DESIGNING HEDGE ALGEBRAIC CONTROLLER AND OPTIMIZING BY GENETIC ALGORITHM FOR SERIAL ROBOTS ADHERING TRAJECTORIES

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Abstract. In recent years, the application of hedge algebras in the field of control has been studied. The results show that this approach has many advantages. In addition, industrial robots are being well-developed and extensively used, especially in the industrial revolution 4.0. Accurate control of industrial robots is a class of problems that many scientists are interested in. In this paper, we design a controller based on hedge algebras for serial robots. The control rule is given by the linguistic rule-based system. The goal is to accurately control the moving robot arm which adheres given trajectories. Optimization of fuzzy parameters for the controller is done by genetic algorithms. The system has been simulated on the Matlab-Simulink software. The simulation results show that the tracking error is very small. Moreover, the controller worked well with correct control quality. This result once presents the simplicity and efficiency of the hedge algebras approach to control.

Keywords. Hedge algebras; Tracking control; Serial robot; Genetic algorithm.

1. INTRODUCTION

Today, men are gradually being replaced by industrial robots in many stages of production, especially in positions of repetitive work. Furthermore, industrial robots can work enduringly and produce product uniformity better than humans. Their activities, especially high-precision activities, always require reasonable control strategies. In recent years, many control methods have been researched by scientists, including some control methods such as sliding control, exact linearization control, optimal control, fuzzy control, etc. Sliding control is a control method that is suitable for nonlinear systems such as robots. However, with this type of control, it is necessary to overcome a major disadvantage that is the phenomenon of chattering [1–4], affecting negatively to the mechanical structure of the system. Controlled by exact linearization, the calculation controller is based on the results of the inverse dynamic problem [5]. With this method, the dynamic parameters of the model are essential to be known exactly [6]. Adaptive control, a robotic control method that differs from conventional control methods presents that the controller's parameters change over time. Moreover, they are calibrated based on the feedback signals in the closed loop [7–9]. This control method is also proper as the dynamic parameters of the robot are incorrect [5]. Optimal control with, control laws is developed for a dynamical robot over a period of time so that an objective function is optimized. In this case, Optimal parameters are found based on the clear dynamic model or dynamic parameters need to know exactly [5, 10].

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Fuzzy controls have been studied and applied for a long time in the field of control [11, 12, 30]. The fuzzy controller shows the ability to work quite effectively with many different types of objects, especially those with strong nonlinearity. The application of the fuzzy controller for robots has also been researched [13, 31]. However, the design of a fuzzy controller also has certain difficulties. That is, every step of designing a fuzzy controller depends much on the knowledge and experience of the designer. In addition, the fuzzy controller requires large computational time [14], so it does not respond well to systems that require real-time response.

In recent years, the application of hedge algebras in the control field has been studied and tested on many different types of objects [14–18]. The results show that the controller based hedge algebras approach has many advantages such as simple structure, design, and fast calculation speed. However, no studies have applied the controller based hedge algebras with control rules given by LRBS to control the robot.

This paper presents the application of hedge algebras theory to design a set of approximate inferences that work as a controller to moves end effector along the desired path. The controlled robot is two degree of freedoms (DOF - Degree of Freedom) serial robot described in Figure 1. Optimization of controller parameters is are found by a genetic algorithm (GA - Genetic Algorithm). Simulation results demonstrated the effectiveness of the hedge algebras controller with optimal parameters.

2. DESIGN OF THE GENERAL CONTROLLER BASED HEDGE ALGEBRA FOR THE SERIAL ROBOT

2.1. Serial robot model

2.1.1. The dynamical model of the serial robot

Let's consider a serial robot with n degrees of freedom with generalized coordinates

$$q = [q_1, \ q_2, \ \ldots, \ q_m]^T = \left[\theta^T, x^T\right]. \tag{1}$$

The differential equation describing the motion of the system can be set by using the Lagrange II method with the following form [19, 20]

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right)^T - \left(\frac{\partial T}{\partial q}\right)^T = Q - \left(\frac{\partial \Pi}{\partial q}\right)^T, \tag{2}$$

where,
   $T = \frac{1}{2}\dot{q}^T M(q)\dot{q}$ is the kinetic energy of the system;
   $M(q)$ is a mass matrix of size $n \times n$;
   $\Pi = \Pi(q)$ is a potential energy expression;
   Q is the generalized force vector of the control forces and the other forces.
The equation (2) can be rewritten as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = B\tau \tag{3}$$

with $Q = B\tau$ and B are matrices associated with the control signal arrangement and τ is the force/torque vector in active joints. Centrifugal matrix and Coriolis $C(q, \dot{q})$ are determined

by the mass matrix $M(q)$ according to Kronecker [21] or based on Christoffel formula as follows

$$C(q, \dot{q}) = c_{ij}(q, \dot{q}),$$
$$c_{ij}(q, \dot{q}) = \frac{1}{2} \sum_{k=1}^{m} \left(\frac{\partial m_{ij}}{\partial q_k} + \frac{\partial m_{ik}}{\partial q_j} - \frac{\partial m_{jk}}{\partial q_i} \right) \dot{q}_k. \tag{4}$$

Performing rotary control of the joints of the robot arm with a brushless DC motor and ignoring the transmission volume and loss of power, we can write the following contact equations

$$\tau = r\tau_m, \tag{5}$$

based on the torque characteristic equation - the speed of a brushless DC motor

$$\tau_m = \frac{K_t}{R} u - \frac{K_t^2}{R} \omega_m, \tag{6}$$

where, τ_m is the torque produced on the rotor shaft of the engine; K_t coefficient of torque; R internal resistance of the motor.

Equations (2) - (6) form the dynamic model of a robot driven by a brushless DC motor. With the voltage $u(t)$ placed on the windings of the motor that generates the rotation of the joints $q(t)$ and corresponds to that of the movement.

2.1.2. The dynamical model of two degrees of freedom serial robot

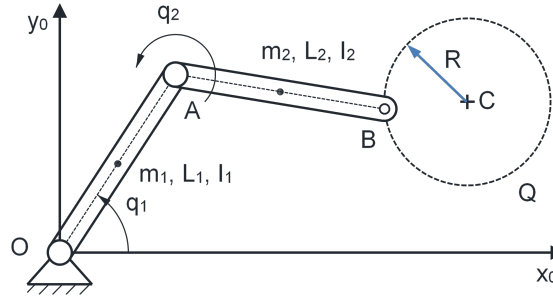Let us consider the 2-DOF serial robot arm driven by two electric motors as shown in Figure 1.

Figure 1. 2-DOF serial robot arm driven by a brushless DC motor

The dynamic model is derived by applying Lagrangian equation (2) shown in the form

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} g_{11} \\ g_{21} \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \tag{7}$$

or written in reduced form matrices as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = B\tau, \tag{8}$$

where the parameters of the system are:

Mass matrix includes elements

$$m_{11} = m_1 L_{c_1}^2 + m_2(L_1^2 + L_{c_2}^2 + 2L_1 L_{c_2} \cos(q_2)) + I_1 + I_2,$$
$$m_{12} = m_2 L_{c_2}^2 + L_1 L_{c_2} \cos(q_2) + I_2,$$
$$m_{21} = m_2 L_{c_2}^2 + L_1 L_{c_2} \cos(q_2) + I_2,$$
$$m_{22} = m_2 L_{c_2}^2 + I_2.$$

Centrifugal matrix and Coriolis include elements

$$c_{11} = -2m_2 L_1 L_{c_1} \dot{q}_2 \cos(q_2),$$
$$c_{12} = -m_2 L_1 L_{c_1} \dot{q}_2 \cos(q_2),$$
$$c_{21} = -\frac{1}{2} m_2 L_1 L_{c_2} (\dot{q}_2 - 2\dot{q}_1) \sin(q_2),$$
$$c_{22} = \frac{1}{2} m_2 L_1 L_{c_2} \dot{q}_1 \sin(q_2).$$

Generalized force vector due to gravity consists of elements

$$g_{11} = (m_1 L_{c_1} + m_2 L_1)g \cos(q_1) + m_2 L_{c_2} g \cos(q_1 + q_2),$$
$$g_{21} = m_2 L_{c_2} g \cos(q_1 + q_2).$$

The mechanical parameters of the robot are given as follows:

- The mass $m_1 = 0.2$ [kg], $m_2 = 0.2$ [kg].

- The inertia moment $I_1 = 0.01$ $[\text{kgm}^2]$, $I_2 = 0.01$ $[\text{kgm}^2]$.

- The length of links $L_1 = 0.65$ [m], $L_2 = 0.45$ [m].

- The center of mass for each link of the robot is determined based on the distance $L_{C_1} = 0.325$ [m], $L_{C_2} = 0.225$ [m].

The dynamics model of the robot is used to simulate control and test the proposed theory in the next section.

2.2. Overview of hedge algebras and approximate reasoning

2.2.1. Overview of hedge algebras

Hedge algebras is a new approach to linguistic elements. They are the linguistic values of a certain linguistic variable. There is a difference from the fuzzy set theory that is the expansion of the concept of the classic collection, hedge algebras is an algebraic structure on the set of linguistic terms. This structure is built on the inherent semantic order of the linguistic terms [22, 23]. For example, considering a linguistic value set as the domain of the linguistic domain of TRUTH's truth variable, there are the following words

$$\begin{aligned} T = dom(TRUTH) = \{ &VeryVeryFalse < MoreVeryFalse < VeryFalse < False \\ &< \ldots < LittleFalse < \ldots < Medium < VeryLittleTrue < MoreLittleTrue < \ldots \quad (9) \\ &< True < VeryTrue < VeryVeryTrue \ldots \} \end{aligned}$$

Assuming that *false <true* are two primitive elements that are opposite each other, acting on them by hedges such as *Very, Little, Rather, More* we will get ordered terms as in (9). It can be considered as the result of a unary operation of operators *Very, Little,* etc. which unlimitedly effects on the number of primitive elements. Then we have an algebraic structure on the specified domain of the linguistic variable defined as follows.

Definition 1. [22] Hedge algebras of the linguistic variable $\mathcal{X}$ is a set of 5 components $\mathcal{AX} = (X, G, C, H, \leq)$, in which

1. X the base set of $\mathcal{AX}$, including the word categories which are linguistic values in $\mathcal{X}$.

2. $G = \{c^-, c^+\}$, $c^- \leq c^+$ is called generating elements (primitive words, for example *false<true*).

3. $C = \{\mathbf{0}, \boldsymbol{W}, \mathbf{1}\}$ is a set of constants, with $\mathbf{0} \leq c^- \leq \boldsymbol{W} \leq c^+ \leq \mathbf{1}$, denotes elements with the smallest semantics, neutral elements, and the biggest semantic elements.

4. H is a set of unary operations, called hedges. $H = H^- \cup H^+$, with $H^- = \{h_j : -q \leq j \leq -1\}$ is a set of negative hedges, $H^+ = h_j : 1 \leq j \leq p$ are positive hedges.

5. $\leq$ presents the order relation on linguistic words (fuzzy concepts) on T, it is "induced" from the natural semantics of the language.

These components have the following:

1. Assume that hedges in H are ordered operators, that is $(\forall h \in H, h: X \rightarrow X)$, $(\forall u \in X)$ $\{hu \leq u$ or $hu \geq u\}$.

2. The two hedges $h, k \in H$ are opposite if $(\forall u \in X)$ $\{hu \leq u$ if and only if $ku \geq u\}$ and they are called compatible if $(\forall u \in X)$ $\{hu \leq u$ if and only if $ku \leq u\}$. The symbol $h \geq k$ if h, k is compatible and $(\forall u \in X)$ $\{hu \leq ku \leq u$ or $hu \geq ku \geq u\}$.

3. In addition, H can be partitioned into two sets H^+ and H^- with hedges in the set H^+ or H^- are compatible, each element in H^+ is opposite to any element in the H^- and vice versa.

4. A positive element h (or negative one) for a hedge k if $(\forall u \in X)$ $\{hku \leq ku \leq u$ or $hku \geq ku \geq u\}$ or $(\forall u \in X)$ $\{ku \leq hku \leq u$ or $ku \geq hku \geq u\}$.

5. X is generated from G by hedges in H. Thus each element of X will be a representation of $x = h_n h_{n-1} \ldots h_1 c$, $c \in G$.

6. The set of all elements generated from the element u has the representation $H(u) \bigcup x = H(hu) | h \in H$.

7. If two concepts u and v are independent, that is, $u \notin H(v)$ and $v \notin H(u)$, then $(\forall x \in H(u))$ $x \notin H(v)$. Moreover, if u and v are unequal, then any $x \in H(u)$ cannot match any $y \in H(v)$. ($H(u)$ is the set of values generated by the effect of hedges of H on u).

8. If $u \notin H(v)$ and $u \leq v$ (or $u \geq v$) then $u \leq hv$ (or $u \geq hv$) to every hedge h.

9. For u, $v \in X$, $u \leq v$ then $u \leq H(v)$, $H(u) \leq v \Rightarrow H(u) \leq H(v)$.

10. Suppose that there is the element V (implicitly Very) in the set H^+ and there is the element L (implicitly Little) in the set H^-, the generating element $c \in G$ is positive if $c \leq Vc$ (denoted as c^+) and negative if $c \geq Vc$ (denoted as c^-) (or $c \in G$ is positive if $c \geq Lc$ and negative if $c \leq Lc$).

11. If G has exactly 2 generating elements, then the first one is called a negative element, c^- and the other is called a positive element c^+, we have $c^- \leq c^+$ (In the above example, c^- corresponds to $false$ as negative, and c^+ corresponds to $true$ as positive and $false < true$).

Based on the properties of increase or decrease under the influence of hedges, we have the definition of the sign function recursively as follows.

Definition 2. [24, 25]. Sign function sgn: $X \rightarrow \{-1, 0, 1\}$ is a mapping defined recursively as follows, where k, $h \in H$, $c \in G$, $x \in X$:

1. $\text{sgn}(c^+) = +1$ and $\text{sgn}(c^-) = -1$.

2. $\{h \in H^+ | \text{sgn}(h) = +1\}$ and $\{h \in H^- | \text{sgn}(h) = -1\}$.

3. $\text{sgn}(hc^+) = +\text{sgn}(c^+)$ if $hc^+ \geq c^+$ or $\text{sgn}(hc^-) = +\text{sgn}(c^-)$ if $hc^- \leq c^-$ and $\text{sgn}(hc^+) = -\text{sgn}(c^+)$ if $hc^+ \leq c^+$ or $\text{sgn}(hc^-) = -\text{sgn}(c^-)$ if $hc^- \geq c^-$. Or $\text{sgn}(hc) = \text{sgn}(h)\text{sgn}(c)$.

4. $\text{sgn}(khx) = +\text{sgn}(hx)$ if k is positive to h ($\text{sgn}(k, h) = +1$) and $\text{sgn}(khx) = -\text{sgn}(hx)$ if k is negative to h ($\text{sgn}(k, h) = -1$).

5. $\text{sgn}(khx) = 0$ if $khx = hx$.

Clause 1 [26]. For $x \in X$, $x = h_n h_{n-1} \ldots h_1 c$, $h_j \in H$, $c \in G$, therehold

$$\text{sgn}(x) = \text{sgn}(h_n, h_{n-1}) \ldots \text{sgn}(h_2, h_1)\text{sgn}(h_1)\text{sgn}(c)$$
$$(\text{sgn}(hx) = +1) \Rightarrow (hx \geq x) \ and \ (\text{sgn}(hx) = -1) \Rightarrow (hx \leq x). \tag{10}$$

The sgn function is used to determine the direction of the impact of increasing or decreasing the semantic value of a hedge on a linguistic value.

The semantic order of the elements in hedge algebras is expressed in the following clause.

Clause 2 [24]. Suppose the hedge algebras $\mathcal{AX} = (X, G, C, H, \leq)$ with H^-, H^+ are sets of linear hedges. Then we have the following assertions:

1. Each $x \in X$, then $H(x)$ is a linear order.

2. If X is generated from G and G is a linearly ordered set, then X is also the linearly ordered one.

3. If $x \in X$ is a fixed element with $h \in H$, that is $hx = x$, then it will be a fixed element for $\forall k \in H, k \neq h$ ($hx = kx$). Furthermore, if $u < v$, and u, v are independent on each other, $u \notin H(v)$ and $v \notin H(u)$, then $H(u) \leq H(v)$.

Theorem 1 [22]. *If the set of hedges H^+ and H^- have a linear ordering order, there exists an isomorphism φ from $\mathcal{AX} = (X, G, C, H, -, \cup, \cap, \Rightarrow, \leq)$ into the multi-valued logical structure based on the range $[0, 1]$ and assure:*

1. *Preserve order relation.*

2. $\varphi(u \cup v) = \max\{\varphi(u), \varphi(u \cup v)\} = \min\{\varphi(u), \varphi(v)\}$.

3. $\varphi(u \Rightarrow v) = \max\{1 - \varphi(u), \varphi(v)\}$ *and* $\varphi(-u) = 1 - \varphi(u)$.

The above theorem allows us to set a Semantically Quantifying Mapping (SQM) function on hedge algebras to convert an element $x \in X$ into a semantic value in the real domain $[0,1]$.

Definition 3 [26]. Let $\mathcal{AX} = (X, G, C, H, \leq)$, a mapping $v: X \to [0, 1]$ is called a semantically quantifying mapping function on X if it satisfies:

1. v is one-to-one mapping from X into $[0, 1]$ and $v(0) = 0, v(1) = 1$, where $0, 1 \in C$.

2. It preserves the order on X, i.e. for all $x, y \in X, x < y$ then $v(x) < v(y)$.

The fuzziness of linguistic values comes from the fact that a linguistic value has a descriptive meaning for many things and phenomena in the real world because the finite set of linguistic values is not enough to reflect the infinite world of phenomena. Thus the concept of fuzziness and fuzziness measurement of a linguistic value is formed and it is a concept that is hard to define, especially in fuzzy set theory. However, in hedge algebras the authors have shown that the fuzzy measurement is reasonably determined: *"the fuzziness of a word term x is interpreted as its semantics can still be changed when acting on it by hedges"*. Therefore, the set of magnetic terms x by hedges will represent the fuzziness of x and $H(x)$ used as a model to represent the fuzziness of x. The set size $H(x)$ is considered as the fuzzy measure of $(fm(x))$ and is calculated recursively from the fuzzy measurement of the fm generating elements $fm(c^-)$, $fm(c^+)$ and the fuzzy measurement of hedges (h), $h \in H$.

Definition 4 [26]. Suppose the given hedge algebras $\mathcal{AX} = (X, G, C, H, \leq)$, $X = H(C)$. The function $fm: X \to [0, 1]$ is called the fuzziness measure function of the elements in X if

$$fm(c^-) + fm(c^+) = 1 \text{ and } \sum_{h \in H}\{fm(hx) = fm(x)\} \text{ with } \forall x \in X, \tag{11}$$

$$fm(x) = 0 \text{ with } \forall x, H(x) = \{x\}, fm(0) = fm(W) = fm(1) = 0, \tag{12}$$

$$\forall x, y \in X, h \in H, \frac{fm(hx)}{fm(x)} = \frac{fm(hy)}{fm(y)}, \tag{13}$$

this ratio (13) does not depend on x, y and it is characteristic for the fuzziness measurement of the element h, denoted by $\mu(h)$. Condition 1) means that the generating elements and hedges are sufficient to model the semantics of the real value domain of physical variables. The set of H and G is enough to cover the entire domain of the linguistic variable. Intuitively, conditions 2) and 3) showing the effect of $h \in H$ hedges on the fuzzy concepts are the same. Properties of $fm(x)$ and $\mu(h)$ are as follows.

Clause 3. Let fm be the function of fuzziness measurement on X. For $x \in X$, $x = h_n h_{n-1} \ldots h_1 c$, $h_j \in H$, $c \in G$, we have

$$fm(hx) = \mu(h)fm(x), \tag{14}$$

$$\sum_{-q < i < p, \ i \neq 0} fm(h_i c) = fm(c), \tag{15}$$

$$\sum_{-q < i < p, i \neq 0} fm(h_i x) = fm(x), \tag{16}$$

$$fm(x) = fm(h_n h_{n-1} \ldots h_1 c) = \mu(h_n)\mu(h_{n-1}) \ldots \mu(h_1)fm(c), \tag{17}$$

$$\sum_{i=-1}^{-q} \mu(h_i) = \alpha \ and \ \sum_{i=1}^{p} \mu(h_i) = \beta \ with \ \alpha, \beta > 0 \ and \ \alpha + \beta = 1. \tag{18}$$

When building models to simulate the fuzzy model given by LRBS, it is necessary to quantify the linguistic values appearing in LRBS to their semantic value and to ensure their inherent order. The reasoning process will be performed on these semantic values. With defined fuzzy parameter sets, semantic value is recursively determined by the function of Semantically Quantifying Mapping (SQM) $\boldsymbol{v}$ as follows.

Definition 5 [26]. Let $\mathcal{AX} = (X, G, C, H, \leq)$, and fm be a fuzziness measure on $\mathcal{X}$. Then, SQM function $v: \mathcal{X} \to [0, \ 1]$ is said to be induced by fm, if it is defined recursively as follows

$$v(W) = \theta = fm(c^-), \tag{19}$$

$$v(c^-) = \theta - \alpha fm(c^-) = \beta fm(c^-), \tag{20}$$

$$v(c^+) = \theta + \alpha fm(c^+) = 1 - \beta fm(c^+), \tag{21}$$

$$v(h_j x) = v(x) + \text{sgn}(h_j x)\Big\{ \sum_{i=\text{sgn}(j)}^{j} fm(h_i x) - \omega(h_j x)fm(h_j x)\Big\}, \tag{22}$$

with $\omega(h_j x) = \dfrac{1}{2}[1 + \text{sgn}(h_p, h_j)(\beta - \alpha)], j \in [-q \wedge p] = [-q, p] \ \{0\}$.

SQMs function can map directly from linguistic value to quantitative semantic value. This is an important tool to build a calculation model, solve approximate reasoning problems for models called by LRBS. The logical correctness of the calculation model on semantic value is that it always ensures the semantic order of language. The types of problems are stated by typical LRBS such as fuzzy association rules; summarize the database in the form of linguistic rules; clustering and fuzzy classification problems; fuzzy database; regression problems, fuzzy time series prediction, etc. In this paper, we build the inference set for LRBS to work as a fuzzy controller.

2.2.2.   The approximate inference based on hedge algebras approach

Consider the fuzzy model given in the form of an LRBS [27]

$$
\begin{aligned}
&\text{If } \mathcal{X}_1 = A_{11} \text{ and } \ldots \text{and } \mathcal{X}_m = A_{m1} \ \text{ then } \ \mathcal{Y} = B_1 \\
&\text{If } \mathcal{X}_1 = A_{12} \text{ and } \ldots \text{and } \mathcal{X}_m = A_{m2} \ \text{ then } \ \mathcal{Y} = B_2 \\
&\ldots \\
&\text{If } \mathcal{X}_1 = A_{1n} \text{ and } \ldots \text{and } \mathcal{X}_m = A_{mn} \ \text{ then } \ \mathcal{Y} = B_p
\end{aligned} \tag{23}
$$

$\mathcal{X}_1$, $\mathcal{X}_2$, ..., $\mathcal{X}_m$ and $\mathcal{Y}$ are linguistic variables; $\mathcal{X}_i, \mathcal{Y}$ belong to the background space U_i, V respectively; A_{ij}, B_k ($i = 1 .. m$, $j = 1..n$, $k = 1..p$) are linguistic values of the corresponding background space.

This rule system has m input components and one output component. According to the hedge algebras approach, we can use the SQMs function to compute the quantitative semantics value of the linguistic terms that appear in the rule system. Then, each rule "If ... then ..." corresponds to a point in the semantic space $[0,1]^{(m+1)}$. The entire rule system will correspond to a "super face" in the semantic space, denoted by $S^{(m+1)}$, called the Quantified Rule Base System (QRBS), respectively with LRBS. Figure 2 depicts the structure of the approximate set of deductions based on the hedge algebras approach.

Suppose that real inputs $(x_{01}, x_{02}, \ldots, x_{0m})$ belong to the corresponding background spaces, normalization will standardize these values to the semantic domain respectively $x_{01s}, x_{02s}, \ldots, x_{0ms})$. We solve the approximate reasoning problem by interpolation method IRMd on S^{m+1}. The interpolation value obtained is the semantically re-interpreted value by denormalization to the real value domain of the output variable y.
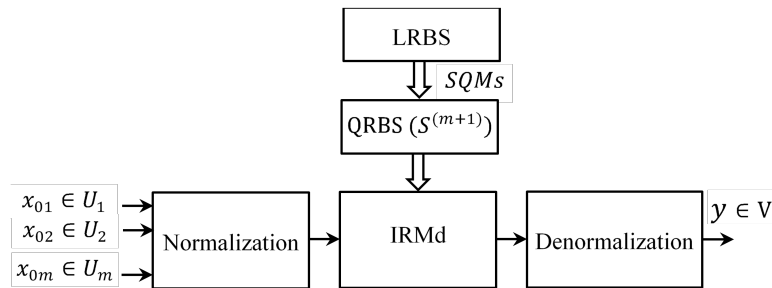

Figure 2. The approximate reasoning set based on hedge algebras approach

As using the approximate reasoning set to work as a controller, we call that the hedge algebras controller (HAC - Hedge Algebra Controller). The design of the controller based on hedge algebras is carried out by the following steps.

Step 1. Determine the input-output variables, their variation domain, and the control rule system with the linguistic elements in hedge algebras. The basic structure of the controller will be determined in this step.

- The input variables of the controller are usually the tracking error, the variable speed of the tracking error, and the component of the tracking error. Depending on the specific objects, the controller may use only 2 or 3 of the above input components. The controller's output is the quantity to control the object directly or indirectly. It is directly related to the output state of the object. The determination of their variable domain is usually done through simulation or empirical measurements. These variable domains can also be adjusted during the optimization process of the controller.

- The control rule system is the most important component of the controller. It describes the qualitative variable relationship between the output quantity and the inputs on the linguistic term. This relationship is often nonlinear and monotonous. In order to be able to calculate the semantic value of linguistic terms, they must have a suitable representation form in hedge algebras, which is an of hedges acting on a generating element.

Step 2. Select the structure of hedge algebras $\mathcal{AX}_i$ ($i = 1, .., m$) and $\mathcal{AY}$ for the variables $\mathcal{X}_i$ and $\mathcal{Y}$. Determine the fuzzy parameters of the generating elements, the hedges, and the

sign relationship between hedges.

   - The structure of hedge algebras should be chosen so that linguistic elements can be generated in accordance with the ones in the rule system. Thus, the modeling of the linguistic rule set is correct.

   - In this step, we can also choose fuzzy parameters for hedges and generating elements. This selection is also qualitative and depends much on the designer. Basing on the specific system, the parameter determination may be manual or optimal algorithms. For complex systems with large simulation time, it is difficult to optimize the parameters with optimal algorithms such as GA, PSO, etc. because the time for calculating the objective function is too large. In this case, parameter correction is usually done manually by the method of trial and error.

Step 3. Calculate the quantitative semantic value for linguistic terms in the rule system. Build the input or output surface relationship $S^{(m+1)}$. Once there are fuzzy parameters, we can calculate the semantic value of the linguistic terms in the rule system and determine the rule system in real space. This step is to convert the rule system from the super face in the linguistic space to super-surface in real space.

Step 4. Select the interpolation method. There are several interpolation methods that can be selected for this step. The selection of a certain interpolation method needs to meet two important criteria. The first thing is to ensure the monotony of the rule system even when interpolating or extrapolating. The second is simple, in terms of computational complexity to be able to meet the real-time computation in control.

Step 5. Optimize the parameters of the controller. For non-complex systems, the computation and simulation time is small, optimizing the fuzzy parameters with optimal algorithms is an effective solution. Therefore, it is necessary to prioritize this option. The optimal parameter is definitely better when manually adjusting. The optimal algorithms are often used by scientists recently to show their superiority such as GA or PSO.

2.3. The controller based on hedge algebra

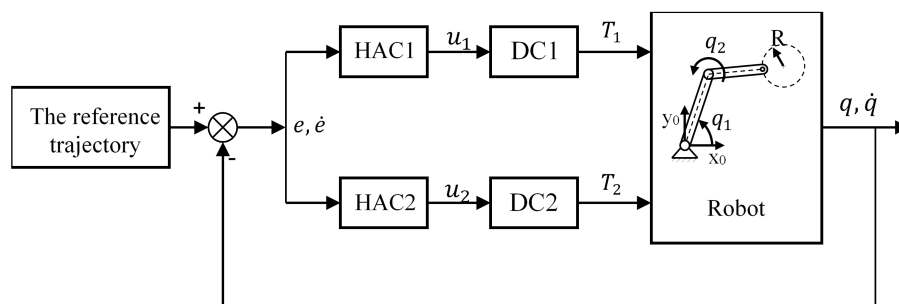### 2.3.1. Structure diagram of the controller in the system

Figure 3. Diagram of 2-DOF serial robot arm controlled by HAC

   Figure 3 shows the HAC for the 2-DOF serial robot described in Figure 1 with the parameters presented in Subsection 2.1.2.

- Each joint of the robot is driven by a motor, which is DC1 and DC2. These motors are controlled by HAC1 and HAC2.

- The reference path block is responsible for calculating inverse kinematic equations to create an instantaneous coordinate to supply to the robot controller. The inverse kinematic problem of 2-DOF are shown

$$\begin{cases} q_1 = \text{atan}\left(\dfrac{y_B}{x_B}\right) + \text{acos}\dfrac{L_1^2 + x_B^2 + y_B^2 - L_2^2}{2L_1\sqrt{x_B^2 + y_B^2}} \, , \\ q_2 = -\pi + \text{acos}\dfrac{L_1^2 + L_2^2 - x_B^2 - y_B^2}{2L_1L_2} \, . \end{cases} \tag{24}$$

The velocity of joints is determined based on the time derivative of the kinematic equations

$$\text{J}_q\dot{q} + \text{J}_x\dot{x} = 0. \tag{25}$$

Then, we can derive a formula that expresses the relation of the joint velocity

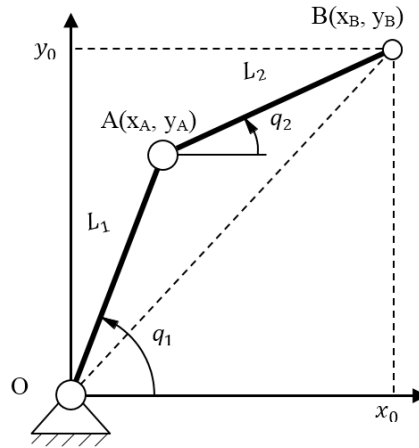$$\dot{q} = -\text{J}_q^{-1}\text{J}_x\dot{x} \tag{26}$$

Figure 4. Kinematic diagram of the 2-DOF serial robot

- The HACs receive input variables including position and velocity error $e, \dot{e}$ to calculate the voltage values u_1 and u_2 used to control the motor DC1 and DC2. T_1, T_2 are the torque values generated by the motor for the robot to work, this is also the input for the inverse dynamic problem in the Robot block.

- Output variables $q, \dot{q}$ are information feedbacked to the input to determine the control errors $e, \dot{e}$.

In order to satisfy optimal control, we need to optimize each HAC with independent parameters. However, for simplicity in this study, we design and optimize both HACs similarly.

2.3.2. Controller designing

HAC has two input variables and one control variable at the output:

1. e (*error*) - control error, which is the difference between the desired instantaneous position of the reference trajectory and the actual position. The variation domain determined through simulation is ranging within the interval [-0.1, 0.1].

2. ce (*change error*) - indicates the variable speed of e. By means of simulations, it is possible to establish that the domain of variation is in the range [-2, 2].

3. The HAC output is u control quantity to control the voltage of the source, varying in the range [-48, 48].

Input-output linguistic variables include the following linguistic values

$$e,\ ce = \{VN<LN<ZE<LP<VP\},$$
$$u = \{VN<N<LN<ZE<LP<P<VP\},$$

where, $N=Negative,\ P=Positive,\ VN=Very\ Negative,\ LN=Little\ Negative,$

$$ZE=Zero,\ LP=Little\ Positive,\ VP=Very\ Positive.$$

The rule of control considering as a LRBS is given in Table1.

Table 1. Rule control system - LRBS

e ce	VN	LN	ZE	LP	VP
VN	VN	VN	N	LN	ZE
LN	VN	N	LN	ZE	LP
ZE	N	LN	ZE	LP	P
LP	LN	ZE	LP	P	VP

Choose the structure for hedge algebras $\mathcal{AX}_{e,ce}$, for the variables e, ce and $\mathcal{AY}$ for u with the following components:

1. Set of generating elements $G=\{N<P\}$.

2. Set of hedges $H^-=\{L\}$ and $H^+=\{V\}$.

3. The fuzzy parameter of the hedge algebras for variables e, ce and u includes the measurement of the fuzziness of the generating elements, the measurement of the fuzziness of the hedges. Based on the hedge algebras structure for the variables built as above, we need to select the fuzzy measurement of the negative generation elements $fm(c^-)=fm(N)$ $(fm(c^+)=1-fm(c^-)=fm(P)=1-fm(N))$ and the fuzzy measurement of negative hedge $\alpha=\mu(L)$ $(\beta=\mu(V)=1-\alpha)$. The fuzzy parameters to be searched by the transmission algorithm are presented in the following section.

4. The sign of the generating elements, hedges, and the sign relationship between the hedges is determined based on the semantic nature of the terms of the language. For example, we have $\text{sgn}(N) = -1$, $\text{sgn}(P) = 1$. In addition, it can be seen that $VVN{<}VN \Rightarrow \text{sgn}(V,V) = 1$. $LVN{>}VN \Rightarrow \text{sgn}(L,V) = -1$. Similar to the other linguistic elements, we determine the sign relationship as shown in Table 2.

Table 2. The sign relationship of hedges and generating elements

	V	L	N	P
V	+	+	−	+
L	−	−	+	−

Interpolation based on the Semantic Distance Weighting method (ISDMd) is selected as in [28]. This is a linear interpolation method that allows interpolation in n-dimensional space. It ensures the systematic monotony in case of interpolation/extrapolation and needs a small number of calculations that meets the requirements of real-time control of the system.

3. OPTIMIZATION OF PARAMETERS BY GENETIC SOLUTION

The genetic algorithm is a method of optimal random search that follows the evolution and selection of biological populations in nature [29]. Operations in algorithm-based evolution include crossover, mutation, and selection. Each individual is expressed simply as a chromosome consisting of multiple gene segments. Each gene segment is encoded for an optimal parameter. Then each individual is a solution of the problem with an optimal set of parameters. After an evolutionary process (repetition) is large enough, individuals will gradually adapt to the adaptive conditions evaluated by the fitness function.

The fuzziness parameters need to be optimized for the controller.

For fuzzy controllers, the variable domain of the input and output is usually symmetric. When expressing the value on this domain in language, the rank of Zero (ZE) has semantics that refers to the real value of 0. This is the element corresponding to the neutral element W of hedge algebra, $v(W) = \theta$. When mapping all linguistic values to the semantic domain in the range [0, 1], semantic value $(ZE) = 0.5$. Therefore, according to (19), we choose fixed variables of values $fm(N) = v(ZE) = 0.5$.

The set of hedges in hedge algebras is built with only 2 hedges, *V (Very)* and *L (Little)*, so we only need to optimize $\mu(L) = \alpha$ (because of $\mu(V) = \beta = 1 - \alpha$). So we only need to optimize 3 parameters α_e, α_{ce} and α_u corresponding to 3 variables e, ce, and u.

The objective function is chosen according to the absolute value integration standard of the tracking error (IAE)

$$fitness = \sum_{k=1}^{n} |e(k)| \to \min \qquad (27)$$

where, $e(k)$ is the sample of deviation data at the simulation cycle k, n is the total number of data samples of a simulation run.

In Matlab, GA is an available tool that allows us to easily use it. In this study, we use the GA() function in Matlab with gene encoding using real numbers of type double. The values set for GA include:

- Population size: Population Size = 100;

- The number of Generation: Generation = 4*Population Size;

- Maximum limit for searching time: Time Limit = 86400, (24 hours);

- The target function is used as in formula (27).

Results are the optimal parameters set for the controller as shown in Table 3.

Table 3. The optimal parameters of HAC based on GA

$\mathcal{AX}_e: \ \alpha_e = \mu(L_e)$	$\mathcal{AX}_{ce}: \ \alpha_{ce} = \mu(L_{ce})$	$\mathcal{AY}: \ \alpha_u = \mu(L_u)$
0.520936	0.508062	0.499074
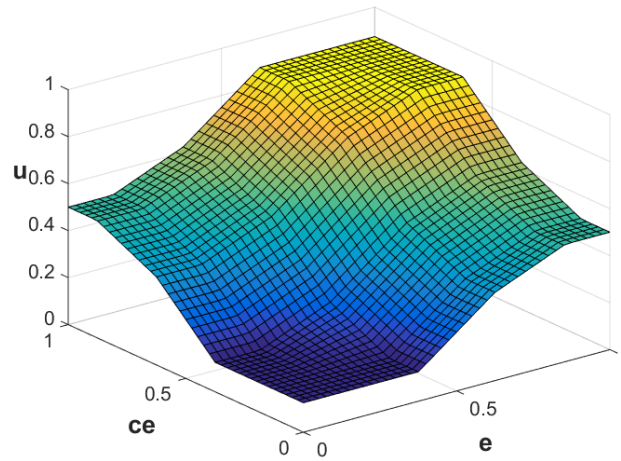
Figure 5. Surface of input/output relationship S^3 of optimal HAC

With optimal fuzziness parameters found in Table 3, the SQMs function (19) - (22) is used to calculate the semantic value of the elements in LRBS in Table 1, we obtain QRBS and the surface S^3 of HAC correspondingly. Figure 5 is an input-output relationship of the rule based system corresponding to the optimal QRBS.

4.   SIMULATION RESULTS AND DISCUSSION

The diagram simulating the control system of 2-DOF serial robot with HAC on the Matlab-Simulink environment as shown in Figure 6. In the simulation diagram, we use 2 sets of HAC with common parameter sets to control for 2 motors with the same parameters. The control program code is written as Level-2 M-file S-Function. Blocks DC1 and DC2 are mathematical representations for brushless DC motors. These selected motor are the same. The "Robot" block calculates the inverse dynamics problem. "Trajectory" is a block that generates reference data for moving robots. The "forward_kinematic" block performs forward kinematic computation to convert data from joint space to workspace.

Some simulations with a 2-DOF serial robot arm which moves in the horizontal plane driven by 2 actuators are implemented. The numerical simulations are carried out in Simulink. These trjectories include circle, spriral and square shapes. The results are shown in Figure 7 to Figure 12.
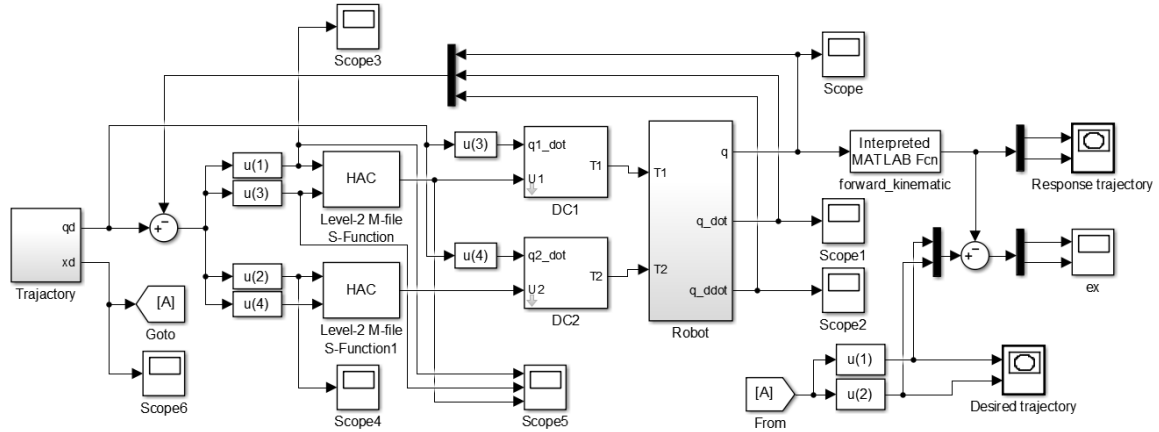

Figure 6. Diagram of 2-DOF serial robot arm simulation to HAC

Circle trajectory

In this simulation, the center of the end effector will be moved along a circular trajectory, The trajectory has a center at $(xC; yC) = (0.45; -0.45)$ [m] and radius $R = 0.2$ [m]. This trajectory is chosen to be in the task space of the robot. The starting point to move is (0.6022; -0.5127). Simulation results are shown in Figure 7 and Figure 8.
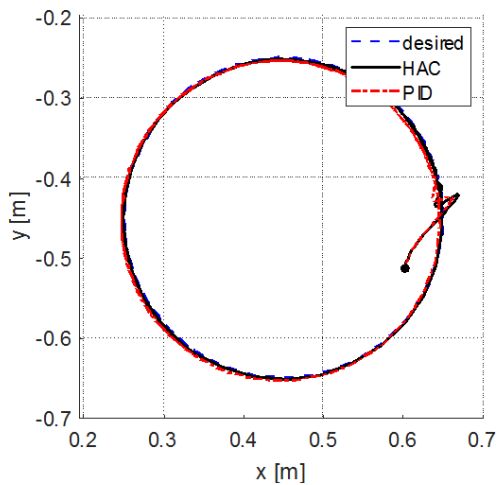
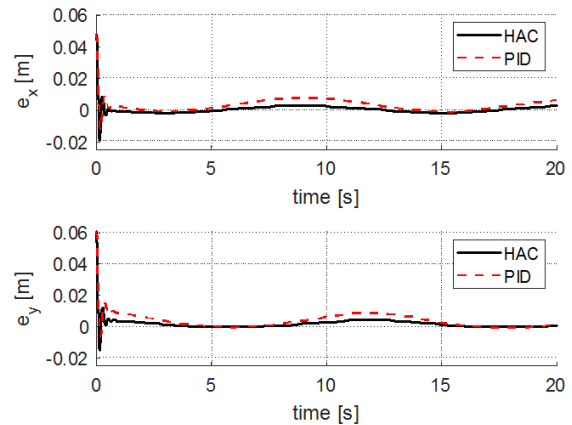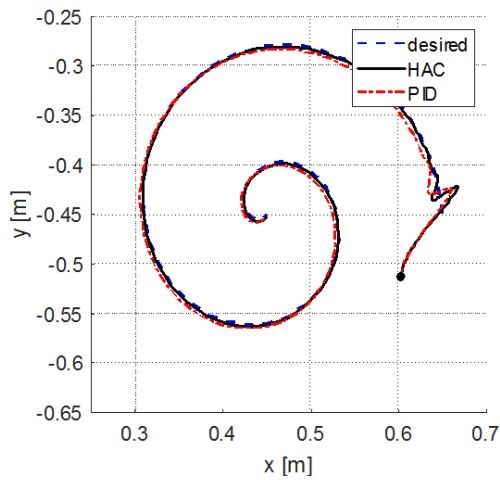*Figure 7.* The desired path vs. the actual path of the end effector with the circle path

Figure 8. Tracking errors of x, y coordinates with the circle path

Spiral trajectory

In this simulation, the center of the end effector will be moved along a spiral trajectory, The trajectory has a center at $(xC; yC) = (0.45; -0.45)$ [m] and the boundary radius $R = 0.2$ [m]. This trajectory is chosen to be in the task space of the robot. The starting point to

move (0.6022; -0.5127). Simulatinon results are shown in Figure 9 and Figure 10.

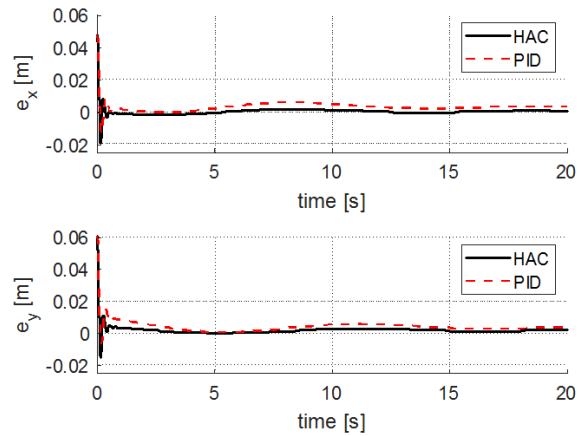*Figure 9.* The desired path vs. the actual path of the end effector with the spiral path

Figure 10. Tracking errors of x, y coordinates with the spiral path

Square trajectory

In this simulation, the center of the end effector will be moved along a square trajectory with the following coordinates

$$A(0.65, -0.45); \ B(0.45, -0.25); \ C(0.25, -0.45); \ D(0.45, -0.65) \tag{28}$$

is at the starting point to move (0.6022; -0.5127). Simulation results are shown in Figure 11 and Figure 12.
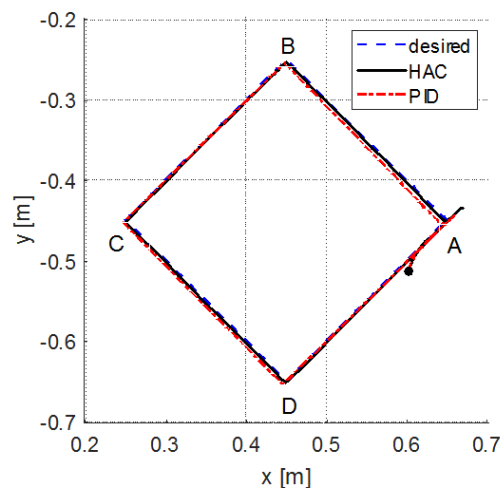

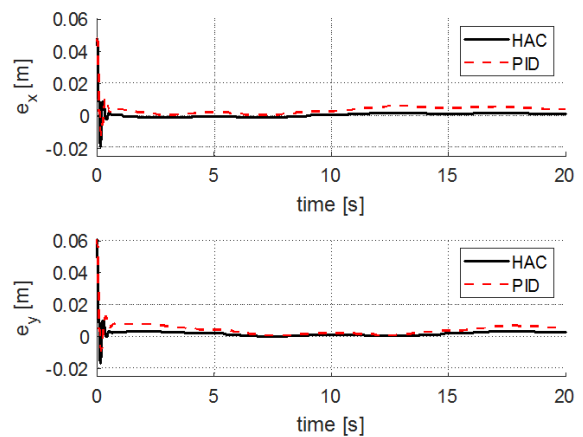*Figure 11.* The desired path vs. the actual path of the end effector with the square path

Figure 12. Tracking errors of x, y coordinates with the square path

Simulation with noise

In this simulation, the center of the end effector will be moved along a spiral trajectory with the effection of noise. The result is shown in Figure 13.
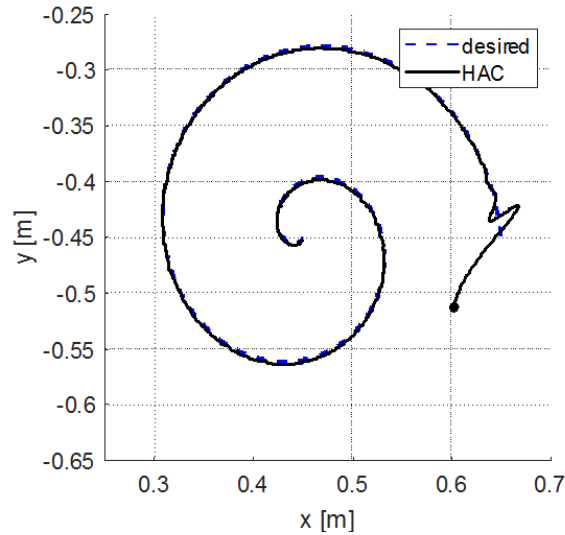

Figure 13. The desired path vs. the actual path of the end effector moves along the spiral path with noise

It can be seen in Figure 7, the end effector of the robot moves from the initial position at the coordinate of (0.6022; -0.5127). The control trajectory of HAC is closer to the reference trajectory than the control trajectory by PID-Controller. Figure 8 shows the tracking error in the x and y directions by time is rapidly decreasing to the value which is close to 0 in the range of 0.5 [s] with one small overshoot/undershoot. Still at the original position at (0.6022; -0.5127), perform simulations with more complex trajectories such as Spiral Trajectory (Figure 9, Figure 10) and Square Trajectory (Figure 11, Figure 12). The control trajectory of HAC always gives deviations and response time is better than the control trajectory of the PID-controller. Simulated with the case of white noise impacting the angles q_1 and q_2 at the output of the robot with the spiral reference trajectory, the control trajectory of the HAC has a small fluctuation but does not lose control and still close to the trajectory reference (see Figure 13). The results show the efficiency of the hedge algebras controller optimized by GA with tracking control 2-DOF serial robot arm.

5.   CONCLUSION

In this study, we designed HAC for the 2-DOF serial robot arm. The controller has a fairly simple structure with a rule control system consisting of only 25 rules. The structure of hedge algebras for the input-output variables has only two hedges, including a negative hedge - *Little* and a positive hedge - *Very*. The controller has only 3 parameters that measure the fuzziness of the hedge. The application of available GA in Matlab to optimize these parameters is very effective. With the optimal set of parameters, through the simulation, it was found that HAC worked very well.
This result shows that the potential of HAC application in robot control is very wide and

promising. In the near future, we will expand the research on HAC applications for more complex robots such as robots with 3 degrees of freedom or higher, parallel robots, etc. Reference trajectories will be set up with many complex forms, with major turning points. Parameter optimization will also be performed in parallel and separately for each controller.

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