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NUMERICAL SOLUTION OF THE PROBLEMS FOR PLATES ON SOME COMPLEX PARTIAL INTERNAL SUPPORTS

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Abstract. In the recent works, Dang and Truong proposed an iterative method for solving some problems of plates on one, two and three line partial internal supports (LPISs), and a cross internal support. In nature they are problems with strongly mixed boundary conditions for biharmonic equation. For this reason the method combines a domain decomposition technique with the reduction of the order of the equation from four to two.

In this study, the method is developed for plates on internal supports of more complex configurations. Namely, we examine the cases of symmetric rectangular and H-shape supports, where the computational domain after reducing to the first quadrant of the plate is divided into three subdomains. Also, we consider the case of asymmetric rectangular support where the computational domain needs to be divided into 9 subdomains. The problems under consideration are reduced to sequences of weak mixed boundary value problems for the Poisson equation, which are solved by difference method. The performed numerical experiments show the effectiveness of the iterative method.

Keywords. Rectangular Plate; Internal Line Supports; Biharmonic Equation, Iterative Method, Domain Decomposition Method.

1. INTRODUCTION

The plates with line partial internal supports (LPIS) play very important role in engineering. Therefore, recently they have attracted attention from many researchers. In the essence, the problems of plates on internal supports are strongly mixed boundary value problems for biharmonic equation. There are some methods for analysis of these plates. It is worthy to mention the Discrete Singular Convolution (DSC) algorithm developed by Xiang, Zhao and Wei in 2002 [15, 16]. Essentially, DSC based on the theory of distributions and the theory of wavelets is an algorithm for the approximation of functions and their derivatives. Its efficiency has been proven in solving many complex engineering problems. To the best of our knowledge a rigorous justification of DSC has not been established yet. Later, in 2007, 2008 Sompornjaroensuk and Kiattikomol [11, 12] transformed the problem with one LPIS to dual series equations, which then by the Hankel transformation are reduced to the form of a Fredholm integral equation. It should be noted that the kernel and the right-hand side of the equation are represented in a series containing Hankel functions of both first and second kinds; therefore, the numerical treatment for this integral equation is very difficult. So, this result is of pure significance. Motivated by these mentioned works, some years ago Dang and Truong [3, 4] proposed a simple iterative method that reduces the problems with one and two LPISs to sequences of boundary value problems for the Poisson equation with weak mixed boundary conditions which can be solved by using the available efficient methods and software for second-order equations. This is achieved due to the combination of a domain decomposition technique and a technique for reduction of the order of differential equations. These techniques were used separately or together in the works [1, 6, 7, 8].

In this study, we develop the method for the problems of rectangular plate with more complex internal supports, namely for a symmetric rectangular support (lying in the center of the plate), asymmetric rectangular support (not lying in the center of the plate) and a symmetric H-shape support.

Suppose that the plates are subjected to a uniformly distributed load (q), their bottom and top edges are clamped, while the left and right edges are simply supported. Then the problems are reduced to the solution of the biharmonic equation $\Delta^2 u = f$ for the deflection $u(x, y)$ inside the plates, where $f = q/D$, D is the flexural rigidity of the plates, with boundary conditions on the plate edges and the conditions on the internal supports. As seen later, in the cases of the symmetric internal supports the problems will be reduced to ones in the domain divided into 3 subdomains. But in the case of asymmetric rectangular plate the domain of the problem must be divided into 9 subdomains. As was shown in [4], the boundary conditions on the fictitious boundary inside the plate are $\dfrac{\partial u}{\partial \nu} = \dfrac{\partial \Delta u}{\partial \nu} = 0$ and the conditions on the internal support are the same as clamped boundary conditions $u = \dfrac{\partial u}{\partial \nu} = 0$. In result of the domain decomposition method the problem for plates on internal supports will be reduced to sequences of boundary value problems for Poisson equation in the rectangles with weakly mixed boundary conditions. The rigorous theoretical proof of the convergence of the iterative method can be done in a similar way as the proof for one LPIS in [4] but due to the complexity of the internal supports we omit it.

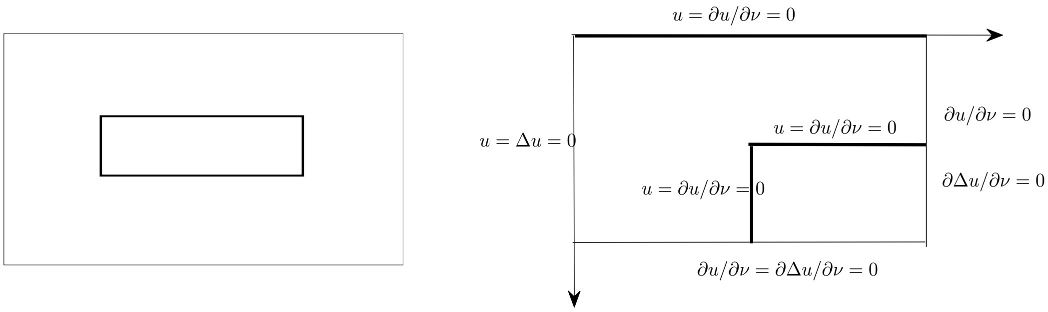
The paper is organized as follows. In Section 2 we consider the plate with a symmetric rectangular internal support. An iterative method for the problem with general boundary conditions is described and the numerical results are reported. In Section 3, omitting the description of iterative method, we briefly present the results of computation for the plate with a symmetric H-shape internal support. In Section 3 we extend the results of Section 2 to the case of asymmetric rectangular internal support. Some concluding remarks are given in the last section.

2. PROBLEM FOR PLATE ON A SYMMETRIC RECTANGULAR INTERNAL SUPPORT

2.1. The problem setting

In this section we consider the problem for plate on a symmetric rectangular internal support, i.e., a rectangular support which lies in the center of the plate as in Figure 1(a). As in [4] and [3], due to the two-fold symmetry it suffices to consider the problem in a quadrant of the plate. Associated conditions are given on the actual and fictitious boundaries, and on the parts of the support inside the quadrant as depicted in Figure 1(b).

Thus, we have to solve the biharmonic equation $\Delta^2 u = f$ in the first quadrant of plate which is denoted by Ω with the boundary conditions given in Figure 1(b). For this purpose



(a) Rectangular support (b) First quadrant of rectangular support

Figure 1. Symmetric rectangular support and its quadrant with boundary conditions

we set the problem with the general boundary conditions, namely, consider the problem

$$\begin{cases} \Delta^2 u = f \quad \text{in} \quad \Omega \setminus (MN \cup MQ), \\ u = g_0 \quad \text{on} \quad S_A \cup S_D \cup MN \cup MQ, \\ \dfrac{\partial u}{\partial \nu} = g_1 \quad \text{on} \quad S_A \cup S_B \cup S_C \cup MN \cup MQ, \\ \Delta u = g_2 \quad \text{on } S_D, \\ \dfrac{\partial}{\partial \nu} \Delta u = g_3 \quad \text{on } S_B \cup S_C, \end{cases} \tag{1}$$

where Ω is the rectangle $(0,a)\times(0,b)$, $S_A, S_B = S_{B1} \cup S_{B2},\ S_C = S_{C1} \cup S_{C2},\ S_D = S_{D1} \cup S_{D2}$ are its sides. See Figure 2.

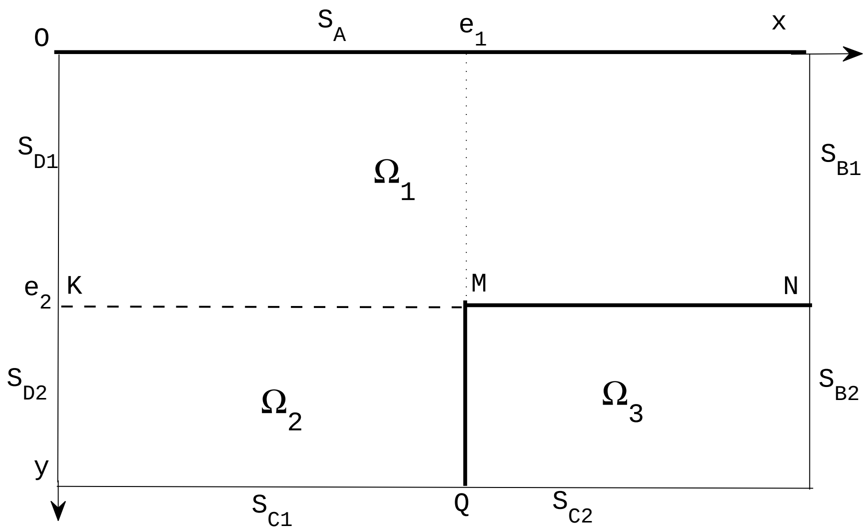


Figure 2. Domain decomposition for the problem considered in a quadrant of plate

In the case if all boundary functions $g_i = 0$ $(i = \overline{0,3})$, the problem models the bending of the first quadrant of a rectangular plate.

2.2. Description of the iterative method

To solve the problem, the domain Ω is divided into three subdomains Ω_1, Ω_2 and Ω_3 as shown in Figure 2. Next, we set $v = \Delta u$ and denote $u_i = u|_{\Omega_i},\ v_i = v|_{\Omega_i}$ and by ν_i denote the outward normal to the boundary of Ω_i $(i = 1, 2, 3)$.

It should be noted that on four sides of the subrectangle Ω_3 there are defined boundary conditions sufficient for solving the biharmonic equation in this subdomain. This is a problem with weakly mixed boundary conditions, which can be performed by iterative method in a similar way as in [1]. After that it remains to solve the biharmonic problem in $\Omega_1 \cup \Omega_2$ by an iterative process. Thus, we must perform two iterative processes in sequence. But we do not handle so. Instead, we suggest the following *combined iterative method* for the problem (2), which is based on the idea of simultaneous update of the boundary functions $\varphi_1 = v_1$ on S_A, $\varphi_2 = v_2$ on MQ, $\varphi_3 = v_3$ on MN, $\xi = \dfrac{\partial v_2}{\partial \nu_2}$ on $KM, \eta = \dfrac{\partial u_2}{\partial \nu_2}$ on KM as follows:
Combined iterative method:

1. Given

$$\begin{aligned} &\varphi_1^{(0)} = 0 \ \text{ on } S_A;\ \varphi_2^{(0)} = 0 \text{ on } MQ,\ \varphi_3^{(0)} = 0 \text{ on } MN,\\ &\xi^{(0)} = 0,\ \eta^{(0)} = 0 \text{ on } KM. \end{aligned} \tag{2}$$

2. Knowing $\varphi_1^{(k)}, \varphi_2^{(k)}, \varphi_3^{(k)}$, $\xi^{(k)}$, $\eta^{(k)}$, $(k = 0, 1, ...)$, solve sequentially problems for $v_3^{(k)}$ and $u_3^{(k)}$ in Ω_3, problems for $v_2^{(k)}$ and $u_2^{(k)}$ in Ω_2, and problems for $v_1^{(k)}$ and $u_1^{(k)}$ in Ω_1:

$$\begin{cases} \Delta v_3^{(k)} &= f \quad \text{in} \quad \Omega_3,\\ v_3^{(k)} &= \varphi_2^{(k)} \quad \text{on} \quad MQ,\\ v_3^{(k)} &= \varphi_3^{(k)} \quad \text{on} \quad MN,\\ \dfrac{\partial v_3^{(k)}}{\partial \nu_3} &= g_3 \quad \text{on} \quad S_{B2} \cup S_{C2}, \end{cases} \qquad \begin{cases} \Delta u_3^{(k)} &= v_3^{(k)} \quad \text{in} \quad \Omega_3,\\ u_3^{(k)} &= g_0 \quad \text{on} \quad MQ \cup MN,\\ \dfrac{\partial u_3^{(k)}}{\partial \nu_3} &= g_1 \quad \text{on} \quad S_{B2} \cup S_{C2}, \end{cases} \tag{3}$$

$$\begin{cases} \Delta v_2^{(k)} &= f \quad \text{in} \quad \Omega_2,\\ v_2^{(k)} &= g_2 \quad \text{on} \quad S_{D2},\\ \dfrac{\partial v_2^{(k)}}{\partial \nu_2} &= \xi^{(k)} \quad \text{on} \quad KM,\\ v_2^{(k)} &= \varphi_2^{(k)} \quad \text{on} \quad MQ,\\ \dfrac{\partial v_2^{(k)}}{\partial \nu_2} &= g_3 \quad \text{on} \quad S_{C1}, \end{cases} \qquad \begin{cases} \Delta u_2^{(k)} &= v_2^{(k)} \quad \text{in} \quad \Omega_2,\\ u_2^{(k)} &= g_0 \quad \text{on} \quad S_{D2} \cup MQ,\\ \dfrac{\partial u_2^{(k)}}{\partial \nu_2} &= \eta^{(k)} \quad \text{on} \quad KM,\\ \dfrac{\partial u_2^{(k)}}{\partial \nu_2} &= g_1 \quad \text{on} \quad S_{C1}, \end{cases} \tag{4}$$

$$\begin{cases} \Delta v_1^{(k)} &= f \quad \text{in} \quad \Omega_1,\\ v_1^{(k)} &= g_2 \quad \text{on} \quad S_{D1},\\ v_1^{(k)} &= \varphi_1^{(k)} \quad \text{on} \quad S_A,\\ \dfrac{\partial v_1^{(k)}}{\partial \nu_1} &= g_2 \text{ on } S_{B1},\\ v_1^{(k)} &= \varphi_3^{(k)} \quad \text{on } MN,\\ v_1^{(k)} &= v_2^{(k)} \quad \text{on } KM, \end{cases} \qquad \begin{cases} \Delta u_1^{(k)} &= v_1^{(k)} \quad \text{in} \quad \Omega_2,\\ u_1^{(k)} &= g_0 \quad \text{on} \quad S_{D1} \cup S_A\\ \dfrac{\partial u_1^{(k)}}{\partial \nu_1} &= g_1 \quad \text{on} \quad S_{B1},\\ u_1^{(k)} &= g_0 \quad \text{on} \quad MN\\ u_1^{(k)} &= u_2^{(k)} \quad \text{on} \quad KM \end{cases} \tag{5}$$

3. Calculate the new approximation

$$\begin{cases} \varphi_1^{(k+1)} = \varphi_1^{(k)} - \tau\Big(\dfrac{\partial u_1^{(k)}}{\partial \nu_1} - g_1\Big) \text{ on } S_A,\ \varphi_2^{(k+1)} = \varphi_2^{(k)} - \tau\Big(\dfrac{\partial u_2^{(k)}}{\partial \nu_2} - g_1\Big) \text{ on } MQ, \\ \varphi_3^{(k+1)} = \varphi_3^{(k)} - \tau\Big(\dfrac{\partial u_3^{(k)}}{\partial \nu_2} - g_1\Big) \text{ on } MN, \\ \xi^{(k+1)} = (1-\theta)\xi^{(k)} - \theta\dfrac{\partial v_2^{(k)}}{\partial \nu_2},\ \eta^{(k+1)} = (1-\theta)\eta^{(k)} - \theta\dfrac{\partial u_2^{(k)}}{\partial \nu_2} \text{ on } KM \end{cases} \tag{6}$$

where τ and θ are iterative parameters to be chosen for guaranteeing the convergence of the iterative process.

The convergence of the above iterative method can be proved in the same way as for the case of one and of two LPIS in [4]. But this is very cumbersome work, therefore we omit it.

2.3. Numerical example

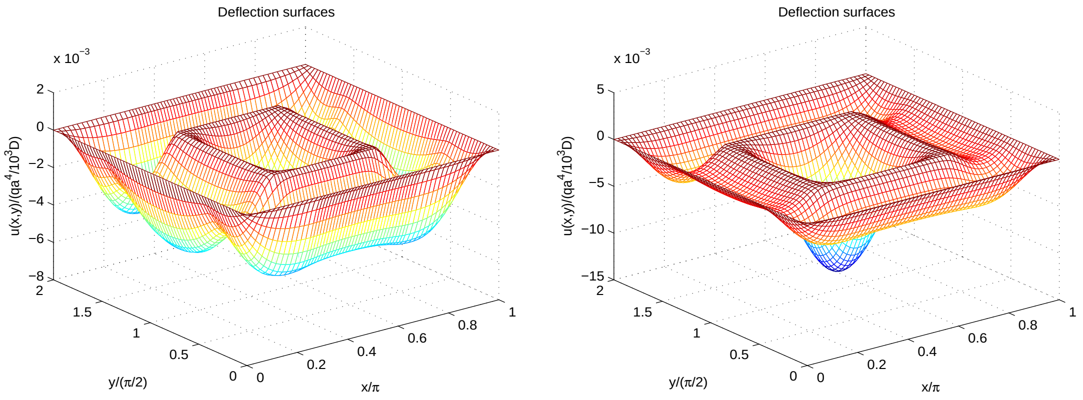
In order to realize the above combined iterative method we use difference schemes of second order of accuracy for mixed boundary value problems (3)-(5) and compute the normal derivatives in (6) by difference derivatives of the same order of accuracy. All computations are performed for uniform grids on rectangles Ω_i $(i = 1, 2, 3)$. The convergence of the discrete analog of the iterative method (2)-(6) was verified on some exact solutions for some sizes of the rectangular support and for some grid sizes. Performed experiments show that the convergence rate depends on the sizes e_1, e_2 (see Figure 2) and the values of the iteration parameters τ and θ. From the results of the experiments we observe that the values $\tau = 0.9$ and $\theta = 0.95$ give good convergence. The number of iterations for achieving the accuracy $\|u^{(k)} - u\|_\infty \le 10^{-4}$, where u is the exact solution, changes from 30 to 45.

Using the chosen above iteration parameters τ and θ we solve the problem for computing the deflection of the symmetric rectangular support. As said in the end of the previous subsection, for the problem of bending of the plate $g_i = 0$ $(i = \overline{0,3})$. We perform numerical experiments for plate of the sizes $\pi \times \pi$ with the flexural rigidity $D = 0.0057$ under the load $q = 0.3$. The surfaces of deflection of the whole plate for some sizes of the support under uniform load are depicted in Figure 3(a) and 3(b).

3. PROBLEMS FOR PLATES ON H-SHAPE INTERNAL SUPPORT

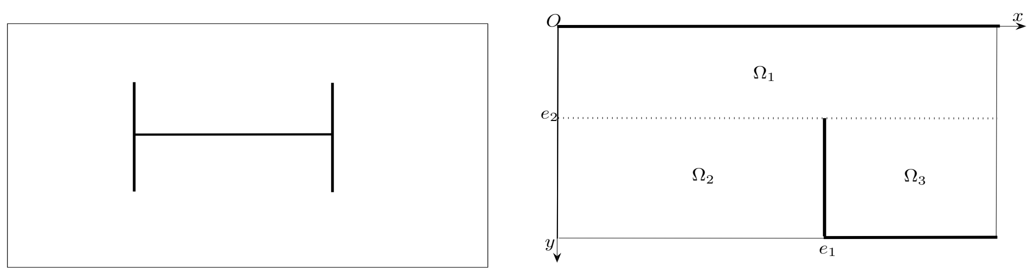
As in the case of a symmetric rectangular internal support, the problem for plate on a H-shape support (see Figure 4(a)) is reduced to boundary value problems with strongly mixed boundary conditions in a quadrant of the plate. For the latter one, the computational domain can also be divided into three subdomains (rectangles) as shown in Figure 4(b).

The iterative method combining decrease of the equation order and domain decomposition for these problems is constructed in an analogous way. The results of computation of deflection surfaces for some sizes of H-shape support are given in Figure 5(a) and 5(b).



(a) (b)

Figure 3. The surfaces of deflection of the whole plate for e_1/π, e_2/π equal $0.30, 0.50$ (a) and $0.25, 0.50$ (b), respectively



(a) H-shape support (b) First quadrant of H-shape support

Figure 4. H-shape support and its first quadrant

4. PROBLEM FOR PLATE ON AN ASYMMETRIC RECTANGULAR INTERNAL SUPPORT

4.1. The problem setting

Now we consider the plate on an asymmetric rectangular internal support when the support lies not in the middle of the plate. In this case the problem has no symmetry, so it cannot be reduced to one in a quadrant of the plate.

In order to construct a solution method for the problem, as in the previous section, we consider it with general boundary conditions. Namely, consider the following BVP

$$\begin{cases} \Delta^2 u = f & \text{in} \quad \Omega, \\ u = g_0 & \text{on} \quad OF \cup HC \cup FH \cup OC \cup MN \cup MP \cup PQ \cup NQ, \\ \dfrac{\partial u}{\partial \nu} = g_1 & \text{on} \quad FH \cup OC \cup MN \cup MP \cup PQ \cup NQ, \\ \Delta u = g_2 & \text{on} \quad OF \cup HC. \end{cases} \tag{7}$$

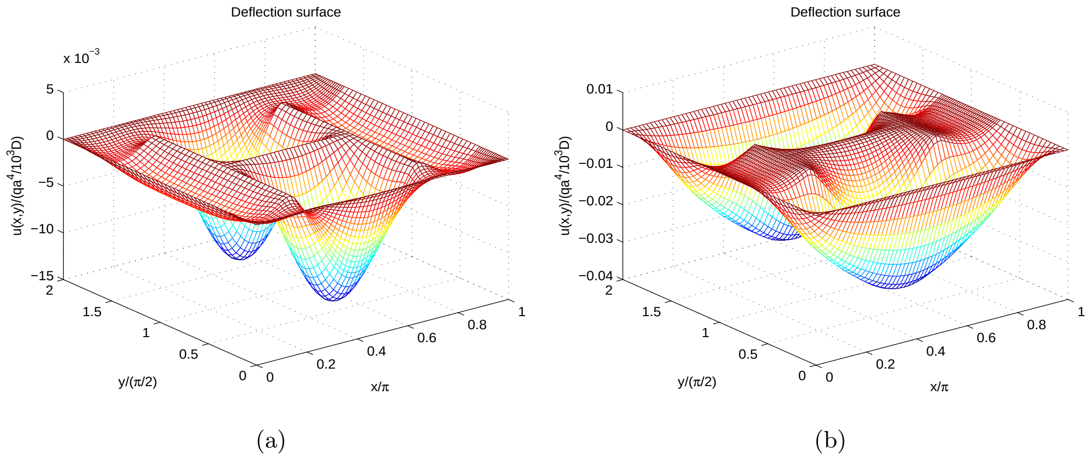


Figure 5. The surfaces of deflection of the plate on a H-shape support for e_1/π, e_2/π equal 0.15, 0.30 (a) and 0.30, 0.40 (b), respectively

where $\Omega = \cup_{i=1}^{9}\Omega_i$ (the interior of the plate excluding the internal support). See Figure 6.

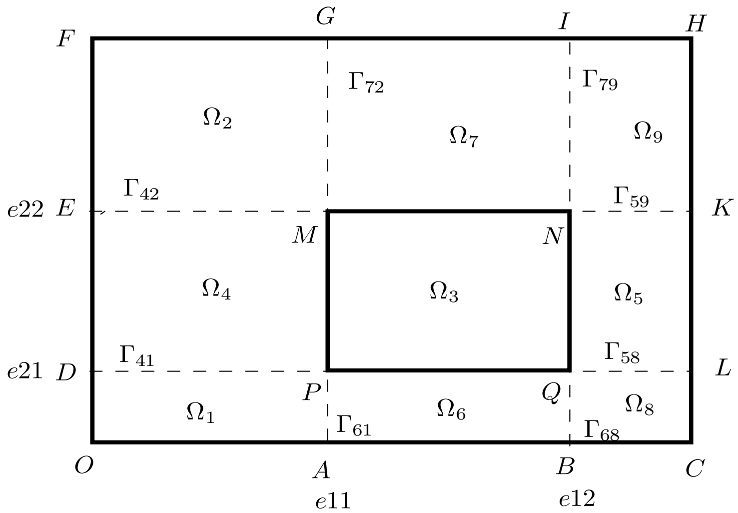


Figure 6. Domain decomposition for the problem for plate on asymmetric rectangular internal support

4.2. Solution method

To solve the problem, the domain Ω is divided into 9 subdomains $\{\Omega_i, i = 1, 2, ...9\}$ by the fictitious boundaries Γ_{41}, Γ_{42}, Γ_{58}, Γ_{59}, Γ_{61}, Γ_{68}, Γ_{72}, Γ_{79} as described in Figure 6.

As usual, we set: $u_i = u|_{\Omega_i}$, $v_i = \Delta u_i|_{\Omega_i}$, $i = 1, 2, ..., 9$.
Further, set

$$\xi_{41} = \frac{\partial v_4}{\partial \nu}|_{\Gamma_{41}}, \quad \eta_{41} = \frac{\partial u_4}{\partial \nu}|_{\Gamma_{41}},$$
$$\xi_{42} = \frac{\partial v_4}{\partial \nu}|_{\Gamma_{42}}, \quad \eta_{42} = \frac{\partial u_4}{\partial \nu}|_{\Gamma_{42}},$$
$$\xi_{58} = \frac{\partial v_5}{\partial \nu}|_{\Gamma_{58}}, \quad \eta_{58} = \frac{\partial u_5}{\partial \nu}|_{\Gamma_{58}},$$
$$\xi_{59} = \frac{\partial v_5}{\partial \nu}|_{\Gamma_{59}}, \quad \eta_{59} = \frac{\partial u_5}{\partial \nu}|_{\Gamma_{59}},$$
$$\xi_{72} = \frac{\partial v_7}{\partial \nu}|_{\Gamma_{72}}, \quad \eta_{72} = \frac{\partial u_7}{\partial \nu}|_{\Gamma_{72}},$$
$$\xi_{79} = \frac{\partial v_7}{\partial \nu}|_{\Gamma_{79}}, \quad \eta_{79} = \frac{\partial u_7}{\partial \nu}|_{\Gamma_{79}},$$
$$\xi_{61} = \frac{\partial v_6}{\partial \nu}|_{\Gamma_{61}}, \quad \eta_{61} = \frac{\partial u_6}{\partial \nu}|_{\Gamma_{61}},$$
$$\xi_{68} = \frac{\partial v_6}{\partial \nu}|_{\Gamma_{68}}, \quad \eta_{68} = \frac{\partial u_6}{\partial \nu}|_{\Gamma_{68}},$$

and set

$$\begin{aligned}
\varphi_1 &= v_1|_{OA}, \\
\varphi_2 &= v_2|_{FG}, \\
\varphi_6 &= v_6|_{AB}, \\
\varphi_7 &= v_7|_{GI}, \\
\varphi_8 &= v_8|_{BC}, \\
\varphi_9 &= v_9|_{IH}, \\
\varphi_{34} &= v_3|_{MP}, \\
\varphi_{43} &= v_4|_{MP}, \\
\varphi_{35} &= v_3|_{NQ}, \\
\varphi_{53} &= v_5|_{NQ}, \\
\varphi_{36} &= v_3|_{PQ}, \\
\varphi_{63} &= v_6|_{PQ}, \\
\varphi_{37} &= v_3|_{MN}, \\
\varphi_{73} &= v_7|_{MN},
\end{aligned}$$

$$I = \{1, 2, 6, 7, 8, 9, 34, 43, 35, 53, 36, 63, 37, 73\},$$
$$J = \{41, 42, 58, 59, 61, 68, 72, 79\}.$$

Consider the following parallel iterative method with the idea of simultaneous update of ξ, η, φ on boundaries:

1. Given starting approximations $\varphi_i^{(0)}$, $i \in I$; $\xi_j^{(0)}$, $\eta_j^{(0)}$, $j \in J$ on respective boundaries, for example, $\varphi_i^{(0)} = 0$, $\xi_j^{(0)} = 0$, $\eta_j^{(0)} = 0$.

2. Knowing $\varphi_i^{(k)}$, $\xi_j^{(k)}$, $\eta_j^{(k)}$ $(k = 0, 1, 2, ...)$, solve in parallel five problems for $v_3^{(k)}$, $u_3^{(k)}$ in Ω_3, problems for $v_4^{(k)}$, $u_4^{(k)}$ in Ω_4, problems for $v_5^{(k)}$, $u_5^{(k)}$ in Ω_5, problems for $v_6^{(k)}$, $u_6^{(k)}$ in Ω_6, problems for $v_7^{(k)}$, $u_7^{(k)}$ in Ω_7:

   On domain Ω_3

$$\begin{cases} \Delta v_3^{(k)} = f & \text{in} \quad \Omega_3, \\ v_3^{(k)} = \varphi_{34}^{(k)} & \text{on} \quad MP, \\ v_3^{(k)} = \varphi_{35}^{(k)} & \text{on} \quad NQ, \\ v_3^{(k)} = \varphi_{36}^{(k)} & \text{on} \quad PQ, \\ v_3^{(k)} = \varphi_{37}^{(k)} & \text{on} \quad MN, \end{cases} \qquad \begin{cases} \Delta u_3^{(k)} = v_3^{(k)} & \text{in} \quad \Omega_3, \\ u_3^{(k)} = g_0 & \text{on} \quad \partial\Omega_3. \end{cases} \tag{8}$$

   On domain Ω_4

$$\begin{cases} \Delta v_4^{(k)} = f & \text{in} \quad \Omega_4, \\ v_4^{(k)} = g_2 & \text{on} \quad ED, \\ v_4^{(k)} = \varphi_{43}^{(k)} & \text{on} \quad MP, \\ \frac{\partial v_4^{(k)}}{\partial \nu} = \xi_{41}^{(k)} & \text{on} \quad \Gamma_{41}, \\ \frac{\partial v_4^{(k)}}{\partial \nu} = \xi_{42}^{(k)} & \text{on} \quad \Gamma_{42}, \end{cases} \qquad \begin{cases} \Delta u_4^{(k)} = v_4^{(k)} & \text{in} \quad \Omega_4, \\ u_4^{(k)} = g_0 & \text{on} \quad ED \cup MP, \\ \frac{\partial u_4^{(k)}}{\partial \nu} = \eta_{41}^{(k)} & \text{on} \quad \Gamma_{41}, \\ \frac{\partial u_4^{(k)}}{\partial \nu} = \eta_{42}^{(k)} & \text{on} \quad \Gamma_{42}. \end{cases} \tag{9}$$

   On domain Ω_5

$$\begin{cases} \Delta v_5^{(k)} = f & \text{in} \quad \Omega_5, \\ v_5^{(k)} = g_2 & \text{on} \quad KL, \\ v_5^{(k)} = \varphi_{53}^{(k)} & \text{on} \quad NQ, \\ \frac{\partial v_5^{(k)}}{\partial \nu} = \xi_{58}^{(k)} & \text{on} \quad \Gamma_{58}, \\ \frac{\partial v_5^{(k)}}{\partial \nu} = \xi_{59}^{(k)} & \text{on} \quad \Gamma_{59}, \end{cases} \qquad \begin{cases} \Delta u_5^{(k)} = v_5^{(k)} & \text{in} \quad \Omega_5, \\ u_5^{(k)} = g_0 & \text{on} \quad NQ \cup KL, \\ \frac{\partial u_5^{(k)}}{\partial \nu} = \eta_{58}^{(k)} & \text{on} \quad \Gamma_{58}, \\ \frac{\partial u_5^{(k)}}{\partial \nu} = \xi_{59}^{(k)} & \text{on} \quad \Gamma_{59}. \end{cases} \tag{10}$$

   On domain Ω_6

$$\begin{cases} \Delta v_6^{(k)} = f & \text{in} \quad \Omega_6, \\ v_6^{(k)} = \varphi_6^{(k)} & \text{on} \quad AB, \\ v_6^{(k)} = \varphi_{63}^{(k)} & \text{on} \quad PQ, \\ \frac{\partial v_6^{(k)}}{\partial \nu} = \xi_{61}^{(k)} & \text{on} \quad \Gamma_{61}, \\ \frac{\partial v_6^{(k)}}{\partial \nu} = \xi_{68}^{(k)} & \text{on} \quad \Gamma_{68}, \end{cases} \qquad \begin{cases} \Delta u_6^{(k)} = v_6^{(k)} & \text{in} \quad \Omega_6, \\ u_6^{(k)} = g_0 & \text{on} \quad PQ \cup AB, \\ \frac{\partial u_6^{(k)}}{\partial \nu} = \eta_{61}^{(k)} & \text{on} \quad \Gamma_{61}, \\ \frac{\partial u_6^{(k)}}{\partial \nu} = \eta_{68}^{(k)} & \text{on} \quad \Gamma_{68}. \end{cases} \tag{11}$$

On domain Ω_7

$$\begin{cases} \Delta v_7^{(k)} & = f \quad \text{in} \quad \Omega_7, \\ v_7^{(k)} & = \varphi_{73}^{(k)} \quad \text{on} \quad MN, \\ v_7^{(k)} & = \varphi_7^{(k)} \quad \text{on} \quad GI, \\ \frac{\partial v_7^{(k)}}{\partial \nu} & = \xi_{72}^{(k)} \quad \text{on} \quad \Gamma_{72}, \\ \frac{\partial v_7^{(k)}}{\partial \nu} & = \xi_{79}^{(k)} \quad \text{on} \quad \Gamma_{79}, \end{cases} \qquad \begin{cases} \Delta u_7^{(k)} & = v_7^{(k)} \quad \text{in} \quad \Omega_7, \\ u_7^{(k)} & = g_0 \quad \text{on} \quad GI \cup MN, \\ \frac{\partial u_7^{(k)}}{\partial \nu} & = \eta_{72}^{(k)} \quad \text{on} \quad \Gamma_{72}, \\ \frac{\partial u_7^{(k)}}{\partial \nu} & = \eta_{79}^{(k)} \quad \text{on} \quad \Gamma_{79}. \end{cases} \tag{12}$$

3. Solve parallel problems for $v_1^{(k)}$, $u_1^{(k)}$ in Ω_1, problems for $v_2^{(k)}$, $u_2^{(k)}$ in Ω_2, problems for $v_8^{(k)}$, $u_8^{(k)}$ in Ω_8, problems for $v_9^{(k)}$, $u_9^{(k)}$ in Ω_9
On domain Ω_1

$$\begin{cases} \Delta v_1^{(k)} & = f \quad \text{in} \quad \Omega_1, \\ v_1^{(k)} & = g_2 \quad \text{on} \quad OD \cup OA, \\ v_1^{(k)} & = v_6^{(k)} \quad \text{on} \quad \Gamma_{61}, \\ v_1^{(k)} & = v_4^{(k)} \quad \text{on} \quad \Gamma_{41}, \end{cases} \qquad \begin{cases} \Delta u_1^{(k)} & = v_1^{(k)} \quad \text{in}\, \Omega_1, \\ u_1^{(k)} & = g_0 \quad \text{on} \quad OD \cup OA, \\ u_1^{(k)} & = u_4^{(k)} \quad \text{on} \quad \Gamma_{41}, \\ u_1^{(k)} & = u_6^{(k)} \quad \text{on} \quad \Gamma_{61}. \end{cases} \tag{13}$$

On domain Ω_2

$$\begin{cases} \Delta v_2^{(k)} & = f \quad \text{in} \quad \Omega_2, \\ v_2^{(k)} & = g_2 \quad \text{on} \quad EF \cup FG, \\ v_2^{(k)} & = v_4^{(k)} \quad \text{on} \quad \Gamma_{42}, \\ v_2^{(k)} & = v_7^{(k)} \quad \text{on} \quad \Gamma_{72}, \end{cases} \qquad \begin{cases} \Delta u_2^{(k)} & = v_2^{(k)} \quad \text{in} \quad \Omega_2, \\ u_2^{(k)} & = g_0 \quad \text{on} \quad EF \cup FG, \\ u_2^{(k)} & = u_4^{(k)} \quad \text{on} \quad \Gamma_{41}, \\ u_2^{(k)} & = u_7^{(k)} \quad \text{on} \quad \Gamma_{72}. \end{cases} \tag{14}$$

On domain Ω_8

$$\begin{cases} \Delta v_8^{(k)} & = f \quad \text{in} \quad \Omega_8, \\ v_8^{(k)} & = g_2 \quad \text{on} \quad BC \cup LC, \\ v_8^{(k)} & = v_6^{(k)} \quad \text{on} \quad \Gamma_{68}, \\ v_8^{(k)} & = v_5^{(k)} \quad \text{on} \quad \Gamma_{58}, \end{cases} \qquad \begin{cases} \Delta u_8^{(k)} & = v_8^{(k)} \quad \text{in}\, \Omega_8, \\ u_8^{(k)} & = g_0 \quad \text{on}\, BC \cup CL, \\ u_8^{(k)} & = u_6^{(k)} \quad \text{on}\, \Gamma_{68}, \\ u_8^{(k)} & = u_5^{(k)} \quad \text{on}\, \Gamma_{58}. \end{cases} \tag{15}$$

On domain Ω_9

$$\begin{cases} \Delta v_9^{(k)} & = f \quad \text{in}\, \Omega_9, \\ v_9^{(k)} & = g_2 \quad \text{on}\, IH \cup HK, \\ v_9^{(k)} & = v_5^{(k)} \quad \text{on}\, \Gamma_{59}, \\ v_9^{(k)} & = v_7^{(k)} \quad \text{on}\, \Gamma_{79}, \end{cases} \qquad \begin{cases} \Delta u_9^{(k)} & = v_9^{(k)} \quad \text{in}\, \Omega_9, \\ u_9^{(k)} & = g_0 \quad \text{on}\, IH \cup HK, \\ u_9^{(k)} & = u_5^{(k)} \quad \text{on}\, \Gamma_{59}, \\ u_9^{(k)} & = u_7^{(k)} \quad \text{on}\, \Gamma_{79}. \end{cases} \tag{16}$$

4. Calculate the new approximations

$$
\begin{cases}
\varphi_1^{(k+1)} = \varphi_1^{(k)} - \tau\Big(\dfrac{\partial u_1^{(k)}}{\partial \nu} - g_1\Big) \text{ on } OA, \ \varphi_2^{(k+1)} = \varphi_2^{(k)} - \tau\Big(\dfrac{\partial u_2^{(k)}}{\partial \nu} - g_1\Big) \text{ on } FG,\\
\varphi_6^{(k+1)} = \varphi_6^{(k)} - \tau\Big(\dfrac{\partial u_6^{(k)}}{\partial \nu} - g_1\Big) \text{ on } AB, \ \varphi_7^{(k+1)} = \varphi_7^{(k)} - \tau\Big(\dfrac{\partial u_7^{(k)}}{\partial \nu} - g_1\Big) \text{ on } GI,\\
\varphi_8^{(k+1)} = \varphi_8^{(k)} - \tau\Big(\dfrac{\partial u_8^{(k)}}{\partial \nu} - g_1\Big) \text{ on } BC, \ \varphi_9^{(k+1)} = \varphi_9^{(k)} - \tau\Big(\dfrac{\partial u_9^{(k)}}{\partial \nu} - g_1\Big) \text{ on } IH,\\
\varphi_{34}^{(k+1)} = \varphi_{34}^{(k)} - \tau\Big(\dfrac{\partial u_3^{(k)}}{\partial \nu} - g_1\Big) \text{ on } MP, \ \varphi_{43}^{(k+1)} = \varphi_{43}^{(k)} - \tau\Big(\dfrac{\partial u_4^{(k)}}{\partial \nu} - g_1\Big) \text{ on } MP,\\
\varphi_{35}^{(k+1)} = \varphi_{35}^{(k)} - \tau\Big(\dfrac{\partial u_3^{(k)}}{\partial \nu} - g_1\Big) \text{ on } NQ, \ \varphi_{53}^{(k+1)} = \varphi_{53}^{(k)} - \tau\Big(\dfrac{\partial u_5^{(k)}}{\partial \nu} - g_1\Big) \text{ on } NQ,\\
\varphi_{37}^{(k+1)} = \varphi_{37}^{(k)} - \tau\Big(\dfrac{\partial u_3^{(k)}}{\partial \nu} - g_1\Big) \text{ on } MN, \ \varphi_{73}^{(k+1)} = \varphi_{73}^{(k)} - \tau\Big(\dfrac{\partial u_7^{(k)}}{\partial \nu} - g_1\Big) \text{ on } MN,\\
\varphi_{36}^{(k+1)} = \varphi_{36}^{(k)} - \tau\Big(\dfrac{\partial u_3^{(k)}}{\partial \nu} - g_1\Big) \text{ on } PQ, \ \varphi_{63}^{(k+1)} = \varphi_{63}^{(k)} - \tau\Big(\dfrac{\partial u_6^{(k)}}{\partial \nu} - g_1\Big) \text{ on } PQ,\\
\xi_{41}^{(k+1)} = \theta\xi_{41}^{(k)} - (1-\theta)\dfrac{\partial v_1^{(k)}}{\partial \nu}, \quad \eta_{41}^{(k+1)} = \theta\eta_{41}^{(k)} - (1-\theta)\dfrac{\partial u_1^{(k)}}{\partial \nu} \text{ on } \Gamma_{41},\\
\xi_{42}^{(k+1)} = \theta\xi_{42}^{(k)} - (1-\theta)\dfrac{\partial v_2^{(k)}}{\partial \nu}, \quad \eta_{42}^{(k+1)} = \theta\eta_{42}^{(k)} - (1-\theta)\dfrac{\partial u_2^{(k)}}{\partial \nu} \text{ on } \Gamma_{42},\\
\xi_{58}^{(k+1)} = \theta\xi_{58}^{(k)} - (1-\theta)\dfrac{\partial v_8^{(k)}}{\partial \nu}, \quad \eta_{58}^{(k+1)} = \theta\eta_{58}^{(k)} - (1-\theta)\dfrac{\partial u_8^{(k)}}{\partial \nu} \text{ on } \Gamma_{58},\\
\xi_{59}^{(k+1)} = \theta\xi_{59}^{(k)} - (1-\theta)\dfrac{\partial v_9^{(k)}}{\partial \nu}, \quad \eta_{59}^{(k+1)} = \theta\eta_{59}^{(k)} - (1-\theta)\dfrac{\partial u_9^{(k)}}{\partial \nu} \text{ on } \Gamma_{59},\\
\xi_{61}^{(k+1)} = \theta\xi_{61}^{(k)} - (1-\theta)\dfrac{\partial v_1^{(k)}}{\partial \nu}, \quad \eta_{61}^{(k+1)} = \theta\eta_{61}^{(k)} - (1-\theta)\dfrac{\partial u_1^{(k)}}{\partial \nu} \text{ on } \Gamma_{61},\\
\xi_{68}^{(k+1)} = \theta\xi_{68}^{(k)} - (1-\theta)\dfrac{\partial v_8^{(k)}}{\partial \nu}, \quad \eta_{68}^{(k+1)} = \theta\eta_{68}^{(k)} - (1-\theta)\dfrac{\partial u_8^{(k)}}{\partial \nu} \text{ on } \Gamma_{68},\\
\xi_{72}^{(k+1)} = \theta\xi_{72}^{(k)} - (1-\theta)\dfrac{\partial v_2^{(k)}}{\partial \nu}, \quad \eta_{72}^{(k+1)} = \theta\eta_{72}^{(k)} - (1-\theta)\dfrac{\partial u_2^{(k)}}{\partial \nu} \text{ on } \Gamma_{72},\\
\xi_{79}^{(k+1)} = \theta\xi_{79}^{(k)} - (1-\theta)\dfrac{\partial v_9^{(k)}}{\partial \nu}, \quad \eta_{79}^{(k+1)} = \theta\eta_{79}^{(k)} - (1-\theta)\dfrac{\partial u_9^{(k)}}{\partial \nu} \text{ on } \Gamma_{79},
\end{cases}
\tag{17}
$$

where τ and θ are iterative parameters to be selected for guaranteeing the convergence of the iterative process.

4.3. Numerical example

For the problem of plate on asymmetric rectangular internal support we verify the convergence of the discrete analog of the parallel iterative process (8)-(17) on some exact solutions for some sizes of the rectangular support. The performed experments show that the convergence rate depends on the sizes (e_{11}, e_{12}), (e_{21}, e_{22}) and the values of the iteration parameters

τ and θ. The numerical results show that the values of the iteration parameters, which give good convergence of the iterative method are $\tau = 0.95$ and $\theta = 0.75$.

Using the chosen above iteration parameters τ and θ we solve the problem for computing the deflection of the asymmetric rectangular support. The sizes of the plate now are normalized as 1×1. The surfaces of deflection $u(x, y)$ of the whole plate for some sizes of the support under uniform constant unit load are depicted in Figure 7 and 8.

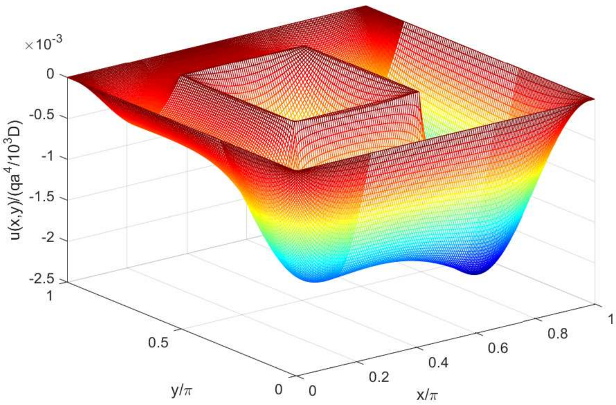


Figure 7. The surfaces of deflection of the whole plate for $e11/\pi = 1/4,\ e12/\pi = 2/3,\ e21/\pi = 1/3,\ e22/\pi = 5/6$

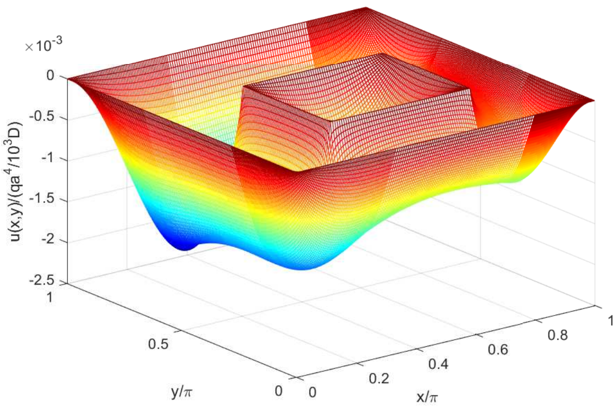


Figure 8. The surfaces of deflection of the whole plate for $e11/\pi = 1/3,\ e12/\pi = 4/5,\ e21/\pi = 2/7,\ e22/\pi = 2/3$

5. CONCLUDING REMARKS

In this study, we have just constructed an iterative method for finding the solutions to the problems for plates having a symmetric or asymmetric rectangular support and H-shape internal support. The idea of the method is to lead the problems to sequences of problems for the Poisson equation with weakly mixed boundary conditions, which can be efficiently solved numerically by the difference method. The performed numerical results demonstrate the effectiveness of the iterative method. This method based on the combination of the reduction of order of differential equation and domain decomposition method can be applied to the plates having combination of some internal supports of the types considered in this paper and in [3, 4].

It is remarked that the studied problems for plates on internal supports are linear boundary value problems for biharmonic equation. Their difficulty and complexity are in conditions on the supports inside the plates. Recently, many authors concentrate their attention to solving nonlinear biharmonic equations (see e.g. [2, 9, 10, 13, 14]). Especially, in [5] an iterative method was studied for the problem of nonlinear biharmonic equation describing the plate rested on nonlinear foundation. So, if the foundation is on internal supports then combine the method in [5] with the method of [3, 4], and the present work, we think that in principle we can solve this complicated problem. This is a topic of our research in the future.

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