A CLOSED-FORM SOLUTION FOR A QUEUEING MODEL OF ENERGY EFFICIENT ETHERNET LINKS

TIEN VAN DO[1,*], CSABA ROTTER[2]

[1]*Department of Networked Systems and Services,*
Budapest University of Technology and Economics
H-1111, Magyar tudósok körútja 2., Budapest, Hungary
[2]*Nokia Bell Labs Hungary*
H-1083, Bókay János utca 36 - 42, Budapest, Hungary

Abstract. To save energy consumption of Ethernet switches, IEEE has standardized a new energy-efficient operation for Ethernet links with a low-power state and transition mechanisms between the high-power state for transporting traffic and the low-power state. In this paper, we propose a queueing model with the Markov Modulated Compound Poisson Process that is able to characterize backbone packet traffic. We derive a closed-form solution for the stationary distribution of the proposed queueing model. We show that our model can capture an entire system where the transition times are constant.

Keywords. Energy efficient Ethernet; Queueing model; Markov modulated compound Poission process.

1. INTRODUCTION

Legacy Ethernet links are active during non-traffic periods, which could lead to the waste of energy [7, 14]. The IEEE 802.3az Energy Efficient Ethernet (EEE) standard specifies four states for the operation of green Ethernet links: Active (A), Sleep (S), Wake (W) and Low Power Idle (L). The energy-efficient operation for Ethernet links standardized by IEEE [6, 5, 8, 16, 17] consists of a low-power state and transition mechanisms between the active state with high power consumption and the low-power state. Reviriego et al. [15] reported 90% power save in state L where there is no traffic. IEEE, however, did not standardize the triggering mechanism on which conditions state L should be reached. As a result, many algorithms have been proposed for better L state usage, like frame transmission [16] and burst transmission [18] algorithm.

Herreria-Alonso et al. [8], Reviriego et al. [16] and Ajmone Marsan et al. [1] reported performance evaluations using simulation models and simple analytical models (with Poisson

---

traffic and a fixed batch packet size) for Ethernet links based on new standards. These models provided a qualitative comparison between algorithms and evaluated the effect of configuration parameters. Herreria-Alonso et al. [9] provided an analytical model for both frames and burst transmission algorithms. Larrabeiti et al. [11] proposed an analytical model to estimate the energy consumption of a two-state link and presented a performance comparison of cooper based versus optical Ethernet. The links' bidirectional behaviour is considered in the $M/G/1$ model described by Chatzipapas et al. [4]. Although their model shows a good approximation, minor discrepancies appear when burst traffic alters from the Poisson assumption. An additional effective triggering mechanism to reach state L is considered using traffic prediction in a work by Cenedese et al. [2]. Similarly, Jiang et al. [10] proposed a generalized predictive control mechanism to adjust the time window parameter automatically. However, there are no available analytical models with stochastic processes (like Markov Modulated Poisson Process [13]) that can characterize IP packet traffic.

In this paper, we propose a new model that assumes packet arrivals according to the Markov Modulated Compound Poisson Process, which is a generalization of the Markov Modulated Poisson Process – an attractive model for characterizing backbone packet traffic. It is worth emphasizing that only approximate Markov models are possible due to the fixed transition times between the modes of energy-efficient Ethernet links. Therefore, we assume that the transition times between the states of Ethernet links are exponentially distributed to obtain a tractable model. We validate our model with the exact simulation of energy-efficient Ethernet links. Furthermore, we derive the exact solution for the proposed analytical model to compare alternatives concerning the operation modes of green Ethernet links in an efficient way.

The rest of this paper is organized as follows. A queueing model is proposed in Section 2. Numerical results are shown in Section 3. Finally, Section 4 concludes our paper.

2.   A NEW ANALYTICAL MODEL

Green Ethernet links have four states: Active (A), Sleep (S), Wake (W) and Low Power Idle (L). The transitions between the states for EEE 10GBASE-T links and for EEE 100BASE-TX or 1000BASE-T links are illustrated in Figure 1 (a) and (b). T_s and T_w are the transition times between the modes of energy efficient Ethernet links.

We assume that packets arrive at an Ethernet link according to the Markov Modulated Compound Poisson Process (MMCPP).
- The modulating Markov process has two states ON and OFF with parameters α and β.
- In the OFF state, there are no packet arrivals.
- In the ON state, packets arrive according to the CPP with parameters (λ, ω). Note that $0 \leq \omega < 1$. The probability distribution function of inter-arrival times τ for customers is defined by $\Pr(\tau = 0) = \omega$ and $\Pr(0 < \tau < t) = (1 - \omega)(1 - e^{-\lambda t})$. Therefore, during ON state, the arrival process can be seen as batch-Poisson, with batches having geometric size distribution. The probability that a batch is of size s is $(1 - \omega)\omega^{s-1}$.

Similarly, to [1], the size of the transmission buffer of a specific link is assumed to be infinite. Let $J(t)$ be the number of packets in the system heading for the specific link (including the number of packet in the transmission buffer and a packet being transmitted) at time instant t.
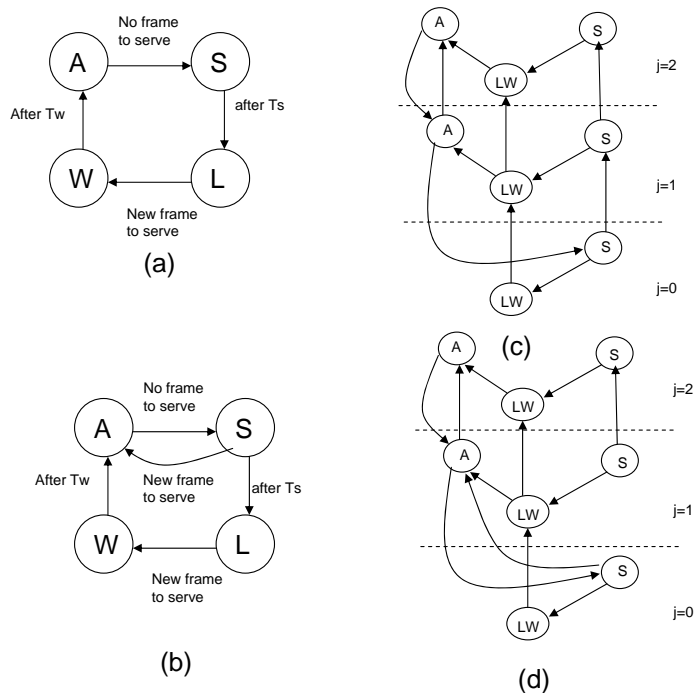
Figure 1: The state transitions

To jointly describe the state of the link and the state of the modulating process at time instant t, random variable $I(t)$ is introduced. From the operation rule [6, 5] (illustrated in Figure 1) it is observed that the link

- is never in state L when there are packets heading for the link (i.e., when $J(t) > 0$),
- is never in state W when no packet is in the system (i.e., when $J(t) = 0$).

Therefore, we join state L and W in our analytical model into state LW. If $J(t) = 0$ and the model is in state LW then the link is in state L. If $J(t) \geq 1$ and the model is in state LW, then the link is in state W. We assume that the transition times between the states of Ethernet links are exponentially distributed to obtain a tractable model. The transition times from the state S to state L and the transition times from the state W to state A are exponentially distributed with rate ν_s and ν_W, respectively. Note that $\nu_s = T_s$ and $\nu_w = T_w$.

Now it is easy to identify the value set of $I(t)$ if we lexicographically sort the states representing the link and the modulating process as in Table 1. The system is described by continuous time Markov chain (CTMC) $\{I(t), J(t)\}$. The possible transitions of the CTMC for EEE 10GBASE-T links when $1 \leq I(t) \leq 3$ are illustrated in Figure 1 (c). The possible transitions of the CTMC for EEE 100BASE-TX or 1000BASE-T links when $1 \leq I(t) \leq 3$ are depicted in Figure 1 (d).

Table 1: Values of $I(t)$

$I(t)$	1	2	3	4	5	6
The state of the link	A	S	LW	A	S	LW
The state of the modulating Markov process	ON			OFF		

Let the steady state probabilities of CTMC $\{I(t), J(t)\}$ be denoted by

$$\pi_{i,j} = \lim_{t \to \infty} \Pr(I(t) = i, J(t) = j).$$

Define the row vector $\mathbf{v}_j = [\pi_{1,j}, \ldots, \pi_{6,j}]$. We have $\pi_{1,0} = \pi_{4,0} = 0$ due to the operation rule of the link.

The infinitesimal matrix Q of CTMC $\{I(t), J(t)\}$ is a block matrix (see Table 4). The blocks contain the transitions of the Markov chain as follows.

(a) Block $Q_{0,s}^{(j)}$ includes element $Q_{0,s}^{(j)}(i,k)$ that is the $s-$step upward transition from state (i,j) to state $(k, j+s)$ $(1 \le i, k \le 6; j = 0, 1, \ldots)$. These transitions are caused by the arrivals of customers.

(b) Block $Q_1^{(j)}$ consists of element $Q_1^{(j)}(i,k)$ that denotes the purely phase transition from state (i,j) to state (k,j) $(1 \le i, k \le 6, k \ne i; j = 0, 1, \ldots)$.

(c) Block $Q_2^{(j)}$ contains element $Q_2^{(j)}(i,k)$ that is the one$-$step downward transition from state (i,j) to state $(k, j-1)$ $(1 \le i, k \le 6; j = 1, \ldots)$. These transitions are caused by the departures of customers from the system.

Based on the operation of the link, we can write the transition matrices and the infinitesimal generator matrix in Tables 2, 3 and 4. Note that matrices $Q_2^{(j)}$, $j \ge 0$, are the same for EEE 10GBASE-T, EEE 100BASE-TX and EEE 1000BASE-T links.

The balance equations and the normalization equation can be written as follows

$$\mathbf{v}_0 Q_1^{(0)} + \mathbf{v}_1 Q_2^{(1)} = 0, \tag{1}$$

$$\mathbf{v}_0 Q_{0,j}^{(0)} + \sum_{s=1}^{j-1} \mathbf{v}_{j-s} Q_{0,s}^{(j-s)} + \mathbf{v}_j Q_1 + \mathbf{v}_{j+1} Q_2 = 0, \quad j \ge 1,$$

$$\sum_{j=0}^{\infty} \sum_{i=1}^{6} \pi_{i,j} = 1. \tag{2}$$

Theorem 1. *For $j > 1$, the following equation holds between the stationary probabilities*

$$\mathbf{v}_{j-1}\widetilde{Q}_0 + \mathbf{v}_j\widetilde{Q}_1 + \mathbf{v}_{j+1}\widetilde{Q}_2 = 0, \tag{3}$$

where $\widetilde{Q}_0 = (Q_{0,1} - Q_1\Omega)$, $\widetilde{Q}_1 = (Q_1 - Q_2\Omega)$ and $\widetilde{Q}_2 = Q_2$.

Proof. Let us define $\omega_s = \lambda(1 - \omega)\omega^{s-1}$, $\Omega = Diag[\omega, \omega, \omega, 0, 0, 0]$, $\Lambda = Diag[\lambda, \lambda, \lambda, 0, 0, 0]$ Then, we obtain

$$Q_{0,s}^{(j)} = \Lambda(I - \Omega)\Omega^{s-1} = Q_{0,s-1}^{(j)}\Omega, \tag{4}$$

$$Q_{0,j}^{(0)} = Q_{0,j-1}^{(0)}\Omega. \tag{5}$$

Multiplying balance equation concerning level $j-1$, $j \ge 2$, with Ω, we get

$$\mathbf{v}_0 Q_{0,j-1}^{(0)}\Omega + \sum_{s=1}^{j-2} \mathbf{v}_{j-1-s} Q_{0,s}^{(j-1-s)}\Omega = -\mathbf{v}_{j-1}Q_1\Omega - \mathbf{v}_j Q_2\Omega = 0, \quad j \ge 2.$$

We substitute (6) into (2), and utilize equations (4) and (5). After some algebraic steps, we get $-\mathbf{v}_{j-1}Q_1\Omega - \mathbf{v}_j Q_2\Omega + \mathbf{v}_{j-1}Q_{0,1}^{(j-1)} + \mathbf{v}_j Q_1 + \mathbf{v}_{j+1}Q_2 = 0$, which yields (3). ∎

Table 2: Transition matrices for modeling an EEE 10GBASE-T link

$$Q_{0,s}^{(0)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_s & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} , \ s \geq 1 \ ;$$

$$Q_{0,s}^{(j)} = Q_{0,s} = \begin{bmatrix} \omega_s & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_s & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} , \ j \geq 1, \ s \geq 1;$$

$$Q_1^{(0)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda - \alpha - \nu_s & \nu_s & 0 & \alpha & 0 \\ 0 & 0 & -\lambda - \alpha & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & -\beta - \nu_s & \nu_s \\ 0 & 0 & \beta & 0 & 0 & -\beta \end{bmatrix} ,$$

$$Q_1^{(j)} = Q_1 = \begin{bmatrix} -\lambda - \alpha - \mu & 0 & 0 & \alpha & 0 & 0 \\ 0 & -\lambda - \alpha - \nu_s & \nu_s & 0 & \alpha & 0 \\ \nu_w & 0 & -\lambda - \alpha - \nu_w & 0 & 0 & \alpha \\ \beta & 0 & 0 & -\beta - \mu & 0 & 0 \\ 0 & \beta & 0 & 0 & -\beta - \nu_s & \nu_s \\ 0 & 0 & \beta & \nu_w & 0 & -\beta - \nu_w \end{bmatrix} ,$$
$$j \geq 1,$$

$$Q_2^{(1)} = \begin{bmatrix} 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} ,$$

$$Q_2^{(j)} = Q_2 = \begin{bmatrix} \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} , \ j > 1.$$

Table 3: Transition matrices for modeling an EEE 100BASE-TX or 1000BASE-T link

$$Q_{0,s}^{(0)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \omega_s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ s \geq 1,$$

$$Q_{0,s}^{(j)} = Q_{0,s} = \begin{bmatrix} \omega_s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ j \geq 1, \ s \geq 1,$$

$$Q_1^{(0)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda - \alpha - \nu_s & \nu_s & 0 & \alpha & 0 \\ 0 & 0 & -\lambda - \alpha & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & -\beta - \nu_s & \nu_s \\ 0 & 0 & \beta & 0 & 0 & -\beta \end{bmatrix},$$

$$Q_1^{(j)} = Q_1 = \begin{bmatrix} -\lambda - \alpha - \mu & 0 & 0 & \alpha & 0 & 0 \\ 0 & -\alpha & 0 & 0 & \alpha & 0 \\ \nu_w & 0 & -\lambda - \alpha - \nu_w & 0 & 0 & \alpha \\ \beta & 0 & 0 & -\beta - \mu & 0 & 0 \\ 0 & \beta & 0 & 0 & -\beta - \nu_s & \nu_s \\ 0 & 0 & \beta & \nu_w & 0 & -\beta - \nu_w \end{bmatrix}, \ j \geq 1,$$

Table 4: Infinitesimal generator matrix

$$Q = \begin{bmatrix} Q_1^{(0)} & Q_{0,1}^{(0)} & Q_{0,2}^{(0)} & \cdots & Q_{0,j-1}^{(0)} & Q_{0,j}^{(0)} & Q_{0,j+1}^{(0)} & Q_{0,j+2}^{(0)} & Q_{0,j+3}^{(0)} & \cdots \\ Q_2^{(1)} & Q_1^{(1)} & Q_{0,1}^{(1)} & \cdots & Q_{0,j-2}^{(1)} & Q_{0,j-1}^{(1)} & Q_{0,j}^{(1)} & Q_{0,j+1}^{(1)} & Q_{0,j+2}^{(1)} & \cdots \\ 0 & Q_2^{(2)} & Q_1^{(2)} & \cdots & Q_{0,j-3}^{(2)} & Q_{0,j-2}^{(2)} & Q_{0,j-1}^{(2)} & Q_{0,j}^{(2)} & Q_{0,j+1}^{(2)} & \cdots \\ 0 & 0 & Q_2^{(3)} & \cdots & Q_{0,j-4}^{(3)} & Q_{0,j-3}^{(3)} & Q_{0,j-2}^{(3)} & Q_{0,j-1}^{(3)} & Q_{0,j}^{(3)} \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & Q_2^{(j)} & Q_1^{(j)} & Q_{0,1}^{(j)} & Q_{0,2}^{(j)} & Q_{0,3}^{(j)} & \\ \cdots & \cdots & \cdots & \cdots & \cdots & Q_2^{(j+1)} & Q_1^{(j+1)} & Q_{0,1}^{(j+1)} & Q_{0,2}^{(j+1)} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & Q_2^{(j+2)} & Q_1^{(j+2)} & Q_0^{(j+2)} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix},$$

The consequence of Theorem 1 is that v_j, $j \geq 1$, is the solution of quadratic matrix equation (3) as

$$\mathbf{v}_j = \sum_{k=1}^{6} b_k \boldsymbol{\psi}_k x_k^{j-1} \quad (j \geq 1); \tag{6}$$

where
- b_k are suitable coefficients to be determined using balance equation (1), balance equation (2) for $j = 1$, and the normalization equation,
- $(x_k, \boldsymbol{\psi}_k)$, $k = 1, \ldots, 6$ are the left eigenvalue-eigenvector pairs of $Q(x) = \widetilde{Q}_0 + \widetilde{Q}_1 x + \widetilde{Q}_2 x^2$ inside the unit circle (see [3]). Let $\boldsymbol{\psi}_k = [\psi_{k,1}, \psi_{k,2}, \psi_{k,3}, \psi_{k,4}, \psi_{k,5}, \psi_{k,6}]$. Note that the left eigenvalue-eigenvector pairs satisfy $\boldsymbol{\psi}_k Q(x_k) = \mathbf{0}$ and $det[Q(x_k)] = 0$, $k = 1, \ldots, 6$.

Remarks. One may observe that equation (3) looks like the balance equation of homogeneous quasi-birth-death processes [12]. Therefore, one may express v_j as the function of the so-called rate matrix R (see [12] for the matrix geometric solution). However, some properties (e.g., the non-negative elements) of the rate matrix do not hold because $\widetilde{Q}_0 + \widetilde{Q}_1 + \widetilde{Q}_2$ is not a stochastic matrix.
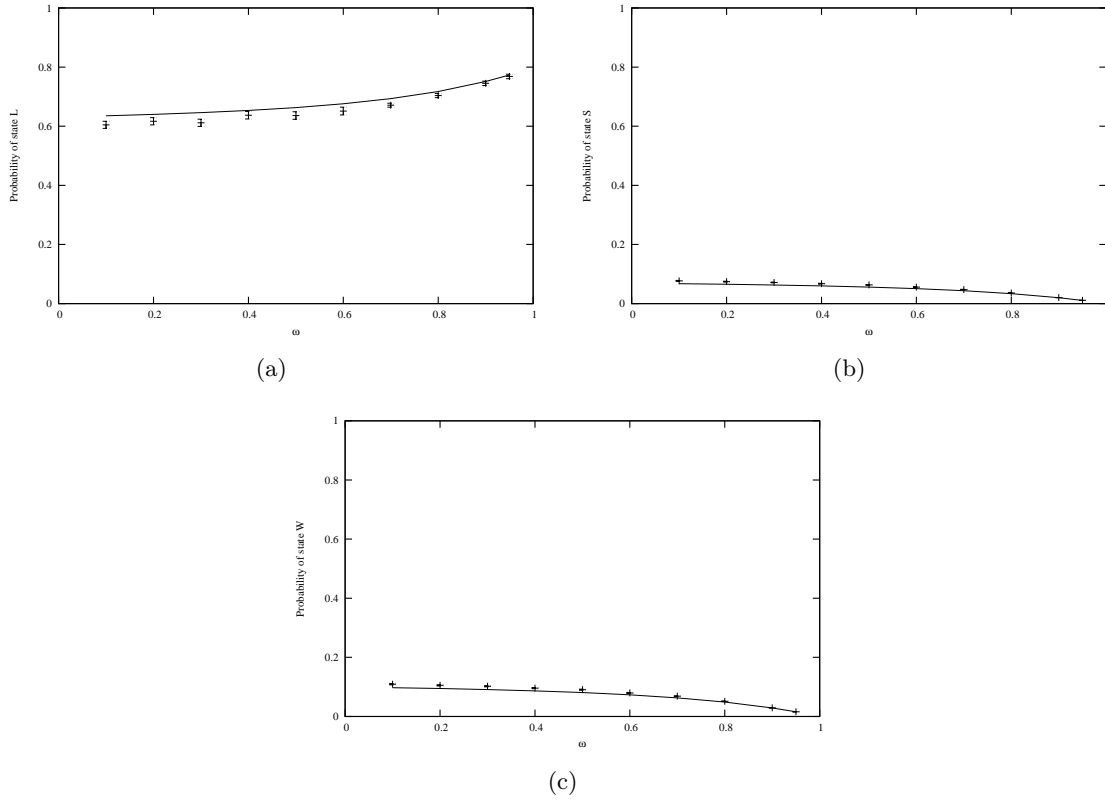

(a)

(b)

(c)
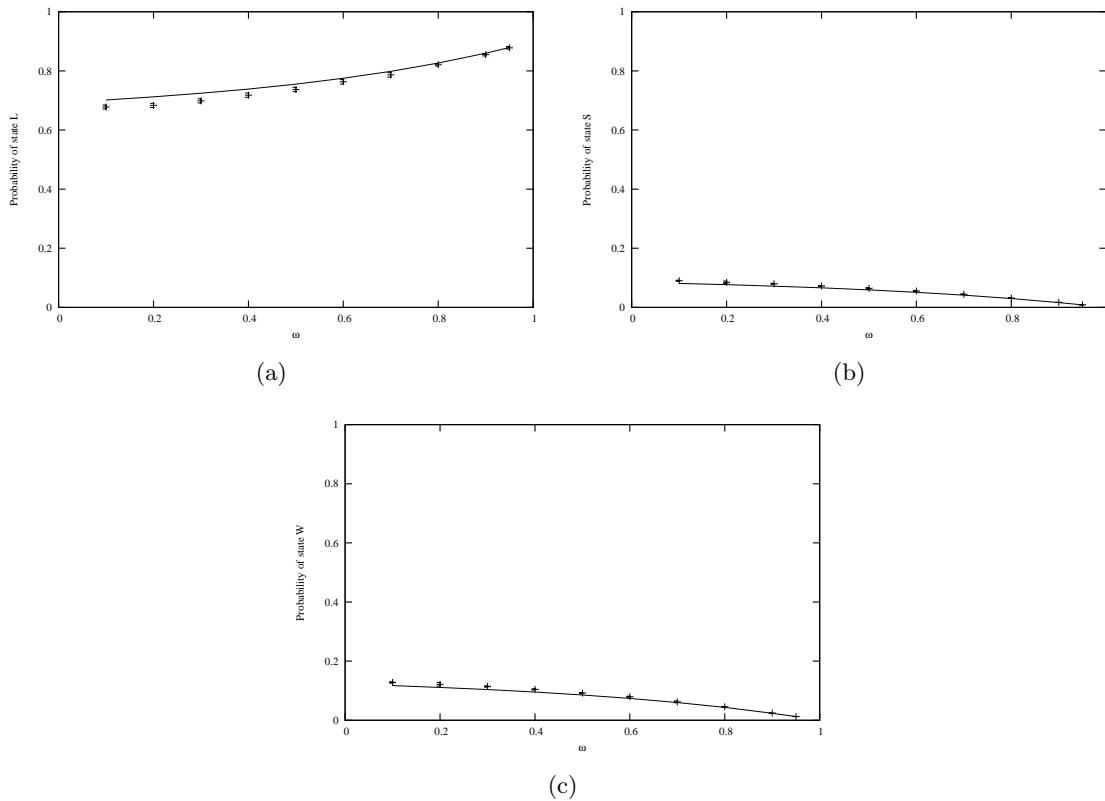
Figure 2: Probability vs ω for $\rho = 0.1$

2.1. The left-eigenvalues for an EEE 10GBASE-T link model

Utilizing Theorem 1, we obtain

$$det[Q(x)] = det[\widetilde{Q}_0 + \widetilde{Q}_1 x + \widetilde{Q}_2 x^2] = (x - 1)x^3 f_1(x) f_2(x) f_3(x),$$

Figure 3: Probability vs ω for $\rho = 0.2$

where

$$
\begin{aligned}
f_1(x) &= \mu^2 x^2 - (\alpha\mu + \beta\mu + \lambda\mu + \mu^2 + \mu^2\omega)x + \beta\lambda + \lambda\mu + \alpha\mu\omega + \beta\mu\omega + \mu^2\omega, \\
f_2(x) &= (\beta\lambda + \alpha\nu_s + \beta\nu_s + \lambda\nu_s + \nu_s^2)x - \beta\lambda - \lambda\nu_s - \alpha\nu_s\omega - \beta\nu_s\omega - \nu_s^2\omega, \\
f_3(x) &= (\beta\lambda + \alpha\nu_w + \beta\nu_w + \lambda\nu_w + \nu_w^2)x - \beta\lambda - \lambda\nu_w - \alpha\nu_w\omega - \beta\nu_w\omega - \nu_w^2\omega.
\end{aligned}
$$

As a consequence, $det[Q(x)]$ has 8 roots. The roots inside the unit circle are

$$
\begin{aligned}
x_1 &= x_2 = x_3 = 0, \\
x_4 &= \frac{\beta\lambda + \lambda\nu_s + \alpha\nu_s\omega + \beta\nu_s\omega + \nu_s^2\omega}{\beta\lambda + \alpha\nu_s + \beta\nu_s + \lambda\nu_s + \nu_s^2}, \\
x_5 &= \frac{\beta\lambda + \lambda\nu_w + \alpha\nu_w\omega + \beta\nu_w\omega + \nu_w^2\omega}{\beta\lambda + \alpha\nu_w + \beta\nu_w + \lambda\nu_w + \nu_w^2}, \\
x_6 &= \frac{\alpha\mu + \beta\mu + \lambda\mu + \mu^2 + \mu^2\omega}{2\mu^2} \\
&\quad - \frac{\sqrt{(\alpha\mu + \beta\mu + \lambda\mu + \mu^2 + \mu^2\omega)^2 - 4\mu^2(\beta\lambda + \lambda\mu + \alpha\mu\omega + \beta\mu\omega + \mu^2\omega)}}{2\mu^2}.
\end{aligned}
$$

2.2. The left-eigenvalues for an EEE 100BASE-TX or 1000BASE-T link model

In this case, $det[Q(x)]$ can be expressed as follows

$$det[Q(x)] = \alpha \nu_s (x - 1) x^3 (x - \omega) f_4(x) f_5(x),$$

where

$$
\begin{aligned}
f_4(x) &= x(\beta\lambda + \alpha\nu_w + \beta\nu_w + \lambda\nu_w + \nu_w^2) - \beta\lambda - \lambda\nu_w - \alpha\nu_w\omega - \beta\nu_w\omega - \nu_w^2\omega, \\
f_5(x) &= \beta\lambda + \lambda\mu + \alpha\mu\omega + \beta\mu\omega + \mu^2\omega - x(\alpha\mu + \beta\mu + \lambda\mu + \mu^2 + \mu^2\omega) + \mu^2 x^2.
\end{aligned}
$$

Thus, the roots inside the unit circle are

$$
\begin{aligned}
x_1 &= x_2 = x_3 = 0, \\
x_4 &= \omega, \\
x_5 &= \frac{\beta\lambda + \lambda\nu_w + \alpha\nu_w\omega + \beta\nu_w\omega + \nu_w^2\omega}{\beta\lambda + \alpha\nu_w + \beta\nu_w + \lambda\nu_w + \nu_w^2}, \\
x_6 &= \frac{\alpha\mu + \beta\mu + \lambda\mu + \mu^2 + \mu^2\omega}{2\mu^2} \\
&\quad - \frac{\sqrt{(\alpha\mu + \beta\mu + \lambda\mu + \mu^2 + \mu^2\omega)^2 - 4\mu^2(\beta\lambda + \lambda\mu + \alpha\mu\omega + \beta\mu\omega + \mu^2\omega)}}{2\mu^2}.
\end{aligned}
$$

The corresponding eigenvectors of zero-eigenvalues are $[0, 0, 0, 1, 0, 0]$, $[0, 0, 0, 0, 1, 0]$, $[0, 0, 0, 0, 0, 1]$. Using $\boldsymbol{\psi}_k Q(x_k) = \mathbf{0}$, we can easily determine the exact formula of $\boldsymbol{\psi}_k$ for non zero eigenvalues. Then, the expressions for v_0 and b_k, $k = 1, \ldots, 6$, can be derived as well (we do not write here because the long formulae).

2.3. Performance measures

The probability of state A, S, L and W can be determined as

$$
\begin{aligned}
P_A &= \pi_{1,0} + \pi_{4,0} + \sum_{k=4}^{6} \frac{b_k(\psi_{k,1} + \psi_{k,4})}{1 - x_k} + \sum_{k=1}^{3} b_k(\psi_{k,1} + \psi_{k,4}), \\
P_S &= \pi_{2,0} + \pi_{5,0} + \sum_{k=4}^{6} \frac{b_k(\psi_{k,2} + \psi_{k,5})}{1 - x_k} + \sum_{k=1}^{3} b_k(\psi_{k,2} + \psi_{k,5}), \\
P_L &= \pi_{3,0} + \pi_{6,0}, \\
P_W &= 1 - P_A - P_S - P_L.
\end{aligned}
$$

3. NUMERICAL RESULTS

To show that the queueing model can evaluate the performance of Energy Efficient Ethernet links, we compare results obtained by a simulation where transition times T_w and T_s between states are fixed. Note that in the proposed queueing model, the transition times follow the exponential distribution. Simulations are performed with a confidence level of 99%. For $T_s = 2.88$, $T_w = 4.16$ μs (see [16]), $\alpha = 0.0001695$ (the average ON period is 5.899

ms), $\beta = 0.000110$ (the average OFF period is 9.09 ms), the average packet size is of 1500 bytes (hence $\mu = 0.833$ $1/\mu s$ in 10Gbits links), $\rho = \frac{r_{ON}\lambda}{(1-\omega)\mu}$, we plot the probabilities obtained with our model and the simulation model in Figures 2, 3 and 4. We can conclude that the agreement between the simulation and analytical results is excellent.
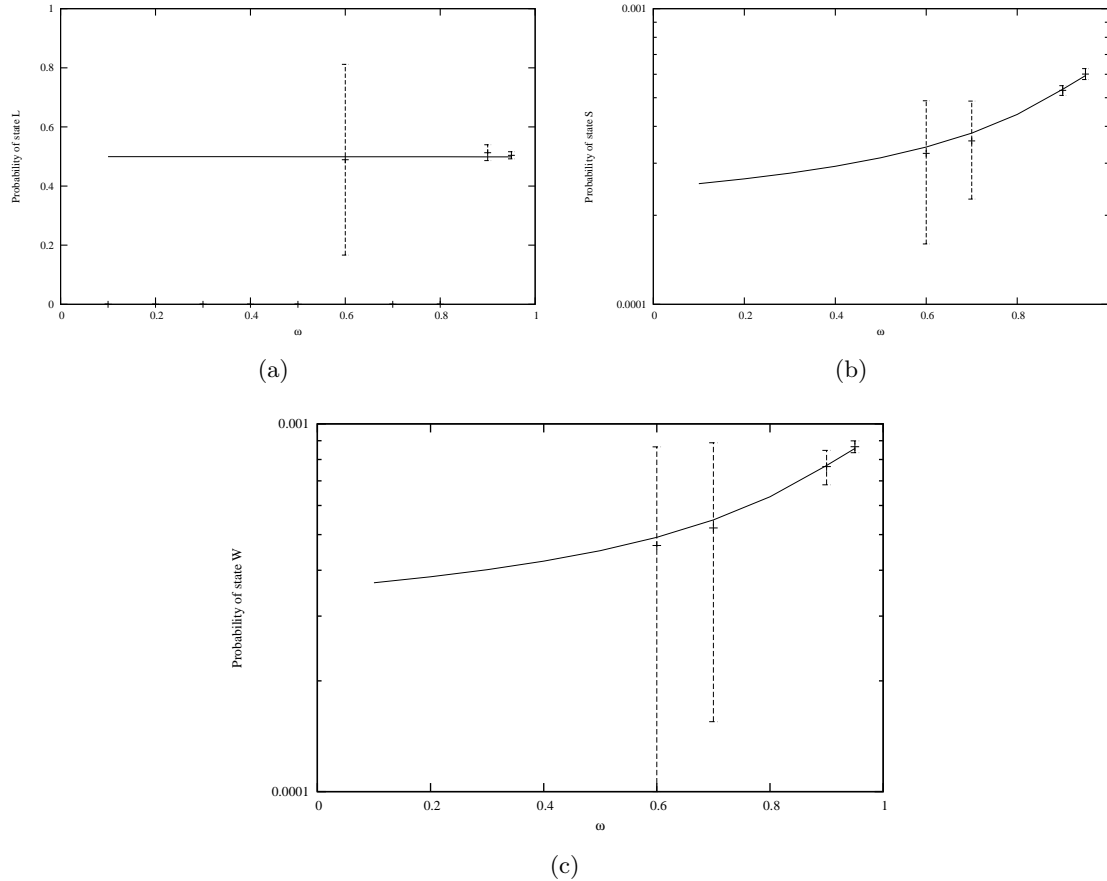

(a)

(b)

(c)

Figure 4: Probability vs ω for $\rho = 0.5$

From Figure 5, when the load is small, the higher is ω is, the better energy saving can be achieved. At the same load (the same amount of packets), the burstiness (packets in batches of more packets) would have a better impact on energy saving.

For an EEE 1000BASE-T link model, we plot the probability in state L versus T_w and T_s in Figure 6. From the curve, it seems that T_w has a minimal impact on P_L.

4.   CONCLUSIONS

We have proposed a queueing model for Energy Efficient Ethernet links. In the model packets arrive according to the Markov Modulated Compound Poisson Process and the transition times between the states of Ethernet links are exponentially distributed. We have derived the exact solution for the steady-state probabilities of the analytical model. The comparison between our model and the simulation of energy-efficient Ethernet links shows that the proposed model can capture the behaviour of Energy Efficient Ethernet links.
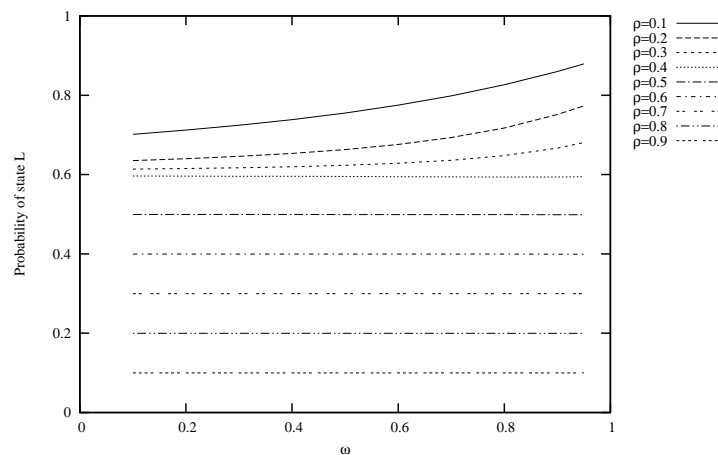
Figure 5: Probability in state L
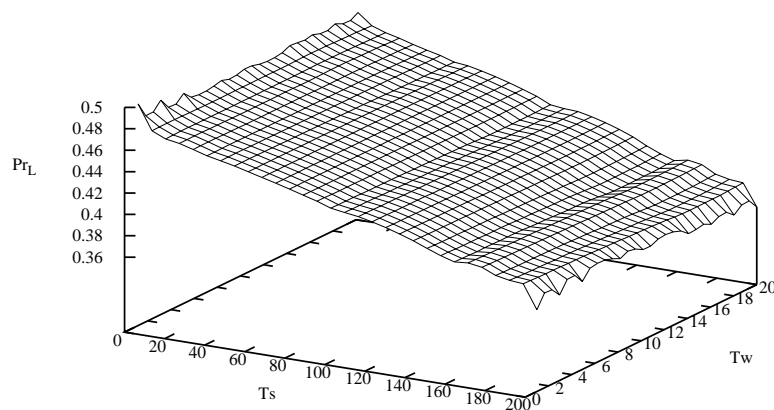
Figure 6: Probability in state L vs T_s and T_w, 1000BASE-T links

REFERENCES

[1] M. Ajmone Marsan, A. Anta, V. Mancuso, B. Rengarajan, P. Vasallo, and G. Rizzo, "A simple analytical model for energy efficient Ethernet," *IEEE Communications Letters*, vol. 15, no. 7, pp. 773 –775, July 2011.

[2] A. Cenedese, F. Tramarin, and S. Vitturi, "An energy efficient Ethernet strategy based on traffic prediction and shaping," *IEEE Transactions on Communications*, pp. 1–1, 2016. [Online]. Available: https://doi.org/10.1109/tcomm.2016.2623702

[3] R. Chakka and T. V. Do, "The MM $\sum_{k=1}^{K} CPP_k/GE/c/L$ G-Queue with Heterogeneous Servers: Steady state solution and an application to performance evaluation," *Performance Evaluation*, vol. 64, no. 3, pp. 191–209, March 2007.

[4] A. Chatzipapas and V. Mancuso, "An M/G/1 model for gigabit energy efficient Ethernet links with coalescing and real-trace-based evaluation," *IEEE/ACM Transactions on Networking*, vol. 24, no. 5, pp. 2663–2675, Oct. 2016. [Online]. Available: https://doi.org/10.1109/tnet.2015.2477090

[5] K. Christensen, P. Reviriego, B. Nordman, M. Bennett, M. Mostowfi, and J. Maestro, "IEEE 802.3az: the road to energy efficient Ethernet," *Communications Magazine, IEEE*, vol. 48, no. 11, pp. 50 –56, November 2010.

[6] K. Christensen, P. Reviriego, B. Nordman, M. Bennett, M. Mostowfi, and J. A. Maestro, "Ieee 802.3az: the road to energy efficient Ethernet," *IEEE Communications Magazine*, vol. 48, no. 11, pp. 50–56, 2010.

[7] M. Gupta, S. Grover, and S. Singh, "A feasibility study for power management in LAN switches," in *Proceedings of the 12th IEEE International Conference on Network Protocols, 2004. ICNP 2004.*, 2004, pp. 361–371.

[8] S. Herreria-Alonso, M. Rodriguez-Perez, M. Fernandez-Veiga, and C. Lopez-Garcia, "A power saving model for burst transmission in energy-efficient Ethernet," *Communications Letters, IEEE*, vol. 15, no. 5, pp. 584 –586, May 2011.

[9] ——, "A GI/G/1 Model for 10 Gb/s Energy Efficient Ethernet Links," *IEEE Transactions on Communications*, vol. 60, no. 11, pp. 3386–3395, Nov. 2012. [Online]. Available: https://doi.org/10.1109/tcomm.2012.081512.120089

[10] W. Jiang, J. Wang, L. Peng, and J. Wang, "Generalized predictive control of the time window for energy efficient Ethernet with prediction," *IEEE Communications Letters*, vol. 25, no. 5, pp. 1491–1495, May 2021. [Online]. Available: https://doi.org/10.1109/lcomm.2021.3051626

[11] D. Larrabeiti, P. Reviriego, J. Hernández, J. Maestro, and M. Urueña, "Towards an energy efficient 10 Gb/s optical Ethernet: Performance analysis and viability," *Optical Switching and Networking*, vol. 8, no. 3, pp. 131–138, Jul. 2011. [Online]. Available: https://doi.org/10.1016/j.osn.2011.03.009

[12] G. Latouche and V. Ramaswami, *Introduction to Matrix Analytic Methods in Stochastic Modeling*. ASA-SIAM Series on Statistics and Applied Probability, 1999.

[13] L. Muscariello, M. Mellia, M. Meo, M. A. Marsan, and R. L. Cigno, "Markov models of internet traffic and a new hierarchical MMPP model," *Computer Communications*, vol. 28, no. 16, pp. 1835–1851, 2005.

[14] B. Nordman, "Eee savings estimate," *IEEE 802.03az Task Force Information",year =2008*.

[15] P. Reviriego, K. Christensen, J. Rabanillo, and J. A. Maestro, "An initial evaluation of energy efficient Ethernet," *IEEE Communications Letters*, vol. 15, no. 5, pp. 578–580, 2011.

[16] P. Reviriego, J. Hernandez, D. Larrabeiti, and J. Maestro, "Performance evaluation of energy efficient Ethernet," *Communications Letters, IEEE*, vol. 13, no. 9, pp. 697 –699, Sept. 2009.

[17] P. Reviriego, J. Maestro, J. Hernándndez, and D. Larrabeiti, "Burst transmission for energy-efficient Ethernet," *Internet Computing, IEEE*, vol. 14, no. 4, pp. 50 –57, july-aug. 2010.

[18] M. Rodriguez-Perez, S. Herreria-Alonso, M. Fernandez-Veiga, and C. Lopez-Garcia, "Improved opportunistic sleeping algorithms for lan switches," in *GLOBECOM 2009 - 2009 IEEE Global Telecommunications Conference*, 2009, pp. 1–6.