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NEW DISSIMILARITY MEASURES ON PICTURE FUZZY SETS AND APPLICATIONS*

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Abstract. The dissimilarity measures between fuzzy sets/intuitionistic fuzzy sets/picture fuzzy sets are studied and applied in various matters. In this paper, we propose some new dissimilarity measures on picture fuzzy sets. These new dissimilarity measures overcome the restrictions of all existing dissimilarity measures on picture fuzzy sets. After that, we apply these new measures to the pattern recognition problems. Finally, we introduce a multi-criteria decision making (MCDM) method that uses the new dissimilarity measures and apply them in the supplier selection problems.

Keywords. Picture fuzzy set; Dissimilarity measure; MCDM.

1. INTRODUCTION

The ranking of subjects is very important in the decision-making process. The ranking can be based on measures such as the similarity measures, the distance measures or dissimilarity measures. In practical problems, fuzzy set and intuitionistic fuzzy set have been widely used [3, 9, 12, 18, 19, 21, 22]. The dissimilarity measures between them were also studied and applied in various matters [10, 14, 16, 17, 20, 23].

In 2014, Picture fuzzy set was introduced by Cuong [4]. It has three memberships: a degree of positive membership, a degree of negative membership, and a degree of neutral membership. Picture fuzzy set is a generality of fuzzy set [42] and intuitionistic fuzzy set [1]. Today, picture fuzzy set has been studied and applied widely in many fields [2, 6, 8, 11, 24, 25, 26, 37], especially in clustering problems [13, 15, 27, 28, 29, 32, 33, 31, 36]. Hoa et al. [13] used picture fuzzy sets to apply for Geographic Data Clustering. Thao and Dinh approximated the picture fuzzy set on the crisp approximation spaces to give results as rough picture fuzzy sets and picture fuzzy topologies [30]. Dinh et al. investigated the picture fuzzy set database [35]. Cuong and Hai [5] studied some fuzzy logic operators for picture fuzzy sets. The cross-entropy and similarity measures on picture fuzzy sets were studied by Wei and applied in MCDM [38, 41, 39, 40]. As opposed to the similarity measures, the dissimilarity measures on picture fuzzy sets were first introduced by Dinh et al. in 2017 [7, 34]. But these

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dissimilarity measures have certain restrictions (detail in Example 1, Section 3). To continue with the idea of the dissimilarity measures on picture fuzzy sets in practical applications, we propose some new dissimilarity measures to overcome the mentioned restrictions and apply them in practical problems (detail in Example 1 and Example 2, Section 3). In the similarity measure, if the value of the similarity measure between two objects is greater, the two objects are more likely to be identical. On the contrary, in the dissimilarity measure, if the value of the dissimilarity measure between two objects is smaller, the two objects are considered to be the same.

In this paper, we introduce some new dissimilarity measures on picture fuzzy sets. The paper is organized as follows: the concept of picture fuzzy set is recalled in Section 2. The dissimilarity measures on PFS-sets are defined in Section 3. After that, we introduce an application of the dissimilarity measures between PFS-sets for the pattern recognition in Section 4. We also propose a multi-criteria decision making using new dissimilarity measures and apply this MCDM to select the supplier in Section 5.

2. BASIC NOTIONS

Definition 1. (see [4]) Picture fuzzy set on a universe U is an object of the form $A = \{(u, \mu_A(u),\ \eta_A(u),\ \gamma_A(u))|u \in U\}$, where μ_A is a membership function, η_A is neutral membership function, γ_A is non-membership function of A and $0 \leq \mu_A(u) + \eta_A(u) + \gamma_A(u) \leq 1$ for all $u \in U$.

Further, we denote by $PFS(U)$ the collection of picture fuzzy sets on U with $U = \{(u, 1, 0, 0)|u \in U\}$ and $\emptyset = \{(u, 0, 0, 1)|u \in U\}$ for all $u \in U$.

For $A, B \in PFS(U)$ and for all $u \in U$ consider some algebraic operators for picture fuzzy sets as follows:

+ Union of A and B: $A \cup B = \{(u, \mu_{A\cup B}(u),\ \eta_{A\cup B}(u),\ \gamma_{A\cup B}(u))|u \in U\}$, where
  $\mu_{A\cup B}(u) = \max\{\mu_A(u),\ \mu_B(u)\}$,
  $\eta_{A\cup B}(u) = \min\{\eta_A(u),\ \eta_B(u)\}$ and
  $\gamma_{A\cup B}(u) = \min\{\gamma_A(u),\ \gamma_B(u)\}$.

+ Intersection of A and B: $A \cap B = \{(u,\ \mu_{A\cap B}(u),\ \eta_{A\cap B}(u),\ \gamma_{A\cap B}(u))|u \in U\}$, where
  $\mu_{A\cap B}(u) = \min\{\mu_A(u),\ \mu_B(u)\}$,
  $\eta_{A\cap B}(u) = \min\{\eta_A(u),\ \eta_B(u)\}$, and
  $\gamma_{A\cap B}(u) = \max\{\gamma_A(u),\ \gamma_B(u)\}$.

+ Subset: $A \subset B$ iff $\mu_A(u) \leq \mu_B(u),\ \eta_A(u) \leq \eta_B(u)$ and $\gamma_A(u) \geq \gamma_B(u)$.

3. NEW DISSIMILARITY MEASURES ON PICTURE FUZZY SETS

In this section, we introduce concept of dissimilarity measure on picture fuzzy sets.

Definition 2. A function $DM : PFS(U) \times PFS(U) \to R$ is a dissimilarity measure on PFS-sets if it satisfies the following properties:

+ PF-Diss 1: $0 \leq DM(A,B) \leq 1$;

+ PF-Diss 2: $DM(A,B) = DM(B,A)$;

+ PF-Diss 3: $DM(A,A) = 0$;

+ PF-Diss 4: If $A \subset B \subset C$ then $DM(A,C) \geq \max\{DM(A,B), DM(B,C)\}$ for all $A,B,C \in PFS(U)$.

In [7, 34] Dinh et al. gave some dissimilarity measures on picture fuzzy sets as follows.

Definition 3. [7, 34] Let $U = \{u_1, u_2, ..., u_n\}$ be a universe set. Given two picture fuzzy sets $A, B \in PFS(U)$. We define some dissimilarity measures on picture fuzzy sets as follows:

$$DM_C(A,B) = \frac{1}{3n}\sum_{i=1}^{n}\left[|S_A(u_i) - S_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)|\right] \quad (1)$$

where $S_A(u_i) = |\mu_A(u_i) - \gamma_A(u_i)|$ and $S_B(u_i) = |\mu_B(u_i) - \gamma_B(u_i)|$.

$$DM_H(A,B) = \frac{1}{3n}\sum_{i=1}^{n}\left[|\mu_A(u_i) - \mu_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)| + |\gamma_A(u_i) - \gamma_B(u_i)|\right]. \quad (2)$$

$$DM_L(A,B) =$$

$$\frac{1}{5n}\sum_{i=1}^{n}\Big[|S_A(u_i) - S_B(u_i)| + |\mu_A(u_i) - \mu_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)| + |\gamma_A(u_i) - \gamma_B(u_i)|\Big]. \quad (3)$$

$$DM_O(A,B) = \frac{1}{\sqrt{3n}}\sum_{i=1}^{n}\Big[|\mu_A(u_i) - \mu_B(u_i)|^2 + |\eta_A(u_i) - \eta_B(u_i)|^2 + |\gamma_A(u_i) - \gamma_B(u_i)|^2\Big]^{\frac{1}{2}}. \quad (4)$$

These measures have a restriction, which is shown in the following example.

Example 1. Assume that there are two patterns denoted by picture fuzzy sets on $U = \{u_1, u_2\}$ as follows: Let

$A_1 = \{(u_1, 0, 0, 0), (u_2, 0.2, 0.2, 0.1)\}$, $A_2 = \{(u_1, 0, 0.1, 0.1), (u_2, 0.1, 0.1, 0.1)\}$ and $B = \{(u_1, 0, 0.1, 0), (u_2, 0, 0.3, 0.1)\}$.

Question: Which class of pattern does B belong to?

+ Case 1: If using $DM_C(A,B)$ in eq.(1) then

$$DM_C(A_1, B) = DM_C(A_2, B) = 0.066666667.$$

+ Case 2: If using $DM_H(A,B)$ in eq.(2) then

$$DM_H(A_1, B) = DM_H(A_2, B) = 0.066666667.$$

+ Case 3: If using $DM_L(A,B)$ in eq.(3) then $DM_L(A_1, B) = DM_L(A_2, B) = 0.06$.

+ Case 4: If using $DM_O(A,B)$ in eq.(4) then

$$DM_O(A_1, B) = DM_O(A_2, B) = 0.132111922.$$

We do not know which class of pattern B belongs to when using these dissimilarity measures.

This drawback suggests us to improve the dissimilarity measure on picture fuzzy sets.

Suppose $U = \{u_1, u_2, ..., u_n\}$ is an universe set. For any $A, B \in PFS(U)$, we denote

$$R_A(u_j) = \mu_A(u_j) - \gamma_A(u_j), R_B(u_j) = \mu_B(u_j) - \gamma_B(u_j),$$

$$S_A(u_j) = \eta_A(u_j) - \gamma_A(u_j), S_B(u_j) = \eta_B(u_j) - \gamma_B(u_j),$$

and

$$D_j(A, B) = \frac{|R_A(u_j) - R_B(u_j)| + |S_A(u_j) - S_B(u_j)|}{4} \quad (5)$$

for all $j = 1, 2, ..., n$.

Definition 4. Let $U = \{u_1, u_2, \ldots, u_n\}$ be an universal set. For any $A, B \in PFS(U)$ the dissimilarity measure $DM_N : PFS(U) \times PFS(U) \to [0, 1]$ is defined by

$$DM_N(A, B) = \frac{1}{n}\sum_{j=1}^{n} D_j(A, B). \quad (6)$$

Theorem 1. *Let $U = \{u_1, u_2, ..., u_n\}$ be a universal set. For any $A, B \in PFS(U)$, a function $DM_N : PFS(U) \times PFS(U) \to R$ defined by $DM_N(A, B) = \frac{1}{n}\sum_{j=1}^{n} D_j(A, B)$ satisfies*

(i) $0 \leq DM_N(A, B) \leq 1$;

(ii) $DM_N(A, B) = DM_N(B, A)$;

(iii) $DM_N(A, A) = 0$;

(iv) If $A \subset B \subset C$ then $DM_N(A, C) \geq \max\{DM_N(A, B), DM_N(B, C)\}$ for all $A, B, C \in PFS(U)$.

Proof.

(i) We have $0 \leq R_A(u_j),\ R_B(u_j),\ S_A(u_j),\ S_B(u_j) \leq 1$. Hence, $0 \leq D_j(A, B) \leq 1$. Therefore, from eq.(6) we have $0 \leq DM_N(A, B) \leq 1$.

(ii) It is obvious.

(iii) It is obvious.

(iv) If $A \subset B \subset C$ then $\mu_A(u_j) \leq \mu_B(u_j) \leq \mu_C(u_j)$, $\eta_A(u_j) \leq \eta_B(u_j) \leq \eta_C(u_j)$ and $\gamma_A(u_j) \geq \gamma_B(u_j) \geq \gamma_C(u_j)$ for all $u_j \in U$.

So that, $R_A(u_j) \leq R_B(u_j) \leq R_C(u_j)$ and $S_A(u_j) \leq S_B(u_j) \leq S_C(u_j)$.

Hence, $|R_C(u_j) - R_A(u_j)| \geq \max\{|R_C(u_j) - R_B(u_j)|, |R_B(u_j) - R_A(u_j)|\}$ and $|S_C(u_j) - S_A(u_j)| \geq \max\{|S_C(u_j) - S_B(u_j)|, |S_B(u_j) - S_A(u_j)|\}$.

Hence, $DM_N(A, C) \geq \max\{DM_N(A, B), DM_N(B, C)\}$. It means PF-Diss 4 is satisfied.

■

Now, we assign to u_j a weight $\omega_j \in [0, 1]$ such that $\sum_{j=1}^{n} \omega_j = 1$. We can define a new dissimilarity measure between two picture fuzzy sets as follows.

Definition 5. Let $U = \{u_1, u_2, ..., u_n\}$ be a universal set. For any $A, B \in PFS(U)$, a dissimilarity measure $DM_N^{\omega} : PFS(U) \times PFS(U) \to [0, 1]$ is defined by

$$DM_N^{\omega}(A, B) = \sum_{j=1}^{n} \omega_j D_j(A, B). \quad (7)$$

Definition 6. Let $U = \{u_1, u_2, ..., u_n\}$ be a universal set. For any $A, B \in PFS(U)$, a dissimilarity measure $DM_P^{\omega} : PFS(U) \times PFS(U) \to [0, 1]$ is defined by

$$DM_P^{\omega}(A, B) = \sum_{j=1}^{n} \omega_j D_j^P, (A, B) \tag{8}$$

where

$$D_j^p(A, B) = \frac{[|R_A(u_j) - R_B(u_j)|^p + |S_A(u_j) - S_B(u_j)|^p]^{\frac{1}{p}}}{4} \tag{9}$$

for all $j = 1, 2, ..., n;\ p \in N^*$.

Theorem 2. *Let $U = \{u_1, u_2, ..., u_n\}$ be a universe set. Then for any $A, B \in PFS(U)$*

$$DM_N^{\omega}(A, B) = \sum_{j=1}^{n} \omega_j D_j(A, B)$$

and

$$DM_P^{\omega}(A, B) = \sum_{j=1}^{n} \omega_j D_j^P(A, B)$$

are the dissimilarity measures on picture fuzzy sets.

Proof. It is easy.

Example 2. We consider the problem in Example 1. In that example, we cannot determine whether sample B belongs to the class of pattern A_1 or A_2 if we use the dissimilarity measures in expressions eq.(1), eq.(2), eq.(3) and eq.(4). Now, we consider this problem with the new dissimilarity measures in eq.(6) and eq.(8) with $\omega_1 = \omega_2 = 0.5$ and $p = 2$.

+ Using the dissimilarity measure in eq.(6), we have

$$DM_N(A_1, B) = 0.05 \text{ and } DM_N(A_2, B) = 0.0375.$$

+ Using the dissimilarity measure in eq.(8), we have

$$DM_P^{\omega}(A_1, B) = 0.04045 \text{ and } DM_P^{\omega}(A_2, B) = 0.03018.$$

We can easily see that using two new measures we can conclude that the sample B belongs to the class of pattern A_2.

4. APPLYING THE PROPOSED DISSIMILARITY MEASURE IN PATTERN RECOGNITION

In this section, we will give some examples using dissimilarity measures in the pattern recognition. Given for m patterns $A_1, A_2, \ldots, A_m$ are picture fuzzy sets in the universal set $U = \{u_1, u_2, \ldots, u_n\}$. If we have a sample B is also a picture fuzzy set on U.

Question: Which class of pattern does B belong to?

To answer this question, we practice the following steps:

Step 1. Compute the dissimilarity measures $DM(A_i, B)$ of $A_i (i = 1, 2, \ldots, m)$ and B.

Step 2. We put B to the class of pattern A^*, in which
$$DM(A^*, B) = \min\{DM(A_i, B)|i = 1, 2, ..., m\}.$$

Example 3. Assume that there are two patterns denoted by picture fuzzy sets on $U = \{u_1, u_2, u_3\}$ as follows

$A_1 = \{(u_1, 0.1, 0.1, 0.1), (u_2, 0.1, 0.4, 0.3), (u_3, 0.1, 0, 0.9)\}$,

$A_2 = \{(u_1, 0.7, 0.1, 0.2), (u_2, 0.1, 0.1, 0.8), (u_3, 0.1, 0.1, 0.7)\}$.

Now, there is a sample $B = \{(u_1, 0.4, 0, 0.4), (u_2, 0.6, 0.1, 0.2), (u_3, 0.1, 0.1, 0.8)\}$.

Question: Which class of pattern does B belong to?

To answer this question, we consider the dissimilarity measures shown in eq.(6), eq.(8) with the weight vector $\omega = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

+ Applying the dissimilarity measure in eq.(6), we have

$$DM_N(A_1, B) = 0.1417,\ DM_N(A_2, B) = 0.1667.$$

It means that B belongs to the class of pattern A_1.

+ Applying the dissimilarity measure in eq.(8) with $p = 2$, we have

$$DM_P^{\omega}(A_1, B) = 0.0982,\ DM_P^{\omega}(A_2, B) = 0.1741.$$

It means that B belongs to the class of pattern A_1.

+ Applying the dissimilarity measure in eq.(8) with $p = 3$, we have

$$DM_P^{\omega}(A_1, B) = 0.0935,\ DM_P^{\omega}(A_2, B) = 0.161.$$

It means that B belongs to the class of pattern A_1.

Example 4. Assume that there are three patterns denoted by picture fuzzy sets on $U = \{u_1, u_2, u_3\}$ as follows

$A_1 = \{(u_1, 0.5, 0, 0.4), (u_2, 0.5, 0.2, 0.25), (u_3, 0.1, 0, 0.9), (u_4, 0.1, 0.1, 0.65)\}$,

$A_2 = \{(u_1, 0.7, 0.1, 0.2), (u_2, 0.1, 0.1, 0.8), (u_3, 0.1, 0.1, 0.7), (u_4, 0.4, 0.1, 0.5)\}$,

$A_3 = \{(u_1, 0.6, 0.1, 0.2), (u_2, 0.6, 0.2, 0.15), (u_3, 0, 0.1, 0.9), (u_4, 0.15, 0.2, 0.6)\}$.

Now, there is a sample

$B = \{(u_1, 0.5, 0.1, 0.4), (u_2, 0.6, 0.15, 0.2), (u_3, 0.1, 0, 0.8), (u_4, 0.1, 0.2, 0.6)\}$.

Question: Which class of pattern does B belong to?

Using the weight vector $\omega = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ and eq.(6), eq.(8), then:

+ Applying the dissimilarity measure in eq.(6), we have

$$DM_N(A_1, B) = 0.0375,\ DM_N(A_2, B) = 0.15,\ DM_N(A_3, B) = 0.0594.$$

It means that B belongs to the class of pattern A_1.

+ Applying the dissimilarity measure in eq.(8) with $p = 2$, we have $DM_P^\omega(A_1, B) = 0.06$, $DM_P^\omega(A_2, B) = 0.303$, $DM_P^\omega(A_3, B) = 0.099$. It means that B belongs to the class of pattern A_1.

+ Applying the dissimilarity measure in eq.(8) with $p = 3$, we have $DM_P^\omega(A_1, B) = 0.073$, $DM_P^\omega(A_2, B) = 0.3598$, $DM_P^\omega(A_3, B) = 0.1154$. It means that B belongs to the class of pattern A_1.

Example 5. Assume that there are three patterns denoted by picture fuzzy sets on $U = \{u_1, u_2, u_3, u_4\}$ as follows

$A_1 = \{(u_1, 0.3, 0.4, 0.1), (u_2, 0.3, 0.4, 0.1), (u_3, 0.6, 0.1, 0.2), (u_4, 0.6, 0.1, 0.2)\}$,

$A_2 = \{(u_1, 0.4, 0.4, 0.1), (u_2, 0.3, 0.2, 0.4), (u_3, 0.6, 0.1, 0.3), (u_4, 0.5, 0.2, 0.2)\}$,

$A_3 = \{(u_1, 0.4, 0.4, 0.1), (u_2, 0.3, 0.1, 0.3), (u_3, 0.6, 0.1, 0.2), (u_4, 0.5, 0.2, 0.1)\}$.

Now, there is a sample

$B = \{(u_1, 0.35, 0.65, 0), (u_2, 0.55, 0.35, 0.1), (u_3, 0.65, 0.1, 0.1), (u_4, 0.6, 0.15, 0.2)\}$.

Question: Which class of pattern does B belong to?

To answer this question, we consider the dissimilarity measures shown in eq.(6), eq.(7), and eq.(8) with the weight vector $\omega = (0.4, 0.3, 0.2, 0.1)$

+ Applying the dissimilarity measure in eq.(6), we have

$$DM_N(A_1, B) = 0.06875,\ DM_N(A_2, B) = 0.125,\ DM_N(A_3, B) = 0.10625.$$

$\Rightarrow DM_N(A_1, B) < DM_N(A_3, B) < DM_N(A_2, B)$. It means that B belongs to the class of pattern A_1.

+ Applying the dissimilarity measure in eq.(7), we have

$$DM_N^\omega(A_1, B) = 0.08625,\ DM_N^\omega(A_2, B) = 0.14125,\ DM_N^\omega(A_3, B) = 0.12375.$$

$\Rightarrow DM_N^\omega(A_1, B) < DM_N^\omega(A_3, B) < DM_N^\omega(A_2, B)$. It means that B belongs to the class of pattern A_1.

+ Applying the dissimilarity measure in eq.(8) with $p = 2$, we have

$$DM_P^\omega(A_1, B) = 0.06746,\ DM_P^\omega(A_2, B) = 0.10744,\ DM_P^\omega(A_3, B) = 0.09585.$$

$$\Rightarrow DM_P^\omega(A_1, B) < DM_P^\omega(A_3, B) < DM_P^\omega(A_2, B).$$

It means that B belongs to the class of pattern A_1.

5. APPLICATION IN MULTI-CRITERIA DECISION MAKING

In the MCDM problem, one has to find an optimal alternative from a set of alternatives $A = \{A_1, A_2, \ldots, A_m\}$. In this section, we introduce a method based on the new dissimilarity measures to solve a MCDM problem.

Step 1. Determine the criteria set $C = \{C_1, C_2, \dots, C_n\}$ for the MCDM.

Step 2. Express each alternative A_i as a picture fuzzy set on the set $C = \{C_1, C_2, \dots, C_n\}$,

$$A_i = \{(C_j, \mu_{ij}, \eta_{ij}, \gamma_{ij})|C_j \in C\}$$

for all $i = 1, 2, \dots, m$.

Step 3. We choose the best alternative A_b to be also a picture fuzzy set on the set $C = \{C_1, C_2, \dots, C_n\}$.

Step 4. Determine the weight ω_j of criteria C_j by considering $C_j = \{(A_i, \mu_{ij}, \eta_{ij}, \gamma_{ij})|A_i \in A\}$ as a picture fuzzy set on $A = \{A_1, A_2, \dots, A_m\}$.
Based on the union of picture fuzzy sets we propose a method to determine the weight ω_j of criteria $C_j (j = 1, 2, \dots, n)$ as follows:

- We calculate

$$d_j = d_{1j} + d_{2j} + d_{3j} \tag{10}$$

where $d_{1j} = \max\limits_{1 \le i \le m} \mu_{ij}$, $d_{2j} = \min\limits_{1 \le i \le m} \eta_{ij}$, and $d_{3j} = \min\limits_{1 \le i \le m} \gamma_{ij}$ for all $j = 1, 2, \dots, n$.
Then, $A^* = \{(C_j, d_{1j}, d_{2j}, d_{3j})|C_j \in C\} = \bigcup\limits_{i=1}^{m} A_i$ and d_j in the eq.(10) is referred to frequency of $C_j (j = 1, 2, \dots, n)$ in A^*.
So that, we can determine the weight ω_j of criteria $C_j (j = 1, 2, \dots, n)$ based on frequency $d_j (j = 1, 2, ..., n)$.

- Put

$$\omega_j^{(k)} = \frac{d_j^{(k)}}{\sum_{j=1}^{n} d_j^{(k)}} \tag{11}$$

for all $j = 1, 2, \dots, n;\ k = 0, 1, 2, \dots$
Note that, when $k = 0$ then we have the weight $\omega_j = \dfrac{1}{n}$ for all $j = 1, 2, \dots, n$.

Step 5. Compute the dissimilarity measures $DM(A_i, A_b)$ between $A_i (i = 1, 2, \dots, m)$ and A_b.

Step 6. Rank the alternatives based on the dissimilarity measures as follows

$$A_i \prec A_k \text{ iff } DM(A_i, A_b) < DM(A_k, A_b)(i, k = 1, 2, \dots, m).$$

Example 6. Consider a supplier section problem. Suppose a construction company wants to procure the material for their upcoming project. The company invites the tenders for procuring the required material. Given five suppliers are $\{A_1, A_2, A_3, A_4, A_5\}$. To find an optimal supply, we apply the six steps for solving this MCDM problem as follows:

Step 1. The company has fixed criteria for supplier selection: C_1: quality of material; C_2: price; C_3: services; C_4: delivery; C_5: technical support if required; C_6: behavior.

Step 2. Alternatives A_i is expressed as a picture fuzzy set on a criteria set $\{C_1, C_2, \ldots, C_6\}$ in Table 1 and Table 2.

Step 3. The best alternative A_b is

$$A_b = \{(C_j, 1, 0, 0)|j = 1, 2, 3, 4, 5, 6\}.$$

Step 4. Using the eq.(1), we get $d_1 = 0.85$, $d_2 = 1$, $d_3 = 0.9$, $d_4 = 1$, $d_5 = 0.95$, $d_6 = 0.9$.
To calculate the weight ω_j of criteria $C_j (j = 1, 2, \ldots, 6)$ we use the eq.(11):
$k = 0$ we have the weight vector is $\omega_0 = (\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$.
$k = 1$ we have the weight vector is $\omega_1 = (0.145, 0.171, 0.171, 0.171, 0.171, 0.171)$.
$k = 2$ we have the weight vector is $\omega_2 = (0.125, 0.175, 0.175, 0.175, 0.175, 0.175)$.

Table 1. The picture fuzzy decision matrix for the supplier selection

	C_1	C_2	C_3	C_4
A_1	$(0.4, 0.05, 0.5)$	$(0.1, 0.1, 0.8)$	$(0.7, 0, 0.3)$	$(0.6, 0.1, 0.2)$
A_2	$(0.7, 0.05, 0.2)$	$(0.5, 0.1, 0.3)$	$(0.3, 0.3, 0.4)$	$(0.8, 0.05, 0.1)$
A_3	$(0.6, 0.2, 0.1)$	$(0.7, 0, 0.3)$	$(0.6, 0.1, 0.2)$	$(0.4, 0.3, 0.1)$
A_4	$(0.5, 0.05, 0.4)$	$(0.4, 0.2, 0.3)$	$(0.8, 0.1, 0.1)$	$(0.7, 0.05, 0.2)$
A_5	$(0.4, 0.3, 0.3)$	$(0.1, 0.15, 0.7)$	$(0.5, 0.25, 0.2)$	$(0.9, 0, 0.1)$

Table 2. The picture fuzzy decision matrix for the supplier selection (cont.)

	C_5	C_6
A_1	$(0.5, 0.1, 0.4)$	$(0.3, 0.2, 0.4)$
A_2	$(0.2, 0.1, 0.6)$	$(0.4, 0, 0.5)$
A_3	$(0.3, 0.2, 0.4)$	$(0.8, 0, 0.2)$
A_4	$(0.6, 0.25, 0.1)$	$(0.7, 0.2, 0.1)$
A_5	$(0.8, 0.05, 0.1)$	$(0.6, 0, 0.4)$

Step 5. Compute the dissimilarity measures $DM(A_i, A_b)$ between $A_i (i = 1, 2, \ldots, m)$ and A_b using the eq.(8) with $p = 1$ and $p = 2$.

Step 6. Rank the alternatives based on the dissimilarity measure.
The results of Step 5 and Step 6 with the various weight vectors are shown in Table 3, 4, 5.

- With the weight vector $\omega_0 = (\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$, we have the dissimilarity measure and ranking of alternatives as in Table 3.

- With the weight vector $\omega_1 = (0.145, 0.171, 0.171, 0.171, 0.171, 0.171)$, we have the dissimilarity measure and ranking of alternatives as in Table 4.

Table 3. The dissimilarity measure and ranking of alternatives with the weight vector ω_0

		A_1	A_2	A_3	A_4	A_5
$p = 1$	$DM(A_i, A_b)$	0.2688	0.2229	0.1833	0.1813	0.1917
	Rank	5	4	2	1	3
$p = 2$	$DM(A_i, A_b)$	0.2651	0.2281	0.1776	0.1693	0.198
	Rank	5	4	2	1	3

Table 4. The dissimilarity measure and ranking of alternatives with the weight vector ω_1

		A_1	A_2	A_3	A_4	A_5
$p = 1$	$DM(A_i, A_b)$	0.2657	0.2245	0.1842	0.1778	0.1908
	Rank	5	4	2	1	3
$p = 2$	$DM(A_i, A_b)$	0.2919	0.2548	0.1957	0.1829	0.216
	Rank	5	4	2	1	3

- With the weight vector $\omega_2 = (0.125, 0.175, 0.175, 0.175, 0.175, 0.175)$, we have the dissimilarity measure and ranking of alternatives as in Table 5.

Table 5. The dissimilarity measure and ranking of alternatives with the weight vector ω_2

		A_1	A_2	A_3	A_4	A_5
$p = 1$	$DM(A_i, A_b)$	0.2632	0.22261	0.1852	0.175	0.1902
	Rank	5	4	2	1	3
$p = 2$	$DM(A_i, A_b)$	0.2902	0.257	0.197	0.1802	0.215
	Rank	5	4	2	1	3

6. CONCLUSION

In this paper, we introduce some new dissimilarity measures on picture fuzzy sets. These new measures overcome the limitations of the previous dissimilarity measures on picture fuzzy sets in [7, 34]. After that, we apply the proposed dissimilarity measures in the pattern recognition. We also use these new dissimilarity measures for a MCDM problem to select an optimal supplier. In the future, we also continue to study about the dissimilarity measures on picture fuzzy sets and the relationship of them and other measures on picture fuzzy sets. Beside, we also find new applications of them to deal with the real problems.

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