. This means that $x_b = 0$ and $y_b = 0$. The COM of the robot can be obtained as follows

$$x_{com} = \frac{m_b x_b + m_1 x_1 + m_2 x_2 + m_c x_c + m_3 x_3 + m_4 x_4 + m_e x_e}{m_b + m_1 + m_2 + m_c + m_3 + m_4 + m_e}; \tag{10}$$

$$y_{com} = \frac{m_b y_b + m_1 y_1 + m_2 y_2 + m_c y_c + m_3 y_3 + m_4 y_4 + m_e y_e}{m_b + m_1 + m_2 + m_c + m_3 + m_4 + m_e}; \tag{11}$$

$$z_{com} = \frac{m_b z_b + m_1 z_1 + m_2 z_2 + m_c z_c + m_3 z_3 + m_4 z_4 + m_e z_e}{m_b + m_1 + m_2 + m_c + m_3 + m_4 + m_e}; \tag{12}$$

where (x_b, y_b, z_b) and (x_e, y_e, z_e) are the coordinates of the ankle joints B_2 and $E, (x_1, y_1, z_1)$ and (x_4, y_4, z_4) are the coordinates of the knee joints B_1 and $K_1, (x_2, y_2, z_2)$ and (x_3, y_3, z_3) are the coordinate of the hip joints B and $K, (x_c, y_c, z_c)$ is the coordinate of the center of pelvis link C, m_b and me are the mass of ankle joints B_2 and E, m_1 and m_4 are the mass of knee joints B_1 and K_1, m_2 and m_3 are the mass of hip joints B and K, and m_c is the mass of the center of pelvis link C.

If the mass of links of legs is negligible compared with mass of the trunk, Eqs. (1)–(3) can be rewritten as follows

$$x_{com} = x_c; \tag{13}$$

$$y_{com} = y_c; \tag{14}$$

$$z_{com} = z_c. \tag{15}$$

It means that the COM is concentrated on the center of the pelvis link.

2.2. Dynamical model of biped robot

When the biped robot is supported by one leg, the dynamics of the robot can be approximated by a simple 3D inverted pendulum whose leg is the foot of biped robot and head is COM of biped robot as shown in Fig. 2.

The length of inverted pendulum r can be expanded or contracted. The position of the mass point $p = [x_{ca}, y_{ca}, z_{ca}]^T$ can be uniquely specified by a set of state variable $q = [\theta_r, \theta_p, r]^T$ as follows [1]

$$x_{ca} = r \sin\theta_p \equiv rS_p; \tag{16}$$

$$y_{ca} = -r \sin\theta_r \equiv -rS_r; \tag{17}$$

$$z_{ca} = r\sqrt{1 - \sin^2\theta_r - \sin^2\theta_p} \equiv rD. \tag{18}$$

$[\tau_r, \tau_p, f]^T$ is defined as actuator torques and force associated with the variables $[\theta_r, \theta_p, r]^T$. The Lagrangian of the 3D inverted pendulum is

$$L = \frac{1}{2} m(\dot{x}_{ca}^2 + \dot{y}_{ca}^2 + \dot{z}_{ca}^2) - mgz_{ca}, \tag{19}$$

where m is the total mass of the biped robot, g is the gravity acceleration.

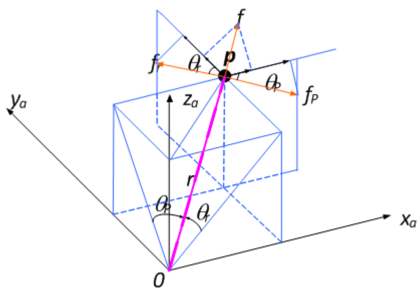


Fig. 2. Three dimension inverted pendulum

Based on the Largange's equation, the dynamics of 3D inverted pendulum can be obtained in the Cartesian coordinate as follows

$$m\begin{bmatrix} 0 & -rC_r & -\dfrac{rC_rS_r}{D} \\ rC_p & 0 & -\dfrac{rC_pS_p}{D} \\ S_p & -S_r & D \end{bmatrix}\begin{bmatrix} \ddot{x}_{ca} \\ \ddot{y}_{ca} \\ \ddot{z}_{ca} \end{bmatrix} = \begin{bmatrix} \tau_r \\ \tau_p \\ f \end{bmatrix} - mg\begin{bmatrix} -\dfrac{rC_rS_r}{D} \\ -\dfrac{rC_pS_p}{D} \\ D \end{bmatrix}. \tag{20}$$

Multiplying the first row of the Eq. (20) by D/C_r yields

$$m(-rD\ddot{y}_{ca} - rS_r\ddot{z}_{ca}) = \frac{D}{C_r}\tau_r + mgrS_r. \tag{21}$$

Substituting Eqs. (16) and (17) into Eq. (21), the dynamical equation of inverted pendulum along y_{ca} axis can be obtained as

$$m(-z_{ca}\ddot{y}_{ca} + y_{ca}\ddot{z}_{ca}) = \tau_x - mgy_{ca}. \tag{22}$$

Using similar procedure, the dynamical equation of inverted pendulum along x_{ca} axis can be derived from the second row of the Eq. (20) as

$$m(z_{ca}\ddot{x}_{ca} - x_{ca}\ddot{z}_{ca}) = \tau_y + mgx_{ca}. \tag{23}$$

The motions of the point mass of inverted pendulum are assumed to be constrained on the plane whose normal vector is $[k_x, k_y, -1]^T$ and z intersection is z_c. The equation of the plane can be expressed as

$$z_{ca} = k_x x_{ca} + k_y y_{ca} + z_c, \tag{24}$$

where k_x, k_y, z_c are constant.

Second order derivative of Eq. (24) is

$$\ddot{z}_{ca} = k_x\ddot{x}_{ca} + k_y\ddot{y}_{ca}. \tag{25}$$

Substituting Eqs. (24) and (25) into Eqs. (22) and (23), the equation of motion of 3D inverted pendulum under constraint can be expressed as

$$\ddot{y}_{ca} = \frac{g}{z_c}y_{ca} - \frac{k_x}{z_c}(x_{ca}\ddot{y}_{ca} - \ddot{x}_{ca}y_{ca}) - \frac{1}{mz_c}\tau_x; \tag{26}$$

$$\ddot{x}_{ca} = \frac{g}{z_c}x_{ca} + \frac{k_y}{z_c}(x_{ca}\ddot{y}_{ca} - \ddot{x}_{ca}y_{ca}) + \frac{1}{mz_c}\tau_y. \tag{27}$$

It is assumed that the biped robot walks on the flat floor and horizontal plane. In this case, k_x and k_y are set to zero. It means that the mass point of inverted pendulum moves on a horizontal plane with the height $z_{ca} = z_c$. Eqs. (26) and (27) can be rewritten as

$$\ddot{y}_{ca} = \frac{g}{z_c}y_{ca} - \frac{1}{mz_c}\tau_x; \tag{28}$$

$$\ddot{x}_{ca} = \frac{g}{z_c}x_{ca} + \frac{1}{mz_c}\tau_y. \tag{29}$$

When inverted pendulum moves on the horizontal plane, the dynamial equation along the x_{ca} axis and y_{ca} axis are independent and linear differential equations[1].

(x_{zmp}, y_{zmp}) is defined as location of ZMP on the floor as shown in Fig. 3.

ZMP is such a point where the net support torque from floor about x_{ca} axis and y_{ca} axis is zero. From D'Alembert's principle, ZMP of inverted pendulum under constraint can be expressed as

$$x_{zmp} = x_{ca} - \frac{z_c}{g}\ddot{x}_{ca}; \tag{30}$$

$$y_{zmp} = y_{ca} - \frac{z_c}{g}\ddot{y}_{ca}; \tag{31}$$

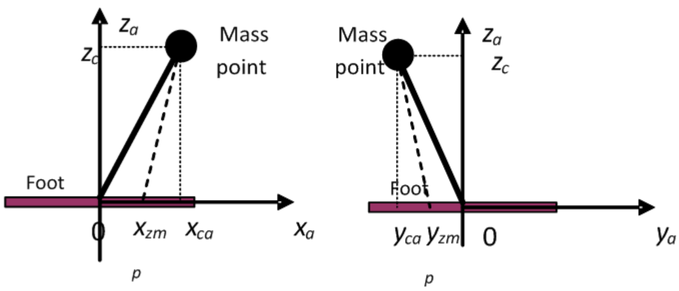


Fig. 3. ZMP of inverted pendulum

Eq. (30) shows that the position of ZMP along x_{ca} axis is a linear differential equation and it depends only on the position of mass point along x_{ca} axis. Similarly, the position of ZMP along y_{ca} axis does not depend on x_{ca} but only on y_{ca} axis.

3. WALKING PATTERN GENERATION

The objective of controlling the biped robot is to realize a stable walking or running. The stable walking or running of the biped robot depends on a walking pattern. The walking pattern generation is used to generate a trajectory for the COM of the biped robot. For the stable walking or running of the biped robot, the walking pattern should satisfy the condition that the ZMP of the biped robot always exists inside the stable region. Since position of the COM of the biped robot has the close relationship with the position of the ZMP as shown in Eqs. (25)–(26), a trajectory of the COM can be obtained from the trajectory of the ZMP. Based on a sequence of the desired footprint and the period time of each step of the biped

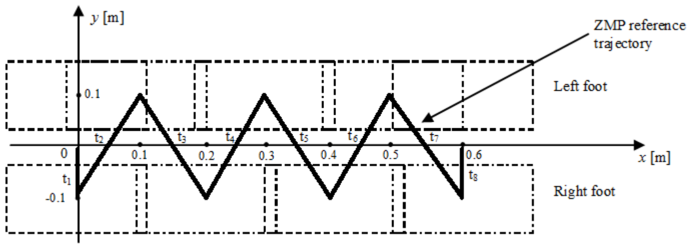


Fig. 31. Footprint and reference trajectory of the ZMP

robot, a reference trajectory of the ZMP can be specified. Fig. 3 illustrates the footprint and the zigzag reference trajectory of the ZMP to guarantee a stable gait.

The x and y ZMP trajectories versus times corresponding to the zigzag reference trajectory of the ZMP in Fig. 3 can be obtained as shown in Figs. 4 and 5.

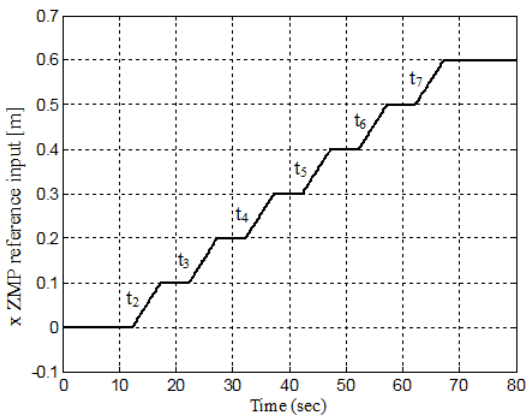


Fig.4. x ZMP reference trajectory versus time

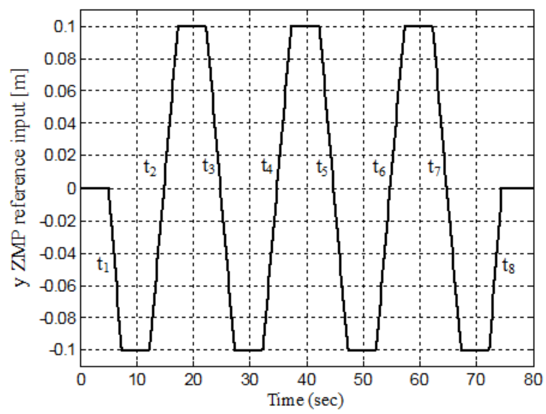


Fig.5. y ZMP reference trajectory versus time

3.1. Walking pattern generation based on optimal tracking control of the ZMP

When the biped robot is modeled as the 3D inverted pendulum which is moved on the horizontal plane, the ZMP's position of the biped robot is expressed by the linear independent equations along x_a and y_a directions which are shown as Eqs. (30)–(31).

$u_x = \dddot{x}_{ca}$ and $u_y = \dddot{y}_{ca}$ are defined as the time derivatives of the horizontal acceleration along x_a and y_a directions of the COM, u_x and u_y are introduced as inputs. Eqs. (30)–(31) can be rewritten in a strictly proper form as follows

$$\begin{bmatrix} \dot{x}_{ca} \\ \ddot{x}_{ca} \\ \dddot{x}_{ca} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{ca} \\ \dot{x}_{ca} \\ \ddot{x}_{ca} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_x, \; x_{zmp} = \begin{bmatrix} 1 & 0 & -\frac{z_{cd}}{g} \end{bmatrix} \begin{bmatrix} x_{ca} \\ \dot{x}_{ca} \\ \ddot{x}_{ca} \end{bmatrix} \tag{32}$$

$$\begin{bmatrix} \dot{y}_{ca} \\ \ddot{y}_{ca} \\ \dddot{y}_{ca} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_{ca} \\ \dot{y}_{ca} \\ \ddot{y}_{ca} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_y, \; y_{zmp} = \begin{bmatrix} 1 & 0 & -\frac{z_{cd}}{g} \end{bmatrix} \begin{bmatrix} y_{ca} \\ \dot{y}_{ca} \\ \ddot{y}_{ca} \end{bmatrix} \tag{33}$$

where x_{zmp} is position of the ZMP along x_a axis as output of the system (32), y_{zmp} is position of the ZMP along y_a axis as output of the system (33), and are positions of the COM with respect to x_a and y_a axes, and $\dot{x}_{ca}$, $\ddot{x}_{ca}$, $\dot{y}_{ca}$, $\ddot{y}_{ca}$ are horizontal velocities and accelerations with respect to x_a and y_a directions, respectively.

The systems (32) and (33) can be generalized as a linear time invariant system as follows

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u; \\ y &= \boldsymbol{C}\boldsymbol{x}. \end{aligned} \tag{34}$$

Instead of solving differential equations (30)–(31), the position of the COM can be obtained by constructing a controller to track the ZMP as the outputs of Eqs. (32)–(33). When x_{zmp} and y_{zmp} are controlled to track the x and y ZMP reference trajectories, the COM trajectories can be obtained from state variables x_{ca} and y_{ca}. According to this pattern, the walking or running of the biped robot are stable.

3.2. Continuous time controller design for ZMP tracking control

The system (34) is assumed to be controllable and observable. The objective designing this controller is to stabilize the closed loop system and to track the output of the system to the reference input. An error signal between the reference input $r(t)$ and the output of the system is defined as follows

$$e(t) = r(t) - y(t). \tag{35}$$

The objective of the control system is to regulate the error signal $e(t)$ equal to zero when time goes to infinity.

The first order and second order derivatives of the error signal are expressed as follows

$$\dot{e}(t) = \dot{r}(t) - \dot{y}(t) = -\boldsymbol{C}\dot{\boldsymbol{x}}. \tag{36}$$

From the time derivative of the first row of Eq. (34) and Eq. (36) the augmented system is obtained as follows

$$\frac{d}{dt}\begin{bmatrix} \dot{\boldsymbol{x}} \\ e \end{bmatrix} \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0}_{n\times 1} \\ -\boldsymbol{C} & 0 \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{x}} \\ e \end{bmatrix} + \begin{bmatrix} \boldsymbol{B} \\ 0 \end{bmatrix} w, \tag{37}$$

where $w = \dot{u}$ is defined as a new input signal.

The control signal w of the system (37) can be obtained as

$$w = \dot{u} = -\boldsymbol{K}_c\boldsymbol{X}_a = \boldsymbol{K}_{1c}\dot{\boldsymbol{x}} + K_{2c}e, \tag{38}$$

where $\boldsymbol{K}_c = [-\boldsymbol{K}_{1c} \quad -K_{2c}] = R_c^{-1}\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{P}_c$ and $\boldsymbol{P}_c \in \Re^{n+1\times n+1}$ is solution of the Ricatti equation.

When the initial conditions are $u_c(0) = 0$ and $\boldsymbol{x}(0) = 0$, Eq. (38) yields

$$u(t) = \boldsymbol{K}_{1c}\boldsymbol{x}(t) + K_{2c}\int_0^t e(t)dt. \tag{39}$$

Block diagram of the closed loop optimal tracking control system is shown as follows

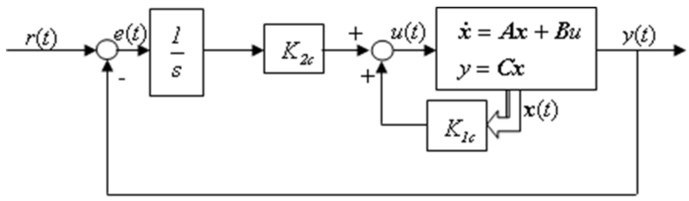


Fig. 6. Block diagram of the closed loop optimal tracking control system

4. WALKING CONTROL

Based on the stable walking pattern generation discussed in previous section, a continuous time trajectory of the COM of the biped robot is generated by the ZMP tracking control system. The continuous time trajectory of the COM of the biped robot is sampled with sampling time Tc and is stored into micro-controller. The ZMP reference trajectory of the ZMP system is chosen to satisfy the stable condition of the biped robot. The control objective for the stable walking of the biped robot is to track the center of the pelvis link to the COM trajectory. The inverse kinematics of the biped robot is solved to obtain the angle of each joint of the biped robot. The walking control of the biped robot is performed based on the solutions of the inverse kinematics which is solved by the solid geometry method.

Solving the inverse kinematics problems directly from kinematics models is complex. An inverse kinematics based on the solid geometry method is presented in this section. During the walking of the biped robot, the following assumptions are supposed:

- Trunk of the biped robot is always kept on the sagittal plane: $\theta_2 = -\theta_4$ and $\theta_9 = -\theta_6$.

- The feet of the biped robot are always parallel with floor: $\theta_3 = \theta_1 + \theta_5$.

- The walking of the biped robot is divided into 3 phase: Two legs supported, right leg supported and left leg supported.

- The origin of the 3D inverted pendulum is located at the ankle of supported leg.

4.1. Inverse kinematics of biped robot in one leg supported

When the biped robot is supported by right leg, left leg swings. A coordinate system Σ_a that takes the origin at the ankle of supported leg is defined. Since the trunk of robot is always kept on the sagittal plane, the pelvis link is always on the horizontal plane as shown in Fig. 7.

The knee joint angle of the biped robot is gotten as follows

$$\theta_3(k) = \pi - \alpha = \pi - \cos^{-1}\Big(\frac{l_1^2 + l_2^2 - h^2(k)}{2l_1l_2}\Big). \tag{42}$$

The ankle joint angle $\theta_2(k)$ can be obtained from Eq. (43). The angle $\theta_1(k)$ can be obtained from Eq. (44)

$$\theta_2(k) = \angle AOB = \sin^{-1}((y_{ca}(k) - l_3/2)/h(k)); \tag{43}$$

$$\theta_1(k) = \angle DOB_1 = \sin^{-1}\Big(\frac{x_{ca}(k)}{h(k)}\Big) + \cos^{-1}\Big(\frac{h^2(k) + l_1^2 - l_2^2}{2h(k)l_1}\Big), \tag{44}$$

where $h(k) = \sqrt{\frac{l_3^2}{4} + r^2(k) - l_3 y_{ca}(k)}$.

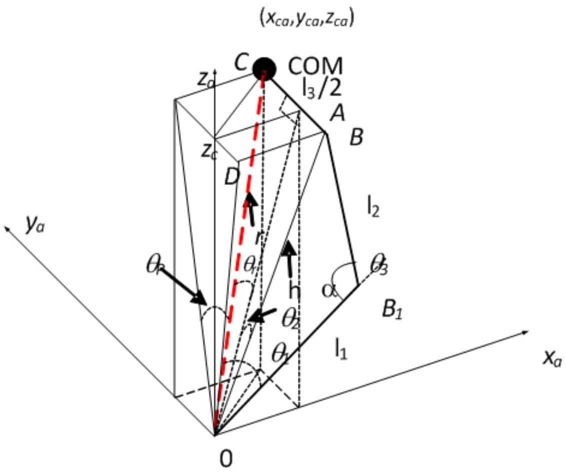


Fig. 7. Inverted pendulum and supported leg

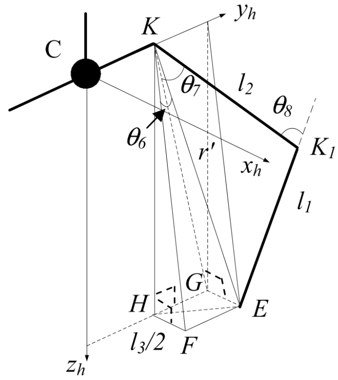


Fig. 8. Swing leg of biped robot

4.2. Inverse kinematics of swing leg

When the biped robot is supported by right leg, left leg is swung as shown in Fig. 8.

A coordinate system Σ_h with the origin that is taken at the middle of pelvis link is defined. During the swing of this leg, the coordinate y_{eh} of the foot of swing leg is constant. $r'(k)$ is defined as the distance between foot and hip joint of swing leg at k^{th} sample time. It is expressed in the coordinate system Σ_h as follows

$$r'(k)^2 = x_{eh}^2(k) + (y_{eh}(k) - \frac{l_3}{2})^2 + z_{eh}^2(k), \tag{45}$$

where $(x_{eh}(k),\ y_{eh}(k),\ z_{eh}(k))$ is the coordinate of the ankle of swing leg in the coordinate at k^{th} sample time.

The hip angle $\theta_6(k)$ of the swing leg is obtained based on the right triangle KEF as

$$\theta_6(k) = \angle EKF = -\sin^{-1}\Big(\frac{y_{eh}(k) - l_3/2}{r'(k)}\Big). \tag{46}$$

The minus sign in (46) means counterclockwise.

The hip angle $\theta_7(k)$ is equal to the angle between link l_2 and KG line. It is can be expressed as

$$\theta_7(k) = \angle GKE + \angle EKK_1 = \sin^{-1}\Big(\frac{x_{eh}(k)}{r'(k)}\Big) + \cos^{-1}\Big(\frac{r'^2(k) + l_2^2 - l_1^2}{2r'(k)l_2}\Big). \tag{47}$$

Using the cosin's law, the angle of knee of swing leg can be obtained as

$$\theta_8(k) = \pi - \angle KK_1E = \pi - \cos^{-1}\Big(\frac{l_1^2 + l_2^2 - r'^2(k)}{2l_1l_2}\Big). \tag{48}$$

When the robot is supported by two legs, the inverse kinematics is calculated by similar proceduce of one leg supported.

5. SIMULATION AND EXPERIMENTAL RESULTS

The walking control method proposed in previous section is implemented in the HUTECH-1 biped robot developed for this paper as shown in Fig. 9.

Fig. 9. HUTECH-1 biped robot

The block diagram of proposed controller for biped robot is shown in Fig. 10.

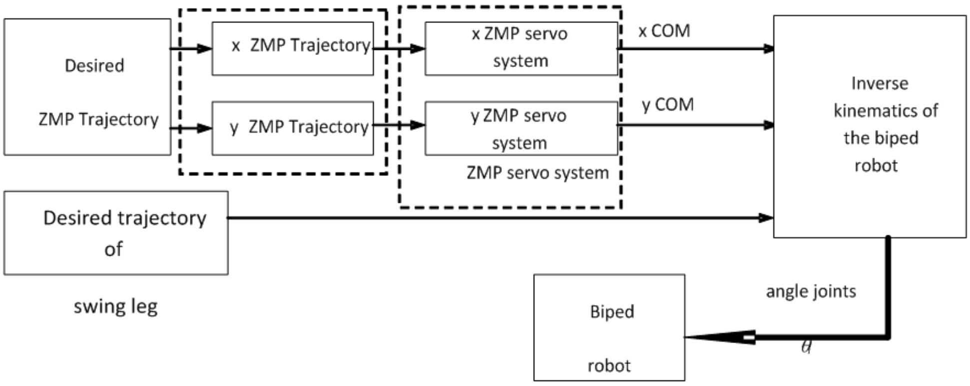


Fig. 10. Block diagram of proposed controller

The footprint and ZMP desired trajectory are shown in Fig. 11.

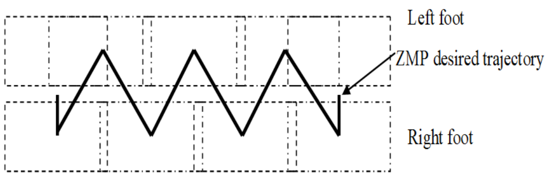
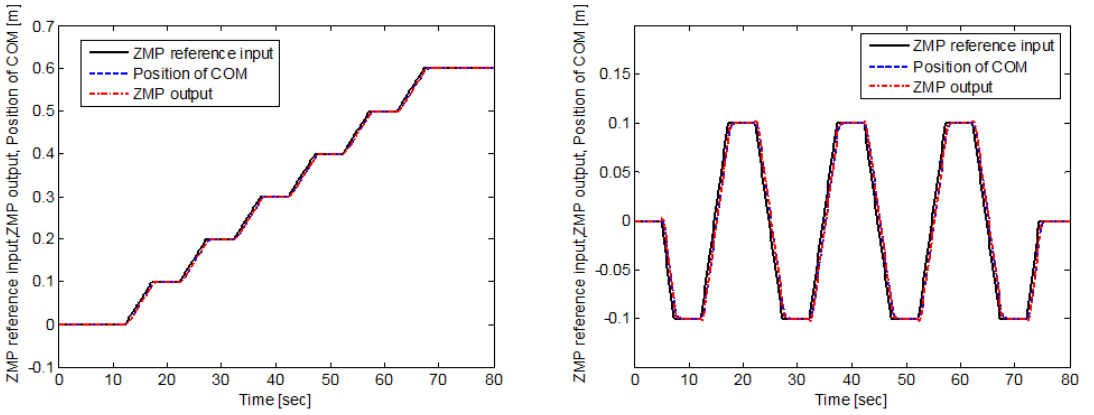


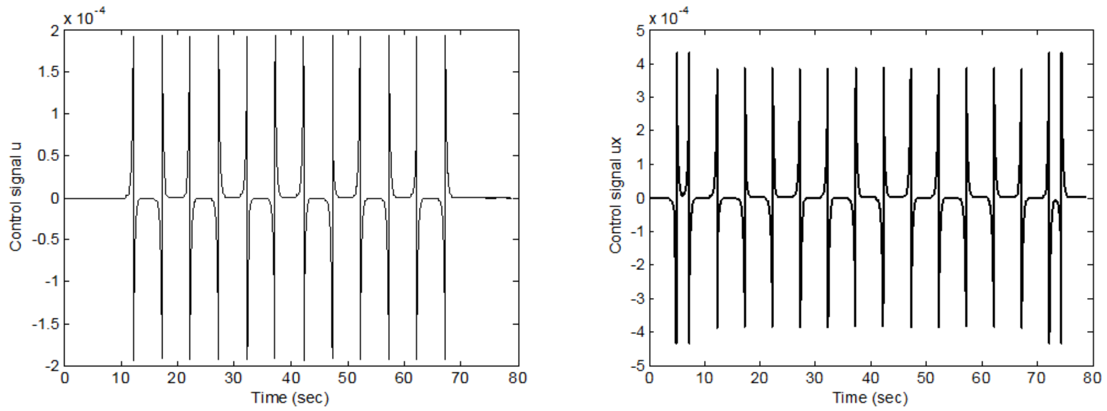
Fig. 11. Footprint and desired trajectory of ZMP

The simulation results are shown in Figs. 12–18. Fig. 12 presents x, y ZMP reference, output and coordinate of COM with respect to time. Figs. 13–14 show control signals and tracking errors. Figs. 15–17 show joints' angle of one leg of the robot, the joints' angle of opposite side leg are similar. Fig. 18 presents movement of the center of pelvis link in the world coordinate system.



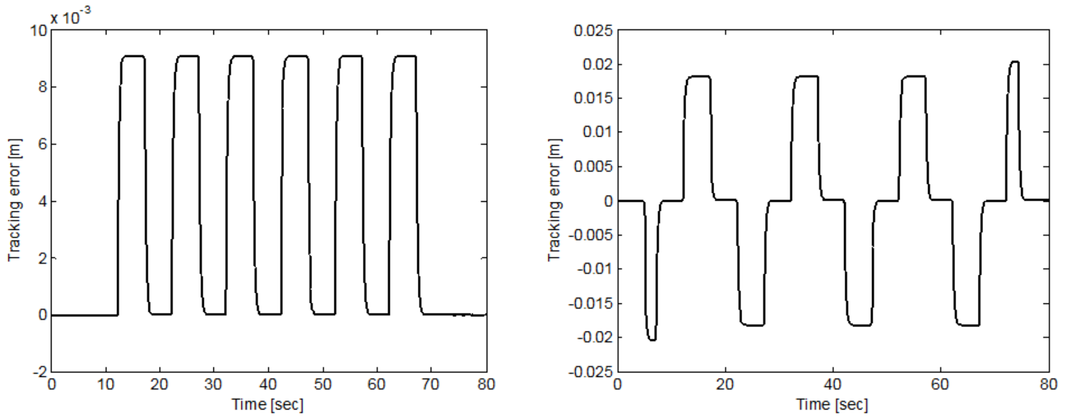
a) x ZMP reference, output and COM b) y ZMP reference, output and COM

Fig. 12. x, y ZMP reference, output and COM



a) Control signal u of y ZMP b) Control signal u of x ZMP

Fig. 13. Control signal input



a) x tracking error b) y tracking error

Fig. 14. Tracking error

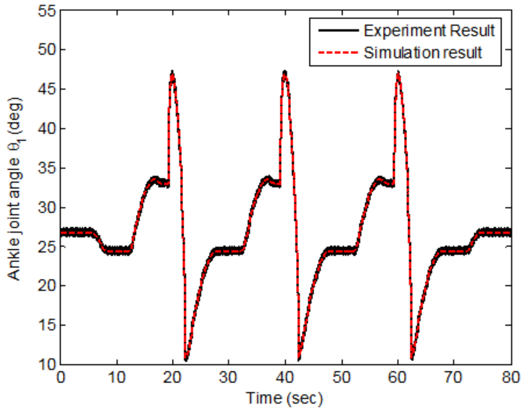


Fig. 15. The ankle joint angle θ_1

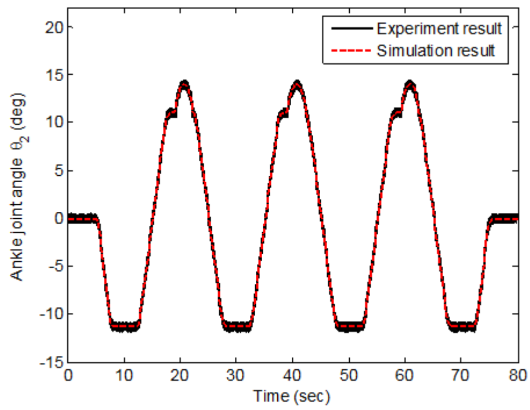


Fig. 16. The ankle joint angle θ_2

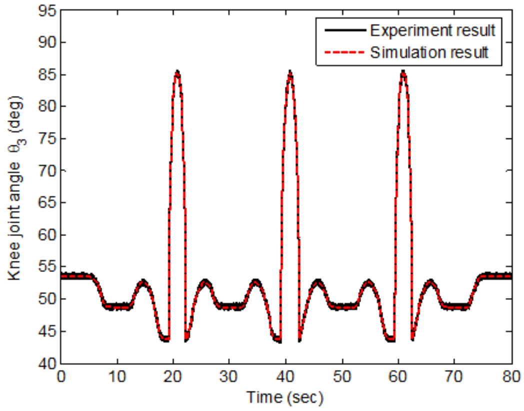


Fig. 17. The knee joint angle θ_3

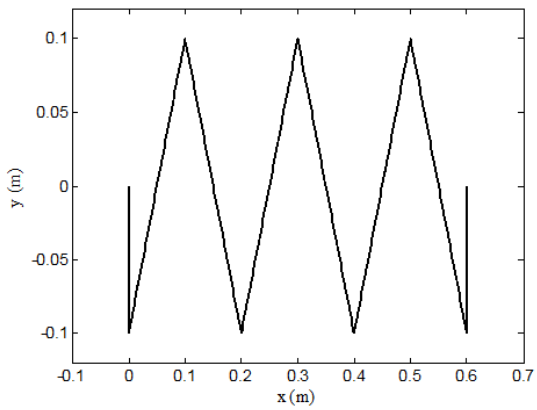


Fig. 18. Coordinate of center of pelvis link

6. CONCLUSION

In this paper, a 10 DOF biped robot has been developed. The kinematic and dynamic models of the biped robot have been proposed. A continuous time optimal tracking controller is designed to generate the trajectory of the COM for its stable walking. The walking control of the biped robot is performed based on the solutions of the inverse kinematics which is solved by the solid geometry method. A simple hardware configuration has been constructed to control the biped robot. The simulation and experimental results are given to illustrate the effectiveness of the proposed controller.

REFERENCES

[1] S. Kajita, F. Kanehiro, K. Kaneko, K, Yokoi and H. Hirukawa, The 3D linear inverted pendulum mode: A simple modeling for a biped walking pattern generation, *Proc. of IEEE/RSJ International Conference on Intelligent Robots and Systems*, Maui, Hawaii, USA, 2001 (239–246).

[2] C. Zhu and A. Kawamara, Walking principle analysis for biped robot with zmp concept, friction constraint, and inverted pendulum model, *Proc. of IEEE/RSJ International conference on Intelligent Robots and Systems*, Las Vegas, Nevada, USA, 2003 (364–369).

[3] S. K. Agrawal, and A. Fattah, Motion control of a novel planar biped with nearly linear dynamics, *IEEE/ASME Transaction on Mechatronics* **11** (2) (2006) 162–168.

[4] J. H. Part, Impedance control for biped robot locomotion, *IEEE Transaction on Robotics and Automation* **17** (3) (2001) 870–882.

[5] Q. Huang and Y. Nakamura, Sensor reflex control for humanoid walking, *IEEE Transaction on Robotics* **21** (5) (2005) 977–984.

[6] B. C. Kou, *Digital Control Systems*, International Edition, 1992.

[7] D. Li, D. Zhou, Z. Hu, and H. Hu, Optimal priview control applied to terrain following flight, *Proc. of IEEE Conference on Decision and Control*, Orlando Florida, USA, 2001 (211–216).

[8] D. Plestan, J. W. Grizzle, E. R. Westervelt, and G. Abba, Stable walking of A 7-DOF biped robot, *IEEE Transaction on Robotics and Automation* bf19 (4) (2003) 653–668.

[9] F. L. Lewis, C. T. Abdallah, and D.M. Dawson, *Control of Robot Manipulator*, Prentice Hall International Edition, 1993.

[10] G. F. Franklin, J. D. Powell, and A. E. Naeini, *Feedback Control of Dynamic System*, Prentice Hall Upper Saddle River, New Jersey (07458), October 3, 2009.

[11] G. A. Bekey, *Autonomous Robots From Biological Inspiration to Implementation and Control*, The MIT Press, 2005.

[12] H. K. Lum, M. Zribi, and Y. C. Soh, Planning and control of a biped robot, *International Journal of Engineering Science ELSEVIER* **37** (1999) 1319–1349.

[13] H. Hirukawa, S. Kajita, F. Kanehiro, K. Kaneko, and T. Isozumi, The human-size humanoid robot that can walk, lie down and get up, *International Journal of Robotics Research* **24** (9) (2005) 755–769.

[14] K. Mitobe, G. Capi, and Y. Nasu, A new control method for walking robots based on angular momentum, *Journal of Mechatronics ELSEVIER* **14** (2004) 164–165.

[15] K. Harada, S. Kajita, K. Kaneko, and H. Hirukawa, Walking motion for pushing manipulation by a humanoid robot, *Journal of the Robotics Society of Japan* **22** (3) (2004) 392—399.

Received on March 06, 2013
Revised on June 14, 2013