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ROBUST FINITE-TIME SUPOPTIMAL CONTROL OF LARGE-SCALE SYSTEMS WITH INTERACTED STATE AND CONTROL DELAYS

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Abstract. This paper concerns with a problem of supoptimal finite-time control for a class of linear large-scale delay systems. The system under consideration is subjected to the state and control delays interacted between subsystems. Based on improved LMI approach combining with new estimation techniques, we derive sufficient conditions for solving H_∞ finite-time control and guaranteed cost control of the system. A numerical example is given to illustrate the validity and effectiveness of the theoretical results.

Keywords. Finite-time control; Supoptimal control; Large-scale system; State and control delay; Lyapunov function; Linear matrix inequality.

1. INTRODUCTION

Over the past decades stability and control theory of large-scale systems has been extensively studied due to its useful applications in various areas of applied science such as signal processing, communications, power systems and telecommunication networks (see, e.g. [1, 2] and the references therein). The concept of large-scale systems has been introduced when it became clear that there are real world control problems, which cannot be solved by using one-shot approaches. In general, a large-scale system can be characterized by a large number of variables representing the system, a strong interaction between subsystem variables. The control and stability analysis of large-scale systems have became complicated owing to the high dimensionality of the system equation, uncertainties, and time-delays. In the framework of large-scale interconnected systems, the problem of H_∞ control has been received considerable attention [3, 4, 5, 6]. On the other hand, the H_∞ finite-time control problem consists of the design of a state feedback control, which stabilizes the closed-loop system and guarantees an adequate level of system performance over a finite-time interval. It is notable

Dedicated to Professor Phan Dinh Dieu on the occasion of his 85th birth anniversary.

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that the aforementioned results are only devoted to asymptotic stability, i.e. the stability criteria are defined on an infinite-time interval. However, in practice we are interested not only in the system behavior in infinite-time interval, but also in finite-time interval. Then, instead of asymptotic stability, it is preferable to use the stability defined over a finite-time interval, i.e. finite-time stability (FTS) [7]. The authors of [8, 9, 10, 11] proposed some sufficient conditions for H_∞ finite-time control of linear large-scale systems, however, either the state delays or control delays were not considered. It is worth pointing out that almost the existing results on finite-time control of large-scale systems were studied for systems without control delays in interconnection. It is difficult to design feedback controllers for large-scale systems when the state and control delays are both interacted between all subsystems. The reason is that the interacted state and control delays are of high dimensions and thus require extensive computations to implement the centralized procedure. In [12, 13, 14] the authors studied problem of finite-time stability and stabilization for large-scale systems with delays, however the control delays are not considered. To the best of our knowledge, the problem of guaranteed cost finite-time control for large-scale systems with delays on both the state and control has not yet been studied in the literature. Therefore, this problem for large-scale systems with control and state delays in interconnection still remains open, which motivates our present research.

In this paper, we consider the problem of robust H_∞ finite-time control for linear large-scale systems with state and control delays in interconnection. Our purpose is to design state feedback controllers which guarantee not only the robust finite-time stability of the closed-loop system, but also provide an optimal level of the cost performance. The contribution of our paper lies in three aspects. (a) The time delays are interacted in both the state and control variables; (b) The disturbance is norm-bounded; (c) Based on Lyapunov function method combining with linear matrix inequality technique, we provide new sufficient conditions for solving the control problem. The conditions are formulated in terms of linear matrix inequalities (LMIs), which can be easily implemented by numerical algorithms [15]. Finally, a numerical example is given to show the validity and effectiveness of the theoretical results.

Notations. $R^{n\times r}$ denotes the space of all $(n \times r)$- matrices; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{max}(A) = \max\{\mathrm{Re}\lambda : \ \lambda \in \lambda(A)\}$; $\lambda_{min}(A) = \min\{\mathrm{Re}\lambda : \ \lambda \in \lambda(A)\}$; $C([a, b], R^n)$ denotes the set of all R^n-valued continuous functions on $[a, b]$; $L_2([0, T], R^r)$ stands for the set of all square-integrable R^r- valued functions on $[0, T]$; A matrix P is symmetric positive definite, $P > 0$, if $P = P^\top$ and $x^\top Px > 0$ for all $x \in R^n, x \neq 0$, $P > Q$ means that $P - Q > 0$. The symmetric terms in a matrix are denoted by $*$; The segment of the trajectory $x(t)$ is denotes by $x_t = \{x(t + s) : \ s \in [-\tau, 0]\}$ with its norm $||x_t|| = \sup\limits_{s\in[-\tau,0]} ||x(t + s)||$.

2. PRELIMINARIES

Consider the following linear large-scale control system with delays

$$\begin{cases} \dot{x}_i(t) = & A_i x_i(t) + \sum\limits_{j=1, j\neq i}^{N} A_{ij} x_j(t-d_{ij}) + B_i u_i(t) \\ & \qquad + \sum\limits_{j=1, j\neq i}^{N} B_{ij} u_j(t-m_{ij}) + D_i w_i(t), \\ z_i(t) = & E_i x_i(t) + F_i x_i(t-d_{ii}), \\ x_i(t) = & \varphi_i(t),\ u_i(t) = \phi_i(t), \quad t \in [-\tau, 0], \end{cases} \tag{1}$$

where $x_i(t) \in R^{n_i}$ is the state vector, $u_i(t) \in R^{m_i}$ is the control vector; $w_i(t) \in R^{r_i}$ is the disturbance vector, $z_i(t) \in R^{n_i}$ is the observation vector; $\tau = \max\{d, m\}$; $A_i, E_i, F_i \in R^{n_i \times n_i}, , B_i \in R^{n_i \times m_i}, A_{ij} \in R^{n_i \times n_j}, B_{ij} \in R^{n_i \times m_j}, D_i \in R^{n_i \times r_i}$ are constant matrices of appropriate dimensions; the delays satisfy

$$0 \le d_{ij} \le d, \quad 0 \le m_{ij} \le m, \quad i, j = \overline{1, N};$$

the disturbance $w_i(t)$ satisfies

$$\exists \eta > 0 : \max_{i=\overline{1,N}} \Big\{ \sup_{t>0} \{ w_i^\top(t) w_i(t) \} \Big\} \le \eta. \tag{2}$$

A solution of large-scale system (1) is a vector function $x(t) = [x_1(t), x_2(t), ..., x_N(t)] \in R^{Nn}$, which satisfies system of differential equations (1). It is well-known from [1], for $u_i(t) \in L_2([0, T], R^{m_i}), w_i(t) \in L_2([0, T], R^{r_i}), i = 1, 2, ..., N, T > 0$, the large-scale system (1) under the initial condition $\varphi_i(.) \in C([-\tau, 0], R^{n_i})$ has a unique solution $x(t)$.

Associate with system (1), we consider the following cost function

$$J(t, u) = \int_0^t \sum_{i=1}^{N} \Big[x_i^\top(t) U_i x_i(t) + \sum_{j=1, j\neq i}^{N} x_i^\top(t - d_{ji}) V_i x_i(t - d_{ji}) + u_i^\top(t) W_i u_i(t) \Big] dt, \tag{3}$$

where $U_i > 0,\ V_i > 0$, and $W_i > 0,\ i = \overline{1, N}$ are given symmetric matrices. Let us set

$$Q = diag\{Q_1, \cdots, Q_N\},\ x^\top(t) = [x_1(t)^\top, \dots, x_N(t)^\top],$$

$$u(t) = [u_1(t), \dots, u_N(t)],\ \varphi^\top(t) = [\varphi_1(t)^\top, \dots, \varphi_N(t)^\top].$$

Definition 1. (Robust finite-time stability) Given positive numbers T, c_1, c_2 and a symmetric matrix $Q > 0$, the unforced system (1) ($u(t) = 0$) is said to be robustly finite-time stable w.r.t. (c_1, c_2, T, Q) if for all disturbances $w(t)$ satisfying (2) the solution $x(t)$ of the system satisfies the following relation

$$\sup_{s \in [-\tau, 0]} \{ \varphi^\top(s) Q \varphi(s) \} \le c_1 \to x^\top(t) Q x(t) < c_2,\ \forall t \in [0, T].$$

Definition 2. (Finite-time stabilization) For given positive numbers T, c_1, c_2, and a symmetric matrix $Q > 0$, system (1) is robustly finite-time stabilizable with respect to (c_1, c_2, T, Q)

if there exist state feedback controllers $u_i(t) = K_i x_i(t),\ i = \overline{1,N}$, such that the closed-loop system is robustly finite-time stable w.r.t. (c_1, c_2, T, Q).

Definition 3. (Robust H_∞ finite-time control). For given positive numbers $T, c_1, c_2, \gamma > 0$, and a symmetric matrix $Q > 0$, the robust H_∞ finite-time control problem for system (1) is solvable if:

(i) System (1) is robustly finite-time stabilizable w.r.t. (c_1, c_2, T, Q).

(ii) There is a number $c_0 > 0$ such that

$$\sup \frac{\int_0^T \sum_{i=1}^N \|z_i(t)\|^2 dt}{c\|\varphi\|^2 + \sum_{i=1}^N \int_0^T \|w_i(t)\|^2 dt} \le \gamma, \tag{4}$$

where the suppremum is taken over all $\varphi_i(t) \in C([-\tau, 0], R^{n_i})$ and non-zero disturbances $w_i(t)$ satisfying (2).

Definition 4. (Guaranteed cost control) For given positive numbers T, c_1, c_2 and symmetric matrices $Q > 0$, the guaranteed cost finite-time control for system (1) is solvable if there exist state feedback controllers $u_i = K_i x_i(t), i = \overline{1,N}$ and a number $J^* > 0$ such that the closed-loop system of (1) is robustly finite-time stable w.r.t (c_1, c_2, T, Q) and $J(T, u) \le J^*$. Number J^* is the guaranteed cost value, control $u(t)$ is the guaranteed cost controller. The following technical lemmas are introduced for the proof of the main result.

Lemma 1. *(Cauchy matrix inequality* [16]*) For given $a, b \in R^n,\ 0 < P \in R^{n\times n}$, we have*

$$2a^\top b \le a^\top P^{-1} a + b^\top P b.$$

Lemma 2. *(Schur complement lemma* [16]*) For given matrices $U,\ V,\ Q$ with appropriate dimensions satisfying $V = V^\top > 0,\ U = U^\top$, we have*

$$U + Q^\top V^{-1} Q < 0 \Leftrightarrow \begin{bmatrix} U & Q^\top \\ Q & -V \end{bmatrix} < 0.$$

Lemma 3. *For given matrices $M_1,\ M_2,\ Z,\ Y,\ Q$ with appropriate dimensions satisfying $M_1 = M_1^\top,\ Q = Q^\top > 0$ and $Y = Y^\top > 0$, we have*

$$\begin{bmatrix} M_1 + Z^\top Y^{-1} Z & M_2^\top \\ M_2 & -Q \end{bmatrix} < 0 \Leftrightarrow \begin{bmatrix} M_1 & M_2^\top & Z^\top \\ M_2 & -Q & 0 \\ Z & 0 & -Y \end{bmatrix} < 0.$$

Proof. We have

$$\begin{bmatrix} M_1 + Z^\top Y^{-1} Z & M_2^\top \\ M_2 & -Q \end{bmatrix} = \begin{bmatrix} M_1 & M_2^\top \\ M_2 & -Q \end{bmatrix} + \begin{bmatrix} Z^\top Y^{-1} Z & 0 \\ 0 & 0 \end{bmatrix}.$$

Since $\begin{bmatrix} Z^\top Y^{-1} Z & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} Z^\top \\ 0 \end{bmatrix} Y^{-1} \begin{bmatrix} Z & 0 \end{bmatrix}$, we apply the Schur complement lemma, Lemma 2, and obtain that

$$\begin{bmatrix} M_1 & M_2^\top \\ M_2 & -Q \end{bmatrix} + \begin{bmatrix} Z^\top \\ 0 \end{bmatrix} Y^{-1} \begin{bmatrix} Z & 0 \end{bmatrix} < 0,$$

which is equivalent to

$$\begin{bmatrix} M_1 & M_2^\top & Z^\top \\ M_2 & -Q & 0 \\ Z & 0 & -Y \end{bmatrix} < 0.$$

■

3. MAIN RESULTS

In this section, we provide sufficient conditions for solving finite-time supoptimal finite-time control of systems (1). Before presenting the theorem, we introduce the following notations of several matrix variables for simplicity.

$P_{i1} = P_i^{-1},\ R_{i1} = P_i^{-1} R_i P_i^{-1},$

$H^i_{1,1} = A_i P_i + P_i A_i^\top + \frac{1}{\gamma\beta} D_i D_i^T + B_i Y_i + Y_i^T B_i^\top + (N-1)R_i + \sum_{j=1, j\neq i}^{N} A_{ij} A_{ij}^\top,$

$H^i_{1+i,1+i} = -I,\ H^i_{1,1+i} = \sqrt{N-1}P_i,\ H^i_{1+j,1+j} = -R_j,\ H^i_{1,1+j} = B_{ij} Y_j,\ i,j = \overline{1,N},\ j \neq i,$

$H^i_{1,N+2} = P_i E_i^\top, H^i_{1,N+3} = P_i F_i^\top, H^i_{N+2,N+2} = H^i_{N+3,N+3} = -\frac{1}{\beta} I,$

$H^i_{ij} = 0$, for all other $i, j,\ j \neq i$,

$\alpha_1 = \min_{i=\overline{1,N}} \{ \frac{\lambda_{min}(P_{i1})}{\lambda_{max}(Q_i)} \},$

$$\alpha_2 = \max_{i=\overline{1,N}} \Big(\frac{\lambda_{max}(P_{i1})}{\lambda_{min}(Q_i)} \Big) + 2\beta\tau \max_{i=\overline{1,N}} \Big(\frac{\lambda_{max}(F_i^\top F_i)}{\lambda_{min}(Q_i)} \Big) + (N-1)\tau \Big[\max_{i=\overline{1,N}} \Big(\frac{1}{\lambda_{min}(Q_i)} \Big) + \max_{i=\overline{1,N}} \Big(\frac{\lambda_{max}(R_{i1})}{\lambda_{min}(Q_i)} \Big) \Big],$$

$$\alpha_3 = \max_{i=\overline{1,N}} \Big\{ \lambda_{max}(P_{i1}) \Big\} + 2\beta\tau \max_{i=\overline{1,N}} \Big\{ \lambda_{max}(F_i^\top F_i) \Big\} + (N-1)\tau \Big[1 + \max_{i=\overline{1,N}} \Big\{ \lambda_{max}(R_{i1}) \Big\} \Big].$$

Next theorem gives sufficient conditions for H_∞ finite-time control of system (1).

Theorem 1. *For given positive numbers T, c_1, c_2, and a symmetric matrix $Q > 0$, the robust H_∞ finite-time control problem for the systems (1) is solvable if there exist symmetric matrices $P_i > 0,\ R_i > 0,\ i = \overline{1,N}$, matrices $Y_i,\ i = \overline{1,N}$, and a number $\beta > 0$ satisfying the following conditions*

$$\begin{bmatrix} H^i_{11} & H^i_{12} & . & . & . & H^i_{1(N+3)} \\ * & H^i_{22} & . & . & . & H^i_{2(N+3)} \\ . & . & . & . & . & . \\ * & * & . & . & . & H^i_{(N+3)(N+3)} \end{bmatrix} < 0,\ i = \overline{1,N}, \tag{5}$$

$$\frac{\alpha_2 c_1 + N\gamma\beta\eta T}{\alpha_1} \leq e^{-\beta T} c_2. \tag{6}$$

Moreover, the state feedback controllers are defined by $u_i(t) = Y_i P_i^{-1} x_i(t),\ i = \overline{1,N}$.

Proof. Consider the following Lyapunov-Krasovskii functional

$$V(t,x_t) = \sum_{i=1}^{N}[V_{i1}(t,x_t) + V_{i2}(t,x_t) + V_{i3}(t,x_t)],$$

where

$$\begin{aligned}
V_{i1}(t,x_t) =& e^{\beta t} x_i(t)^\top P_{i1} x_i(t),\\
V_{i2}(t,x_t) =& e^{\beta t} \int\limits_{t-d_{ii}}^{t} 2\beta x_i(s)^\top F_i^\top F_i x_i(s)ds,\\
V_{i3}(t,x_t) =& e^{\beta t} \sum_{j=1, j\neq i}^{N} \Big(\int\limits_{t-d_{ji}}^{t} x_i(s)^\top x_i(s)ds + \int\limits_{t-m_{ji}}^{t} x_i(s)^\top R_{i1} x_i(s)ds \Big).
\end{aligned}$$

Taking the derivative of $V(t,x_t)$ in t, we have

$$\begin{aligned}
\dot{V}_{i1}(t,x_t) =& \beta V_{i1}(t,x_t) + e^{\beta t} 2 x_i(t)^\top P_{i1}\Big[A_i x_i(t) \\
&+ \sum_{j=1,j\neq i}^{N} [A_{ij} x_j(t-d_{ij}) + B_{ij} Y_j P_{j1} x_j(t-m_{ij})] + B_i Y_i P_{i1} x_i(t) + D_i w(t)\Big], \qquad (7)\\
\dot{V}_{i2}(t,x_t) =& \beta V_{i2}(t,x_t) + e^{\beta t} 2\beta \Big[x_i(t)^\top F_i^\top F_i x_i(t) - x_i(t-d_{ii})^\top F_i^\top F_i x_i(t-d_{ii})\Big], \qquad (8)\\
\dot{V}_{i3}(t,x_t) =& \beta V_{i3}(t,x_t) + e^{\beta t}(N-1)\Big[x_i(t)^\top x_i(t) + x_i(t)^\top R_{i1} x_i(t)\Big]\\
&- e^{\beta t} \sum_{j=1,j\neq i}^{N} \Big[x_i(t-d_{ji})^\top x_i(t-d_{ji}) + x_i(t-m_{ji})^\top R_{i1} x_i(t-m_{ji})\Big]. \qquad (9)
\end{aligned}$$

Applying Cauchy matrix inequality, Lemma 1, for the following inequalities

$$\begin{aligned}
2x_i(t)^T P_{i1}\Big[\sum_{j=1,j\neq i}^{N} A_{ij} x_j(t-d_{ij})\Big] \leq& \sum_{j=1,j\neq i}^{N} [x_i^\top(t) P_{i1} A_{ij} A_{ij}^\top P_{i1}^\top x_i(t)\\
&+ x_j(t-d_{ij})^\top x_j(t-d_{ij})],\\
2x_i^\top(t) P_{i1}\Big[\sum_{j=1,j\neq i}^{N} B_{ij} Y_j P_{j1} x_j(t-m_{ij})\Big] \leq& \sum_{j=1,j\neq i}^{N} [x_i^\top(t) P_{i1} B_{ij} Y_j R_j^{-1} Y_j^\top B_{ij}^\top P_{i1}^\top x_i(t)\\
&+ x_j(t-m_{ij})^\top R_{j1} x_j(t-m_{ij})],\\
2x_i^\top(t) P_{i1} D_i w_i(t) \leq& \frac{1}{\gamma\beta} x_i^\top(t) P_{i1} D_i D_i^\top P_{i1} x_i(t) + \gamma\beta w_i^\top(t) w_i(t),
\end{aligned}$$

we obtain that

$$\begin{aligned}\dot{V}_{i1}(t,x_t) \le& \beta V_{i1}(t,x_t) + e^{\beta t}x_i(t)^\top[P_{i1}A_i + A_i^\top P_{i1}^\top + \frac{1}{\gamma\beta}P_{i1}D_iD_i^\top P_{i1} + P_{i1}(B_iY_i \\ &+ Y_i^\top B_i^\top)P_{i1}]x_i(t) \\ &+ e^{\beta t}\Big\{\sum_{j=1,j\neq i}^{N}\Big[x_i(t)^\top P_{i1}A_{ij}A_{ij}^\top P_{i1}x_i(t) + x_j(t-d_{ij})^\top x_j(t-d_{ij}) \\ &+ x_i(t)^\top P_{i1}B_{ij}Y_jR_j^{-1}Y_j^\top B_{ij}^\top P_{i1}x_i(t) + x_j(t-m_{ij})^\top R_{j1}x_j(t-m_{ij})\Big]\Big\} \\ &+ e^{\beta t}\gamma\beta\sum_{i=1}^{N}w_i^\top(t)w_i(t).\end{aligned}$$

Moreover, using the following identities

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}x_j(t-d_{ij})^\top x_j(t-d_{ij}) = \sum_{i=1}^{N}\sum_{j=1,i\neq j}^{N}x_i(t-d_{ji})^\top x_i(t-d_{ji}), \\ &\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}x_j(t-m_{ij})^\top R_{j1}x_j(t-m_{ij}) = \sum_{i=1}^{N}\sum_{j=1,i\neq j}^{N}x_i(t-m_{ji})^\top R_{i1}x_i(t-m_{ji}), \\ &\|z_i(t)\|^2 \le 2x_i^T(t)E_i^TE_ix_i(t) + 2x_i^T(t-d_{ii})F_i^TF_ix_i(t-d_{ii}).\end{aligned}$$

we obtain from (7)-(9) that

$$\begin{aligned}\dot{V}(t,x_t) - \beta V(t,x_t) \le& e^{\beta t}\sum_{i=1}^{N}\Big[x_i(t)^\top[P_{i1}A_i + A_i^\top P_{i1} + \frac{1}{\gamma\beta}P_{i1}D_iD_i^\top P_{i1} \\ &+ P_{i1}(B_iY_i + Y_i^TB_i^\top)P_{i1} + 2\beta E_i^\top E_i + 2\beta F_i^\top F_i]x_i(t) \\ &+ \sum_{j=1,i\neq j}^{N}x_i(t)^\top P_{i1}A_{ij}A_{ij}^\top P_{i1}x_i(t) \\ &+ \sum_{j=1,i\neq j}^{N}x_i(t)^\top P_{i1}B_{ij}Y_j\ R_j^{-1}\ Y_j^\top B_{ij}^\top P_{i1}x_i(t) \\ &+ (N-1)[x_i(t)^\top x_i(t) + x_i(t)^\top R_{i1}x_i(t)]\Big] \\ &+ \gamma\beta e^{\beta t}\sum_{i=1}^{N}w_i^\top(t)w_i(t) - \beta e^{\beta t}\sum_{i=1}^{N}z_i^\top(t)z_i(t) \\ \le& e^{\beta t}\sum_{i=1}^{N}y_i(t)^T\mathcal{M}^iy_i(t) + \gamma\beta e^{\beta t}\sum_{i=1}^{N}w_i^\top(t)w_i(t) \\ &- \beta e^{\beta t}\sum_{i=1}^{N}z_i^\top(t)z_i(t),\end{aligned} \tag{10}$$

where $y_i(t) = P_{i1}x_i(t),\ i = \overline{1,N}$, and

$$\mathcal{M}^i = A_iP_i + P_i^\top A_i^\top + B_iY_i + \frac{1}{\gamma\beta}D_iD_i^\top + Y_i^T B_i^\top + 2\beta P_i^\top E_i^\top E_iP_i + 2\beta P_i^\top F_i^\top F_iP_i + (N-1)[R_i + P_i^2] + \sum_{j=1,j\neq i}^{N}[A_{ij}A_{ij}^\top + B_{ij}Y_jR_j^{-1}Y_j^\top B_{ij}^\top].$$

Applying Lemma 3, the condition (5) is equivalent to $\mathcal{M}^i < 0,\ i = \overline{1,N}$, and hence from (10) it follows that

$$\dot{V}(t,x_t) - \beta V(t,x_t) \leq \gamma\beta e^{\beta t}\sum_{i=1}^{N} w_i^\top(t)w_i(t).$$

Moreover, since

$$\frac{d}{dt}\Big[e^{-\beta t}V(t,x_t)\Big] = e^{-\beta t}\dot{V}(t,x_t) - \beta e^{-\beta t}V(t,x_t),$$

we get

$$V(t,x_t) \leq (V(0,x_0) + N\gamma\beta\eta T)e^{\beta T},\ \forall t \in [0,T]. \tag{11}$$

Next, in the view of $V(t,x_t)$ we have

$$\begin{aligned} V(t,x_t) \geq& \sum_{i=1}^{N} x_i(t)^\top P_{i1}x_i(t) \geq \sum_{i=1}^{N}\lambda_{min}(P_{i1})x_i(t)^\top x_i(t) \\ \geq& \alpha_1\sum_{i=1}^{N} x_i(t)^\top Q_ix_i(t) = \alpha_1 x(t)^\top Qx(t). \end{aligned} \tag{12}$$

On the other hand, the following estimation for $V(0,x_0)$ can be derived

$$\begin{aligned} V(0,x_0) =& \sum_{i=1}^{N} x_i(0)^\top P_{i1}x_i(0) + \sum_{i=1}^{N}\int_{-d_{ii}}^{0} 2\beta x_i(s)^\top F_i^\top F_ix_i(s)ds \\ &+ \sum_{i=1}^{N}\sum_{j=1,\ j\neq i}^{N}\Big(\int_{-d_{ji}}^{0} x_i(s)^\top x_i(s)ds + \int_{-m_{ji}}^{0} x_i(s)^\top R_{i1}x_i(s)ds\Big) \\ \leq& \alpha_2 \sup_{s\in[-\tau,0]}\{\varphi(s)^\top Q\varphi(s)\} \leq \alpha_2 c_1. \end{aligned} \tag{13}$$

Therefore, combining the derived inequalities (10)-(13) with the condition (6) gives

$$\begin{aligned} x(t)^TQx(t) &\leq \frac{1}{\alpha_1}(V(0,x_0) + N\beta\gamma\eta T)e^{\beta t} \\ &\leq \frac{1}{\alpha_1}(\alpha_2c_1 + N\gamma\beta\eta T)e^{\beta T} \leq c_2,\ \forall t \in [0,T]. \end{aligned}$$

To complete the proof of the theorem, it remains to show the $\gamma-$optimal level condition (4). For this, we consider the following relation

$$\sum_{i=1}^{N}\int_0^T \beta\|z_i(t)\|^2-\gamma\beta\|w_i(t)\|^2dt=\int_0^T\{\sum_{i=1}^{N}\Big[\beta\|z_i(t)\|^2-\gamma\beta\|w_i(t)\|^2\Big]+\frac{d}{dt}\Big[e^{-\beta t}V(t,x_t)\Big]\}dt$$
$$-\int_0^T\frac{d}{dt}\Big[e^{-\beta t}V(t,x_t)\Big]dt.$$

Since $V(t,x_t)\geq 0$, we have

$$-\int_0^T\frac{d}{dt}\Big[e^{-\beta t}V(t,x_t)\Big]dt=-e^{-\beta T}V(T,x_T)+V(0,x_0)\leq\alpha_3\|\varphi\|^2.$$

It follows from (10) that

$$\sum_{i=1}^{N}\Big[\beta||z_i(t)\|^2-\gamma\beta\|w_i(t)\|^2\Big]+\frac{d}{dt}[e^{-\beta}V(t,x_t)]<0,$$

hence

$$\sum_{i=1}^{N}[\int_0^T(\beta\|z_i(t)\|^2-\gamma\beta\|w_i(t)\|^2)dt]\leq\alpha_3\|\varphi\|^2.$$

Setting $c_0=\dfrac{\alpha_3}{\gamma\beta}>0$, the above inequality yields

$$\sup\frac{\int_0^T\sum\limits_{i=1}^{N}\|z_i(t)\|^2dt}{c_0\|\varphi\|^2+\sum\limits_{i=1}^{N}\int_0^T\|w_i(t)\|^2dt}\leq\gamma.$$

This condition holds for all non-zero $w_i\in L_2([0,T],R^{r_i}),\ \varphi_i\in C([-\tau,0];R^{n_i})$, and then the condition (4) is derived. This completes the proof of the theorem. ■

Next theorem gives sufficient conditions for solving the guaranteed cost finite-time control of system (1). Let us denote

$$H^i_{1,1}=A_iP_i+P_iA_i^\top+D_iD_i^T+B_iY_i+Y_i^TB_i^\top+(N-1)R_i+\sum_{j=1,j\neq i}^{N}A_{ij}A_{ij}^\top,$$
$$H^i_{1+i,1+i}=-I,\ H^i_{1,1+i}=\sqrt{N-1}P_i,\ H^i_{1+j,1+j}=-R_j,\ H^i_{1,1+j}=B_{ij}Y_j,\ i,j=\overline{1,N},\ j\neq i,$$
$$\alpha_2=\max_{i=\overline{1,N}}\left(\tfrac{\lambda_{max}(P_{i1})}{\lambda_{min}(Q_i)}\right)+(N-1)\tau\Big[\max_{i=\overline{1,N}}\left(\tfrac{\lambda_{max}(I_i+V_i)}{\lambda_{min}(Q_i)}\right)+\max_{i=\overline{1,N}}\left(\tfrac{\lambda_{max}(R_{i1})}{\lambda_{min}(Q_i)}\right)\Big].$$

Theorem 2. *For given positive numbers $T,\ c_1,\ c_2$, and symmetric matrices $Q_i>0,\ W_i>0,\ U_i\geq 0,\ V_i\geq 0,\ i\in\overline{1,N}$, the guaranteed cost finite-time control for system* (1) *is solvable if there exist symmetric matrices $P_i>0,R_i>0,i=\overline{1,N}$, matrices $Y_i,\ i=\overline{1,N}$, and a number $\beta>0$ satisfying the following conditions*

$$\begin{bmatrix} H^i_{11} & H^i_{12} & \cdots & H^i_{1,N+1} & Y_i^\top W_i^{1/2} & P_i U_i^{1/2} & \sqrt{N-1}P_iV_i^{1/2} \\ * & H^i_{22} & \cdots & H^i_{2,N+1} & 0 & 0 & 0 \\ . & . & \cdots & . & . & . & . \\ * & * & \cdots & H^i_{N+1,N+1} & 0 & 0 & 0 \\ * & * & \cdots & * & -I & 0 & 0 \\ * & * & \cdots & * & * & -I & 0 \\ * & * & \cdots & * & * & * & -I \end{bmatrix} < 0, \qquad (14)$$

$$\frac{\alpha_2 c_1 + N\eta T}{\alpha_1} \le e^{-\beta T} c_2. \qquad (15)$$

Moreover, the guaranteed cost controllers are defined by $u_i(t) = Y_i P_i^{-1} x_i(t),\ i = \overline{1,N}$ *and the guaranteed cost value is defined by* $J^* = \alpha_2 c_1 + N\eta T$.

Proof. We take the following Lyapunov-Krasovskii functional

$$V(t,x_t) = \sum_{i=1}^N \Big[V_{i1}(t,x_t) + V_{i2}(t,x_t)\Big],$$

where $V_{i1}(t,x_t) = e^{\beta t} x_i(t)^\top P_{i1} x_i(t)$,

$$V_{i2}(t,x_t) = e^{\beta t} \sum_{j=1,\ j\neq i}^N \Big(\int_{t-d_{ji}}^t x_i(s)^\top (I+V_i) x_i(s) ds + \int_{t-m_{ji}}^t x_i(s)^\top R_{i1} x_i(s) ds\Big).$$

By the same arguments used in the proof of Theorem 1 we have derived the following estimation:

$$\begin{aligned} \dot V(t,x_t) - \beta V(t,x_t) \le & e^{\beta t} \sum_{i=1}^N \Big[x_i(t)^\top [P_{i1}A_i + A_i^\top P_{i1} + P_{i1} D_i D_i^\top P_{i1} \\ & + P_{i1}(B_i Y_i + Y_i^T B_i^\top) P_{i1}] x_i(t) + \sum_{j=1,\ j\neq i}^N x_i(t)^\top P_{i1} A_{ij} A_{ij}^\top P_{i1} x_i(t) \\ & + \sum_{j=1,\ j\neq i}^N x_i(t)^\top P_{i1} B_{ij} Y_j\ R_j^{-1}\ Y_j^\top B_{ij}^\top P_{i1} x_i(t) \\ & + (N-1) x_i(t)^\top [I + V_i + R_{i1}] x_i(t)\Big] + e^{\beta t} \sum_{i=1}^N w_i^\top(t) w_i(t) \\ & - e^{\beta t} \sum_{i=1}^N \sum_{j=1,\ j\neq i}^N x_i(t-d_{ji})^\top V_i x_i(t-d_{ji})^\top. \end{aligned} \qquad (16)$$

Adding and substituting the value $e^{\beta t} f^0(t)$, where $f^0(t)$ is denoted by

$$f^0(t) := \sum_{i=1}^N [x_i^\top(t) U_i x_i(t) + \sum_{j=1, j\neq i}^N x_i^\top (t-d_{ji}) V_i x_i(t-d_{ji}) + u_i^\top(t) W_i u_i(t)],$$

on the right hand-side of (16), we have

$$\dot V(t,x_t) - \beta V(t,x_t) \le e^{\beta t} \sum_{i=1}^N y_i(t)^T \mathbb{M}^i y_i(t) + e^{\beta t} \sum_{i=1}^N w_i^\top(t) w_i(t) - e^{\beta t} f^0(t), \qquad (17)$$

where $y_i(t) = P_{i1}x_i(t)$ and

$$\mathbb{M}^i = A_iP_i + P_iA_i^\top + B_iY_i + D_iD_i^\top + P_iU_iP_i + Y_i^TB_i^\top + (N-1)[P_iV_iP_i + R_i + P_i^2] + Y_i^\top W_iY_i + \sum_{j=1,j\neq i}^{N}[A_{ij}A_{ij}^\top + B_{ij}Y_jR_j^{-1}Y_j^\top B_{ij}^\top].$$

Applying the Schur complement lemma, Lemma 2, and Lemma 3, the condition (14) is equivalent to $\mathbb{M}^i < 0,\ \forall i = \overline{1,N}$, and hence from the inequality (17) it follows that

$$\dot{V}(t,x_t) - \beta V(t,x_t) \leq -e^{\beta t}f^0(t) + e^{\beta t}\sum_{i=1}^{N} w_i^\top(t)w_i(t),\ \forall t \in [0,T]. \tag{18}$$

Therefore, using the same arguments used in the proof in Theorem 1, we can prove the robust finite-time stabilization of the closed system by the feedback controllers $u_i(t) = Y_iP_i^{-1}x_i(t)$. To obtain the guaranteed cost value, we derive from (18) that

$$e^{-\beta t}\dot{V}(t,x_t) - \beta e^{-\beta t}V(t,x_t) \leq -f^0(t) + \sum_{i=1}^{N} w_i^\top(t)w_i(t). \tag{19}$$

Integrating both sides of (19) from 0 to T leads to

$$\begin{aligned}\int_0^T f^0(t)dt \leq & V(0,x_0) - e^{-\beta t}V(t,x_t) + N\eta T\\ \leq & V(0,x_0) + N\eta T,\end{aligned}$$

due to $V(t,x_t) \geq 0$. On the other hand, we see that

$$\begin{aligned}V(0,x_0) = & \sum_{i=1}^{N} x_i(0)^\top P_{i1}x_i(0) + \sum_{i=1}^{N}\sum_{j=1,\ j\neq i}^{N}\Big(\int_{-d_{ji}}^{0} x_i(s)^\top[I+V_i]x_i(s)ds\\ & + \int_{-m_{ji}}^{0} x_i(s)^\top R_{i1}x_i(s)ds\Big)\\ \leq & \alpha_2c_1,\end{aligned}$$

hence, we obtain that

$$J(u) \leq \alpha_2c_1 + N\eta T = J^*.$$

This completes the proof of the theorem. ■

Remark 1. Note that the condition (6) (or (15)) is not a LMI w.r.t. $\beta > 0$. Since β is not included in LMI (5) (or in LMI (14)), we first find solutions P_i, R_i, Y_i from the LMI (5) (or from the LMI (14)), and then define $\beta > 0$ from condition (6) (or from (15)).

4. A NUMERICAL EXAMPLE

In this section, an illustrative example is given to show the validity and effectiveness of the theoretical result.

Consider the dynamic of the mechanical machine (see, e.g. [2]) described by the large-scale complex system (1), where $N = 3$ and the absolute rotor angle, acceleration and angular velocity in each subsystem are respectively denoted by $x_i = (x_{i1}, x_{i2})^\top,\ i = 1, 2, 3$; the observation vector $z_i,\ i = 1, 2, 3$; the coefficient matrices A_i, A_{ij}, the perturbation coefficient matrices D_i, the time-delays d_{ij} are given by

$$\begin{cases} \dot{x}_1(t) = & A_1x_1(t) + A_{12}x_2(t-d_{12}) + A_{13}x_3(t-d_{13}) + B_1u_1(t) + B_{12}u_2(t-m_{12}) \\ & +B_{13}u_3(t-m_{13}) + D_1w_1(t), \\ z_1(t) = & E_1x_1(t) + F_1x_1(t-d_{11}), \\ x_1(t) = & \varphi_1(t),\ u_1(t) = \phi_1(t), \quad t \in [-\tau, 0], \end{cases}$$

$$\begin{cases} \dot{x}_2(t) = & A_2x_2(t) + A_{21}x_1(t-d_{21}) + A_{23}x_3(t-d_{23}) + B_2u_2(t) + B_{21}u_1(t-m_{21}) \\ & +B_{23}u_3(t-m_{23}) + D_2w_2(t), \\ z_2(t) = & E_2x_2(t) + F_2x_2(t-d_{22}), \\ x_2(t) = & \varphi_2(t),\ u_2(t) = \phi_2(t), \quad t \in [-\tau, 0], \end{cases}$$

$$\begin{cases} \dot{x}_3(t) = & A_3x_3(t) + A_{31}x_1(t-d_{31}) + A_{32}x_2(t-d_{32}) + B_2u_2(t) + B_{31}u_1(t-m_{31}) \\ & +B_{32}u_2(t-m_{32}) + D_3w_3(t), \\ z_3(t) = & E_3x_3(t) + F_3x_3(t-d_{33}), \\ x_3(t) = & \varphi_3(t),\ u_3(t) = \phi_3(t), \quad t \in [-\tau, 0], \end{cases}$$

and $d_{12} = 0.4$; $d_{13} = 0.09$; $d_{21} = 0.06$; $d_{23} = 0.2$; $d_{31} = 0.3$; $d_{32} = 0.04; m_{12} = 0.42$; $m_{13} = 0.5$; $m_{21} = 0.28$; $m_{23} = 0.37$; $m_{31} = 0.45$; $m_{32} = 0.1$; $\tau = 0.5$.

$$A_1 = \begin{bmatrix} -2 & 0.1 \\ -0.1 & -4 \end{bmatrix}, A_2 = \begin{bmatrix} -3 & 0 \\ -1 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} -5 & 0.1 \\ 0 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 0 \\ 2 & 0.1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -1 & 1 \\ 1.9 & -5 \end{bmatrix}, B_3 = \begin{bmatrix} 2 & 0.1 \\ 0.4 & -2 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.2 & 1 \\ 0.1 & 0 \end{bmatrix}, A_{13} = \begin{bmatrix} 0.5 & 0.1 \\ 0 & 3 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 0.5 & 0 \\ -0.1 & 0.5 \end{bmatrix}, A_{23} = \begin{bmatrix} 0.4 & -0.1 \\ 0 & 1 \end{bmatrix}, A_{31} = \begin{bmatrix} -0.02 & -1 \\ 0.1 & -0.2 \end{bmatrix},$$

$$A_{32} = \begin{bmatrix} 1 & 0.4 \\ 0 & -2 \end{bmatrix}, B_{12} = \begin{bmatrix} -1 & 0 \\ 0.1 & 3 \end{bmatrix}, B_{13} = \begin{bmatrix} -2 & 0 \\ 1 & 0.2 \end{bmatrix}, B_{21} = \begin{bmatrix} -1 & 0.1 \\ 0 & -1 \end{bmatrix},$$

$$B_{23} = \begin{bmatrix} -1 & 0 \\ -0.5 & 0 \end{bmatrix}, B_{31} = \begin{bmatrix} -2 & 0 \\ 0.2 & -3 \end{bmatrix}, B_{32} = \begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix}, D_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.05 \\ 0.1 \end{bmatrix}, D_3 = \begin{bmatrix} -0.1 \\ 0.01 \end{bmatrix}, E_1 = \begin{bmatrix} 0.1 & -0.2 \\ 0.2 & 0.1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0.2 \\ 0 & -0.1 \end{bmatrix},$$

$$E_3 = \begin{bmatrix} -1 & 0 \\ -1 & -2 \end{bmatrix}, F_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, F_2 = \begin{bmatrix} 1 & 0 \\ 0 & -0.1 \end{bmatrix}, F_3 = \begin{bmatrix} -0.1 & 0 \\ -0.5 & 0.5 \end{bmatrix},$$

and the matrices

$$Q = diag(Q_1, Q_2, Q_3), \quad Q_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 0.64 & 0 \\ 0 & 0.64 \end{bmatrix}, \quad Q_3 = \begin{bmatrix} 0.42 & 0 \\ 0 & 0.42 \end{bmatrix}.$$

For $\eta = 0.1;\ \gamma = 4,\ \beta = 0.01,\ c_1 = 1,\ c_2 = 115,\ T = 10$, using the LMI algorithm in Matlab [15] to find solutions of the LMI (5), we have

$$P_1 = \begin{bmatrix} 0.9327 & 0.0107 \\ 0.0107 & 1.8988 \end{bmatrix}, P_2 = \begin{bmatrix} 1.5631 & 0.0416 \\ 0.0416 & 0.2447 \end{bmatrix}, P_3 = \begin{bmatrix} 2.3185 & -0.0769 \\ -0.0769 & 1.7110 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.6955 & 1.0743 \\ 1.0743 & 2.2041 \end{bmatrix}, R_2 = \begin{bmatrix} 1.1333 & -1.0559 \\ -1.0559 & 3.1772 \end{bmatrix}, R_3 = \begin{bmatrix} 0.4498 & 0.4507 \\ 0.4507 & 60.3971 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} -0.9953 & -2.0142 \\ -0.2314 & -0.5320 \end{bmatrix}, Y_2 = \begin{bmatrix} 0.4243 & -1.1587 \\ -0.1651 & 0.4800 \end{bmatrix}, Y_3 = \begin{bmatrix} -0.0363 & -2.2345 \\ 0.3305 & 37.5152 \end{bmatrix}.$$

By Theorem 1, the system is robust finite-time H_∞ stabilizable, and the state feedback control $u_i(t) = Y_i P_i^{-1} x_i(t)$ are given by

$$u_1(t) = \begin{bmatrix} -1.0550 & -1.0548 \\ -0.2449 & -0.2788 \end{bmatrix} x_1(t),$$
$$u_2(t) = \begin{bmatrix} 0.3992 & -4.8024 \\ -0.1585 & 1.9883 \end{bmatrix} x_2(t),$$
$$u_3(t) = \begin{bmatrix} -0.0590 & -1.3086 \\ 0.8708 & 21.9651 \end{bmatrix} x_3(t).$$

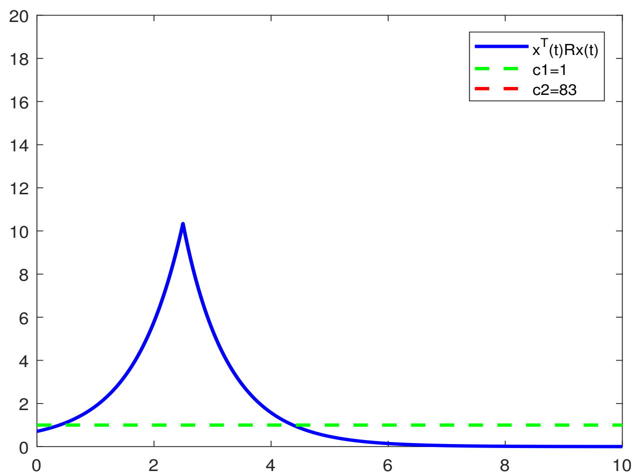

Figure 1. Time history of $x^\top(t)Rx(t)$ for the system

For the guaranteed cost control, we take $c_1 = 1,\ c_2 = 83,\ T = 10$ and the cost matrices

$$V_1 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, V_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.04 \end{bmatrix}, V_3 = \begin{bmatrix} 0.04 & 0 \\ 0 & 0.04 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.0225 \end{bmatrix}, U_2 = \begin{bmatrix} 0.04 & 0 \\ 0 & 0.01 \end{bmatrix}, U_3 = \begin{bmatrix} 0.04 & 0 \\ 0 & 0.0625 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 0.0225 & 0 \\ 0 & 0.0225 \end{bmatrix}, W_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, W_3 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.04 \end{bmatrix},$$

the solutions of the inequalities (14), (15) are defined as

$$P_1 = \begin{bmatrix} 0.9743 & 0.0060 \\ 0.0060 & 1.9313 \end{bmatrix}, P_2 = \begin{bmatrix} 1.5824 & -0.0107 \\ -0.0107 & 0.3980 \end{bmatrix}, P_3 = \begin{bmatrix} 2.3362 & -0.0215 \\ -0.0215 & 1.2289 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.7025 & 0.9511 \\ 0.9511 & 1.9673 \end{bmatrix}, R_2 = \begin{bmatrix} 1.5972 & -1.6245 \\ -1.6245 & 5.5039 \end{bmatrix}, R_3 = \begin{bmatrix} 1.2160 & 0.0388 \\ 0.0388 & 14.3699 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} -0.8592 & -1.7635 \\ -0.1222 & -0.3062 \end{bmatrix}, Y_2 = \begin{bmatrix} 0.6132 & -1.7502 \\ -1.7502 & 0.8320 \end{bmatrix}, Y_3 = \begin{bmatrix} -0.0447 & -0.6375 \\ -0.5078 & 14.6352 \end{bmatrix}.$$

By Theorem 2, the guaranteed cost control problem is solvable and the guaranteed cost value is $J^* = 66.7$, the guaranteed cost controllers are

$$u_1(t) = \begin{bmatrix} -0.8763 & -0.9104 \\ -0.1245 & -0.1582 \end{bmatrix} x_1(t),$$
$$u_2(t) = \begin{bmatrix} 0.3578 & -4.3874 \\ -0.1566 & 2.0862 \end{bmatrix} x_2(t),$$
$$u_3(t) = \begin{bmatrix} -0.0239 & -0.5192 \\ -0.1077 & 11.9073 \end{bmatrix} x_3(t).$$

Figure 1 shows the time history $x^\top(t)Rx(t)$ of the system with the initial conditions $\varphi_1(t) = (0.5\sin t, 0.5)$; $\varphi_2(t) = (e^t, 0.02t)$; $\varphi_3(t) = (0.01e^t, 0.1\cos t)$, $t \in (-0.5; 0)$ and $c_1 = 1,\ c_2 = 83$.

5. CONCLUSIONS

In this paper, we have studied the problem of supoptimal finite-time control for linear large-scale systems with the state and control delays in interconnection. Exploiting the Lyapunov function method and linear matrix inequality technique, we have proposed new LMI conditions for the robust H_∞ finite-time control and guaranteed cost finite-time control of such systems. The conditions are presented in terms of tractable LMIs, which can be solved by standard computational LMI toolbox algorithms. The validity and effectiveness of the proposed results have been illustrated by a numerical example.

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