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NOVEL APPROACH OF ROBUST H_∞ TRACKING CONTROL FOR UNCERTAIN FUZZY DESCRIPTOR SYSTEMS USING FIXED LYAPUNOV FUNCTION

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Abstract. This paper proposes a novel uncertain fuzzy descriptor system which is an extension from standard T-S fuzzy system. A fixed Lyapunov function-based approach is considered and controller design for this rich class of fuzzy descriptor systems is formulated as a problem of solving a set of LMIs. The design conditions for the descriptor fuzzy system are more complicated than the standard state-space-based systems. However, the descriptor fuzzy system-based approach has the advantage of possessing fewer number of matrix inequality conditions for certain special cases. Hence, it is suitable for complex systems represented in descriptor form which is often observed in highly nonlinear mechanical systems.

Keywords. Descriptor fuzzy system; Lyapunov function; Uncertain nonlinear mechanical systems; Robust H_∞ tracking control; LMI matrix inequality.

1. INTRODUCTION

Nowadays fuzzy logic-based control has proven to be a successful approach for controlling uncertain nonlinear systems [1, 2, 3, 4, 5]. The fuzzy-model proposed by Takagi and Sugeno [6], known as the T-S fuzzy model, is becoming a popular type of fuzzy model representation. Up to now there have been numerous successful applications of the T-S fuzzy model-based approach in uncertain nonlinear control systems. Linear matrix inequality (LMI)-based T-S fuzzy control is an important and successful approach used in uncertain nonlinear control. Up to now adequate studies are available that discusses linear matrix inequality (LMI)-based T-S fuzzy control system design using the fixed Lyapunov function [7, 8, 9, 10]. Although LMI-based approach gained popularity and great success, conservatism is still dominant in fixed quadratic Lyapunov function-based approach due to the limited choice of Lyapunov function [11].

In the robust control approaches discussed in [12], a T-S fuzzy model is employed, where its consequent parts are described via linear state-space systems. The description system improved from a standard state-space form successfully describes a wider class of systems and then can be used in certain mechanical and electrical systems. Then the T-S fuzzy model will be a special case of the descriptor fuzzy model. The advantage of choosing the descriptor representation over the state-space model is that the amount of LMI inequalities for designing the controller can be reduced for certain problems [13]. Compared with the standard state-space based system representation, descriptor representation holds more complicated structure and hence the controller design is also more complex [14].

Up to now, considerable work has been done involving stability control, H stabilization and model following control for fuzzy descriptor systems [13]. The necessity for such control techniques is principally improved via the increasingly experimental interest for a generalized system descriptor taking the intrinsically physical structure into consideration. Furthermore, the conventional state-space system problem can be considered as a special case of descriptor systems and then is able to be efficiently resolved by applying descriptive system computational methods [15].

Recently, numerous results obtained for robust H_∞ stabilization with parametric Lyapunov function have been presented in reviewing the results from literature for fixed Lyapunov function based on robust H_∞ stabilization for fuzzy descriptor systems [16, 17, 18, 19]. Zhi et al. (2018) in [16] proposed a new robust H_∞ control for T-S fuzzy descriptor systems with state and input time-varying delays. Xue et al. in [17] introduced a robust sliding mode control for T-S fuzzy descriptor systems via quantized state feedback. Ge et al. in [18] (2019) proposed a robust H_∞ stabilization for T-S fuzzy descriptor systems with time-varying delays and memory sampled-data control. Nasiri et al. in [19] introduced a new method for reducing conservatism in an H_∞ robust state-feedback control design of T-S fuzzy descriptor systems.

A model following control is considered in [13] and observer using H tracking control problem is introduced in [14]. For a state feedback H_∞ tracking control problem, this proposed approach yielded the conditions in terms of bilinear matrix inequalities (BMI) usually resolved by a two-step process. Based on this approach, the sufficient condition for implementing a state-feedback controller cannot be framed as LMIs.

Based on results abovementioned, this paper innovatively proposes an LMI formulation with respect to design conditions using fixed Lyapunov function for a model reference trajectory tracking problem responding to H_∞ performance criteria. Next these results are combined with the concepts presented in [15] and parametric Lyapunov function-based design for controlling using uncertain descriptor fuzzy systems is proposed here.

The rest of this paper is structured as follows. Section 2 introduces the T-S fuzzy descriptor system and constant Lyapunov function-based stability conditions. Section 3 presents the performance of H trajectory tracking control for the T-S fuzzy descriptor system. Section 4 proposes the novel T-S fuzzy descriptor for uncertain nonlinear system. Section 5 presents and analyses the simulation of proposed robust H_∞ tracking control implementation with fixed Lyapunov function using T-S fuzzy descriptor system. Finally, Section 6 includes the conclusions.

2. PROPOSED T-S FUZZY DESCRIPTOR SYSTEM

This paper starts with introduction to T-S fuzzy model and then H tracking control problem is formulated. The T-S fuzzy model initially introduced by Takagi and Sugeno [6] describes the dynamics of an uncertain nonlinear plant based on fuzzy IF-THEN laws. Let us investigate the descriptor fuzzy model of a nonlinear system in the form as follows.

Plant law $k-i$: IF $z_1^e(t)$ is $N_{k_1}^e$,..., $z_{p^k}^e(t)$ is N_{kpk}^e and $z(t)$ is N_{i1},...,$z_p(t)$ is N_{ip} THEN

$$\begin{aligned} E_k\dot{x}(t) &= A_i x(t) + B_i u(t), \\ y(t) &= C_i x(t),\ i=1,2,...,r,\ k=1,2,...r_e, \end{aligned} \tag{1}$$

where $z_1(t),...,z_p(t)$ represent premise variables, p represents the amount of premise variables, N_{kj}^e $(j=1...p^k)$, N_{ij} $(j=1...p)$ are the fuzzy sets and r represents the number of laws. Furthermore, $x(t)\in R^{n\times 1}$ represents the state vector, $y(t)\in R^{n_y\times 1}$ represents the controlled output and $u(t)\in R^{m\times 1}$ is the input vector. $A_i\in R^{n\times n}$, $B_i\in R^{n\times m}$, $C_i\in R^{n_y\times n}$, $E_k\in R^{n\times n}$ are constant real matrices. The necessary assumptions are that rank$(E)\le n$, $\triangle A_i\in R^{n\times n}$ represent the uncertainties and are bounded, i.e., $\|\triangle A_i\| < \delta_i$, where $\|.\|$ denotes spectral norm and δi represents positive value. Other specific constraints can be consulted in [14]. From input $x(t)$ and output $u(t)$, the eventual output of the fuzzy descriptor system is determined as follows

$$\sum_{k=1}^{r_e}\mu_k^e(z(t))E_k\dot{x}(t) = \sum_{i=1}^{r}\mu_i(z(t))\{A_i x(t) + B_i u(t) + D_i w(t)\},$$

$$y(t) = \sum_{k=1}^{r}\mu_i(z(t))C_i x(t), \tag{2}$$

where

$$\mu_i(z(t)) = \frac{\zeta_i(z(t))}{\sum_{j=1}^{r}\zeta_j(z(t))},\quad \zeta_i(z(t)) = \prod_{j=1}^{p}N_{ij}(z_j(t)),$$

$$\mu_k^e(z(t)) = \frac{\zeta_k^e(z^e(t))}{\sum_{j=1}^{r^e}\zeta_j^e(z^e(t))},\quad \zeta_k^e(z^e(t)) = \prod_{j=1}^{p^e}N_{kj}^e(z_j^e(t)),$$

and $N_{ij}(z_j(t))$, $N_{kj}^e(z_j^e(t))$ are the degrees of membership of $z_j(t)$ and $z_j^e(t)$ in the fuzzy set N_{ij} and N_{kj}^e, respectively. Here $\sum_{i=1}^{r}\mu_i(z(t))=1$ and $\sum_{k=1}^{r}\mu_k(z(t))=1$. We investigate a referential model described as [20]

$$\dot{x}_r(t) = A_r x_r(t) + D_r r(t), \tag{3}$$

with $x_r(t)$ represents the reference state, A_r represents specific asymptotically stable matrix, $r(t)$ represents a bounded referential input.

The trajectorial tracking error is defined as

$$e(t) = x(t) - x_r(t). \tag{4}$$

We investigate the H_∞ tracking performance with respect to the tracking error $e(t)$ as [21]

$$\int_0^{t_f} e^T(t)Qe(t)dt \le \rho^2\int_0^{t_f}\omega^T(t)\omega(t)dt, \tag{5}$$

where Q represents a positive definite weight matrix, t_f represents the finished time of control and ρ represents the preset disturbance alleviation level.

Let us consider the Parallel Distributed Compensation (PDC) provided from fuzzy controller [12] as

$$u(t) = \sum_{i=1}^{r}\sum_{k=1}^{r_e} \mu_i\mu_k(K_{1jk}e(t) + K_{2jk}x_r(t)), \tag{6}$$

where K_{1jk}, K_{2jk} are the controller gains. Then the proposed fuzzy controller is to be designed with the feedback gains K_{1jk}, K_{2jk} $(j = 1, ..., r,\ k = 1, ..., r^e)$ such that the resulting closed-loop fuzzy system is asymptotically stable and also satisfies the H performance criterion given in (5).

Combining (2) and (3), the enhanced fuzzy system is to be described as

$$E^*\dot{x}^*(t) = \sum_{i=1}^{r}\sum_{k=1}^{r_e} \mu_i\mu_k(A^*_{ik}x^*(t) + B^*_i u(t) + D^*_i\omega^*(t)), \tag{7}$$

where,

$$x^*(t) = \begin{bmatrix} e(t) \\ x_r(t) \\ \dot{e}(t) \end{bmatrix}, \quad \omega^*(t) = \begin{bmatrix} \omega(t) \\ r(t) \end{bmatrix}, \quad E^* = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$A^*_{ik} = \begin{bmatrix} 0 & 0 & I \\ 0 & A_r & 0 \\ A_i & (A_i - E_kA_r) & -E_k \end{bmatrix}, \quad B^*_i = \begin{bmatrix} 0 \\ 0 \\ B_i \end{bmatrix}, \quad D^*_i = \begin{bmatrix} 0 & 0 \\ 0 & D_r \\ D_i & -E_kD_r \end{bmatrix}.$$

3. H_∞ TRAJECTORY TRACKING CONTROL

For the enhanced fuzzy system proposed in (7), the performance of the H_∞ trajectory tracking control is demonstrated in the following theorem.

Theorem 1. *Let us investigate the fuzzy descriptor system (2) with respect to the control rule (6). In case it obtains the matrices X_{11}, X_{21}, X_{22}, X_{31}, X_{32}, X_{33} and W_{1jk}, W_{2jk} $(j = 1, ..., r,\ k = 1, ..., r^e)$ in order to satisfy the following matrix inequalities*

$$S = S^T > 0, \tag{8}$$

$$\phi_{iik} < 0,\ i = 1, 2, ..., r,\ k = 1, 2, ..., r^e, \tag{9}$$

$$\frac{1}{r-1}\phi_{ijk} + \frac{1}{2}(\phi_{ijk} + \phi_{jik}) < 0,\ i \neq j \leq r,\ k = 1, 2, ..., r^e, \tag{10}$$

with

$$S = \begin{bmatrix} X_{11} & X_{21}^T \\ X_{21} & X_{22} \end{bmatrix},$$

$$\phi_{ijk} = \begin{bmatrix} H^{11} & * & * & * & * & * \\ H^{21} & H^{22} & * & * & * & * \\ H^{31}_{ijk} & H^{32}_{ijk} & H^{33}_k & * & * & * \\ 0 & 0 & D_i^T & -\rho^2 I & * & * \\ 0 & D_r^T & -D_r^TE_k^T & 0 & -\rho^2 I & * \\ X_{11} & X_{21}^T & 0 & 0 & 0 & -Q^{-1} \end{bmatrix},$$

$$\begin{aligned} H^{11} &= X_{31}^T + X_{31}, \quad H^{21} = X_{31}^T + A_r X_{21}, \\ H^{22} &= A_r X_{22} + X_{22}^T A_r, \quad H_{ijk}^{31} = X_{33}^T + A_i, \\ H_{ijk}^{32} &= A_i X_{21}^T + (A_i - E_k A_r) X_{22} - E_k X_{32} + B_i W_{1jk}, \\ H_k^{33} &= -X_{33}^T E_k^T - E_k^T X_{33}. \end{aligned}$$

Here and after the symbols '' in matrices denote the transposed elements in symmetric positions.*

Then the closed loop system with the controller gain matrices $[K_{1jk}, K_{2jk}] = [W_{1jk}, W_{2jk}] \times [X_{11}, X_{21}^T; X_{21}, X_{22}]^{-1}$ *satisfy the given H performance criteria.*

Proof. Let us consider a candidate of Lyapunov function

$$V(t) = x^{*T}(T) E^{*T} X^{-1} x^*(t), \tag{11}$$

with $X = \begin{bmatrix} X_{11} & X_{21}^T & 0 \\ X_{21} & X_{22} & 0 \\ X_{31} & X_{32} & X_{33} \end{bmatrix}$ and $E^{*T} X^{-1} = X^{-T} E^* \geq 0$.

If the inequalities in (9) and (10) are satisfied then

$$\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r_e} \mu_i \mu_j \mu_k \phi_{ijk} < 0. \tag{12}$$

The above inequality can be written as

$$\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r_e} \mu_i \mu_j \mu_k \begin{bmatrix} X^T \Omega_{ijk} X + X^T Q^* X & * \\ D_i^{*T} & -\rho^2 I \end{bmatrix} < 0, \tag{13}$$

with $\Omega_{ijk} = (A_{ik}^* + B_i^* K_{jk}^*)^T X^{-1} + X^{-1} \left(A_{ik}^* + B_i^* K_{jk}^* \right)$; and $Q^* = \mathrm{diag}\{Q, 0, 0\}$.

Pre-multiplying and post multiplying the above inequality by block diag$[X^{-T}, 0]$ and block diag$[X^{-1}, 0]$, the following parameterized matrix inequality is obtained

$$\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r_e} \mu_i \mu_j \mu_k \begin{bmatrix} \Omega_{ijk} + Q^* & * \\ D_i^{*T} X^{-1} & -\rho^2 I \end{bmatrix} < 0. \tag{14}$$

Let us consider the candidate of Lyapunov function (11)

$$V(t) = x^{*T}(t) E^{*T} X^{-1} x^*(t). \tag{15}$$

Let $K_{ik}^* = \begin{bmatrix} K_{1ik} & K_{2ik} & 0 \end{bmatrix}$. Then from the derivative of the Lyapunov function, it gives

$$\begin{aligned} &\dot{V}(t) + x^{*T}(t) Q^* x^*(t) - \rho^2 \omega^{*T}(t) \omega^*(t) = \\ &\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r_e} \mu_i \mu_j \mu_k \{ x^{*T}(t) ((A_{ik}^* + B_i^* K_{jk}^*)^T X^{-1} + X^{-1} (A_{ik}^* + B_i^* K_{ik}^*) + Q^*) x^*(t) \} \qquad (16) \\ &+ x^{*T}(t) X^{-T} D_i^* \omega^*(t) + \omega^{*T}(t) D_i^{*T} X^{-1} x^*(t) - \rho^2 \omega^{*T}(t) \omega^*(t) \end{aligned}$$

$$= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e}\mu_i\mu_j\mu_k \begin{bmatrix} x^{*T}(t) & \omega^{*T}(t) \end{bmatrix} \begin{bmatrix} \Omega_{ijk}+Q^* & * \\ D_i^{*T}X^{-1} & -\rho^2 I \end{bmatrix} \begin{bmatrix} x^{*(t)} \\ \omega^*(t) \end{bmatrix}, \quad (17)$$

where $x^*(t)$, $\omega^*(t)$ are matrices and have been defined in Eq. (7); $x^{*T}(t)$, $\omega^*(t)$ are transposed matrices of $x^*(t)$, $\omega^*(t)$.

From (17) and (14), the following inequality is obtained

$$\dot{V}(t) + x^{*T}(t)Q^*x^*(t) - \rho^2\omega^{*T}(t)\omega^*(t) < 0. \quad (18)$$

Integrating the above inequality from 0 to ∞ on both sides, it yields

$$V(\infty) - V(0) + \int_0^\infty (x^{*T}(t)Q^*x^*(t) - \rho^2\omega^{*T}(t)\omega^*(t))dt < 0. \quad (19)$$

With zero initial condition, $V(0) = 0$ and hence

$$\int_0^\infty x^{*T}(t)Q^*x^*(t)dt < \int_0^\infty \rho^2\omega^{*T}(t)\omega^*(t)dt, \quad (20)$$

$$\int_0^\infty e^{*T}(t)Q^*x^*(t)dt < \int_0^\infty \rho^2\omega^{*T}(t)\omega^*(t)dt. \quad (21)$$

Eventually the proof is complete. ■

3.1. Stability analysis

Let us consider (18). If $w^*(t) = 0$, then $\dot{V}(t) < 0$, which implies that the closed loop system seems asymptotically stable.

3.2. Common B matrix case

In this subsection, the case related to common B matrix is considered, where $B_i = B$ $(i = 1, 2, ..., r)$. The LMI conditions for designing the controller are given via the theorem as follows.

Theorem 2. *Let us investigate the fuzzy descriptor system (2) with respect to the control rule (6). In case it obtains some matrices* $X_{11}, X_{21}, X_{22}, X_{31}, X_{32}, X_{33}$ *and* W_{1ik}, W_{2ik} $(i = 1, ..., r,\ k = 1, ..., r^e)$ *as to satisfied the matrix inequalities as follows,*

$$S = S^T > 0, \quad (22)$$

$$\begin{bmatrix} M_{11} & * & * & * & * & * \\ M_{21} & M_{22} & * & * & * & * \\ M_{31} & M_{32} & M_{33} & * & * & * \\ 0 & 0 & D_i^T & -\rho^2 I & * & * \\ 0 & D_r^T & -D_r^T E_k^T & 0 & -\rho^2 I & * \\ X_{11} & X_{21}^T & 0 & 0 & 0 & -Q^{-1} \end{bmatrix} < 0,\ i = 1, ..., r,\ k = 1, ..., r^e,$$

(recall that '' represents the transposed elements in symmetric positions).*

$$
\begin{aligned}
S &= \begin{bmatrix} X_{11} & X_{21}^T \\ X_{21} & X_{22} \end{bmatrix}, \\
M_{11} &= X_{31}^T + X_{31}, \\
M_{21} &= X_{32}^T + A_r X_{21}, \\
M_{22} &= A_r X_{22} + X_{22}^T A_r, \\
M_{31} &= X_{33}^T + A_i X_{11} + (A_i - E_k A_r) X_{21} - E_k X_{31} + BW_{1ik}, \\
M_{32} &= A_i X_{21}^T + (A_i - E_k A_r) X_{22} - E_k X_{32} + BW_{2ik}, \\
M_{33} &= -X_{33}^T E_k^T - E_k^T X_{33}.
\end{aligned}
$$

Then the closed loop system with the controller gain matrices $[K_{1ik}, K_{2ik}] = [W_{1ik}, W_{2ik}]$, $[X_{11}X_{21}^T; X_{21}X_{22}]^{-1}$ *satisfy the given* H_∞ *performance criteria.*

In this case, the LMI conditions for controller design are simpler and number of LMI conditions is also less than that of the general case.

3.3. Simulation results

Let us consider the simple uncertain nonlinear system introduced in [13] with some external disturbance. The system is represented by

$$
(1 + a\cos(\theta(t)))\ddot{\theta}(t) = -b\dot{\theta}^3(t) + c\theta(t) + du(t) + 0.1\omega(t), \tag{23}
$$

with $a = 0.2,\ b = 1,\ c = -1,\ d = 10,\ w(t) = \sin(5t)$ and the range of $\dot{\theta}(t)$ is $|\dot{\theta}(t)| < \phi$, $\phi = 4$. The newly proposed descriptor fuzzy model is improved from [13] as follows

$$
\sum_{k=1}^{2} \mu_k^e(z(t)) E_k \dot{x}(t) = \sum_{k=1}^{2} \mu_k^e(z(t)) \{A_i x(t) + B_i u(t) + D_i \omega(t)\},
$$

$$
y(t) = \sum_{k=1}^{2} \mu_k^e(z(t)) C_i x(t), \tag{24}
$$

with $x(t) = [x_1(t), x_2(t)]T = [\theta(t), \dot{\theta}(t)]^T$. The parameters of the constant matrices are as

$$
E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1+a \end{bmatrix},\ E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1-a \end{bmatrix},
$$

$$
A_1 = \begin{bmatrix} 0 & 1 \\ c & -b\phi^2 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 1 \\ c & 0 \end{bmatrix},\ B_1 = B_2 = \begin{bmatrix} 0 \\ d \end{bmatrix},\ D_i = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},\ i = 1, 2,
$$

$$
\mu_1(x_2(t)) = \frac{x_2^2(t)}{2},\ \mu_2(x_2(t)) = 1 - \frac{x_2^2(t)}{2},
$$

$$
\mu_1^e(x_1(t)) = \frac{1 + \cos(x_1(t))}{2},\ \mu_2^e(x_1(t)) = \frac{1 - \cos(x_1(t))}{2}.
$$

Then the referential model and referential input were considered as follows

$$
\begin{bmatrix} \dot{x}_{r1} \\ \dot{x}_{r2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_{r1} \\ x_{r2} \end{bmatrix} + \begin{bmatrix} 0 \\ 2\sin\left(\frac{t}{2}\right) \end{bmatrix}.
$$

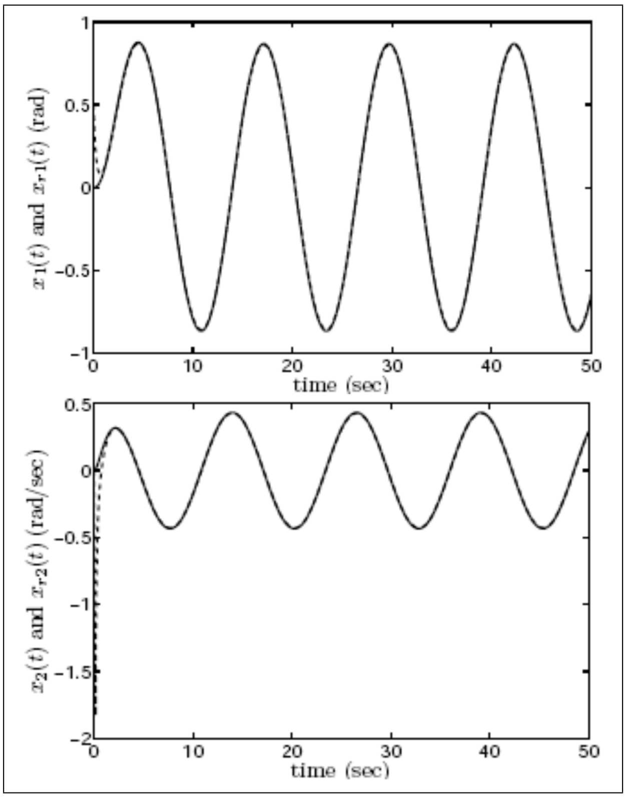

Figure 1. Trajectorial results of state variables $x(t)$ (dashed line) and the referencing trajectories $x_r(t)$ (solid line)

The H_∞ tracking controller is implemented based on the LMI requirements in Theorem 2. With $Q = 0.1\text{I}$ and $\rho^2 = 0.01$, the parameters of Lyapunov function and the feedback gain matrices K_{1ik}, K_{2ik} obtained are given below

$$X_{11} = \begin{bmatrix} 3.6783 & -9.2633 \\ -9.2633 & 109.4131 \end{bmatrix},\ X_{21} = \begin{bmatrix} 0.5044 & 0.9124 \\ -0.5114 & -2.8609 \end{bmatrix},\ X_{22} = \begin{bmatrix} 362.01 & -71.42 \\ -71.42 & 227.28 \end{bmatrix},$$

$$X_{31} = \begin{bmatrix} -4.92 \times 10^8 & 112.29 \\ -0.6444 & -81775 \end{bmatrix},\ X_{32} = \begin{bmatrix} 1.0239 & -2.8364 \\ 79.828 & -352.48 \end{bmatrix},$$

$$X_{33} = \begin{bmatrix} 4.92 \times 10^8 & -3.107 \\ -3.1071 & 81291 \end{bmatrix},$$

$$K_{111} = \begin{bmatrix} -8.5519 & -1.7772 \end{bmatrix},\quad K_{112} = \begin{bmatrix} -8.7643 & -1.8713 \end{bmatrix},$$
$$K_{121} = \begin{bmatrix} -6.9749 & -2.7477 \end{bmatrix},\quad K_{122} = \begin{bmatrix} -6.8766 & -2.7285 \end{bmatrix},$$

$$K_{211} = \begin{bmatrix} -0.0271 & 1.0333 \end{bmatrix},\quad K_{212} = \begin{bmatrix} -0.0201 & 1.0119 \end{bmatrix},$$
$$K_{221} = \begin{bmatrix} -0.0111 & -0.2714 \end{bmatrix},\quad K_{222} = \begin{bmatrix} 0.0302 & -0.1605 \end{bmatrix}.$$

State and reference trajectories $x(t)$ and $x_r(t)$ with the initial condition $x(0) = [0.5\ 0]^T$ and $x_r(0) = [0\ 0]^T$ are presented in Fig. 1.

4. NOVEL T-S FUZZY DESCRIPTOR FOR UNCERTAIN NONLINEAR SYSTEM

This section starts with introduction to uncertain T-S descriptor fuzzy model and then the robust H_∞ tracking control requirement is formulated.

The continuous T-S fuzzy model [6] denotes nonlinear system dynamics based on fuzzy IF-THEN laws. It is possible to present the newly proposed descriptor fuzzy model of an uncertain nonlinear system presented as follows.

Plant law:

IF $z_1^e(t)$ is $N_{k1}^e, ..., z_{p^k}^e(t)$ is $N_{kkp}^e k$ and $z_1(t)$ is $N_{i1}, ..., z_p(t)$ is N_{ip} THEN

$$\begin{gathered}(E_k(\theta) + \triangle E_k(t))\dot{x}(t) = (A_i(\theta) + \triangle A_i(t))x(t) + (B_i(\theta) + \triangle B_i(t))u(t) + D_i w(t),\\ y(t) = C_i x(t),\ i = 1, 2, ...r,\ k = 1, 2, ...r^e,\end{gathered} \tag{25}$$

where, $A_i(\theta) = A_{i0} + \sum_{l=1}^{L} \theta_l(t) A_{il},\ B_i(\theta) = B_{i0} + \sum_{l=1}^{L} \theta_l(t) B_{il},\ E_k(\theta) = E_{k0} + \sum_{l=1}^{L} \theta_l(t) E_{kl}$ $z_1(t), ..., z_p(t)$ are premise variables, p is the number of premise variables, N_{kj}^e $(j = 1...p^k)$, N_{ij} $(j = 1...p)$ are the fuzzy sets and r represents the amount of laws. For simplicity $\theta(t)$ is denoted as θ. Here, $x(t) \in R^{n\times 1}$ is the state vector, $y(t)$ is the controlled output and $u(t)$ is the input vector. $A_{i0} \in R^{n\times n}$, $A_{il} \in R^{n\times n}$, $B_{i0} \in R^{n\times m}$, $B_{il} \in R^{n\times m}$, $E_{k0} \in R^{n\times n}$, $E_{kl} \in R^{n\times n}$, $C_i \in R^{n_y\times n}$ are constant real matrices, $\theta_l(t)$ represents time varying parametric uncertainties; $\triangle A_i(t)$, $\triangle B_i(t)$ and $\triangle E_k(t)$ are time-varied matrices of dimensions available, which represent modelling errors. The necessary assumptions prove that rank$(E_k) \le n$; $\triangle A_i \in R^{n\times n}$, $\triangle B_i \in R^{n\times m}$, $\triangle E_i \in R^{n\times m}$ represent the uncertainties and are bounded, i.e., $\|\triangle A_i\| < \delta_i$, $\|\triangle B_i\| < \beta_i$, $\|\triangle E_i\| < \phi_i$ where $\|.\|$ denotes spectral norm and $\delta_i, \beta_i, \phi_i$ represent any positive values. Other specific constraints can be consulted in [14].

From input $x(t)$ and output $u(t)$, the eventual state-space output of the proposed fuzzy system is described as

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t))\{(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t)\},$$

$$\sum_{k=1}^{r_e} \mu_k^e(E_k(\theta) + \Delta E_k(t))\dot{x}(t) = \sum_{i=1}^{r} \mu_i\{(A_i(\theta) + \Delta A_i(t))x(t) + (B_i(\theta) + \Delta B_i(t))u(t) + D_i\omega(t)\},$$

$$y(t) = \sum_{i=1}^{r} \mu_i C_i x(t), \tag{26}$$

with $\mu_i = \dfrac{\zeta_i(z(t))}{\sum_{j=1}^{r} \zeta_j(z(t))}$, $\quad \zeta_i(z(t)) = \prod_{j=1}^{p} N_{ij}(z_j(t))$,

$\mu_k^e = \dfrac{\zeta_k^e(z^e(t))}{\sum_{j=1}^{r^e} \zeta_j^e(z^e(t))}$, $\quad \zeta_k^e(z^e(t)) = \prod_{j=1}^{p^e} N_{kj}^e(z_j^e(t))$,

$N_{ij}(z_j(t))$ and $N_{kj}^e(z_j^e(t))$ represent the degrees of membership of $z_j(t)$ and $z_j^e(t)$ in the fuzzy set N_{ij} and N_{kj}^e, respectively. Here $\sum_{i=1}^{r} \mu_i(z(t)) = 1$ and $\sum_{k=1}^{r^e} \mu_k(z(t)) = 1$. For simplicity, $\mu_k^e(z(t))$ and $\mu_i(z(t))$ were represented as $\mu_k^e(z(t))$ and $\mu_i(z(t))$ respectively.

The uncertain matrices $\triangle A_i(t)$, $\triangle B_i(t)$ and $\triangle E_k(t)$ were assigned to be norm-limited and improved from [2] as follows

$$[\Delta A_i(t) \quad \Delta B_i(t)] = \sum_{l=1}^{L_a} M_{il}^a \Delta_{il}^a(t)\,[N_{i1l}^a \quad N_{i2l}^a],$$
$$\Delta E_k(t) = \sum_{l=1}^{L_e} M_{kl}^e \Delta_{kl}^e(t) N_{kl}^e, \tag{27}$$

with $M_{il}^a, M_{kl}^e, N_{i1l}^a, N_{i2l}^a$ and N_{k1l}^e represent actual constant matrices with dimension available and $\triangle_{il}^a(t)$, $\triangle_{il}^e(t)$ represent time-varied equations, satisfying $|\Delta_{il}^a(t)| < 1$, $|\Delta_{kl}^e(t)| < 1$, $\forall t > 0$.

Let us consider a reference model and the H performance measure as given in Section 2 with the Parallel Distributed Compensation (PDC) fuzzy controller improved from [12],

$$u(t) = \sum_{i=1}^{r}\sum_{k=1}^{r^e} \mu_i \mu_k^e (K_{1ik} e(t) + K_{2ik} x_r(t)), \tag{28}$$

where K_{1ik} and K_{2ik} are the controller gains. Newly proposed fuzzy controller is implemented with the feedback gains K_{1ik} and K_{2ik} $(i = 1, ..., r,\ k = 1, ..., r^e)$ such that the resulting closed-loop system ensures asymptotically stable and responds the H_∞ performance given in (5).

Combining (26) and (3) and relating to the control rule (28), the augmented fuzzy descriptor system is to be expressed as

$$E^* \dot{x}^*(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r^e} \mu_i \mu_j \mu_k^e \{(A_{ik}^*(\theta) + \Delta A_{ik}^*(t) + (B_i^*(\theta) + \Delta B_i^*(t))K_{jk}^*)x^*(t) + D_i^* \omega^*(t)\}, \tag{29}$$

where $x^*(t) = \begin{bmatrix} e(t) \\ x_r(t) \\ \dot{e}(t) \end{bmatrix}$, $\omega^*(t) = \begin{bmatrix} \omega(t) \\ r(t) \end{bmatrix}$,

$A_{ik}^*(\theta) = \begin{bmatrix} 0 & 0 & I \\ 0 & A_r & 0 \\ A_i(\theta) & A_i(\theta) - E_k(\theta)A_r & -E_k(\theta) \end{bmatrix}$,

$\Delta A_{ik}^*(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \Delta A_i(t) & \Delta A_i(t) - \Delta E_k(t)A_r & -\Delta E_k(t) \end{bmatrix}$,

$B_i^*(\theta) = \begin{bmatrix} 0 \\ 0 \\ B_i(\theta) \end{bmatrix}$, $\Delta B_i^* = \begin{bmatrix} 0 \\ 0 \\ \Delta B_i(t) \end{bmatrix}$,

$K_{jk}^* = \begin{bmatrix} K_{1jk} & K_{2jk} & 0 \end{bmatrix}$,

$D_{jk}^* = \begin{bmatrix} 0 & 0 \\ 0 & D_r \\ D_i & -E_k(\theta)D_r \end{bmatrix}$, $E^* = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

$$\begin{bmatrix} \Delta A_{ik}^*(t) & \Delta B_i^*(t) \end{bmatrix} = \sum_{l=1}^{L_a} M_{il}^{a*} \Delta_{il}^a(t) \begin{bmatrix} N_{i1l}^{a*} & N_{i2l}^{a*} \end{bmatrix} + \sum_{l=1}^{L_e} M_{kl}^{e*} \Delta_{kl}^e(t) N_{kl}^{e*}, \tag{30}$$

$$M_{il}^{a*} = \begin{bmatrix} 0 \\ 0 \\ M_{il}^{a} \end{bmatrix}, \quad M_{kl}^{e*} = \begin{bmatrix} 0 \\ 0 \\ M_{kl}^{e} \end{bmatrix},$$
$$N_{i1l}^{a*} = \begin{bmatrix} N_{i1l}^{a} & N_{i1l}^{a} & 0 \end{bmatrix}, \quad N_{i2l}^{a*} = N_{i2l}^{a}, \quad N_{kl}^{e*} = \begin{bmatrix} 0 & N_{kl}^{e} & 0 \end{bmatrix}.$$

The proposed fuzzy descriptor system (29) affinely depends on the parametric vector. As in [21] and [22], both lower/upper bounds of the uncertain coefficient and their rates of variation are assumed to be known. Specifically:

1. Each parameter θ_l ranges within the known lower $\underline{\theta}_l$ and upper $\overline{\theta}_l$ bounds, i.e.,

$$\theta_l \in [\underline{\theta}_l, \overline{\theta}_l]. \tag{31}$$

2. The speed of variation $\dot{\theta}_l$ is precisely calculated at all times and satisfies

$$\dot{\theta}_l \in [\underline{\upsilon}_l, \overline{\theta}_l], \tag{32}$$

where $\underline{\upsilon}_l$ and $\overline{\theta}_l$ represent known lower/upper bounds of $\dot{\theta}_l$, respectively.

With these assumptions, the parameter vecto θ_l takes values within the hyper-rectangle called parameter box and the rate vector $\dot{\theta}_l$ takes values in another hyper-rectangle called rate box. It is denoted as,

$$V := \{(\nu_1, \nu_2, ..., \nu_L)^T : \nu_l \in \{ \underline{\theta}_l \quad \overline{\theta}_l \}\}, \tag{33}$$
$$W := \{(\omega_1, \omega_2, ..., \omega_L)^T : \omega_l \in \{ \underline{\upsilon}_l \quad \overline{\upsilon}_l \}\}, \tag{34}$$

which are the set of 2^L vertices of the parameter box and the rate box, respectively.

5. PROPOSED ROBUST H_∞ TRACKING CONTROL IMPLEMENTATION WITH FIXED LYAPUNOV FUNCTION

In this section, Lyapunov function-based robust H_∞ tracking controller design for proposed fuzzy descriptor system is presented. First it investigates the fixed Lyapunov function described as,

$$V(t) = x^{*T}(t)E^{*T}X^{-1}x^*(t) \tag{35}$$

with

$$X = \begin{bmatrix} X_{11} & X_{21}^T & 0 \\ X_{21} & X_{22} & 0 \\ X_{31} & X_{32} & X_{33} \end{bmatrix}, \quad E^{*T}X^{-1} = X^{-T}E^* \geq 0.$$

Theorem 3. *Let us consider the fuzzy descriptor system (29) and the control rule (28). In case it obtains certain matrices X as defined in (35) and W_{jk} $(j = 1, ..., r,\ k = 1, ..., r^e)$ as to satisfy the matrix inequalities presented as follows,*

$$S^T = S > 0, \tag{36}$$
$$\phi_{iik}^*(\nu) < 0,\ \forall \nu \in V,\ i = 1, 2, ..., r,\ k = 1, 2, ..., r^e, \tag{37}$$
$$\frac{1}{r-1}\phi_{iik}^*(\nu) + \frac{1}{2}(\phi_{ijk}^*(\nu) + \phi_{jik}^*(\nu)) < 0,\ \forall \nu \in V,\ 1 \leq i \neq j \leq r,\ k = 1, 2, ..., r^e, \tag{38}$$

with

$$S=\begin{bmatrix} X_{11} & X_{21}^T \\ X_{21} & X_{22} \end{bmatrix},$$

$$\phi_{ijk}^{*}(\nu)=\begin{bmatrix} \mathcal{A}_{ijk}^{11}(\nu) & * & * & * & * & * & * \\ D_{ik}^{*T} & -\rho^2 I & * & * & * & * & * \\ \mathcal{Y} & 0 & -Q^{-1} & * & * & * & * \\ \mathcal{A}_{i}^{31} & 0 & 0 & -\epsilon_i^a & * & * & * \\ \mathcal{A}_{ijk}^{41} & 0 & 0 & 0 & -\epsilon_i^a & * & * \\ \mathcal{A}_{k}^{51} & 0 & 0 & 0 & 0 & -\epsilon_k^e & * \\ \mathcal{A}_{k}^{61} & 0 & 0 & 0 & 0 & 0 & -\epsilon_k^e \end{bmatrix},$$

$$\mathcal{A}_i^{31}=\begin{bmatrix} {}^{a}_{i1}M_{i1}^{a*T} \\ \vdots \\ {}^{a}_{iL_a}M_{iL_a}^{a*T} \end{bmatrix}, \quad \mathcal{A}_{ijk}^{41}=\begin{bmatrix} (N_{i11}^{a*}X+N_{i21}^{a*}W_{jk}) \\ \vdots \\ (N_{i1L_a}^{a*}X+N_{i2L_a}^{a*}W_{jk}) \end{bmatrix},$$

$$\mathcal{A}_k^{51}=\begin{bmatrix} {}^{e}_{k1}M_{k1}^{e*T} \\ \vdots \\ {}^{e}_{kL_a}M_{kL_e}^{e*T} \end{bmatrix}, \quad \mathcal{A}_k^{61}=\begin{bmatrix} N_{k1}^{e*}X \\ \vdots \\ N_{kL_e}^{e*}X \end{bmatrix}, \quad \mathcal{Y}=X\begin{bmatrix} I & 0 & 0 \end{bmatrix},$$

$$\mathcal{A}_{ijk}^{11}(\nu)=X^T\mathcal{A}_{ik}^{*T}(\nu)+W_{jk}^{*T}B_i^{*T}(\nu)+B_i^{*T}(v)W_{jk}^{*},$$

$$\epsilon_i^a=\mathrm{diag}(\epsilon_{i1}^a,...,\epsilon_{iL_a}^a), \quad \epsilon_k^e=\mathrm{diag}(\epsilon_{k1}^e,...,\epsilon_{kL_e}^e)$$

and $W_{jk}^{*}=K_{jk}^{*}X$, *then the closed loop system ensures asymptotically stable and satisfies the given* H_∞ *performance criteria.*

Proof. If (37) and (38) are satisfied, the following parameterized inequality is obtained

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r^e}\mu_i\mu_j\mu_k^e\phi_{ijk}^{*}(\nu)<0,\ \forall\nu\in V. \tag{39}$$

If the above inequality is satisfied in the vertices of the parameter box V, then the inequality holds for the range of defined in the parameter box improved from [23]. Hence,

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r^e}\mu_i\mu_j\mu_k^e\phi_{ijk}^{*}(\theta)<0. \tag{40}$$

Based on (27), using the Schur complement Lemma and the inequality $Y^TZ+Z^TY\leq Y^TY+Z^TZ$ improved from [24], the matrices related to $\triangle A_i(t)$, $\triangle B_i(t)$ and $\triangle E_k(t)$ can be rewritten as follows

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r^e}\mu_i\mu_j\mu_k^e\Upsilon_{ijk}^{*}(t,\theta)<0 \tag{41}$$

with $\Upsilon_{ijk}^{*}(t,\theta)=\begin{bmatrix} \Omega_{ijk}^{*}(t,\theta) & * & * \\ D_i^{*T} & -\rho^2 I & * \\ \mathcal{Y} & 0 & -Q^{-1} \end{bmatrix}$, W_{jk}^{*T}, $A_{ik}^{*T}(\theta)$, $B_i^{*T}(\theta)$, $\Delta A_{ik}^{*T}(t)$, $\Delta B_i^{*T}(t)$ have been defined in (30),

$$\Omega^*_{ijk}(t,\theta) = X^T A^{*T}_{ik}(\theta) + W^{*T}_{jk} B^{*T}_i(\theta) + X^T \Delta A^{*T}_{ik}(t) + W^{*T}_{jk} \Delta B^{*T}_i(t) + \Delta B^{*T}_i(t) W^*_{jk}.$$

Again using Schur complement, the above inequality can be expressed as

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r^e} \mu_i \mu_j \mu_k^e \begin{bmatrix} \Omega^*_{ijk}(t,\theta) + X^T Q^* X & * \\ D_i^{*T} & -\rho^2 I \end{bmatrix} < 0, \tag{42}$$

where $Q^* = \mathrm{diag}(Q, 0, 0)$. Pre-multiplying (42) with $\mathrm{diag}(X^{-T}, I)$ and post-multiplying with $\mathrm{diag}(X^{-1}, I)$, it gives,

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r^e} \mu_i \mu_j \mu_k^e \begin{bmatrix} X^{-T}\Omega^*_{ijk}(t,\theta)X^{-1} + Q^* & * \\ D_i^{*T} X^{-1} & -\rho^2 I \end{bmatrix} < 0. \tag{43}$$

Let us introduce a Lyapunov function candidate $V(t) = x^*(t) E^{*T} X^{-1} x^*(t)$. Then from the derivative of $V(t)$, it gives,

$$\begin{aligned} &\dot V(t) + x^{*T}(t) Q^* x^*(t) - \rho^2 \omega^{*T}(t)\omega^*(t) = \\ &\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r^e} \mu_i\mu_j\mu_k^e \{x^{*T}(t)(X^{-T}\Omega^*_{ijk}(t,\theta)X^{-1} + Q^*)x^*(t)\} \\ &+ x^{*T}(t) X^{-T} D_i^* \omega^*(t) + \omega^{*T}(t) D_i^{*T} X^{-1} x^*(t) - \rho^2 \omega^{*T}(t)\omega^*(t) \end{aligned} \tag{44}$$

$$= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r^e} \mu_i\mu_j\mu_k^e \begin{bmatrix} x^{*T}(t) & \omega^{*T}(t) \end{bmatrix} \begin{bmatrix} X^{-T}\Omega^*_{ijk}(t,\theta)X^{-1} + Q^* & * \\ D_i^{*T} X^{-1} & -\rho^2 I \end{bmatrix} \begin{bmatrix} x^*(t) \\ \omega^*(t) \end{bmatrix}. \tag{45}$$

From (43) and (45), the following inequality can be obtained,

$$\dot V(t) + x^{*T}(t) Q^* x^*(t) - \rho^2 \omega^{*T}(t)\omega^*(t) < 0. \tag{46}$$

Integrating the above inequality from 0 to ∞, it gives,

$$V(\infty) - V(0) + \int_0^\infty (x^{*T}(t) Q^* x^*(t) - \rho^2 \omega^{*T}(t)\omega^*(t))dt < 0. \tag{47}$$

With zero initial condition, $V(0) = 0$ and hence

$$\int_0^\infty x^{*T}(t) Q^* x^*(t) dt < \int_0^\infty \rho^2 \omega^{*T}(t)\omega^*(t) dt, \tag{48}$$

$$\int_0^\infty e^T(t) Q^* e(t) dt < \int_0^\infty \rho^2 \omega^{*T}(t)\omega^*(t) dt. \tag{49}$$

Thus the proof is completed. ■

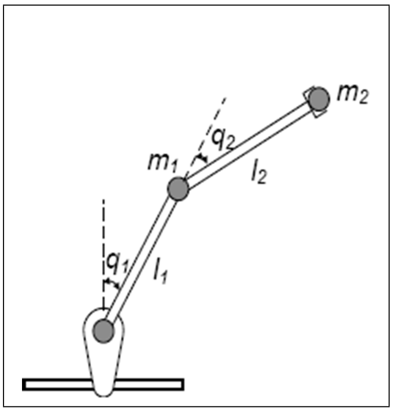

Figure 2. Set-up diagram of a two-joint robot arm

Table 1. Premise variables for the fuzzy rules - two-joint robot arm

Rules i	N_{i1}	N_{i2}
1	Negative	Negative
2	Negative	Zero
3	Negative	Positive
4	Zero	Negative
5	Zero	Zero
6	Zero	Positive
7	Positive	Negative
8	Positive	Zero
9	Positive	Positive

6. SIMULATION RESULTS

Let us consider the two-joint robot arm (see Fig. 2). The dynamics of the two-joint robotic manipulator [20] is expressed as,

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{50}$$

with

$$M(q) = \begin{bmatrix} (m_1+m_2)l_i^2 & m_2 l_1 l_2 (s_1 s_2 + c_1 c_2) \\ m_2 l_1 l_2 (s_1 s_2 + c_1 c_2) & m_2 l_2^2 \end{bmatrix},$$

$$C(q,\dot{q}) = m_2 l_1 l_2 (c_1 s_2 - s_1 c_2) \begin{bmatrix} 0 & -\dot{q}_2 \\ -\dot{q}_1 & 0 \end{bmatrix}, \; G(q) = \begin{bmatrix} -(m_1+m_2) l_1 g s_1 \\ m_2 l_2 g s_2 \end{bmatrix}.$$

The nominal parameters of the system are the link masses $m_1 = m_2 = 1$kg, link lengths $l_1 = l_2 = 1$m and the gravitational acceleration $g_r = 9.81\text{m/s}^2$. In this example, structural uncertainties in masses are considered and the perturbation is assumed to be within $\pm 5\%$ from their nominal value.

The operating domain is considered as $x_1(t) \in [-\pi/3, \pi/3]$, $x_3(t) \in [-\pi/3, \pi/3]$, $x_2(t) \in [-5, 5]$, $x_4(t) \in [-5, 5]$ and the input $u_1(t) \in [-25, 25]$ and $u_2(t) \in [-15, 15]$.

Equidistant triangular membership functions with centers $-\pi/3$, 0 and $\pi/3$ are assumed for both of $x_1(t)$ and $x_3(t)$. With the uncertainties in masses m_1 and m_2, the uncertainties in the fuzzy model can be derived as $\theta_1(t) \in [-0.05, 0.05]$ and $\theta_2(t) \in [-0.05, 0.05]$.

Table 2. Coefficients of matrices $A_i(\theta)$ and $B_i(\theta)$ - Two-joint manipulator

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$
a_{i021}	13.85	14.52	14.50	19.28	18.35	19.28	14.50	14.52	13.85
a_{i023}	2.41	1.155	-0.085	2.01	2.488	2.01	-0.085	1.155	2.41
a_{i041}	-2.34	-1.88	-1.259	-1.91	-1.78	-1.91	-1.259	-1.88	-2.34
a_{i043}	10.49	12.08	8.531	10.10	12.19	10.10	8.531	12.08	10.49
a_{i121}	9.244	7.775	7.25	10.29	10.98	10.29	7.25	7.775	9.244
a_{i123}	-0.975	-0.963	-0.881	0.196	0.385	0.196	-0.881	-0.963	-0.975
a_{i221}	19.99	4.654	2.719	13.68	10.95	13.68	2.719	4.654	19.99
a_{i223}	-11.72	-8.13	-3.93	3.913	6.144	3.913	-3.93	-8.13	-11.72
a_{i241}	17.37	-6.08	-0.069	5.564	-5.65	5.564	-0.069	-6.08	17.37
a_{i243}	-8.967	3.193	9.147	13.92	14.44	13.92	9.147	3.193	-8.967

Table 3. Parameters of matrices δA_i - Two-joint manipulator

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$
δa_{i21}	0.835	1.489	1.152	1.331	1.588	1.311	1.152	1.489	0.835
δa_{i22}	0.067	0	0	0	0	0	0	0	0.067
δa_{i23}	0.209	0.894	0.66	1.489	1.678	1.489	0.66	0.894	0.209
δa_{i24}	0.057	0	0	0	0	0	0	0	0.057
δa_{i41}	0.463	1.502	0.925	1.13	1.464	1.13	0.925	1.502	0.463
δa_{i42}	0.05	0	0	0	0.66	0	0	0	0.05
δa_{i43}	0.68	1.283	0.94	1.508	1.516	1.508	0.94	1.283	0.68
δa_{i44}	0.068	0	0	0	0	0	0	0	0.068

Table 4. Parameters of matrices $E_i(\theta)$ and $\triangle E_i(t)$ - Two-joint manipulator

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$
$e_{240i}, ei420i$	1.061	0.641	-0.574	0.641	1.206	0.641	-0.574	0.641	1.061
$e_{i224}, ei242$	1.024	0.653	0.037	0.653	1.149	0.653	0.037	0.653	1.024
$\triangle e_{i24}, \triangle ei42$	0.256	0.386	0.386	0.386	0.386	0.386	0.386	0.386	0.256

The fuzzy rules are considered as follows.
Plant law i: IF x_1 is N_{i1} and x_3 is N_{i2} THEN

$$(E_i(\theta) + \triangle E_i(t))\dot{x}(t) = (A_i(\theta) + \triangle A_i(t))x(t) + Bu(t) + D_i w(t),$$

$y(t) = C_i x(t),\ i = 1, ..., 9,$

where

$$A_{i0}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21i0} & 0 & a_{23i0} & 0 \\ 0 & 0 & 0 & 1 \\ a_{41i0} & 0 & a_{43i0} & 0 \end{bmatrix}, \quad A_{i1}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21i1} & 0 & a_{23i1} & 0 \\ 0 & 0 & 0 & 1 \\ a_{41i1} & 0 & a_{43i1} & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_{i0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & e_{24i0} \\ 0 & 0 & 1 & 0 \\ 0 & e_{42i0} & 0 & 1 \end{bmatrix},$$

$$\triangle A_i(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \triangle a_{21i}(t) & \triangle a_{22i}(t) & \triangle a_{23i}(t) & \triangle a_{24i}(t) \\ 0 & 0 & 0 & 0 \\ \triangle a_{41i}(t) & \triangle a_{42i}(t) & \triangle a_{43i}(t) & \triangle a_{44i}(t) \end{bmatrix}, \quad D_i = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$E_{i1}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad E_{i2}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & e_{24i}(t) \\ 0 & 0 & 0 & 0 \\ 0 & e_{42i}(t) & 0 & 1 \end{bmatrix},$$

$$\triangle E_i(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \triangle e_{24i}(t) \\ 0 & 0 & 0 & 0 \\ 0 & \triangle e_{42i}(t) & 0 & 0 \end{bmatrix}.$$

The fuzzy sets N_{i1} and N_{i2} for rules $i = 1, ..., 9$ are shown in Table 1. The parameters of the proposed fuzzy model are successfully computed via the linear programming method discussed in Section 2. The values of $A_i(\theta)$ and $B_i(\theta)$ coefficients are described in Table 2.

The parameters of $\triangle A_i(t)$ are shown in Table 3. For $E_i(\theta)$ and $\triangle E_i(t)$, the parameters are shown in Table 4.

Let us investigate the referential model as follows:

$$\dot{x}_r(t) = A_r x_r(t) + r(t) \tag{51}$$

where

$$A_r = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -6 & -5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -6 & -5 \end{bmatrix},$$

and

$$r(t) = \begin{bmatrix} 0 & 7\sin(t) & 0 & 7\cos(t) \end{bmatrix}^T.$$

The H_∞ tracking controller design problem is considered with the above referential model given by (51). In this benchmark test, the fuzzy descriptor model satisfies the condition $\mu_i = \mu_i^e$ and $r = r_e$. Let us assume the mass of the links as $m_1 + \triangle m_1 = 1 + 0.05\sin(2t)$ and $m_2 + \triangle m_2 = 1 + 0.05\cos(2t)$. Here $0.05\sin(2t)$ and $0.05\cos(2t)$ represent the uncertainties.

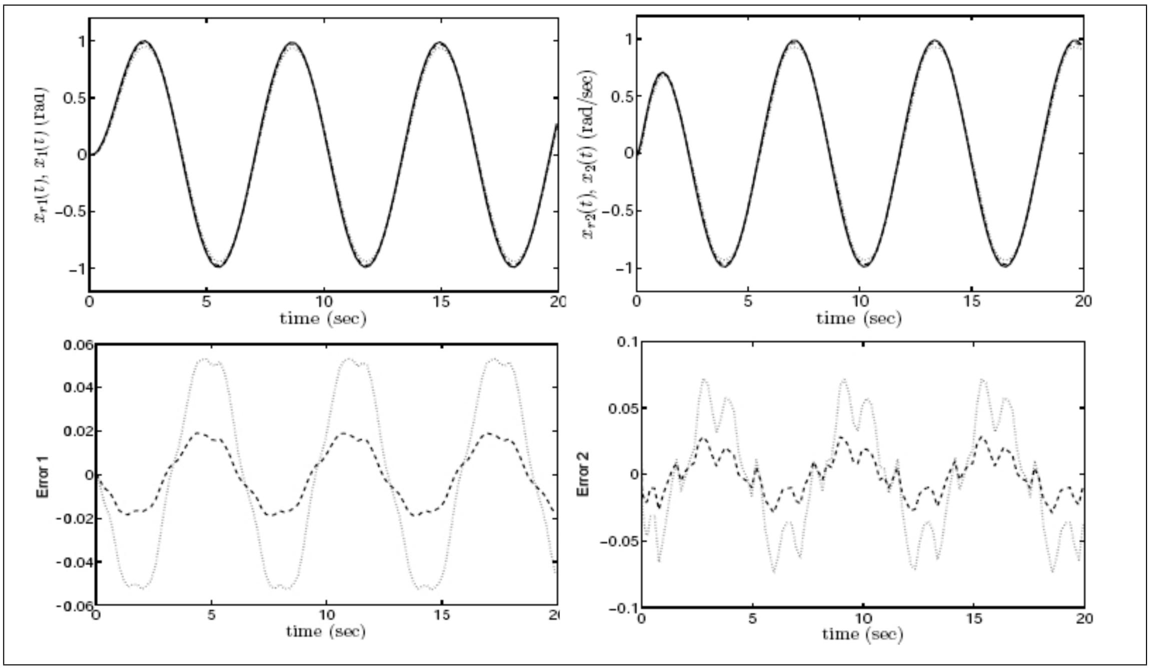

Figure 3. Trajectorial results with zooming tracking error $e(t)$ of state variables $x(t)$ (dashed line & dotted line for $\rho^2 = 0.001$ and $\rho^2 = 0.01$) and the referential trajectories $x_r(t)$ (solid line)

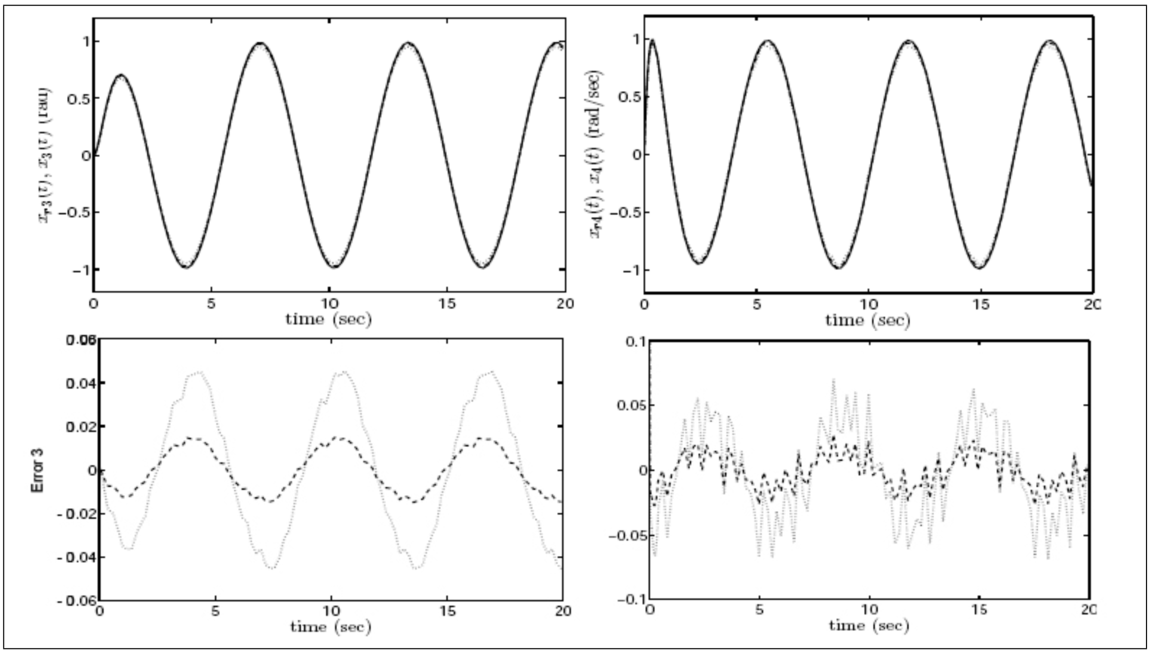

Figure 4. Trajectorial results with zooming tracking error $e(t)$ of state variables $x(t)$ (dashed line & dotted line for $\rho^2 = 0.001$ and $\rho^2 = 0.01$) and the referential trajectories $x_r(t)$ (solid line)

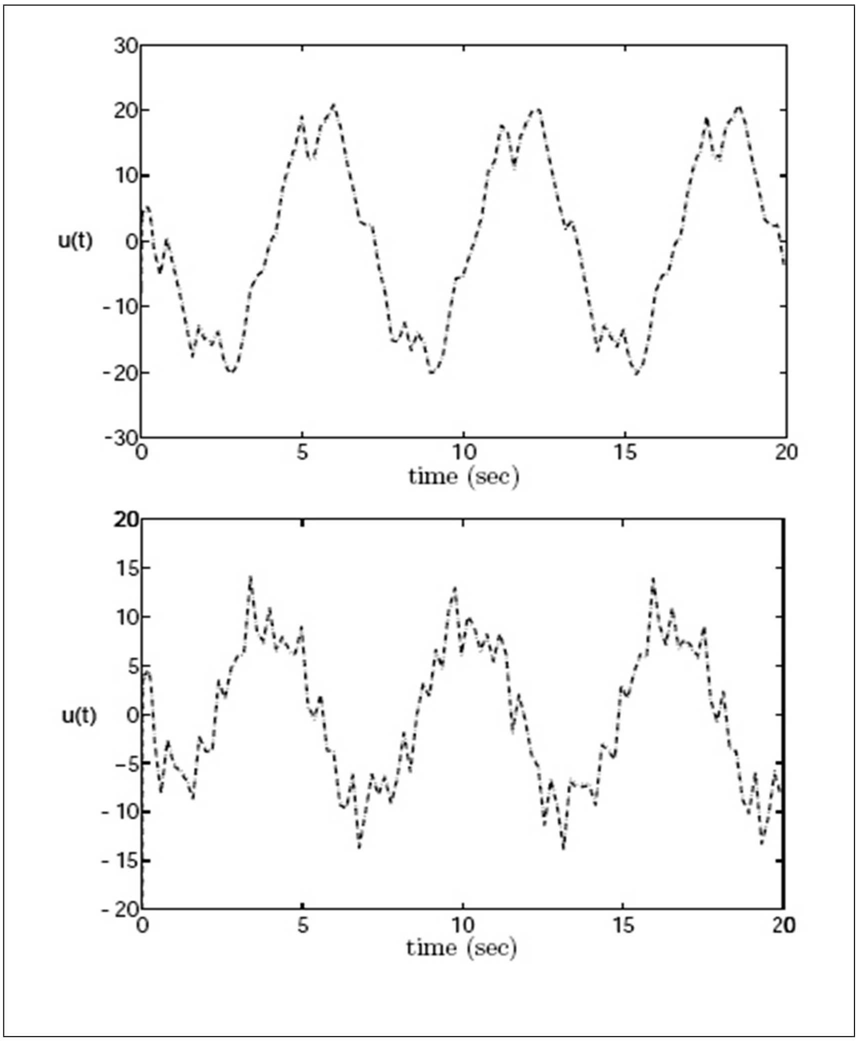

Figure 5. Control input $u(t)$ (dashed line and dotted line for $\rho^2 = 0.001$ and $\rho^2 = 0.01$, respectively)

The external disturbances (e.g., cogging torque in the actuator) are assumed to be $w_1(t) = 0.4\cos(10t)\cos(2t) + 0.2\exp(-t)\sin(4t)$ and $w_2(t) = 0.3\sin(5t) + 0.25\exp(-2t)$.

With $Q = 0.01I$, the H_∞ tracking controller is designed for different values by using the proposed Algorithm 1. With zero initial condition, the simulation results are presented in Fig. 3, Fig. 4, Fig. 5 for $\rho^2 = 0.001$ and $\rho^2 = 0.01$. In Fig. 3, the trajectorial results of $x(t)$ and the referential trajectories $x_r(t)$ for $= 0.001$ and $= 0.01$ are shown. The tracking error plots for these two values of ρ^2 are presented in Fig.3, Fig. 4. The control inputs $u(t)$ are plotted in Fig. 5.

7. CONCLUSIONS

This paper proposes a T-S fuzzy model-based reference trajectory controller satisfying H_∞ performance criterion for uncertain fuzzy descriptor systems. Sufficient conditions for controller design satisfying the given H_∞ performance criterion are formulated via LMI matrix inequalities. The proposed fuzzy descriptor system approach yields lesser number of inequality conditions than those obtained using the standard state-space approach. It is convincingly shown that, by the newly proposed design approach, the required tracking controller can be successfully implemented by resolving a set of inequalities and the specified H_∞ disturbance attenuation level can be obtained. In order to demonstrate the effectiveness of the novel fuzzy controller approach, tracking control benchmark test of a two-joint robot arm under external disturbances is investigated and the simulation results show that the proposed fuzzy system robustly and precisely track the referential trajectory.

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REFERENCES

[1] C. Hua, L. Zhang, and X. Guan, "Robust adaptive control for time-delay system via T-S fuzzy approach," *Robust Control for Nonlinear Time-Delay Systems*, pp. 93–112, 2018.

[2] K. Tanaka, T. Ikeda, and H. O. Wang, "Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs," *IEEE Transactions on Fuzzy Systems*, vol. 6, no. 2, pp. 250–265, 1998.

[3] S. Tong, T. Wang, and H.-X. Li, "Fuzzy robust tracking control for uncertain nonlinear systems," *International Journal of Approximate Reasoning*, vol. 30, no. 2, pp. 73–90, 2002.

[4] B. Zhang, S. Zhou, and T. Li, "A new approach to robust and non-fragile H_∞ control for uncertain fuzzy systems," *Information Sciences*, vol. 177, no. 22, pp. 5118–5133, 2007.

[5] F. Cuesta, F. Gordillo, J. Aracil, and A. Ollero, "Stability analysis of nonlinear multivariable Takagi-Sugeno fuzzy control systems," *IEEE Transactions on Fuzzy Systems*, vol. 7, no. 5, pp. 508–520, 1999.

[6] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *Readings in Fuzzy Sets for Intelligent Systems*, pp. 387–403, 1993.

[7] J.-C. Lo and M.-L. Lin, "Robust H_∞ control for fuzzy systems with Frobenius norm-bounded uncertainties," *IEEE Transactions on Fuzzy Systems*, vol. 14, no. 1, pp. 1–15, 2006.

[8] X.-M. Zhang, Q.-L. Han, and X. Ge, "A novel finite-sum inequality-based method for robust H_∞ control of uncertain discrete-time Takagi-Sugeno fuzzy systems with interval-like time-varying delays," *IEEE Transactions on Cybernetics*, vol. 48, no. 9, pp. 2569–2582, 2017.

[9] L. Xiaodong and Z. Qingling, "New approaches to H_∞ controller designs based on fuzzy observers for T-S fuzzy systems via LMI," *Automatica*, vol. 39, no. 9, pp. 1571–1582, 2003.

[10] S. Ghosh, S. K. Das, and G. Ray, "Stabilizing adaptive controller for uncertain dynamical systems: An LMI approach," *International Journal of Control, Automation and Systems*, vol. 7, no. 2, pp. 311–317, 2009.

[11] A. Sala and C. Arino, "Asymptotically necessary and sufficient conditions for stability and performance in fuzzy control: Applications of polya's theorem," *Fuzzy Sets and Systems*, vol. 158, no. 24, pp. 2671–2686, 2007.

[12] K. Tanaka and H. O. Wang, *Fuzzy control systems design and analysis: a linear matrix inequality approach.* John Wiley & Sons, 2004.

[13] T. Taniguchi, K. Tanaka, and H. O. Wang, "Fuzzy descriptor systems and nonlinear model following control," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 4, pp. 442–452, 2000.

[14] W.-S. Yu, "Observer-based fuzzy tracking control design for nonlinear uncertain descriptor dynamic systems," in *2007 IEEE International Conference on Systems, Man and Cybernetics*, pp. 127–132, IEEE, 2007.

[15] W. Tian, H. Zhang, and X. Yang, "Robust H_∞ control for fuzzy descriptor systems with time-varying delay and parameter uncertainties," in *Third International Conference on Natural Computation (ICNC 2007)*, vol. 3, pp. 18–22, IEEE, 2007.

[16] Z. Lian, Y. He, C.-K. Zhang, P. Shi, and M. Wu, "Robust H_∞ control for T-S fuzzy systems with state and input time-varying delays via delay-product-type functional method," *IEEE Transactions on Fuzzy Systems*, 2019.

[17] Y. Xue, B.-C. Zheng, and X. Yu, "Robust sliding mode control for T-S fuzzy systems via quantized state feedback," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 4, pp. 2261–2272, 2017.

[18] C. Ge, Y. Shi, J. H. Park, and C. Hua, "Robust h_∞ stabilization for T-S fuzzy systems with time-varying delays and memory sampled-data control," *Applied Mathematics and Computation*, vol. 346, pp. 500–512, 2019.

[19] A. Nasiri, S. K. Nguang, A. Swain, and D. J. Almakhles, "Reducing conservatism in an H_∞ robust state-feedback control design of T-S fuzzy systems: A nonmonotonic approach," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 1, pp. 386–390, 2017.

[20] C.-S. Tseng, B.-S. Chen, and H.-J. Uang, "Fuzzy tracking control design for nonlinear dynamic systems via T-S fuzzy model," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 3, pp. 381–392, 2001.

[21] P. Gahinet, P. Apkarian, and M. Chilali, "Affine parameter-dependent Lyapunov functions and real parametric uncertainty," *IEEE Transactions on Automatic Control*, vol. 41, no. 3, pp. 436–442, 1996.

[22] Y.-Y. Cao and Z. Lin, "A descriptor system approach to robust stability analysis and controller synthesis," *IEEE Transactions on Automatic Control*, vol. 49, no. 11, pp. 2081–2084, 2004.

[23] P. Apkarian and H. D. Tuan, "Parameterized LMIs in control theory," *SIAM Journal on Control and Optimization*, vol. 38, no. 4, pp. 1241–1264, 2000.

[24] S. Tong, T. Wang, and H.-X. Li, "Fuzzy robust tracking control for uncertain nonlinear systems," *International Journal of Approximate Reasoning*, vol. 30, no. 2, pp. 73–90, 2002.

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