Journal of Computer Science and Cybernetics, Vol.20, No.4 (2004), 368–372

TRANSLATION OF RELATION SCHEMES AND SOME RELATED PROBLEMS

NGUYEN HOANG SON

Department of Mathematics, College of Sciences, Hue University

Abstract. The keys play important roles in the relational databases design theory. The results of keys have been widely investigated, they can be seen in [4, 5, 6]. In [4] to find minimal keys of relation scheme $S = (U, F)$, we translate relation scheme S to relation scheme $\widetilde{S}$, which has a less number of attributes and shorter functional dependencies. In this translation relation scheme, finding minimal keys becomes much more simple. The aim of this article is to investigate some properties of the translation relation scheme $\widetilde{S}$ and some related problems.

Tóm tắt. Khóa đóng vai trò quan trọng trong lý thuyết thiết kế cơ sở dữ liệu quan hệ. Các kết quả về khóa đã được nghiên cứu khá nhiều, có thể tìm thấy các kết quả này trong [4, 5, 6]. Trong [4] để tìm khoá tối tiểu của sơ đồ quan hệ $S = (U, F)$, chúng ta chuyển dịch sơ đồ quan hệ S về sơ đồ quan hệ $\widetilde{S}$, là sơ đồ có ít thuộc tính hơn và các phụ thuộc hàm ngắn gọn hơn. Trong sơ đồ quan hệ chuyển dịch này việc tìm khóa tối tiểu trở nên đơn giản hơn. Mục đích của bài báo này là nghiên cứu một số tính chất của sơ đồ quan hệ chuyển dịch $\widetilde{S}$ và một số vấn đề liên quan.

1. INTRODUCTION

Let us give some necessary definitions and results that are used in the next section. The concepts are given in this section can be found in [1, 2, 3, 5].

Definition 1.1. Let $U = \{a_1, \ldots, a_n\}$ be a nonempty finite set of attributes. A functional dependency (FD) is a statement of form $X \to Y$, where $X, Y \subseteq U$. The FD $X \to Y$ holds in a relation $R = \{h_1, \ldots, h_m\}$ over U if

$$(\forall h_i, h_j \in R)((\forall a \in X)(h_i(a) = h_j(a)) \Rightarrow (\forall b \in Y)(h_i(b) = h_j(b))).$$

We also say that R satisfies the FD $X \to Y$.

Let F_R be a family of all FDs that holds in R.

Definition 1.2. Then $F = F_R$ satisfies

(F1) $X \to X \in F$,

(F2) $(X \to Y \in F, Y \to Z \in F) \Rightarrow (X \to Z \in F)$,

(F3) $(X \to Y \in F, X \subseteq V, W \subseteq Y) \Rightarrow (V \to W \in F)$,

(F4) $(X \to Y \in F, V \to W \in F) \Rightarrow (X \cup V \to Y \cup W \in F)$.

A family of FDs satisfying (F1) - (F4) is called an f-family over U.

Clearly, F_R is an f-family over U. It is known [1] that if F is an arbitrary f-family, then there is a relation R over U such that $F_R = F$.

Given a family F of FDs over U, there exists a unique minimal f-family F^+ that contains F. It can be seen that F^+ contains all FDs which can be derived from F by the rules (F1)-(F4).

A relation scheme S is a pair (U, F), where U is a set of attributes, and F is a set of FDs over U.

Denote $X^+ = \{a \in U : X \to \{a\} \in F^+\}$. X^+ is called the closure of X over S.

It is clear that $X \to Y \in F^+$ iff $Y \subseteq X^+$.

Definition 1.3. Let $S = (U, F)$ be a relation scheme over U, $K \subseteq U$. K is called a minimal key of S, if it satisfies the following two conditions:
(1) $K \to U \in F^+$,
(2) $\nexists K' \subset K$ such that $K' \to U \in F^+$.

The subset K which satisfies only (1) is called a key of S.

2. RESULTS

Let $S = (U, F)$ be a relation scheme, where $U = \{a_1, a_2, \ldots, a_n\}$ is a set of attributes, and $F = \{L_i \to R_i : L_i, R_i \subseteq U, L_i \cap R_i = \emptyset, i = 1, 2, \ldots, m\}$ is a set of FDs over U. Denote

$$L = \bigcup_{i=1}^{m} L_i, R = \bigcup_{i=1}^{m} R_i.$$

The following theorem is known [6].

Theorem 2.1. ([6]) *Let $S = (U, F)$ be a relation scheme over U and K be a minimal key of S. Then*

$$(U - R) \subseteq K \subseteq (U - R) \cup ((L \cap R) - a(L, R)),$$

where $a(L, R) = (L \cap R) \cap (L - R)^+$.

Definition 2.2. Let $S = (U, F)$ be a relation scheme over U. Set $\widetilde{U} = (L \cap R) - (L - R)^+$, and $\widetilde{F} = \{L_i \cap \widetilde{U} \to R_i \cap \widetilde{U} : L_i \cap \widetilde{U} \neq \emptyset, R_i \cap \widetilde{U} \neq \emptyset, L_i \to R_i \in F\}$. Then $\widetilde{S} = (\widetilde{U}, \widetilde{F})$ is called a translation relation scheme of S over $\widetilde{U}$.

In [4] we proved the following result.

Theorem 2.3. *Let $S = (U, F)$ be a relation scheme over U, $\widetilde{S} = (\widetilde{U}, \widetilde{F})$ is a translation relation scheme of S over $\widetilde{U}$, and $K \subseteq \widetilde{U}$. Then, K is a minimal key of $\widetilde{S}$ if and only if $K \cup (U - R)$ is a minimal key of S.*

Denote K_S the set of all minimal keys of S. From Theorem 2.3 we obtain the following corollaries.

Corollary 2.4. *If $K \in K_{\widetilde{S}}$, then there exists $K' \in K_S$ such that $K \subseteq K'$.*

Corollary 2.5. *If $U - R = \emptyset$ then $K_{\widetilde{S}} = K_S$.*

The following corollary is also clear.

Corollary 2.6. *Let $S = (U, F)$ be a relation scheme, where $U = \{K_1, K_2, \dots, K_m\}$ and $F = \{K_1 \to U, K_2 \to U, \dots, K_m \to U\}$. Then, $\widetilde{U} = U$, $\widetilde{F} = F$ and hence $K_{\widetilde{S}} = K_S$.*

Remark 1. For every $L'_i \to R'_i \in \widetilde{F}$, $(L'_i)^+_{\widetilde{F}} = \widetilde{U}$ is not hold, i.e. L'_i is not the key of $\widetilde{S}$ and so it is not the minimal key. For example, we consider $F = \{\{a,b\} \to \{c\}, \{d\} \to \{a\}, \{c\} \to \{b,d\}\}$ over $U = \{a,b,c,d\}$. Then, we have $L = \{a,b,c,d\}, R = \{a,b,c,d\}, L \cap R = \{a,b,c,d\}, L - R = \emptyset$, and hence $\widetilde{U} = \{a,b,c,d\}, \widetilde{F} = \{\{a,b\} \to \{c\}, \{d\} \to \{a\}, \{c\} \to \{b,d\}\}$. It is obvious that, with a FD $\{d\} \to \{a\} \in \widetilde{F}$ we have $\{d\}^+_{\widetilde{F}} = \{a,d\} \neq \widetilde{U}$.

In translation relation schemes $\widetilde{S} = (\widetilde{U}, \widetilde{F})$, FDs and attributes have some rather interesting properties as follows.

Theorem 2.7. *Let $\widetilde{S} = (\widetilde{U}, \widetilde{F})$ be a translation relation scheme of $S = (U, F)$ trên $\widetilde{U}$. Then*
(i) *If $a \in \widetilde{U}$, then there exists $L'_i \to R'_i \in \widetilde{F}$ such that $a \in R'_i$.*
(ii) *If $a \in \widetilde{U}$, then there exists $L'_j \to R'_j \in \widetilde{F}$ such that $a \in L'_j$.*

Proof. (i) Since $a \in \widetilde{U}$, it is obvious that $a \in L \cap R$. Thus there exists a FD $L_i \to R_i \in F$ such that $a \in R_i$. Therefore we have $a \in R_i \cap \widetilde{U}$, i.e. $R_i \cap \widetilde{U} \neq \emptyset$. Furthermore, $L_i \cap \widetilde{U} \neq \emptyset$. In fact, if $L_i \cap \widetilde{U} = \emptyset$, then

$$L_i \subseteq (L - R)^+,$$

or

$$L - R \to L_i \in F^+. \tag{1}$$

On the other hand, we have

$$L_i \to R_i \in F,\ R_i \to \{a\} \in F^+. \tag{2}$$

From (1) and (2) we have $L - R \to \{a\} \in F^+$, i.e. $a \in (L - R)^+$, which contradicts the hypothesis $a \in \widetilde{U}$. Hence $L_i \cap \widetilde{U} \neq \emptyset$. Set $L'_i = L_i \cap \widetilde{U}, R'_i = R_i \cap \widetilde{U}$ we have (i), i.e. there exists a FD $L'_i \to R'_i \in \widetilde{F}$ such that $a \in R'_i$.

(ii) Because $a \in \widetilde{U}$, we have $a \in L \cap R$, i.e. there exists a FD $L_j \to R_j \in F$ such that $a \in L_j$. Therefore

$$a \in L_j \cap \widetilde{U}.$$

Moreover, we have

$$(L_j)^+_F \cap \widetilde{U} \subseteq (L_j \cap \widetilde{U})^+_{\widetilde{F}}. \tag{3}$$

In fact, according to the algorithm for finding the closure L_j^+ of L_j with $(L_j)^{(0)}_F = L_j, (L_j \cap \widetilde{U})^{(0)}_{\widetilde{F}} = L_j \cap \widetilde{U}$, we have

$$(L_j)^{(0)}_F \cap \widetilde{U} \subseteq (L_j \cap \widetilde{U})^{(0)}_{\widetilde{F}}$$

is trivial. Assume that

$$(L_j)^{(k)}_F \cap \widetilde{U} \subseteq (L_j \cap \widetilde{U})^{(k)}_{\widetilde{F}}. \tag{4}$$

Then

$$\begin{aligned}(L_j)^{(k+1)}_F \cap \widetilde{U} &= ((L_j)^{(k)}_F \cup \{b : L_i \to R_i \in F, b \in R_i, L_i \subseteq (L_j)^{(k)}_F\}) \cap \widetilde{U} \\ &= ((L_j)^{(k)}_F \cap \widetilde{U}) \cup (\{b : L_i \to R_i \in F, b \in R_i, L_i \subseteq (L_j)^{(k)}_F\} \cap \widetilde{U}) \\ &\subseteq (L_j \cap \widetilde{U})^{(k)}_{\widetilde{F}} \cup (\{b : L_i \to R_i \in F, b \in R_i, L_i \subseteq (L_j)^{(k)}_F\} \cap \widetilde{U}).\end{aligned}$$

On the other hand, from assumption (4) and $L_i \subseteq (L_j)^{(k)}_F$ we have

$$L_i \cap \widetilde{U} \subseteq (L_j)^{(k)}_F \cap \widetilde{U} \subseteq (L_j \cap \widetilde{U})^{(k)}_F.$$

So

$$(L_j)^{(k+1)}_F \cap \widetilde{U} \subseteq (L_j \cap \widetilde{U})^{(k)}_{\widetilde{F}} \cup (\{b : L_i \to R_i \in F, b \in R_i, L_i \subseteq (L_j)^{(k)}_F\} \cap \widetilde{U}) \\ \subseteq (L_j \cap \widetilde{U})^{(k+1)}_{\widetilde{F}}.$$

Hence, (3) has been proved, i.e.

$$(L_j)^+_F \cap \widetilde{U} \subseteq (L_j \cap \widetilde{U})^+_{\widetilde{F}}.$$

Moreover $L_j \to R_j \in F$, thus $R_j \subseteq (L_j)^+_F$. Consequently

$$R_j \cap \widetilde{U} \subseteq (L_j)^+_F \cap \widetilde{U} \subseteq (L_j \cap \widetilde{U})^+_{\widetilde{F}}.$$

It shows that

$$L_j \cap \widetilde{U} \to R_j \cap \widetilde{U} \in \widetilde{F}.$$

Set $L'_j = L_j \cap \widetilde{U}, R'_j = R_j \cap \widetilde{U}$, we have (ii), i.e. there exists a FD $L'_j \to R'_j \in \widetilde{F}$ such that $a \in L'_j$.
The theorem is proved. ∎

From Theorem 2.7, we have the following corollaries.

Corollary 2.8. *For each $L'_i \to R'_i \in \widetilde{F}$, if $a \in R'_i$ then $a \in L'_j$, where $L'_j \to R'_j \in \widetilde{F}$.*

Corollary 2.9. *For each $L'_i \to R'_i \in \widetilde{F}$, if $a \in L'_i$ then $a \in R'_j$, where $L'_j \to R'_j \in \widetilde{F}$.*

Theorem 2.10. *Let $S = (U, F)$ be a relation scheme over U and $\widetilde{S} = (\widetilde{U}, \widetilde{F})$ be a translation relation scheme of S over $\widetilde{U}$. Then*
(i) *If $L_i \to R_i \in F$ such that $L_i \cap \widetilde{U} = \emptyset$, then*

$$\forall a \in \widetilde{U} : a \notin R_i \text{ and hence } L_i \cap \widetilde{U} \to R_i \cap \widetilde{U} \notin \widetilde{F}.$$

(ii) *If $L_i \to R_i \in F$ such that $R_i \cap \widetilde{U} = \emptyset$, then*

$$\forall a \in \widetilde{U} : a \notin L_i \text{ and hence } L_i \cap \widetilde{U} \to R_i \cap \widetilde{U} \notin \widetilde{F}.$$

Proof. (i) Since $L_i \cap \widetilde{U} = \emptyset$, we have $L_i \subseteq (L - R)^+$. Thus

$$L - R \to L_i \in F^+.$$

Assume $a \in R_i$, it implies that $R_i \to \{a\} \in F^+$. On the other hand, we have $L_i \to R_i \in F$. Hence, by (F2) in the Definition 1.2 we have

$$L - R \to \{a\} \in F^+,$$

or

$$a \in (L - R)^+,$$

which contradicts the hypothesis $a \in \widetilde{U}$. Thus $a \notin R_i$, and $R_i \cap \widetilde{U} = \emptyset$, i.e.

$$L_i \cap \widetilde{U} \to R_i \cap \widetilde{U} \notin \widetilde{F}.$$

(ii) Suppose $a \in L_i$, which implies that $a \in L_i \cap \widetilde{U}$. With the similar provement like Theorem 2.7, we also obtain

$$L_i \cap \widetilde{U} \to R_i \cap \widetilde{U} \in \widetilde{F},$$

i.e.

$$R_i \cap \widetilde{U} \neq \emptyset,$$

which contradicts the hypothesis $R_i \cap \widetilde{U} = \emptyset$. So $a \notin L_i$, and hence we have

$$L_i \cap \widetilde{U} \to R_i \cap \widetilde{U} \notin \widetilde{F}.$$

The theorem is proved. ■

Note that, if an attribute $a \in U$ appears only in either the left side or the right side or none of the FDs in F, then a will not be in $\widetilde{U}$, i.e., if $a \in L - R$ or $a \in R - L$ or $a \notin L \cup R$ then $a \notin \widetilde{U}$.

REFERENCES

[1] Armstrong W. W., *Dependency Structure of Database Relationship*, Information Processing 74, North-Holland Pub. Co., 1974, 580–583.

[2] Codd E. F., A relational model for large shared data banks, *Comm. ACM* **13** (1970) 337–387.

[3] Demetrovics J., Thuan H. , Bao L. V., and Huy N. X., Translation of relation schemes, Balanced relation schemes and the problem of key representation, *J. Inf. Process. Cybern. EIK* **23** (1987) 81–97.

[4] Son N. H., Hung N. V., Some result s about keys of relation schemes, *Journal of Computer Science and Cybernetics* **18** (2002) 285–289.

[5] Thi V. D., Minimal keys and Antikeys, *Acta Cybernetica* **7** (1986) 361–371.

[6] Thuan H., Bao L. V., Some results about key of relational schemas, *Acta Cybernetica* (1985) 99–113.

Received on August 10, 2004
Revised on December 13, 2004