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MODELING MULTI-CRITERIA DECISION-MAKING IN DYNAMIC NEUTROSOPHIC ENVIRONMENTS BASED ON CHOQUET INTEGRAL

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Abstract. Multi-attributes decision-making problem in dynamic neutrosophic environment is an open and highly-interesting research area with many potential applications in real life. The concept of the dynamic interval-valued neutrosophic set and its application for the dynamic decision-making are proposed recently, however the inter-dependence among criteria or preference is not dealt with in the proposed operations to well treat inter-dependence problems. Therefore, the definitions, mathematical operations and its properties are mentioned and discussed in detail. Then, Choquet integral-based distance between dynamic inteval-valued neutrosophic sets is defined and used to develop a new decision making model based on the proposed theory. A practical application of proposed approach is constructed and tested on the data of lecturers' performance collected from Vietnam National University (VNU) to illustrate the efficiency of new proposal.

Keywords. Multi-attributes decision-making; Dynamic interval-valued neutrosophic environment; Choquet integral.

1. INTRODUCTION

Dynamic decision-making (DDM) problem has attracted many researchers thanks to its potential application in real life. One successful approach for this problem is applying neutrosophic set that has the capability of solving indeterminacy in DDM [2, 5, 16]. Recently, Thong NT et al. [16] has introduced a model that deals with dynamic decision-making problems with time constraints. The authors proposed the new concept called dynamic interval-valued neutrosophic set (DIVNS), and developed a decision-making model based on new neutrosophic set concept [16]. However, a very common DDM which is dynamic multi-criteria decision-making (DMCDM) is not well treated, particularly inter-dependent among criteria or preference is not dealt with, etc. [11].

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The limitation of legacy aggregation operators based on additive measurements within a set of criteria is that they did not handle the impact of the interdependent attributes in criteria set. This fact leads to new approximate aggregation operators that use the fuzzy measurement to handle the dependency between multiple criteria [7]. Choquet integral-based aggregation operator has been applied [8, 9], and it has improved the weakness of simple weighted sum method. For example, if we consider a set of four alternatives $\{x_1, x_2, x_3, x_4\}$ where each alternative x_i is evaluated with three criteria to maximize: $x_1 = (18; 10; 10)$, $x_2 = (10, 18, 10)$, $x_3 = (10, 10, 18)$, $x_4 = (14, 11, 12)$, in truth, the alternative x_4 is not a selected solution with a weighted sum operator, however this alternative is the most balanced alternative and it would likely be a good option. This shortcoming has been overcome by defining a new operator using Choquet integral to make fuzzy measurement [6].

This study utilises the Choquet integral on DIVNS to improve decision making model. A novel aggregation operator named dynamic interval-valued neutrosophic Choquet operator aggregation (DIVNCOA) is proposed, that solves the problem of inter-dependent among criteria in dynamic interval-valued neutrosophic set. DIVNCOA improves the legacy aggregation operator introduced in [11]. Particularly, the definitions, mathematical operations and its properties are proposed and discussed in detail firstly. Then, Choquet integral-based aggregate operator between dynamic interval-valued neutrosophic sets is defined; and a decision making model is developed based on the proposed measure. A practical application was constructed and tested on data of lecturers' performance collected from Vietnam National University (VNU), to illustrate the efficiency of new proposal.

The rest of this document is structured as follows: Section 2 reviews briefly the DIVSNs concept and Choquet integral fundamental. Section 3 presents the Choquet integral-based operators. Section 4 expresses a new decision-making model for DDM and a practical application and Section 5 summarizes the findings.

2. PRELIMINARIES

At first, the definitions of Choquet integral and DIVNSs are reminded as the fundamental for further discussion. Besides, an important fuzzy measure based on Choquet integral is also defined and this measure is applied for decision making model mentioned in the next section.

2.1. Dynamic interval-valued neutrosophic set

Definition 1. [17] Let U be a universe of discourse. A is an Interval Neutrosophic set expressed by

$$A = \left\{x, \langle [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)]\rangle | x \in U\right\} \tag{1}$$

where $[T_A^L(x), T_A^U(x)] \subseteq [0,1]$, $[I_A^L(x), I_A^U(x)] \subseteq [0,1]$, $[F_A^L(x), F_A^U(x)] \subseteq [0,1]$ represent truth, indeterminacy, and falsity membership functions of an element.

Definition 2. [16] Let U be a universe of discourse. A is a dynamic interval-valued neutrosophic set (DIVNS) expressed by

$$A = \left\{x, \langle [T_x^L(t), T_x^U(t)], [I_x^L(t), I_x^U(t)], [F_x^L(t), F_x^U(t)]\rangle | x \in U\right\}, \tag{2}$$

where,
$t = \{t_1, t_2, ..., t_k\}$,
$T_x^L(t) < T_x^U(t)]$,
$I_x^L(t) < I_x^U(t)$,
$F_x^L(t) < F_x^U(t)$
and $[T_x^L(t),\ T_x^U(t)]$,
$[I_x^L(t), I_x^U(t)],\ [F_x^L(t), F_x^U(t)] \subseteq [0,1]$.

And for convenience, we call $\widetilde{n} = \langle [T_x^L(t), T_x^U(t)],\ [I_x^L(t), I_x^U(t)],\ [F_x^L(t), F_x^U(t)] \rangle$ a dynamic interval-valued neutrosphic element (DIVNE).

2.2. Choquet integral

The Choquet integral has been introduced as the useful operator to overcome the limitation of additive measure for fuzzy information. In DMCDM, a fuzzy measure based on Choquet integral is presented as follows.

Definition 3. [8] Let (x, P, μ) be a measurable space and $\mu : P \to [0,1]$ be fuzzy measure if the following conditions are satisfied:

1. $\mu(\varnothing) = 0$;
2. $\mu(A) \leq \mu(B)$ whenever $A \subset B$;
3. If $A_1 \subset A_2 \subset ... \subset A_n; A_n \in P$ then $\mu(\bigcup_{A_n}^{\infty}) = \lim_{n\to\infty} \mu(A_n)$;
4. If $A_1 \supset A_2 \supset ... \supset A_n; A_n \in P$ then $\mu(\bigcup_{A_n}^{\infty}) = \lim_{n\to\infty} \mu(A_n)$.

In practice, Sugeno [3] has proposed a refinement by adding a property, and the simplification of g_λ fuzzy measure is as follows

$$\mu(A \cup B) = \mu(A) + \mu(B) + g_\lambda \mu(a)\mu(b),\ g_\lambda \in (-1, \infty)$$

for all $A, B \in P$ and $A \cap B = \varnothing$.

Definition 4. ([8]) Let $X = \{x_1, x_2, ..., x_v\}$ be a set, λ-fuzzy measure defined on X is shown by Eq. (3)

$$\mu(X) = \begin{cases} \dfrac{1}{\lambda}\Big(\prod\limits_{x_l \in X} \big(1 + \lambda\mu(x_l)\big) - 1\Big),\ \text{if } \lambda \neq 0, \\ \qquad\qquad\qquad \sum\limits_{x_l \in X} (x_l),\ \text{if } \lambda = 0 \end{cases} \tag{3}$$

where $x_i \cap x_j = \varnothing$, $\forall i \neq j | i,\ j = 1, 2, 3, \ldots, v$.

Definition 5. ([15]) Let $X = \{x_1, x_2, \ldots, x_v\}$ be a finite set and μ is a fuzzy measure. The Choquet integral of a function $g : X \to [0,1]$ with respect to fuzzy measure μ can be shown by Eq. (4)

$$\int g d\mu = \sum_{l=1}^{v} \Big(\mu\big(G_{\xi(l)}\big) - \mu\big(G_{\xi(l-1)}\big)\Big) \oplus g\big(x_{\xi(l)}\big), \tag{4}$$

where $\xi(1), \xi(2), \ldots, \xi(l), \ldots, \xi(v)$ is a permutation of $1, 2, \ldots, v$ such that

$$g(x_{\xi(1)}) \leq \ldots \leq g(x_{\xi(l)}) \leq \ldots \leq g(x_{\xi(v)}),\ G_{\xi(l)} = x_{\xi(1)}, x_{\xi(2)}, \ldots, x_{\xi(l)},\ \text{and}\ \ G_{\xi(0)} = \varnothing.$$

3. SCORE FUNCTION AND DYNAMIC INTERVAL VALUED NEUTROSOPHIC CHOQUET AGGREGATION OPERATOR

In this section, a new score function for DIVNEs is proposed and new dynamic interval - valued Choquet aggregation operators are developed based on the previous operations and fuzzy measure above.

3.1. Score function for DIVNS

Definition 6. The score function of DIVNE $\widetilde{n}$ is defined as

$$\text{score}(\widetilde{n}) = \frac{1}{k}\sum_{l=1}^{k}\left(\left(\frac{T^L(t_l)+T^U(t_l)}{2}+\left(1-\frac{I^L(t_l)+I^U(t_l)}{2}\right)+\left(1-\frac{F^L(t_l)+F^U(t_l)}{2}\right)\right)\Big/3\right) \tag{5}$$

where $t = t_1, t_2, \dots, t_k$.

3.2. Weighted score function for DIVNS

Definition 7. The weighted score function of DIVNE $\widetilde{n}$ is defined as

$$\text{score}(\widetilde{n}) = \frac{1}{k}\sum_{l=1}^{k}w_l\times\left(\left(\frac{T^L(t_l)+T^U(t_l)}{2}+\left(1-\frac{I^L(t_l)+I^U(t_l)}{2}\right)+\left(1-\frac{F^L(t_l)+F^U(t_l)}{2}\right)\right)\Big/3\right) \tag{6}$$

where $t = t_1, t_2, \dots, t_k$, w_l is weight of times and $\sum_{l=1}^{k} w_l = 1$.

Obviously, $\text{score}(\widetilde{n}) \in [0,1]$. If $\text{score}(\widetilde{n}_1) \geq \text{score}(\widetilde{n}_2)$ then $\widetilde{n}_1 \geq \widetilde{n}_2$.

3.3. The DIVNCOA operator

DIVNCOA is proposed as an aggregation operator that considers the inter-dependence among elements in dynamic interval-valued neutrosophic environment. This operator is defined based on Choquet integral mentioned in Section 2.2.

Definition 8. Let $\widetilde{n}_l (l = 1, 2, \dots, v)$ be a collection of DIVNEs, $X = \{x_1, x_2, \dots, x_v\}$ be a set of attributes and μ be a measure on X, the DIVNCOA operator is defined as

$$\text{DIVNCOA}_{\mu,\lambda} = \widetilde{n}_1, \widetilde{n}_2, \dots, \widetilde{n}_v = \left(\oplus_1^v \left(\mu\left(G_{\xi(l)}\right) - \mu\left(G_{\xi(l-1)}\right)\right)\widetilde{n}^{\lambda}_{\xi(l)}\right)^{\frac{1}{\lambda}}, \tag{7}$$

where $\lambda > 0$, $\mu_{\xi(l)} = \mu\left(G_{\xi(l)}\right) - \mu\left(G_{\xi(l-1)}\right)$. And $\xi(1), \xi(2), \dots, \xi(l), \dots, \xi(v)$ is a permutation of $l = 1, 2, \dots, v$ such that $g(x_{\xi(1)}) \leq g(x_{\xi(2)}) \leq, \dots, \leq g(x_{\xi(l)} \leq, \dots, \leq g(x_{\xi(v)}$, $G_{\xi(0)} = \varnothing$ and $G_{\xi(l)} = \{x_{\xi(1)}, x_{\xi(2)}, \dots, x_{\xi(l)}\}$.

Theorem 1. *When $\widetilde{n}_l$ $(l = 1, 2, \dots, v)$ is a collection of DIVNEs, then the aggregated value obtained by the DIVNCOA operator is also a DIVNE, and*

$$\mathrm{DIVNCOA}_{\mu,\lambda} = \left(\oplus_1^v \left(\mu\left(G_{\xi(l)}\right) - \mu\left(G_{\xi(l-1)}\right)\right) \widetilde{n}^{\lambda}_{\xi(l)} \right)^{\frac{1}{\lambda}}$$

$$= \left\{ \left[\left(1 - \prod_{l=1}^{v} \left(1 - (T^L_{\xi(l)}(t))^{\lambda} \right)^{\mu_{\xi(l)}} \right)^{\frac{1}{\lambda}}, \left(1 - \prod_{l=1}^{v} \left(1 - (T^U_{\xi(l)}(t))^{\lambda} \right)^{\mu_{\xi(l)}} \right)^{\frac{1}{\lambda}} \right], \right.$$
$$\left[1 - \left(1 - \prod_{l=1}^{v} \left(1 - (1 - I^L_{\xi(l)}(t))^{\lambda} \right)^{\mu_{\xi(l)}} \right)^{\frac{1}{\lambda}}, 1 - \left(1 - \prod_{l=1}^{v} \left(1 - (1 - I^U_{\xi(l)}(t))^{\lambda} \right)^{\mu_{\xi(l)}} \right)^{\frac{1}{\lambda}} \right],$$
$$\left. \left[1 - \left(1 - \prod_{l=1}^{v} \left(1 - (1 - F^L_{\xi(l)}(t))^{\lambda} \right)^{\mu_{\xi(l)}} \right)^{\frac{1}{\lambda}}, 1 - \left(1 - \prod_{l=1}^{v} \left(1 - (1 - F^U_{\xi(l)}(t))^{\lambda} \right)^{\mu_{\xi(l)}} \right)^{\frac{1}{\lambda}} \right] \right\}. \tag{8}$$

Proof. Theorem 1 is proven by inductive method.

When $v = 1$, the result is trivial outcome of Definiton 8.

When $v = 2$, from the operation relations of DIVNE [11], one has:

$$\left(\mu_{\xi(1)} \widetilde{n}^{\lambda}_{\xi(1)} \right)^{\frac{1}{\lambda}} =$$
$$\left\{ \left[\left(1 - \left(1 - (T^L_{\xi(1)}(t))^{\lambda} \right)^{\mu_{\xi(1)}} \right)^{\frac{1}{\lambda}}, \left(1 - \left(1 - (T^U_{\xi(1)}(t))^{\lambda} \right)^{\mu_{\xi(1)}} \right)^{\frac{1}{\lambda}} \right], \right.$$
$$\left[1 - \left(1 - \left(1 - (1 - I^L_{\xi(1)}(t))^{\lambda} \right)^{\mu_{\xi(1)}} \right)^{\frac{1}{\lambda}}, 1 - \left(1 - \left(1 - (1 - I^U_{\xi(1)}(t))^{\lambda} \right)^{\mu_{\xi(1)}} \right)^{\frac{1}{\lambda}} \right],$$
$$\left. \left[1 - \left(1 - \left(1 - (1 - F^L_{\xi(1)}(t))^{\lambda} \right)^{\mu_{\xi(1)}} \right)^{\frac{1}{\lambda}}, 1 - \left(1 - \left(1 - (1 - F^U_{\xi(1)}(t))^{\lambda} \right)^{\mu_{\xi(1)}} \right)^{\frac{1}{\lambda}} \right] \right\}.$$

$$\left(\mu_{\xi(2)} \widetilde{n}^{\lambda}_{\xi(2)} \right)^{\frac{2}{\lambda}} =$$
$$\left\{ \left[\left(1 - \left(1 - (T^L_{\xi(2)}(t))^{\lambda} \right)^{\mu_{\xi(2)}} \right)^{\frac{2}{\lambda}}, \left(1 - \left(1 - (T^U_{\xi(2)}(t))^{\lambda} \right)^{\mu_{\xi(2)}} \right)^{\frac{2}{\lambda}} \right], \right.$$
$$\left[1 - \left(1 - \left(1 - (1 - I^L_{\xi(2)}(t))^{\lambda} \right)^{\mu_{\xi(2)}} \right)^{\frac{2}{\lambda}}, 1 - \left(1 - \left(1 - (1 - I^U_{\xi(2)}(t))^{\lambda} \right)^{\mu_{\xi(2)}} \right)^{\frac{2}{\lambda}} \right],$$
$$\left. \left[1 - \left(1 - \left(1 - (1 - F^L_{\xi(2)}(t))^{\lambda} \right)^{\mu_{\xi(2)}} \right)^{\frac{2}{\lambda}}, 1 - \left(1 - \left(1 - (1 - F^U_{\xi(2)}(t))^{\lambda} \right)^{\mu_{\xi(2)}} \right)^{\frac{2}{\lambda}} \right] \right\}.$$

Assume that Equation (8) holds for $v = j$, we have

$$\mathrm{DIVNCOA}_{\mu,\lambda}\{\widetilde{n}_1, \widetilde{n}_2, \dots, \widetilde{n}_l\} =$$
$$\Bigg\{\Bigg[\Big(1-\prod_{l=1}^{j}\Big(1-(T^L_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}, \Big(1-\prod_{l=1}^{j}\Big(1-(T^U_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}\Bigg],$$
$$\Bigg[1-\Big(1-\prod_{l=1}^{j}\Big(1-(1-I^L_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}, 1-\Big(1-\prod_{l=1}^{j}\Big(1-(1-I^U_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}\Bigg],$$
$$\Bigg[1-\Big(1-\prod_{l=1}^{j}\Big(1-(1-F^L_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}, 1-\Big(1-\prod_{l=1}^{j}\Big(1-(1-F^U_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}\Bigg]\Bigg\}.$$

For $m = j + 1$, according to the inductive hypothesis, we have

$$\mathrm{DIVNCOA}_{\mu,\lambda}\{\widetilde{n}_1, \widetilde{n}_2, \dots, \widetilde{n}_l\} =$$
$$\Bigg\{\Bigg[\Big(1-\prod_{l=1}^{j}\Big(1-(T^L_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}, \Big(1-\prod_{l=1}^{j}\Big(1-(T^U_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}\Bigg],$$
$$\Bigg[1-\Big(1-\prod_{l=1}^{j}\Big(1-(1-I^L_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}, 1-\Big(1-\prod_{l=1}^{j}\Big(1-(1-I^U_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}\Bigg],$$
$$\Bigg[1-\Big(1-\prod_{l=1}^{j}\Big(1-(1-F^L_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}, 1-\Big(1-\prod_{l=1}^{j}\Big(1-(1-F^U_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}\Bigg]\Bigg\}.$$

$$\oplus\Bigg\{\Bigg[\Big(1-\Big(1-(T^{j+1}_{\xi(j+1)}(t))^\lambda\Big)^{\mu_{\xi(j+1)}}\Big)^{\frac{1}{\lambda}}, \Big(1-\Big(1-(T^U_{\xi(j+1)}(t))^\lambda\Big)^{\mu_{\xi(j+1)}}\Big)^{\frac{1}{\lambda}}\Bigg],$$
$$\Bigg[1-\Big(1-\Big(1-(1-I^{j+1}_{\xi(j+1)}(t))^\lambda\Big)^{\mu_{\xi(j+1)}}\Big)^{\frac{1}{\lambda}}, 1-\Big(1-\Big(1-(1-I^U_{\xi(j+1)}(t))^\lambda\Big)^{\mu_{\xi(j+1)}}\Big)^{\frac{1}{\lambda}}\Bigg],$$
$$\Bigg[1-\Big(1-\Big(1-(1-F^{j+1}_{\xi(j+1)}(t))^\lambda\Big)^{\mu_{\xi(j+1)}}\Big)^{\frac{1}{\lambda}}, 1-\Big(1-\Big(1-(1-F^U_{\xi(j+1)}(t))^\lambda\Big)^{\mu_{\xi(j+1)}}\Big)^{\frac{1}{\lambda}}\Bigg]\Bigg\}$$
$$=\Bigg\{\Bigg[\Big(1-\prod_{l=1}^{j+1}\Big(1-(T^L_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}, \Big(1-\prod_{l=1}^{j+1}\Big(1-(T^U_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}\Bigg],$$
$$\Bigg[1-\Big(1-\prod_{l=1}^{j+1}\Big(1-(1-I^L_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}, 1-\Big(1-\prod_{l=1}^{j+1}\Big(1-(1-I^U_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}\Bigg],$$
$$\Bigg[1-\Big(1-\prod_{l=1}^{j+1}\Big(1-(1-F^L_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}, 1-\Big(1-\prod_{l=1}^{j+1}\Big(1-(1-F^U_{\xi(l)}(t))^\lambda\Big)^{\mu_{\xi(l)}}\Big)^{\frac{1}{\lambda}}\Bigg]\Bigg\}.$$

From above equations, we have that equation (8) holds for all natural numbers m, and Theorem 1 is proved. ■

Theorem 2. *The DIVNCOA operator has the following desirable properties:*

1. (Idempotency) *Let* $\widetilde{n}_l = \widetilde{n}$ $(\forall l = 1, 2, \ldots, v)$ *and*

$$\widetilde{n} = \left\{ \left[T^L(t), T^U(t) \right], \left[I^L(t), I^U(t) \right], \left[F^L(t), F^U(t) \right] \right\}$$

then

$$\mathrm{DIVNCOA}_{\mu,\lambda}\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_v\} = \left\{ \left[T^L(t), T^U(t) \right], \left[I^L(t), I^U(t) \right], \left[F^L(t), F^U(t) \right] \right\}.$$

2. (Boundedness) *Let* $\widetilde{n}^- = \left\{ \left[T^{L^-}(t), T^{U^-}(t) \right], \left[I^{L^+}(t), I^{U^+}(t) \right], \left[F^{L^+}(t), F^{U^+}(t) \right] \right\}$;
$\widetilde{n}^+ = \left\{ \left[T^{L^+}(t), T^{U^+}(t) \right], \left[I^{L^-}(t), I^{U^-}(t) \right], \left[F^{L^-}(t), F^{U^-}(t) \right] \right\}$ *then*

$$\widetilde{n}^- \leq \mathrm{DIVNCOA}_{\mu,\lambda}\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_v\} \leq \widetilde{n}^+.$$

3. (Commutativity) *If* $\{\approx{n}_1, \approx{n}_2, \ldots, \approx{n}_v\}$ *is a permutation of* $\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_v\}$

$$\mathrm{DIVNCOA}_{\mu,\lambda}\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_v\} = \mathrm{DIVNCOA}_{\mu,\lambda}\{\approx{n}_1, \approx{n}_2, \ldots, \approx{n}_v\}.$$

4. (Monotonity) *If* $\widetilde{n}_l \leq \approx{n}_l$ for $\forall l \in \{1, 2, \ldots, v\}$, *then*

$$\mathrm{DIVNCOA}_{\mu,\lambda}\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_v\} \leq \mathrm{DIVNCOA}_{\mu,\lambda}\{\approx{n}_1, \approx{n}_2, \ldots, \approx{n}_v\}.$$

Proof. Suppose $(1, 2, ..., v)$ is a permutation such that $\widetilde{n}_1 \leq \widetilde{n}_2 \leq ... \leq \widetilde{n}_v$.

1. For $\widetilde{n} = \left\{ \left[\widetilde{T}^L(t), \widetilde{T}^U(t) \right], \left[\widetilde{I}^L(t), \widetilde{I}^U(t) \right], \left[\widetilde{F}^L(t), \widetilde{F}^U(t) \right] \right\}$, according to Definition 4, it follows that

$$\begin{aligned}
&\mathrm{DIVNCOA}_{\mu,\lambda}\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_v\} = \\
&\Bigg\{ \Bigg[\left(1 - \prod_{l=1}^{v} \left(1 - \left(T^L_{\xi(l)}(t) \right)^\lambda \right)^{\sum_{l=1}^{v} \mu(G_{\xi(l)} - G_{\xi(l-1)})} \right)^{\frac{1}{\lambda}}, \\
&\left(1 - \prod_{l=1}^{v} \left(1 - \left(T^U_{\xi(l)}(t) \right)^\lambda \right)^{\sum_{l=1}^{v} \mu(G_{\xi(l)} - G_{\xi(l-1)})} \right)^{\frac{1}{\lambda}} \Bigg], \\
&\Bigg[\left(1 - \left(1 - \prod_{l=1}^{v} \left(1 - \left(1 - I^L_{\xi(l)}(t) \right)^\lambda \right)^{\sum_{l=1}^{v} \mu(G_{\xi(l)} - G_{\xi(l-1)})} \right)^{\frac{1}{\lambda}} \right), \\
&\left(1 - \left(1 - \prod_{l=1}^{v} \left(1 - \left(1 - I^U_{\xi(l)}(t) \right)^\lambda \right)^{\sum_{l=1}^{v} \mu(G_{\xi(l)} - G_{\xi(l-1)})} \right)^{\frac{1}{\lambda}} \right) \Bigg], \\
&\Bigg[\left(1 - \left(1 - \prod_{l=1}^{v} \left(1 - \left(1 - F^L_{\xi(l)}(t) \right)^\lambda \right)^{\sum_{l=1}^{v} \mu(G_{\xi(l)} - G_{\xi(l-1)})} \right)^{\frac{1}{\lambda}} \right), \\
&\left(1 - \left(1 - \prod_{l=1}^{v} \left(1 - \left(1 - F^U_{\xi(l)}(t) \right)^\lambda \right)^{\sum_{l=1}^{v} \mu(G_{\xi(l)} - G_{\xi(l-1)})} \right)^{\frac{1}{\lambda}} \right) \Bigg] \Bigg\}.
\end{aligned}$$

Since $\sum_{l=1}^{v}\mu(G_{\xi(l)}-G_{\xi(l-1)})=1$, thus,

$$\text{DIVNCOA}_{\mu,\lambda}\{\widetilde{n}_1,\widetilde{n}_2,\dots,\widetilde{n}_v\}=\left\{\left[T^L(t),T^U(t)\right],\left[I^L(t),I^U(t)\right],\left[F^L(t),F^U(t)\right]\right\}.$$

2. For any $\widetilde{T_l}=[\widetilde{T}_l^L,\ \widetilde{T}_l^U]$, $\widetilde{I_l}=[\widetilde{I}_l^L,\widetilde{I}_l^U]$ and $\widetilde{F_l}=[\widetilde{F}_l^L,\widetilde{F}_l^U]$, $l=1,2,...,v$, we have

$$\widetilde{T}^{L^-}\le\widetilde{T}_l^L\le\widetilde{T}^{L^+};\ \widetilde{I}^{L^-}\le\widetilde{I}_l^L\le\widetilde{I}^{L^+};\ \widetilde{F}^{L^-}\le\widetilde{F}_l^L\le\widetilde{F}^{L^+};$$
$$\widetilde{T}^{U^-}\le\widetilde{T}_l^U\le\widetilde{T}^{U^+};\ \widetilde{I}^{U^-}\le\widetilde{I}_l^U\le\widetilde{I}^{U^+};\ \widetilde{F}^{U^-}\le\widetilde{F}_l^U\le\widetilde{F}^{U^+}.$$

Since $f=x^\theta$ $(0<\theta<1)$ is a monotone increasing function when $x>0$ and values in the DIVNCOA operator are all valued in $[0,1]$, therefore,

$$\begin{aligned}
&\left(1-\left(1-(\widetilde{T}^{L^-}_{\xi(l)}(t))^\lambda\right)^{\sum_{l=1}^{v}\mu(G_{\xi(l)}-G_{\xi(l-1)})}\right)^{\frac{1}{\lambda}}+\left(1-\left(1-(\widetilde{T}^{U^-}_{\xi(l)}(t))^\lambda\right)^{\sum_{l=1}^{v}\mu(G_{\xi(l)}-G_{\xi(l-1)})}\right)^{\frac{1}{\lambda}}\\
\le&\left(1-\prod_{l=1}^{v}\left(1-(\widetilde{T}^{L}_{\xi(l)}(t))^\lambda\right)^{\sum_{l=1}^{v}\mu(G_{\xi(l)}-G_{\xi(l-1)})}\right)^{\frac{1}{\lambda}}+\\
&\left(1-\prod_{l=1}^{v}\left(1-(\widetilde{T}^{U}_{\xi(l)}(t))^\lambda\right)^{\sum_{l=1}^{v}\mu(G_{\xi(l)}-G_{\xi(l-1)})}\right)^{\frac{1}{\lambda}}\\
\le&\left(1-\left(1-(\widetilde{T}^{L^+}_{\xi(l)}(t))^\lambda\right)^{\sum_{l=1}^{v}\mu(G_{\xi(l)}-G_{\xi(l-1)})}\right)^{\frac{1}{\lambda}}+\left(1-\left(1-(\widetilde{T}^{U^+}_{\xi(l)}(t))^\lambda\right)^{\sum_{l=1}^{v}\mu(G_{\xi(l)}-G_{\xi(l-1)})}\right)^{\frac{1}{\lambda}}.
\end{aligned}$$

Since $\sum_{l=1}^{v}\mu(G_{\xi(l)}-G_{\xi(l-1)})=1$, the above equation is equivalent to

$$\begin{aligned}
&\widetilde{T}^{L^-}+\widetilde{T}^{U^-}\\
&\le\left(1-\prod_{l=1}^{v}\left(1-(\widetilde{T}^{L}_{\xi(l)}(t))^\lambda\right)^{\sum_{l=1}^{v}\mu(G_{\xi(l)}-G_{\xi(l-1)})}\right)^{\frac{1}{\lambda}}\\
&\quad+\left(1-\prod_{l=1}^{v}\left(1-(\widetilde{T}^{U}_{\xi(l)}(t))^\lambda\right)^{\sum_{l=1}^{v}\mu(G_{\xi(l)}-G_{\xi(l-1)})}\right)^{\frac{1}{\lambda}}\\
&\le\ \widetilde{T}^{L^+}+\widetilde{T}^{U^+}.
\end{aligned}$$

Analogously, we have

$$\begin{aligned}
&\widetilde{I}^{L^-}+\widetilde{I}^{U^-}\\
&\le\left(1-\left(1-\prod_{l=1}^{v}\left(1-(1-\widetilde{I}^{L}_{\xi(l)}(t))^\lambda\right)^{\sum_{l=1}^{v}\mu(G_{\xi(l)}-G_{\xi(l-1)})}\right)^{\frac{1}{\lambda}}\right)\\
&\quad+\left(1-\left(1-\prod_{l=1}^{v}\left(1-(1-\widetilde{I}^{U}_{\xi(l)}(t))^\lambda\right)^{\sum_{l=1}^{v}\mu(G_{\xi(l)}-G_{\xi(l-1)})}\right)^{\frac{1}{\lambda}}\right)\\
&\le\ \widetilde{I}^{L^+}+\widetilde{I}^{U^+}.
\end{aligned}$$

and

$$\begin{aligned}
&\tilde{F}^{L^-} + \tilde{F}^{U^-} \\
&\quad \leq \left(1 - \left(1 - \prod_{l=1}^{v}\left(1 - \left(1 - \tilde{F}^{L}_{\xi(l)}(t)\right)^{\lambda}\right)^{\sum_{l=1}^{v}\mu(G_{\xi(l)} - G_{\xi(l-1)})}\right)^{\frac{1}{\lambda}}\right) \\
&\qquad + \left(1 - \left(1 - \prod_{l=1}^{v}\left(1 - \left(1 - \tilde{F}^{U}_{\xi(l)}(t)\right)^{\lambda}\right)^{\sum_{l=1}^{v}\mu(G_{\xi(l)} - G_{\xi(l-1)})}\right)^{\frac{1}{\lambda}}\right) \\
&\quad \leq \tilde{F}^{L^+} + \tilde{F}^{U^+}.
\end{aligned}$$

Since score$(\widetilde{n}^-) \leq$ score$(\widetilde{n}) \leq$ score$(\widetilde{n}^+)$, thus, $\widetilde{n}^- \leq \mathrm{DIVNCOA}_{\mu,\lambda}\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_v\} \leq \widetilde{n}^+$.

3. Suppose $(\xi(1), \xi(2), ..., \xi(v))$ is a permutation of both $\{\widetilde{\widetilde{n}}_1, \widetilde{\widetilde{n}}_2, ..., \widetilde{\widetilde{n}}_v\}$ and $\{\widetilde{n}_1, \widetilde{n}_2, ..., \widetilde{n}_v\}$ such that $\widetilde{n}_{\xi(1)} \leq \widetilde{n}_{\xi(2)} \leq, ..., \leq \widetilde{n}_{\xi(v)}$, $G_{\xi(l)} = x_{\xi(1)}, x_{\xi(2)}, ..., x_{\xi(l)}$, then

$$\begin{aligned}
\mathrm{DIVNCOA}_{\mu,\lambda}\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_v\} &= \mathrm{DIVNCOA}_{\mu,\lambda}\{\widetilde{\widetilde{n}}_1, \widetilde{\widetilde{n}}_2, \ldots, \widetilde{\widetilde{n}}_v\} \\
&= \oplus_{l=1}^{v}\left(\left(\mu(G_{\xi(l)}) - \mu(G_{\xi(l-1)})\widetilde{n}_{\xi(l)}\right)\right).
\end{aligned}$$

4. It is easily observed from Theorem 1.

Theorem 2 is proved. ∎

4. APPLICATION IN DMCDM UNDER DYNAMIC INTERVAL VALUED NEUTROSOPHIC ENVIRONMENT

The operators have been blueimplemented for the DMCDM problem to illustrate its potential application. blueExtending from the existing DMCDM methods on dynamic interval valued neutrosophic environment, herein the interaction relationship among attributes is considered. It is to remind that the characteristics of the alternatives are represented by DIVNEs. In this case, the correctness of a DMCDM problem is verified based on new Choquet aggregation operators and its practicality is considered.

4.1. Approaches based on the DIVNCOA operator for DMCDM

Assume $A = \{A_1, A_2, \ldots, A_v\}$ and $C = \{C_1, C_2, \ldots, C_n\}$ and $D = \{D_1, D_2, \ldots, D_h\}$ are sets of alternatives, attributes, and decision makers. For a decision maker $D_q,\ q = 1, 2, \ldots, h$ the evaluation characteristic of an alternative $A_m,\ m = 1, 2, \ldots, v$, on an attribute $C_p,\ p = 1, 2, \ldots, n$, in time sequence $t = \{t_1, t_2, \ldots, t_k\}$ is represented by a decision matrix $D^q\big(t_l\big) = \big(d^q_{mp}(t)\big)_{v\times n},\ l = 1, 2, \ldots, k$, where $d^q_{mp}(t) = \Big\langle x^q_{d_{mp}}(t), \big(T^q(d_{mp}, t), I^q(d_{mp}, t), F^q(d_{mp}, t)\big)\Big\rangle$, $t = \{t_1, t_2, \ldots, t_k\}$ taken by DIVNSs evaluated by decision maker D_q.

Step 1. Reorder the decision matrix.

With respect to attributes $C = \{C_1, C_2, \ldots, C_n\}$, reorder DIVNEs d^q_{mp} of $A = \{A_1, A_2, \ldots, A_v\}$ rated by decision makers $D = \{D_1, D_2, \ldots, D_h\}$ from smallest to largest, according to their score function values calculated by Equation (9)

$$\text{score}(\widetilde{n}) = \frac{1}{h} \times \frac{1}{k}\sum_{r}^{h}\omega_r \times \sum_{l=1}^{k} w_l \times \Bigg(\Big(\frac{T^L(t_l)+T^U(t_l)}{2} + \Big(1 - \frac{I^L(t_l)+I^U(t_l)}{2}\Big) + \Big(1 - \frac{F^L(t_l)+F^U(t_l)}{2}\Big)\Big)\Big/3\Bigg),$$

the reorder sequence for $A_m,\ m = 1, \ldots, v$, is $\big(\xi(1), \xi(2), \ldots, \xi(v)\big)$.

Step 2. Calculate fuzzy measures of n attributes.

Use the formula measurement stated in the Equation (3) to calculate the fuzzy measure of C, where the interaction among all attributes is taken into account.

Step 3. Aggregate decision information by the DIVNCOA operator and score values for alternatives.

Aggregate DIVNEs of $A_m,\ m = 1, \ldots, v$, stated in Equation (8), with consideration of all attributes $C = \{C_1, C_2, \ldots, C_n\}$ as proved by theorem, the average values obtained by the DIVNCOA operator are also DIVNEs; and score values for alternatives calculated by (9).

Step 4. Place all alternatives in order.

Rank all alternatives by selecting the best fit by their score function values between $A_m,\ m = 1, \ldots, v$, described in Equation (9).

4.2. Practical application

This section presents an application of the new method proposed in previous sections, particularly it is used to evaluate the performance of lecturers in a Vietnamese university, ULIS-VNU. This problem is DMCDM problem, that includes five alternatives present to five lecturers $A_1, \ldots, A_5$, and three decision makers $D_1, \ldots, D_3$, each lecturer's performance is estimated by six criteria: The total of publications, the teaching student evaluations, the personality characteristics, the professional society, teaching experience and the fluency of foreign language, are symbolized as, $(C_1), (C_2), (C_3)$, $(C_4), (C_5), (C_6)$ respectively.

The set of linguistic label $S = \{\text{VeGo, Go, Me, Po, VePo}\}$ in $t = \{t_1, t_2, t_3\}$ is

VeGo = VeryGood = $([0.6, 0.7], [0.2, 0.3], [0.2, 0.3])$,

Go = Good = $([0.5, 0.6], [0.4, 0.5], [0.3, 0.4])$,

Me = Medium = $([0.3, 0.5], [0.4, 0.6], [0.4, 0.5])$,

Po = Poor = $([0.2, 0.3], [0.5, 0.6], [0.6, 0.7])$,

VePo = VeryPoor = $([0.1, 0.2], [0.6, 0.7], [0.7, 0.8])$.

And Table 1 presents rating of decision makers to lecturers by criteria at three periods.

Step 1. Using Equation (9) to calculate score function values. Values shown in Table 2.

According to score function between criteria and alternatives, the reordered decision is given by Table 3.

Step 2. First, if all inter-related attributes from the fuzzy measures are given as follows: $\mu(C_1) = 0.2$, $\mu(C_2) = 0.42$, $\mu(C_3) = 0.22$, $\mu(C_4) = 0.3$, $\mu(C_5) = 0.1$, $\mu(C_6) = 0.15$, According to Equation (3), the value of λ is obtained $\lambda = -0.60$. Thus, we have:

Table 1. Rating of decision makers for criteria to lecturers

Criteria	**Lec**	**Decision makers**								
		t_1			t_2			t_3		
		D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
C_1	A_1	Me	Go	Go	Go	Go	Go	Go	VeGo	Go
	A_2	Go	Go	VeGo	VeGo	Go	VeGo	VeGo	Go	VeGo
	A_3	Me	Go	Go	Go	Go	Go	Go	Go	VeGo
	A_4	Go	Me	Go	Go	Go	Go	Go	Go	Go
	A_5	Me	Go	Me	Go	Go	Me	Go	Go	Go
C_2	A_1	Go	Go	Go	VeGo	Go	Go	Go	Go	Go
	A_2	VeGo	Go	VeGo	Me	Go	Go	VeGo	Go	Go
	A_3	VeGo	Go	Go	Go	Me	Go	Go	Me	Go
	A_4	Go	Go	Go	Go	VeGo	Go	Go	Go	VeGo
	A_5	VeGo	Go	Go	Go	VeGo	Go	Go	Go	Me
C_3	A_1	VeGo	VeGo	Go	Go	VeGo	Go	Go	Me	Go
	A_2	Go	VeGo	Go	VeGo	Go	VeGo	Go	Go	VeGo
	A_3	Go	VeGo	VeGo	Go	Go	Go	Go	VeGo	Go
	A_4	Go	Go	Go	VeGo	Go	Go	VeGo	Go	Go
	A_5	VeGo	Go	Go	Go	VeGo	Go	Go	Go	Go
C_4	A_1	Me	Go	Me	Go	Go	Me	Me	Go	Me
	A_2	Go	Me	Go	Go	Me	Go	Go	Me	Go
	A_3	Go	Go	Go	Go	Go	Me	Go	Go	VeGo
	A_4	Me	Po	Me	Go	Me	Me	Go	Go	Me
	A_5	Me	Me	Po	Me	Me	Me	Me	Go	Me
C_5	A_1	Me	Go	Me	Me	Go	Go	Go	Me	Go
	A_2	Go	VeGo	Go	VeGo	Go	Go	Go	VeGo	Go
	A_3	Go	Go	Me	Go	Go	Go	Go	VeGo	Go
	A_4	VeGo	Go	Go	VeGo	Go	Go	VeGo	Go	Go
	A_5	Go	Go	Go	Go	Go	Go	Go	VeGo	Go
C_6	A_1	VeGo	Go	Go	VeGo	Go	VeGo	VeGo	Go	VeGo
	A_2	Go	Go	Go	Go	VeGo	Go	Go	Go	VeGo
	A_3	VeGo	Go	VeGo	VeGo	Go	VeGo	VeGo	Go	VeGo
	A_4	Go	VeGo	Go	Go	VeGo	Go	Go	Go	Go
	A_5	Go	Go	Go	VeGo	Go	Go	Go	VeGo	Go

$\mu(C_1, C_2) = 0.5696$,
$\mu(C_1, C_2, C_3) = 0.7144$,
$\mu(C_1, C_2, C_3, C4) = 0.8858$,
$\mu(C_1, C_2, C_3, C_4, C_5) = 0.9327$,
$\mu(C_1, C_2, C_3, C_4, C_5, C_6) = 1$.

And $\mu_{\xi(1)} = 0.2$, $\mu_{\xi(2)} = 0.3696$, $\mu_{\xi(3)} = 0.1448$, $\mu_{\xi(4)} = 0.1714$, $\mu_{\xi(5)} = 0.0469$, $\mu_{\xi(6)} = 0.0673$.

Table 2. The Score for Lecturers - Criterias

A\C	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.587	0.598	0.617	0.527	0.54	0.657
A_2	0.657	0.617	0.657	0.55	0.628	0.613
A_3	0.587	0.576	0.628	0.587	0.587	0.672
A_4	0.572	0.613	0.613	0.502	0.628	0.613
A_5	0.55	0.602	0.613	0.479	0.598	0.613

Table 3. The recordered decision

Criteria	**Lec**	**Decision makers**								
		t_1			t_2			t_3		
		D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
C_1	A_1	Me	Go	Me	Go	Go	Me	Me	Go	Me
	A_2	Go	Me	Go	Go	Me	Go	Go	Me	Go
	A_3	VeGo	Go	Go	Go	Me	Go	Go	Me	Go
	A_4	Me	Po	Me	Go	Me	Me	Go	Go	Me
	A_5	Me	Me	Po	Me	Me	Me	Me	Go	Me
C_2	A_1	Me	Go	Me	Me	Go	Go	Go	Me	Go
	A_2	Go	Go	Go	Go	VeGo	Go	Go	Go	VeGo
	A_3	Me	Go	Go	Go	Go	Go	Go	Go	VeGo
	A_4	Go	Me	Go	Go	Go	Go	Go	Go	Go
	A_5	Me	Go	Me	Go	Go	Me	Go	Go	Go
C_3	A_1	Me	Go	Go	Go	Go	Go	Go	VeGo	Go
	A_2	VeGo	Go	VeGo	Me	Go	Go	VeGo	Go	Go
	A_3	Go	Go	Go	Go	Go	Me	Go	Go	VeGo
	A_4	Go	Go	Go	Go	VeGo	Go	Go	Go	VeGo
	A_5	Go	Go	Go	Go	Go	Go	Go	VeGo	Go
C_4	A_1	Go	Go	Go	VeGo	Go	Go	Go	Go	Go
	A_2	Go	VeGo	Go	VeGo	Go	Go	Go	VeGo	Go
	A_3	Go	Go	Me	Go	Go	Go	Go	VeGo	Go
	A_4	Go	Go	Go	VeGo	Go	Go	VeGo	Go	Go
	A_5	VeGo	Go	Go	Go	VeGo	Go	Go	Go	Me
C_5	A_1	VeGo	VeGo	Go	Go	VeGo	Go	Go	Me	Go
	A_2	Go	Go	VeGo	VeGo	Go	VeGo	VeGo	Go	VeGo
	A_3	Go	VeGo	VeGo	Go	Go	Go	Go	VeGo	Go
	A_4	Go	VeGo	Go	Go	VeGo	Go	Go	Go	Go
	A_5	VeGo	Go	Go	Go	VeGo	Go	Go	Go	Go
C_6	A_1	VeGo	Go	Go	VeGo	Go	VeGo	VeGo	Go	VeGo
	A_2	Go	VeGo	Go	VeGo	Go	VeGo	Go	Go	VeGo
	A_3	VeGo	Go	VeGo	VeGo	Go	VeGo	VeGo	Go	VeGo
	A_4	VeGo	Go	Go	VeGo	Go	Go	VeGo	Go	Go
	A_5	Go	Go	Go	VeGo	Go	Go	Go	VeGo	Go

Step 3. With $\lambda = 1$, we have following score values of lecturers depicted in Table 4.

Table 4. The Scores of Lecturers

Lecturers	Proposed Method	Ranking
A_1	0.999998824	4
A_2	0.999999293	1
A_3	0.999999262	2
A_4	0.99999895	3
A_5	0.999998584	5

Step 4. From the values in Table 4, we have the ranking of lecturers as $A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$.

Compare proposed method with TOPSIS method [11], it is able to demonstrate the advantages and to show the proposed method's application. Table 5 shows that the hierarchical order of the five lectures by TOPSIS method is $A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$ then A_2 is the best lecturer. The result is identical to our method. This means that the method in simplest form can handle DMCDM problem. Moreover, it is more flexible than the method introduced by Thong et al. [11] because the new method considers the inter-dependence among criteria or preference.

Table 5. Closeness coefficient

Lecturers	TOPSIS method	Ranking
A_1	0.339	4
A_2	0.367	1
A_3	0.351	2
A_4	0.345	3
A_5	0.338	5

5. CONCLUSIONS

This paper introduced a new modification of Choquet aggregation operator under the Dynamic inteval valued neutrosophic environment in which the interdependency between criteria are observed and two score function have also been defined for DIVNSs. Furthermore, we have presented a new decision making method based on proposed theories and have tested its potential application by evaluating lecturers' performance in the ULIS-VNU. The testing result shows the efficiency of decision making model using new proposal measures for dynamic decision-making problem under dynamic neutrosophic environment constrained by time.

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