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A HYBRID PSO-SA SCHEME FOR IMPROVING THE ACCURACY OF FUZZY TIME SERIES FORECASTING MODELS

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Abstract. Forecasting methods based on fuzzy time series have been examined intensively during the last few years. Three main factors which affect the accuracy of those forecasting methods are the length of intervals, the way of establishing fuzzy logical relationship groups, and defuzzification techniques. Many researchers focus on studying the methods of optimizing the length of intervals to improve forecasting accuracies by utilizing various optimization techniques. In line with that research trend, this paper proposes a hybrid algorithm combining particle swarm optimization with the simulated annealing technique (PSO-SA) to optimize the length of intervals to improve forecasting accuracies. The experimental results on the datasets of the "enrolments of the University of Alabama," "killed in car road accidents in Belgium," and the "spot gold in Turkey" have shown that the proposed forecasting model is more effective than their counterparts.

Keywords. Fuzzy time series; Particle swarm optimization; Simulated annealing.

1. INTRODUCTION

Time series (TS) modeling and forecasting have attracted the research community's attention over the last few years. Some TS forecasting models based on the probabilistic approach, such as ARMA, MA, and ARIMA [1], etc., have been proposed. Those models have good forecasting results on the large observations (greater than 50) and cannot forecast the TS whose values are linguistic terms such as "slow", "medium", "quick", "very quick", and so on.

In 1993, Song and Chissom proposed the fuzzy time series forecasting model (FTS-FM), in which the values of demand variables are linguistic values, and applied it to the forecasting problem of the "enrollments of the University of Alabama" (EUA) [3, 4]. That model uses the min-max composition operation in fuzzy relations leading to a large amount of computational time. Chen enhanced that model using simple fuzzy reasoning and defuzzification methods [5]. Yu proposed weighted fuzzy time series models for forecasting TAIEX [6] by assigning

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weights to fuzzy relationships to resolve the recurrence of fuzzy relationships and reflect their different importance. Those forecasting models include three main phases: 1) Fuzzify the universe of discourse (UD) of TS using fuzzy sets; 2) Establish fuzzy logical relationship groups (FLRGs) for fuzzy reasoning. 3) Forecast to get fuzzy outputs and then defuzzify the fuzzy outputs to get crisp data. Since then, there have been many studies to improve the effectiveness of FTS-FMs and applied them to many forecasting problems in the real world.

The accuracy of FTS-FMs depends much on the three phases described above. The first factor is the length of intervals. In [2-4], Song and Chen partitioned the UD of historical data of the EUA into seven equal-length intervals without expressing any reason. Huarng [7] recognized that the interval length greatly influences the accuracy of FTS-FMs. The interval length should be neither too small nor too large. The interval length is too small, resulting in meaningless fuzzy time series (FTS). Whereas the interval length is too large, there is no fluctuation in FTS. Then, he introduced two approaches to determine the effective lengths of intervals: average- and distribution-based length. Later, Chen [8], Bas [9], and Lee [10, 11] applied the genetic algorithm to adjust the length of intervals. A genetic algorithm integrated with an automatic clustering technique to tune interval length was proposed by Wang [12]. Kuo et al. proposed two hybrid forecasting models, HPSO [13] and NPSO [14], based on the integration between FTS and PSO. The difference between HPSO and NPSO is only the forecasting rule. More information on each next state of FLRGs is considered in the NPSO model, so it is more accurate than the HPSO. A new model aggregating both the global information and the local information was proposed by Huang et al. [15]. Some other methods of partitioning the UD based on information granules [16, 17], ant colony optimization and auto-regression [18], fuzzy clustering [19, 20], rough-fuzzy [21, 22], etc., were proposed.

The second factor which affects forecasting accuracy is establishing FLRGs for forecasting. Chen introduced the high-order model in [23]. Two-factors high-order models were introduced by Lee [11] and Wang [12]. Yu introduced refinement relation [24] and weighted scheme [6] models. A method to construct the FLRs into FLRGs by using the K-means clustering algorithm was proposed by Cheng et al. [25]. We can easily see that in most of the mentioned research, the time-invariant FLRGs are constructed for forecasting. It means that all FLRs with the same left-hand side (LHS) are grouped into an FLRG regardless of whether they occurred in the past or future. The concept of time-variant FLRG was applied by Tinh and Dieu in [20]. Phong proposed linguistic time series with linguistic forecasting rules in [26].

The third factor affecting forecasting accuracy is the defuzzification technique. Chen proposed the average of the mid-points of fuzzy intervals corresponding to fuzzy sets of the right-hand side (RHS) of FLRG as the crisp forecasted value of the current forecasting time [5]. Yu granted the weights in chronological order to fuzzy sets in the RHS of FLRGs [6]. In [14], Kuo used the information of sub-intervals of each fuzzy interval of the next state in the RHS to compute crisp forecasted values. A new model aggregating the global information of FLRs with the local information of the latest fuzzy fluctuation to find forecasting value was proposed by Huang et al. [15]. A quite efficient defuzzification technique based on the proportions of intervals was proposed by Chen et al. in [27].

There are numerous optimization techniques applied to solve FTS forecasting problems. The intensively examined one is particle swarm optimization (PSO). However, PSO is efficient for global search but weak for local search. Therefore, it is easily trapped into the local optimums and becomes premature convergence. This paper applies a hybrid algorithm combining PSO with the simulated annealing (SA) technique to optimize the length of intervals of the UD to enhance the forecasting accuracies of the FTS-FMs. The SA technique helps PSO jump out of the local optimums to continue its searching process. The experimental results on three datasets of the EUA, the "killed in car road accidents in Belgium" (CAB), and the "spot gold in Turkey" (SGT) show that our proposed forecasting model has better-forecasted accuracy than the existing ones.

The organization of the paper is as follows: Section 2 is some basic concepts of FTS and PSO. The proposed FTS-FM is presented in Section 3. Section 4 shows the experimental results and discussion. Conclusions are written in Section 5.

2. PRELIMINARIES

2.1. Some basic concepts

Song and Chissom introduced FTS-FM in 1993 [2–4]. In 1996, Chen enhanced that model using a simple defuzzification technique. Those FTS-FMs were developed based on the following concepts.

Definition 1 [2-4]. Let $T(t)$ (t= ..., 0, 1, 2,...) be a subset of R^1, where t is the temporal variable. $T(t)$ is the *UD* on which the fuzzy sets $f_i(t)$, $i = 1, 2, \ldots$ are defined. If $F(t)$ is a series of fuzzy sets $f_i(t)$ (i = 1, 2,...), then $F(t)$ is called a fuzzy time series on $T(t)$.

Definition 2 [5]. Fuzzy logical relationship (FLR). At the times $t-1$ and t, if there exists a fuzzy relationship $R(t-1\ ,\ t)$ between $F(t-1\ )$ and $F(t)$ such that

$$F(t) = F(t-1) * R(t-1, t),$$

where * is an operator, $F(t)$ is said to be inferred from $F(t-1\ )$. The relationship between $F(t-1\ )$ and $F(t)$ is defined as $F(t-1) \to F(t)$. If $F(t-1\ ) = X_i$ and $F(t) = X_j$, the logical relationship between $F(t-1\ )$ and $F(t)$ is denoted by $X_i \to X_j$, where X_i is the LHS (current state) and X_j is the RHS (next state) of the fuzzy relation.

Definition 3 [5]. The FLRs with the same LHS are grouped together to form fuzzy logical relationship groups. For example, there are FLRs $X_i \to X_{j1}, X_i \to X_{j2}, \ldots, X_i \to X_{jn}$ that can be put into a group denoted by $X_i \to X_{j1}, X_{j2}, \ldots, X_{jn}$.

2.2. Fuzzy time series forecasting models

2.2.1. The FTS-FM of Chen

In the FTS-FM of Chen, FLRGs are established, and simple arithmetic operations are used to compute crisp forecasted values instead of complex min-max composition operations in FLRs. Hereafter is a brief description of the forecasting model of Chen [5]:

Step 1. Specify the UD of TS and partition it into equal-length intervals u_1, u_2, $\ldots$, u_n.

Step 2. Design fuzzy sets on UD.

Step 3. Fuzzify historical data.

Step 4. Generate FLRs, then establish FLRGs.

Step 5. Forecast to get fuzzy forecasted values. Then, defuzzify fuzzy forecasted values to get crisp ones using defuzzification principles as follows:

Principle 1. If there exists FLRG $X_i \to X_j$ and the midpoint of u_j is v_j, the crisp value of forecasting time is v_j.

Principle 2. If there exists FLRG $X_i \to X_{j1}, X_{j2}, ..., X_{jk}$, where X_i is the fuzzy set of a time, say t, and the midpoints of $u_{j1}, u_{j2}, ..., u_{jk}$ are $v_{j1}, v_{j2}, ..., v_{jk}$, respectively, the crisp forecasted value of time $t + 1$ is specified as

$$CFV_{t+1} = \frac{v_{j1} + v_{j2} + \ldots + v_{jk}}{k}. \tag{1}$$

Principle 3. If there exists FLRG $X_i \to \emptyset$, where the notion of empty set denotes that the RHS of this FLRG is empty, and v_i is the midpoint u_i, the crisp forecasted value is v_i.

2.2.2. The FTS-FM of Yu

Unlike the FTS-FM of Chen, a fuzzy set can be repeated on the RHS of an FLRG of the FTS-FM of Yu. Therefore, fuzzy sets in the RHS of FLRGs are granted weights in chronological order to reflect their different importance. The defuzzification principle 2 of FTS-FM of Chen described above is modified as follows: if there exists FLRG $X_i \to X_{j1}, X_{j2}, ..., X_{jk}$, where X_i is the fuzzy set of a time, say t, and the midpoints of $u_{j1}, u_{j2}, ..., u_{jk}$ are $v_{j1}, v_{j2}, ..., v_{jk}$, respectively, the crisp forecasted value of time $t + 1$ is specified as

$$CFV_{t+1} = \frac{1 \times v_{j1} + 2 \times v_{j2} + \ldots + k \times v_{jk}}{1 + 2 + \ldots + k}. \tag{2}$$

2.3. Particle swarm optimization

Particle swarm optimization (PSO), which was introduced by Kennedy and Eberhart in 1995 [28, 29], mimics the behavior of birds flying to find food sources. Hereafter is a brief description of basic PSO.

Assume that we have a swarm $S = \{x_1, x_2, \ldots, x_N\}$, where x_i is a particle having its position Y_i^t at cycle t computed as

$$Y_i^{t+1} = Y_i^t + V_i^{t+1}, \tag{3}$$

where V_i^{t+1} is the velocity of particle x_i at cycle $t + 1$, which is computed as

$$V_i^{t+1} = \omega V_i^t + c_1 r_1 \left(P_i^t - Y_i^t\right) + c_2 r_2 \left(P_g^t - Y_i^t\right), \tag{4}$$

where P_g^t and P_i^t are the global and local solutions that are found up to cycle t, respectively; c_1 and c_2 are self-cognitive and social cognitive factors; r_1 and r_2 are two random numbers which uniformly distribute in [0, 1]; ω is inertia weight. Hereafter is the basic PSO procedure:

Step 1: Randomly initialize a swarm S with its vector of velocity V and vector of position Y, the iterative variable t, and the number of cycles G_{max}.

Step 2: Calculate the value of objective function $f(Y_i^t)$ of particle x_i.

Step 3: Compare the value of $f(Y_i^t)$ with the one of $f(P_i^t)$. Update P_i^t if $f(Y_i^t)$ is better.

Step 4: Update the global best position P_g^t.
Step 5: Update V_i^t and Y_i^t by Eq. (4) and Eq. (3), respectively.
Step 6: Terminate if $t > G_{max}$. Otherwise, increase variable t and go to Step 2.

2.4. Simulated annealing algorithm

Simulated annealing (SA) [30] is an algorithm operating based on the process of metal cooling in metal annealing. SA begins at a high temperature (T_0) when the metal is in a molten state. The temperature of metal begins to gradually decrease to the ambient temperature (T_{min}) after removing the heating source. At this temperature, the energy of metal reaches the minimal value, and the metal is in a solid state.

The brief description of SA with minimizing energy E is as follows:
Step 1. Initialize an energy state E_j with the cooling rate $\alpha \in [0, 1]$, $T = T_0$], where T_0 is the initial temperature.
Step 2. Calculate the energy change between the present state i and the previous one j of the configuration

$$\Delta E = E_i - E_j.$$

Step 3. If $\Delta E < 0$, new state E_i is accepted (up-hill). Otherwise, the new state E_i is accepted (down-hill) with probability $P = e^{-\frac{\Delta E}{k_B T}}$, where k_B is the Boltzman constant.
Step 4. Terminate if reaching the termination condition. Otherwise, decrease the temperature $T = \alpha T$ and go to Step 2.

3. 3. THE PROPOSED FUZZY TIME SERIES FORECASTING MODEL

In PSO, particles inside the swarm are considered solutions to the problems and explored throughout the solution space to seek the best solutions. Therefore, PSO is very effective in global search but weak in local search. In fact, particles easily get stuck in local optimums, and it is difficult for them to jump out to continue their searching process because of the update mechanism of the velocity equation. Whereas SA has the ability to jump out of local optimums to continue the search process with the help of the "Metropolis law." In [31], PSO-SA is applied efficiently to classification problems. This section presents the proposed FTS-FM with the application of PSO-SA to improve forecasting results.

3.1. A hybrid PSO-SA algorithm

The hybrid PSO-SA algorithm is a combination of PSO with SA, so-called PSO-SA. It makes use of the global search and local search made by PSO and SA, respectively. Hereafter is a brief description of PSO-SA:
Step 1: Initialize a random swarm with n particles and all necessary variables, including cycle step t, initial temperature T_0, and cooling rate α. Evaluate the objective value of each particle.
Step 2: For each particle x_i in the swarm.
Step 2.1: Compute particle's velocity V_i^{t+1} by formula (4).
Step 2.2: Compute new particle position Y_i^{t+1} by formula (3).

Step 2.3: Evaluate objective values of particle x_i.
Step 2.4: Compare the fitness values at the new position Y_i^{t+1} and the old one Y_i^t. If the objective value at Y_i^{t+1} is better than the one at Y_i^t, meaning that the new position is better, then accept Y_i^{t+1} as the new position of x_i. Otherwise, compute the surplus of objective functions ΔF between Y_i^{t+1} and Y_i^t as the following formula

$$\Delta F = \left(\text{fitness}_i^{t+1} - \text{fitness}_i^t\right). \tag{5}$$

Step 2.5: Generate a random number $\sigma \in [0,\, 1]$. Accept new position if $\sigma > e^{-\left(\frac{\Delta F}{T^t}\right)}$ or the number of rejects exceeds 100. Go to Step 2.6 in case the new position is accepted. Otherwise, go to Step 2.1.
Step 2.6: Update the local best position P_i^t of all particles and the global best position P_g^t.
Step 3: If the termination condition is satisfied, terminate and output the best solution. Otherwise, modify annealing temperature $T^{k+1} = \alpha T^k$, $t = t + 1$, and jump to Step 2.

The SA algorithm does the exploitation by repeating from Step 2.1 to Step 2.5 until the appearance of a better position (uphill) or accepting a worse position with the probability $P = e^{-\left(\frac{\Delta F}{T^t}\right)}$ (accepting downhill). By accepting the downhill, the SA takes the opportunity to help PSO jump out of that local optimum to continue the search in other locations.

As the structure organization, the PSO-SA algorithm should take longer to run because it tries to do exploitation to get a better solution. The running time analysis is mentioned in Sub-section 4.4.

3.2. The proposed fuzzy time series forecasting model

In this subsection, a new FTS-FM is proposed in which the PSO-SA algorithm is applied to optimize the interval length to enhance forecasting accuracy. Each interval can be defined by its start and end points, forming a split point set. Therefore, it is necessary to determine the split points so that they form an interval set that minimizes the value of the mean square error (*MSE*) function used as the objective function, calculated by the formula (12).

Suppose the number of intervals of the UD is n. Then, split the UD $U = [p_0,\, p_n]$ into n intervals with the split points $p_1,\, p_2,\, \ldots,\, p_{n-2},\, p_{n-1}$. Therefore, we have interval set $u_1 = [p_0,\, p_1]$, $u_2 = [p_1,\, p_2]$, $\ldots$, $u_n = [p_{n-1},\, p_n]$. Each particle position in PSO-SA is represented by a vector with $n-1$ elements $Y_i = [p_1,\, p_2,\, \ldots,\, p_{n-2},\, p_{n-1}]$, where each p_i ($i = 1, \ldots, n-1$) is a split point. PSO-SA will find Y_i, which generates minimal *MSE* value.

The proposed FTS-FM in detail is as follows.
Step 1. Apply PSO-SA to optimize the length of intervals $u_1,\, u_2,\, \ldots,\, u_n$ of U.

Let D_{min} and D_{max} be the minimal and maximal values of the UD, respectively, and they are defined *as* $D_{min} = F_{min} - N_l$ and $D_{max} = F_{max} + N_h$, where F_{min} and F_{max} are the minimal and maximal values of the historical data, respectively, N_l and N_h are two positive integers used to adjust the lower and upper bounds of U so that U should cover all values that occur in the future. PSO-SA is applied to find the optimal interval set of U, where vector Y_i represents the position of particle x_i.
Step 2. Design the fuzzy sets on U.

As with the other models, each interval of U is assigned a fuzzy set X_i associated with a linguistic label. The fuzzy set X_i is defined as follows

$$X_i = \frac{a_{i1}}{u_1} + \frac{a_{i2}}{u_2} + \ldots + \frac{a_{in}}{u_n}, \tag{6}$$

where $0 \le a_{ij} \le 1,\ 1 \le i,\ j \le n$, is the grade of membership of u_j to X_i. For simplicity, a_{ij} just takes three different membership values of 0, 0.5, and 1. For example, u_2 belongs to X_1, X_2, and X_3 with the membership degrees of 0.5, 1, and 0.5, respectively, and belongs to the rest with the membership degree of 0. The symbol + means the set union operator and the division operator indicates the membership degree of u_j ($1 \le j \le n$) to X_i, respectively.

Step 3. Fuzzy all historical data.

In this step, all actual data of TS is fuzzified by converting it into fuzzy data. Each actual data is assigned a fuzzy set with the largest membership degree. For example, the historical data of the EUA is partitioned into seven equal-length intervals. The intervals from 1 to 7 are assigned the linguistic labels, namely X_1, X_2, ..., and X_7, respectively. The fuzzified data is shown in Table 1.

Table 1. The fuzzified data of the EUA in case the historical data is partitioned into seven equal-length intervals.

Year	**Actual data**	**Fuzzy set**	**Year**	**Actual data**	**Fuzzy set**
1971	13055	X_1	1982	15433	X_3
1972	13563	X_1	1983	15497	X_3
1973	13867	X_1	1984	15145	X_3
1974	14696	X_2	1985	15163	X_3
1975	15460	X_3	1986	15984	X_3
1976	15311	X_3	1987	16859	X_4
1977	15603	X_3	1988	18150	X_6
1978	15861	X_3	1989	18970	X_6
1979	16807	X_4	1990	19328	X_7
1980	16919	X_4	1991	19337	X_7
1981	16388	X_4	1992	18876	X_6

Step 4. Create FLRs and establish FLRGs.

FLRs are created based on the above Definition 2 in such a way that an FLR is created by a fuzzy set associated with time t on the LHS and a fuzzy set associated with time $t+1$ on the RHS. For an example of the EUA in Step 3, the created FLRs are shown in Table 2. The procedure for establishing FLRGs is as follows:

For $t = 1$ to $N - 1$ do begin $//N$ is the number of historical data

Establish the FLR $X_t \to X_{t+1}$; // X_t and X_{t+1} are the fuzzy sets at the time t and $t + 1$, respectively

Find the FLRG whose LHS is X_t. If found then append X_{t+1} to the end of its RHS, otherwise create a new FLRG whose LHS and RHS are X_t and X_{t+1}, respectively;

End;

Once FLRs are created, FLRGs are established by grouping all FLRs with the same LHSs. Chen's forecasting model is applied, so a fuzzy set cannot be repeated in the RHS of an LLRG. All FLRGs in case of seven equal-length intervals are shown in Table 3.

Step 5. Forecast to get fuzzy outputs and then defuzzify the fuzzy outputs to get crisp data.

Table 2. The FLRs of the EUA in case the historical data is partitioned into seven equal-length intervals.

Year	**Actual data**	**Fuzzy set**	**F(t)**	**First-order relationships**
1971	13055	X_1		
1972	13563	X_1	$F(1971) \to F(1972)$	$X_1 \to X_1$
1973	13867	X_1	$F(1972) \to F(1973)$	$X_1 \to X_1$
1974	14696	X_2	$F(1973) \to F(1974)$	$X_1 \to X_2$
1975	15460	X_3	$F(1974) \to F(1975)$	$X_2 \to X_3$
1976	15311	X_3	$F(1975) \to F(1976)$	$X_3 \to X_3$
1977	15603	X_3	$F(1976) \to F(1977)$	$X_3 \to X_3$
1978	15861	X_3	$F(1977) \to F(1978)$	$X_3 \to X_3$
1979	16807	X_4	$F(1978) \to F(1979)$	$X_3 \to X_4$
1980	16919	X_4	$F(1979) \to F(1980)$	$X_4 \to X_4$
1981	16388	X_4	$F(1980) \to F(1981)$	$X_4 \to X_4$
1982	15433	X_3	$F(1981) \to F(1982)$	$X_4 \to X_3$
1983	15497	X_3	$F(1982) \to F(1983)$	$X_3 \to X_3$
1984	15145	X_3	$F(1983) \to F(1984)$	$X_3 \to X_3$
1985	15163	X_3	$F(1984) \to F(1985)$	$X_3 \to X_3$
1986	15984	X_3	$F(1985) \to F(1986)$	$X_3 \to X_3$
1987	16859	X_4	$F(1986) \to F(1987)$	$X_3 \to X_4$
1988	18150	X_6	$F(1987) \to F(1988)$	$X_4 \to X_6$
1989	18970	X_6	$F(1988) \to F(1989)$	$X_6 \to X_6$
1990	19328	X_7	$F(1989) \to F(1990)$	$X_6 \to X_7$
1991	19337	X_7	$F(1990) \to F(1991)$	$X_7 \to X_7$
1992	18876	X_6	$F(1991) \to F(1992)$	$X_7 \to X_6$

Table 3. The FLRGs of the EUA in case the historical data is partitioned into seven equal-length intervals.

Group	**FLRGs**
Group 1	$X_1 \to X_1,\ X_2$
Group 2	$X_2 \to X_3$
Group 3	$X_3 \to X_3,\ X_4$
Group 4	$X_4 \to X_4,\ X_3,\ X_6$
Group 5	$X_6 \to X_6,\ X_7$
Group 6	$X_7 \to X_7,\ X_6$

In this step, each fuzzy forecasted value is implicated by the RHS (next states) of FLRG associated with time t. Then, that fuzzy forecasted value is defuzzified to get a crisp forecasted value. Defuzzification techniques have a strong effect on forecasting accuracy.

Principle 1. If there is FLRG $X_i \to X_{j1}, X_{j2}, ..., X_{jk}$ $(k \geq 1)$, where X_i is the fuzzy set of a time, say t, then the fuzzy forecasted value of time $t + 1$ is $X_{j1}, X_{j2}, ..., X_{jk}$, and it should be defuzzified to get its crisp forecasted value.

In [6], Yu used different weights in chronological order (Eq. 4)) to reflect the different importance of repeated fuzzy sets, and forecasting accuracy is improved in most of the experimental cases. In [20], Tinh et al. applied Eq. (3) to first-order and applied Eq. (7) to high-order time-variant FTS.

$$CFV_{t+1} = \frac{1}{k}\sum_{j=1}^{k} subm_{jl} \tag{7}$$

where $1 \leq j \leq k$, k is the number of next states; $subm_{jl}$ is the midpoint of one of p equal sub-intervals within interval u_{jl} of the next state, which the actual datum of the forecasting time having the maximum value of membership function of X_{jl} falls into. This defuzzification technique does not reflect the different importance of repeated fuzzy sets leading to the forecasting accuracy is not good in some cases (e.g., first-order time series models). In [14], Kuo added the midpoint of intervals of the next state v_{jl} to equation (7) as follows

$$CFV_{t+1} = \frac{1}{k}\sum_{j=1}^{k} \frac{subm_{jl} + v_{jl}}{2}. \tag{8}$$

In [27], Chen proposed a new defuzzification formula based on proportions of intervals as follows

$$CFV_{t+1} = \frac{\{[p_t \times (u_{k1max} - m_{k1min}) + u_{k1min}] + \ldots + [p_t \times (u_{kpmax} - m_{k1min}) + u_{kpmin}]\}}{p}, \tag{9}$$

where f_t is the actual datum at time t, $p_t = (f_t - u_{jmin})\ /\ (u_{jmax} - u_{jmin}) \in [0,\ 1]$, which f_t falls into, u_{jmin} and u_{jmax} are the lower and upper bounds of u_j, respectively, and p is the number of next states.

In [20], Tinh and Dieu proposed a new defuzzification formula which was applied to high-order FTS-FM as follows

$$CFV_{t+1} = \frac{1}{2 * n}\sum_{j=1}^{n} (subm_{jl} + LB_{jl})\,, \tag{10}$$

where LB_{jl} is the lower bound of one of p equal sub-intervals within interval u_{jl} of the next state, the actual datum of the forecasting time having the maximum value of membership function of X_{jl} falls into in case the actual value is less than $subm_{jl}$. Otherwise, LB_{jl} is the upper bound of it. This defuzzification formula will be applied to our proposed first-order FTS-FM.

In this paper, we will apply all the above-mentioned defuzzification techniques to the proposed FTS-FMs to evaluate them to show the best.

Principle 2. If there is FLRG $A_i \rightarrow \emptyset$, Kuo's master voting scheme [13] is applied to compute the crisp forecasted values of the testing patterns. This voting scheme lets us put the weight for the latest past linguistic value

$$CFV_{t+1} = \frac{v_{i1} \times w + v_{i2} + \ldots + v_{i\lambda}}{w + (\lambda - 1)}, \tag{11}$$

where w is the voting weight pre-specified by the user, λ is the order of FLR, and

v_{il} $(1 \leq l \leq \lambda)$ are the mid-points of the corresponding intervals of the λ latest past fuzzy sets.

The mean square error (MSE) measure used to evaluate the forecasting models is defined as follows

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(FD_i - RD_i)^2, \tag{12}$$

where n is the number of forecasted data, FD_i and RD_i are the forecasted data and the historical training datum at the time i, respectively. The smaller MSE value indicates the better solution. Besides, the root mean square error ($RMSE$) and the mean absolute percentage error ($MAPE$) are also used to evaluate the forecasting model and are defined as the formulas (13) and (14), respectively:

$$RMSE = \sqrt{MSE}. \tag{13}$$

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{|FD_i - RD_i|}{FD_i} \times 100. \tag{14}$$

4. EXPERIMENTAL RESULTS AND DISCUSSION

This section presents the experimental results of our proposed forecasting model on three datasets of the EUA, the CAB, and the SGT, as well as compares its experimental results with the ones of state-of-the-art forecasting models. The MSE value is used to evaluate the accuracy of the forecasting models.

In the first step of our proposed model, PSO-SA is applied to optimize the interval lengths of the UD by minimizing the MSE value. The diversity of the population is important. Therefore, in our experiments, the number of particles is 30, the number of cycles is 100, the Inertia coefficient ω is 0.4, the self-cognitive factor c_1 and the social cognitive factor c_2 are 0.2, the cooling rate *alpha* is 0.995, the initial temperature T_0 is 120.

All experiments are implemented by C# and performed using an Intel Core i5-8250U 1.6GHz CPU with 8 GB of memory and running Microsoft Windows 10 64-bit. There are three runs for each experiment, so we get three MSE values, and the smallest one is chosen as forecasting performance.

4.1. Experimental results on the "enrolments of the University of Alabama"

The UD of the historical data of the EUA observed from 1971 to 1992 is defined so that it should cover all data that may occur in the future. Therefore, D_{min} is set to 13,000 and D_{max} is set to 20,000 leading to U = [13,000, 20,000]. The number of intervals is 16, as in [12, 30].

First, to show the impact of the defuzzification technique on the accuracy of FTS-FMs, the experiments of the proposed FTS-FM are implemented and executed with different defuzzification techniques. Then, the experimental results are compared to show the best defuzzification technique. Last, to show the efficiency of the application of PSO-SA in improving forecasting results, the experimental results of the proposed FTS-FM are compared with the ones of the existing forecasting models.

The proposed FTS-FM with the application of defuzzification techniques (1), (2), (7), (8), (9), and (10) are denoted by M1, M2, M7, M8, M9, and M10, respectively. Each experiment of FTS-FM is executed 15 times, so we receive 15 *MSE* values. Then, the best one is selected as an experimental result. The experimental results of M1, M2, M7, M8, M9, and M10 are shown in Table 4. It is easy to see that the *MSE* value of M10 is the smallest, indicating the best value. Therefore, when comparing by the *MSE* values, we have the ranking order: M10 is the best FTS-FM, M9 is the second, M7 is the third, M8 is the fourth, M1 is the fifth, and M2 is the worst. We do further comparisons with the *MAPE* values, and the ranking order is changed as M9 is the best, M7 is the second, M10 is the third, M8 is the fourth, M1 is the fifth, and M2 is the worst.

To show the efficiency of the application of PSO-SA, our proposed FTS-FMs with the application of different defuzzification techniques are compared with the FTS-FM proposed by Chen&Zou in [27], Uslu in [32], and linguistic time series (LTS) proposed by Phong in [33]. The comparison results are shown in Table 4 and visualized in Figure 1. In [27], the defuzzification technique (9) proposed by Chen&Zou is the same as M9. It is seen in Table 4 that the *MSE* value of M9 is 6174, decreased by 73.96%, compared to the value of 23710 in Chen&Zou's model. In addition, the *MAPE* value of M9 is 0.25% smaller than that of Chen&Zou, which is 0.73%. The difference between M9 and Chen&Zou's model is only the optimization algorithm. In [33], Phong et al. applied PSO to optimize the fuzziness parameter values of the LTS forecasting model. The *MSE* of it is better than those of Chen&Zou and Uslu, but worse than ours. We receive the same result when comparing by the *MAPE* values. Therefore, we can state that PSO-SA is the main factor that helps to improve the forecasting performance of the proposed forecasting models.

4.2. Experimental results on "killed in car road accidents in Belgium"

The UD of the historical data of CAB observed from 1974 to 2004 is defined as follows: D_{min} is set to 900 and D_{max} is set to 1700, so we have $U = [900, 1700]$, the number of intervals is 17, as in [12, 30]. M9 and M10 are applied to the forecasting problem of the CAB, and their forecasted results are compared with the existing ones. The forecasted values of M9 and M10 compared to those of Uslu [32] and Chen&Zou [27] are shown in Table 5. The *MSE* value of M9 and M10 are 880 and 861, respectively, decreased, in turn, 14.06% and 15.92%, compared to the one of Chen&Zou. Similarly, the *MSE* values of M9 and M10 decreased, in turn, 49.16% and 50.26%, compared to the one of Uslu. Besides, when comparing by the *MAPE* values, the one of M9 is 1.39% smaller than those of M10, Uslu, and Chen&Zou, which are 1.49%, 2.29%, and 1.77%, respectively. These comparison results show that both our proposed FTS-FMs, M9 and M10, are better than those of Uslu and Chen&Zou. Recall that the difference between M9 and the model of Chen&Zou is just the optimization algorithm.

Table 4. The forecasted values of the Enrolments of Alabama of our proposed FTS-FM with different defuzzification formulas compared with the existing ones.

Year	Actual data	M1	M2	M7	M8	M9	M10	Uslu	Chen&Zou	LTS
1971	13055									
1972	13563	13675	13731	13564	13714	13563	13720	13650	13469	13515
1973	13867	13675	13731	13901	13714	13867	13720	13650	13952	14001
1974	14696	14611	14673	14697	14698	14696	14696	14836	14596	14800
1975	15460	15389	15468	15462	15462	15460	15316	15332	15439	15509
1976	15311	15365	15277	15356	15385	15353	15214	15447	15241	15509
1977	15603	15365	15803	15356	15385	15353	15547	15447	15925	15509
1978	15861	15973	15892	15863	15886	15861	15847	15447	15880	15752
1979	16807	16781	16833	16834	16827	16833	16829	16746	16801	16693
1980	16919	16910	17073	17039	16934	16919	16920	17075	17009	16949
1981	16388	16404	16417	16412	16381	16388	16384	16380	16260	16779
1982	15433	15389	15468	15462	15423	15433	15425	15504	15435	15553
1983	15497	15365	15277	15356	15385	15353	15547	15431	15212	15509
1984	15145	15365	15277	15356	15385	15353	15214	15077	15282	15132
1985	15163	15210	15281	15168	15193	15163	15316	15297	15344	15132
1986	15984	15973	15803	16014	15985	15984	15982	15848	15714	15752
1987	16859	16781	16833	16834	16827	16833	16829	16835	16833	16693
1988	18150	18206	17987	18142	18151	18150	18145	18145	18016	17888
1989	18970	19060	19012	18994	18989	18970	18960	18880	18937	18911
1990	19328	19364	19315	19343	19339	19333	19318	19418	19345	19439
1991	19337	19364	19315	19343	19339	19333	19372	19260	19147	19307
1992	18876	19060	19012	18858	18914	18876	18890	19031	19152	19043
MSE		**12290**	**13169**	**7044**	**8315**	**6174**	**5406**	**31684**	**23710**	**22403**
RMSE		**110.86**	**114.76**	**83.93**	**91.19**	**78.57**	**73.53**	**178.0**	**153.98**	**149.68**
MAPE		**0.56%**	**0.57%**	**0.31%**	**0.37%**	**0.22%**	**0.32%**	**0.90%**	**0.73%**	**0.72%**

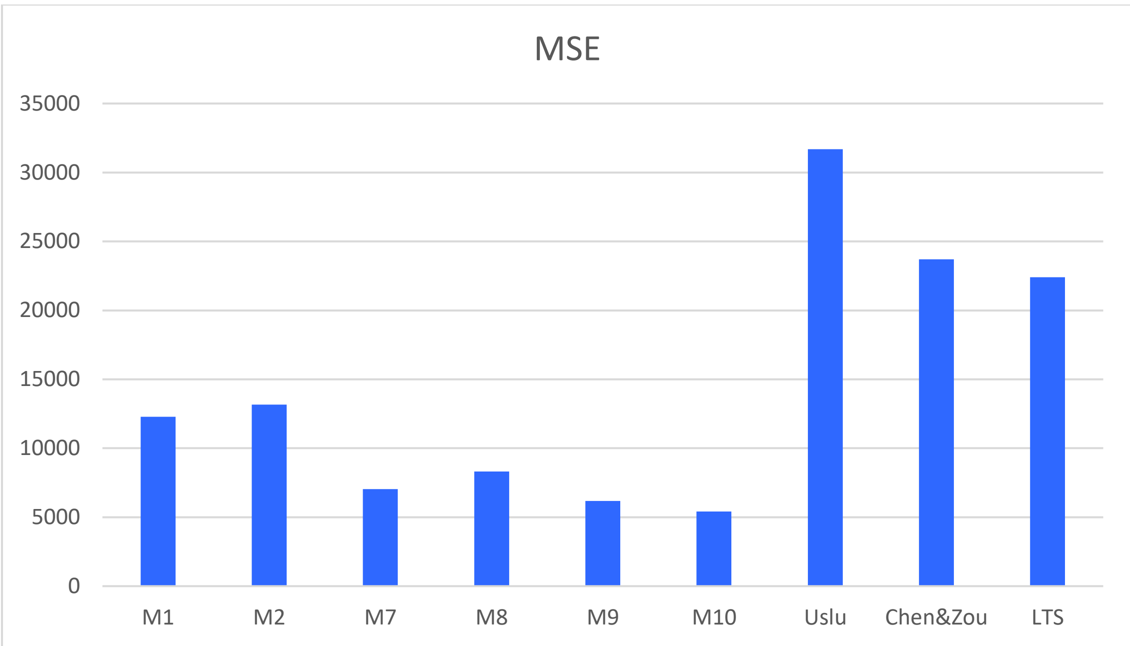

Figure 1. The *MSE* values of the proposed FTS-FMs compared with those of Uslu, Chen&Zou, and LTS.

Table 5. The forecasted values of CAB of our proposed FTS-FM are compared with those of the existing FTS-FMs.

Year	**Actual data**	**Uslu**	**Chen&Zou**	**M9**	**M10**
1974	1574				
1975	1460	1506	1451	1466	1465
1976	1536	1453	1490	1468	1470
1977	1597	1598	1622	1586	1587
1978	1644	1584	1575	1593	1592
1979	1572	1584	1593	1593	1592
1980	1616	1506	1585	1616	1614
1981	1564	1584	1582	1593	1592
1982	1464	1506	1513	1464	1465
1983	1479	1453	1494	1468	1470
1984	1369	1375	1393	1369	1371
1985	1308	1383	1336	1370	1312
1986	1456	1454	1419	1436	1435
1987	1390	1453	1485	1468	1470
1988	1432	1383	1384	1370	1433
1989	1488	1509	1459	1468	1470
1990	1574	1598	1585	1586	1587
1991	1471	1506	1451	1466	1465
1992	1380	1375	1369	1369	1371
1993	1346	1383	1361	1370	1312
1994	1415	1383	1437	1436	1435
1995	1228	1231	1217	1228	1229
1996	1122	1135	1152	1148	1147
1997	1150	1180	1172	1150	1093
1998	1224	1245	1211	1239	1238
1999	1173	1135	1147	1148	1147
2000	1253	1245	1245	1239	1238
2001	1288	1284	1280	1288	1288
2002	1145	1143	1148	1145	1143
2003	1035	970	1028	1035	1093
2004	953	970	953	953	952
MSE		**1731**	**1024**	**880**	**861**
RMSE		**41.61**	**32.0**	**29.66**	**29.34**
MAPE		**2.29%**	**1.77%**	**1.39%**	**1.49%**

4.3. Experimental results on the "spot gold in Turkey"

To show the efficiency of our proposed FTS-FM in a wide variety of forecasting problems, it is applied to the forecasting problem of the SGT with the historical data observed from December 7^{th} to November 10^{th}. The minimum and maximum values of the historical data are 30,503 and 62,450, respectively. Therefore, the UD is determined as $U = [30000, 63000]$. The number of intervals is 16, as in [27, 32]. The forecasting results of M9 and M10 compared with the ones of Uslu and Chen&Zou are shown in Table 6. By a simple calculation, we see that the *MSE* values of M9 and M10 are 848.51 and 840.66, respectively decreased, in

Table 6. The forecasted values of the SGT of our proposed FTS-FM are compared with those of the existing FTS-FMs.

Date	Actual spot gold	Uslu	Chen&Zou	M9	M10
7-Dec	30503				
8-Jan	33132	32740.18	32341.38	34166.50	33271.26
8-Feb	35201	34882.78	34479.36	34166.50	35136.85
8-Mar	38529	37409.66	38605.47	38529.00	38578.87
8-Apr	38300	39894.23	38203.34	37706.33	37717.49
8-May	36142	37023.88	37406.67	37706.33	37717.49
8-Jun	35837	37409.66	36749.36	36455.50	36447.30
8-Jul	37074	37409.66	36452.85	36455.50	36447.30
8-Aug	32955	32740.18	31805.51	32955.00	32865.10
8-Sep	33277	34882.78	34335.42	34166.50	33271.26
8-Oct	38295	37409.66	38120.71	38529.00	38578.87
8-Nov	38677	37023.88	37402.31	37706.33	37717.49
8-Dec	40724	39894.23	40726.33	40724.00	40797.13
9-Jan	43985	43666.21	44515.67	43985.00	43836.54
9-Feb	49931	49662.40	49800.77	49931.00	48984.80
9-Mar	50823	51971.99	50962.66	50823.00	52317.31
9-Apr	46167	45938.07	45869.80	46167.00	46186.90
9-May	46716	46435.40	46548.24	47384.50	46239.92
9-Jun	47337	46435.40	47067.02	47384.50	46239.92
9-Jul	46088	46435.40	47653.83	47384.50	46239.92
9-Aug	45839	46435.40	46473.59	47384.50	46239.92
9-Sep	48053	46435.40	46238.30	47384.50	48984.80
9-Oct	49592	49662.40	48330.41	49592.00	49599.79
9-Nov	53693	51971.99	54338.06	53693.00	52317.31
9-Dec	54553	54188.41	54509.96	54867.00	54362.80
10-Jan	53022	54188.41	53663.01	54867.00	54362.80
10-Feb	53613	54188.41	54183.79	54867.00	53497.13
10-Mar	55031	54188.41	54471.07	54867.00	54362.80
10-Apr	55181	54188.41	55887.68	54867.00	54362.80
10-May	60300	60069.32	60030.78	61200.00	60532.56
10-Jun	62100	60069.32	59888.46	61200.00	60532.56
10-Jul	60500	59849.50	61610.89	61475.00	60532.56
10-Aug	59200	60069.32	60079.84	61200.00	60532.56
10-Sep	61250	60069.32	61520.74	61200.00	60532.56
10-Oct	62450	62437.15	60797.52	61475.00	60532.56
10-Nov	61600	59849.50	61945.80	61475.00	60532.56
MSE		**1030692**	**805291**	**719964**	**706706**
RMSE		**1015.23**	**897.38**	**848.51**	**840.66**
MAPE		**1.80%**	**1.55%**	**1.33%**	**1.29%**

turn, by 10.6% and 12.24%, compared to the one of Chen&Zou. Similarly, the *MSE* values of M9 and M10 decreased, in turn, by 30.15% and 31.43%, compared to the one of Uslu. It is also seen that M10 is better than M9. When comparing by the *MAPE* values, we can see that the one of M10 is the best, the one of M9 is the second, and those of them are better than those of Uslu and Chen&Zou. These results state that our proposed FTS-FMs have the best forecasting performance compared with Uslu and Chen&Zou, and M10 is slightly better than M9.

4.4. The analysis of running time

In this subsection, the running time of the proposed forecasting models with the application of PSO-SA are compared with that of PSO. Both PSO-SA and PSO are executed with the number of cycles and particles are 100 and 30, respectively, and by a single-threaded program. The comparison results of training time between PSO-SA and PSO on the datasets of EUA, CAB, and SGT are shown in Table 7. We can see that PSO-SA takes longer to run on all datasets because it tries to do exploitation with the help of SA to get a better solution, but in turn, we get better forecasted results.

Table 7. The training time of PSO-SA and PSO on the datasets of EUA, CAB, and SGT

Datasets	**EUA**		**CAB**		**SGT**	
Algorithms	PSO	PSO-SA	PSO	PSO-SA	PSO	PSO-SA
Training time (second)	5	7	4	12	4	8

4.5. The analysis of result variation

Table 8. The statistic of 15 execution times of the proposed forecasting models with the application of formula (10) on the datasets of EUA, CAB, and SGT

Run	**Dataset**		
	EUA	**CAB**	**SGT**
1	20644	1469	778213.8
2	17245	964	914344.4
3	16978	861	706706.0
4	5359	1093	1057949.0
5	5406	1129	1006000.0
6	16735	1211	908568.4
7	17015	1354	869023.8
8	13286	1308	996662.1
9	10430	1184	789337.4
10	8406	1221	709327.3
11	13454	1040	1083637.0
12	10438	860	865035.8
13	8447	1147	934467.0
14	16047	1070	745490.5
15	13738	969	778243.3
Average	**12908.5**	**1125.3**	**876200.4**
Variance	**162434021.7**	**1158805.6**	**7.42253E+11**
Standard deviation	**12745.0**	**1076.5**	**861540.8**

To illustrate the variation of the experimental results, we execute the proposed forecasting models with the application of formula (10) on the datasets of EUA, CAB, and SGT 15 times, and the *MSE* values of the executions are shown in Table 8. It is easy to calculate that the difference between the smallest value and the average one of the datasets EUA, CAB, and SGT are 7549.53, 265.33, and 169494.39, respectively. The standard deviations of all datasets are less than the average values.

5. CONCLUSIONS

FTS-FM plays an essential role in the forecasting research field because of its numerous practical applications. Three main factors which have a strong effect on the forecasting accuracy of FTS-FMs are partitioning historical data, establishing fuzzy logical relationship groups, and defuzzification techniques. Among those factors, the study of the methods of optimizing the interval length of the UD to improve forecasting accuracy has attracted many researchers. This paper presents our proposed hybrid FTS-FMs combined with PSO-SA to optimize the length of intervals of the universe of discourse to improve forecasting accuracy. The characteristic of PSO-SA is that it makes use of the local search performed by SA and the global search performed by PSO to improve the search result. The experimental results on the datasets of the "enrolments of the University of Alabama," "killed in car road accidents in Belgium," and the "spot gold in Turkey" have shown that our proposed FTS-FM with the help of PSO-SA outperforms its counterparts. Furthermore, the experimental results also evaluate the influence and efficiency of different defuzzification techniques to show the best one. In specifically, in the experiment of the EUA dataset, when comparing by the *MSE* values, the formula (10) is the best and when comparing by the *MAPE* values, the formula (9) is the best. We get the same comparison result with the EUA dataset. However, the formula (10) is the best on SGT datasets when compared by both the *MSE* and the *MAPE* values.

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