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AGGREGATION OF SYMBOLIC POSSIBILISTIC KNOWLEDGE BASES FROM THE POSTULATE POINT OF VIEW

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Abstract. Aggregation of knowledge bases in the propositional language was soon investigated and the requirements of aggregation processes of propositional knowledge bases basically are unified within the community of researchers and applicants. Aggregation of standard possibilistic knowledge bases where the weight of propositional formulas being numeric has also been investigated and applied in building the intelligent systems, in multi-criterion decision-making processes as well as in decision-making processes implemented by many people.

Symbolic possibilistic logic (SPL for short) where the weight of the propositional formulas is symbols was proposed, and recently it was proven that SPL is soundness and completeness. In order to apply SPL in building intelligent systems as well as in decision-making processes, it is necessary to solve the problem of aggregation of symbolic possibilistic knowledge bases (SPK bases for short). This problem has not been researched so far.

The purpose of this paper is to investigate aggregation processes of SPK bases from the postulate point of view in propositional language. These processes are implemented via impossibility distributions defined from SPK bases. Characteristics of merging operators, including hierarchical merging operators, of symbolic impossibility distributions (SIDs for short) from the postulate point of view will be shown in the paper.

Keywords. Aggregation; Hierarchical aggregation; Merging operator; Impossibility distribution; Symbolic possibilistic logic; Postulate point of view.

1. INTRODUCTION

Aggregation of knowledge bases is always an important research subject in the field of artificial intelligence and has been researched for a long time [1, 5, 8, 9, 10, 11, 12, 17, 18, 19]. It is applied in multi-criteria decision-making processes, decision-making processes implemented by many people and to develop intelligent systems.

Standard possibilistic logic where the truth state (or weight) of sentences in the classical propositional language to be numeric values was rather completely developed [6, 7]. In [6], one proved that this logic is soundness and completenes. In other words, the standard possibilistic logic under the syntactic and semantic approaches is the same.

It means that if a possibilistic formula is received by applying the rules of inference in a standard possibilistic knowledge base (syntactic approach) then it is also received by calculating its weight via the least specificity possibility distribution among possibility distributions satisfying the given knowledge base (semantic approach) and vice versa.

This suggests that the aggregation of standard possibilistic knowledge bases can be implemented via the aggregation of their least specificity possibility distributions. It is very different in terms of comparing with the aggregation of knowledge bases in the propositional logic, where the aggregation is only implemented under the syntactic approach.

The first researches of the aggregation of standard possbilistic knowledge bases carried out via possibility distributions were introduced in the works [13, 14, 15, 16]. The author of these works proposed some conditions which aggregation processes of possibility distributions need to be satisfied (called the axiomatic approach) as well as proposed some merging operators (or aggregation operators) satisfying these conditions. These merging operators were also developed under some different strategies such as respecting majority's opinions where each knowledge base is considered as an agent, respecting differences as well as reliability levels of knowledge bases [13, 16].

Works [2, 3] also researched aggregation processes of standard knowledge bases via possibility distributions but under another way. Here its authors based on the conditions (called postulates) which aggregation processes of propositional knowledge bases need to be satisfied to investigate properties of merging operators of standard possibilistic knowledge bases [1, 3]. The postulates of aggregation processes of knowledge bases in the classical propositional language were proposed by Konieczny & Perez [9], and then they were adjusted by Benferhat et al. to fit aggregation processes of knowledge bases in the standard possibilistic logic [3]. The properties of merging operators from the postulate point of view are important suggestions to propose appropriate merging operators for specific applications in standard possibilistic logic.

Possibilistic logic has been continually developed in the direction of being able to express and build the mechanism of reasoning for symbolic knowledges. Over time, many researchers attempted to build SPL where the weights measuring the truth state of propositional formulas are symbols. In a recent paper [4], its authors showed that SPL is also soundness and completeness.

From the work [4], similarly to the standard possibilistic logic, one question arises as whether the aggregation of SPK bases can be implemented via symbolic possibility distributions? and how to aggregate?

The purpose of this paper is to answer these questions. Namely, this paper will focus on proposing solutions to aggregate SPK bases via special impossibility distributions of SPK bases from the postulate point of view [2, 3]. In SPL, calculations performing on the symbols are only min, max, or a combination of these two calculations under a way, so in this logic, there is no merging operators satisfying all the postulates as in the standard possibilistic logic [3, 6]. Which postulates can be satisfied by merging operators in SPL will be shown in the paper.

The paper is structured as follows, after this section, Section 2 will briefly introduces some preliminaries for next sections such as the standard possibilistic logic and the aggregation of knowledge bases in this logic, SPL and the adjusted postulates of aggregation processes of SPK bases. Sections 3, 4 introduce about the aggregation and the hierarchical aggregation of SIDs from the postulate point of view, respectively. Section 5 presents some conclusions and further research directions.

2. PRELIMINARIES

2.1. Standard possibilistic knowledge bases

Suppose that $\mathcal{L}$ is a propositional language on a limit $\mathcal{H}$, Ω is the set of all possible words (or set of interpretations) of $\mathcal{L}$ on $\mathcal{H}$; $\equiv$ is denotes logical equivalence and the logical operations are denoted by $\wedge$, $\vee$. The logical consequence relation is $\vdash$. For $\omega \in \Omega$, if a formula ϕ (or sentence) in the language $\mathcal{L}$ is true in this possible world then we say ω is the model of the formula ϕ and denoted by $\omega \vdash \phi$.

On the semantics, the standard possibilistic logic can be built on possibility distributions π, that is a mapping from Ω to $[0,\ 1]$, $\pi(\omega)$ represents the uncertain degree of knowledge about (or satisfaction degree) ω. $\pi(\omega) = 1$ means that it is totally possible for ω to be the real world, $1 > \pi(\omega) > 0$ means that ω is only somewhat possible, while $\pi(\omega) = 0$ means that ω does not satisfy at all. From the possibility distribution π, the necessity measure N on the language $\mathcal{L}$ is defined as follows: For each formula ϕ in $\mathcal{L}$, $N(\phi) = 1 - \Pi(\neg\phi)$, here $\Pi(\phi) = \max\{\pi(\omega) : \omega \in \Omega \textit{ and } \omega \vdash \phi\}$; Π is called possibility measure. The relation between the possibility and necessity measures as well as details about these measures can be referenced in [6].

Standard possibilistic knowledge base is the set $B = \{(\phi_i, a_i) : i = 1, \ldots,\ n\}$, where ϕ_i is a propositional formula and $a_i \in [0,\ 1]$. The pair (ϕ_i, a_i) means that the certainty degree of ϕ_i is at least a_i $(N(\phi_i) \geq a_i)$. Denoting $B^* = \{\phi_i,\ i = 1,\ \ldots,\ n\}$ and $\mathcal{C}_{n_p}(B^*) = \{\phi \in \mathcal{L} : B^* \vdash \phi\}$. A standard possibilistic knowledge base B is consistent if and only if $\mathcal{C}_{n_p}(B^*)$ is consistent [3, 6]. The degree of inconsistent of the standard possibilistic knowledge base B is denoted by $Inc(B)$ and is defined as follows

$$Inc(B) = N_B(\perp) = \max\{\ a : B \vdash (\perp, a)\}\ , \tag{2.1}$$

there $\perp$ is the inconsistent element (tautology) of the language $\mathcal{L}$. If $N(\perp) = 0$, the knowledge base B is consistent, if $N(\perp) = \alpha$, the knowledge base B is consistent with degree α and this knowledge base is completely inconsistent if $N(\perp) = 1$.

For a possibilistic knowledge base, generally, there may be many possibility distributions π on the set of representations Ω so that the necessity measure determined from this possibilistic distribution satisfies $N(\phi_i) \geq a_i$ for every formula ϕ_i. Among these possibility distributions, there is a special possibility distribution that is defined as follows [3, 6]

$$\pi_B(\omega) = \begin{cases} 1 \quad \text{if} \quad \omega\ \vdash \phi_i \\ 1 - \max\{a_i\} \quad \text{otherwise,} \end{cases} \tag{2.2}$$

$\forall \omega \in \Omega$ and $(\phi_i, a_i) \in B$.

This possibility distribution in fact is found out by the principle of minimal specificity [13]. This principle is proposed by R.Yager by basing on the idea of the maximal entropy principle in information theory. In [13], its author proved that the two principles really have relations together under a sense.

In [6] it was proven that

$$\mathcal{C}_{n_p}(B) = \{(\phi, a) : B \vdash (\phi, a)\} = \{(\phi, a) : B \models_\pi (\phi, a)\} = \mathcal{C}_{n_\pi}(B). \tag{2.3}$$

Here $\vdash$ and $\models_\pi$ are notations of the classical syntactic and semantic inferences, respectively. In other words, the system of reasoning in the standard possibilistic logic is soundness and completeness for the semantic of this logic.

2.2. SPL base

2.2.1. The syntax of SPL

Definition 2.1. [4] (about SPL base) The set $\wp$ of symbolic expressions a_i acting as weights is recursively obtained using a finite set of variables (called elementary weights) $H = \{p_1, \ldots, p_k, \ldots\}$ and the max / min operators built on H as follows

1. $H \subset \wp$, 0, $1 \in \wp$;
2. If $a_i,\ a_j \in \wp$ then $\max(a_i,\ a_j)$ and $\min(a_i,\ a_j) \in \wp$, here assume that $1 \geq p_i \geq 0\ \forall i$.

SPK base $B = \{(\phi_i, a_i),\ i = 1, \ldots,\ n\}$ is a set of formulas ϕ_i in the propositional language $\mathcal{L}$ and the a_i attached to ϕ_i, is called a weight, that is a symbolic expression of max, min and is built on H. In SPL, (ϕ_i, a_i) is defined as $N(\phi_i) \geq a_i$, where N is the necessity measure.

The min and max operations are commutative, [4] indicates that any symbolic expression can also be presented in the form of

$$\min_{i=1,\ r}\max_{j=1,\ n}x_{ji} \ \text{ or } \max_{h=1,\ m}\min_{k=1,\ s}x_{hk}, \tag{2.4}$$

there x_{ji}, x_{hk} are single variables on [0, 1].

Definition 2.2. ([4]) Valuation is a positive mapping, $v : H \to (0, 1]$, it instantiates all elementary weights in H.

Its domain is extended to all max / min operators and a combination of these two operators in H. The notation $\mathcal{V}$ is the set of all valuation on H, we say that $a_i \geq a_j$ if and only if $\forall v \in \mathcal{V}$ then $v(a_i) \geq v(a_j)$.

Definition 2.3. ([4]) The rules of inference in SPL is defined as follows:

1. Fusion: $\{(\varphi, p), (\varphi, p')\} \vdash (\varphi, \max(p, p')\ )$;
2. Weakening: $(\varphi, p) \vdash (\varphi, p')$ if $p \geq p'$;
3. Modus Ponens: $\{(\varphi \to \psi, p), (\varphi, p)\} \vdash (\psi, p)$;

   From the above rules, it can be inferred.
4. The rule of Modus Ponens extension: $\{(\varphi \to \psi, p), (\varphi, p')\} \vdash (\psi, \min(p, p'))$.

2.2.2. The semantic of SPL

Definition 2.4. ([4]) Suppose $B = \{(\phi_i, a_i) : i = 1, \ldots, n\}$ is a SPK base. The special impossibility distribution τ_B is defined as follows

$$\tau_B(\omega) = \begin{cases} \max_{j:\phi_j \notin B(\omega)} a_j \\ 0, \quad \text{if} \quad B(\omega) = B^*, \end{cases} \tag{2.5}$$

$\forall \omega \in \Omega,\ B(\omega) = \{\phi \in B^* : \omega \vdash \phi\}$ and necessity measure N_B corresponding to this distribution is

$$N_B(\phi_i) = \min_{\omega \notin [\phi_i]} \tau_B(\omega) = \min_{\omega \notin \phi_i} \max_{j:\phi_j \notin B(\omega)} a_j, \tag{2.6}$$

there $[\phi_i] = \{\omega \in \Omega : \ \omega \vdash \phi_i\}$.

In essence, the determination formula of impossibility distributions according to the formula (2.5) is similar to the determination formula of possibility distributions according to the formula (2.2). Because in SPL there is no term "1 -", hence the formula (2.2) is adjusted to fit this context and $\tau_B(\omega)$ is defined by the formula (2.5). Thus, τ_B is not a symbolic possibility distribution and it is called SID.

Similar to the standard possibilistic logic, for each SPK base, in general, there are many different impossibility distributions so that necessity measures generated from these distributions according to the formula (2.6) satisfy the given SPK base. It is easy to see that all impossibility distributions τ always satisfy $\tau(\omega) \geq \tau_B(\omega)\ \forall\ \omega \in \Omega$. In other words, $\tau_B(\omega)$ is the most specificity impossibility distribution. This is contrasts with the least specificity possibility distribution $\tau_B(\omega)$ in the standard possibilistic logic [6, 13]. Soundness and completeness of SPL were also proven in [4], i.e. the formula (2.3) is true for every SPK base. Example 2.5 below illustrates SPK base.

Example 2.5. (Improved from [4]) Assume that different agents exchange information about potential participants in an upcoming meeting.

- Agent A_1 says: Albert, Chris will not come together; if Albert and David arrive, the meeting will not be quiet;
- Agent A_2 says: If the meeting starts late, it will not be quiet; if David comes, then Chris comes.
- Agent A_3 says: if Albert arrives, the meeting will begin late; Chris can not attend the meeting if it starts late.

Here, it is assumed that the agents $A_1,\ A_2$ are known to be more reliable than the agent A_3, but it is not known whether the agent A_1 is more reliable than the agent A_2. This assumption can be expressed by assigning a symbol to each agent. Assume that $a_1,\ a_2,\ a_3$ are symbolic weights attached to these agents. For example, a_1 = "*High reliability*", a_2 = "*reliable*", a_3 = "*moderate trust*". We can say a_1 and $a_2 > a_3$, but a_1 and a_2 are not comparable. Therefore, symbol values are only partially ordered.

Notations α, β, γ are propositional variables corresponding to Albert, Chris, David come to the meeting, κ is a quiet meeting, λ is the meeting started late. With the note that the logical implication "if A then B" is logically equivalence to the logical expression $\neg A \vee B$, so three SPK bases corresponding to the three agents aforementioned are defined as follows:

(A_1) $\quad (\neg(\alpha \wedge \beta),\ a_1),\ (\neg(\alpha \wedge \gamma) \vee \neg\kappa,\ a_1);$

(A_2) $(\neg\lambda \ \vee \neg\kappa,\ a_2),\ (\neg\beta \ \vee \ \gamma \ ,\ a_2);$

(A_3) $(\neg\alpha \ \vee \ \lambda,\ a_3),\ (\neg\lambda \ \vee \neg\gamma \ ,\ a_3).$

2.3. Postulates of merging SPK bases

Assume $B_1, \ldots, B_n$ are n standard possibilistic knowledge bases, $B_i^* \subset \mathcal{L}, i = 1, \ldots, n$. For every knowledge base, we can determine the least specificity possibility distribution according to formula (2.2) so that its necessity measure satisfies this knowledge base. So, the aggregation of standard possibilistic knowledge bases can be implemented via their least specificity possibility distributions.

Definition 2.6. ([3, 14]) Denote by $\oplus$ a merging operator of possibility distributions. It is a mapping $\oplus : [0,\ 1]^n \to [0,\ 1]$, where n is the number of possibilistic knowledge bases, satisfies two following conditions:

$$\begin{aligned} &\bullet\ \oplus(0, \ldots, 0) = 0; \\ &\bullet\ \text{If } a_i \ \geq b_i \ \forall\ i = 1, \ldots, n \text{ then } \oplus(a_1, \ldots, a_n) \geq \ \oplus(b_1, \ldots, b_n). \end{aligned} \tag{2.7}$$

Each possibilistic knowledge base is considered as an agent and the aggregation of possibility distributions is in fact the aggregation of agents to create a new agent from given agents and an aggregated agent is a fusion of these given agents.

Assume that SPK bases $B_i,\ i = 1, \ldots, n$ are consistent. In the context of SPL, the postulates of merging standard possibilistic knowledge bases in [3] are adjusted appropriately as in the Definition 2.7 below.

Definition 2.7. The postulates of aggregation processes of SPL bases are as follows:

W_1 : $\mathcal{C}_{n_\pi}(\mathfrak{B}_\oplus)$ is consistent, here the $\mathfrak{B}_\oplus$ is SPK base aggregated from given consistent SPK bases.

In SPL, the inconsistent degree of SPK base B (denoted as Inc (B)) is also defined by the formula (2.1).

W_2 : If $B_1 \cup \ B_2 \ \cup \cdots \cup \ B_n$ is consistent then $\mathcal{C}_{n_\pi}(\mathfrak{B}_\oplus) \equiv \mathcal{C}_{n_\pi}(B_1 \cup \ B_2 \ \cup \cdots \cup \ B_n)$, here $\equiv$ means that $\forall(\phi,\ a) \ \in \ \mathcal{C}_{n_\pi}(\mathfrak{B}_\oplus)$ then $(\phi,\ a) \ \in \mathcal{C}_{n_\pi}(B_1 \cup \ B_2 \ \cup \cdots \cup \ B_n)$ and vice versa.

Let B_i be a SPK base, $\mathfrak{B} = \{B_1,\ B_2,\ \ldots,\ B_n\}$ is called a multi-set (or a set of sets). The notation $\bigsqcup$ is a union of multi-sets.

W_3 : Suppose $\mathfrak{B}$, $\mathfrak{B}'$ are multi-sets, if $\mathfrak{B} \Leftrightarrow \mathfrak{B}'$ then $\mathcal{C}_{n_\pi}(\mathfrak{B}_\oplus) \equiv \mathcal{C}_{n_\pi}(\mathfrak{B}'_\oplus)$, here $\mathfrak{B} \Leftrightarrow \mathfrak{B}'$ means $\forall B_i \in \ \mathfrak{B},\ \exists! B'_j \in \ \mathfrak{B}'$ so that $\mathcal{C}_{n_\pi}(B_i) \ \equiv \ \mathcal{C}_{n_\pi}(B'_j)$ and reverse $\forall B'_j \in \mathfrak{B}',\ \exists! B_i \in \mathfrak{B} :\ \mathcal{C}_{n_\pi}(B_i) \ \equiv \ \mathcal{C}_{n_\pi}(B'_j)$, here $B_i,\ B'_j$ are SPK bases.

Let $\mathcal{A},\ B$ be SPK bases; $\mathcal{A}$ is called conflict set of B if $\mathcal{A}^* \subset B^*,\ \mathcal{A}$ is inconsistent, and for $\forall(\phi, a) \in \mathcal{A},\ \mathcal{A} - \{(\phi, a)\}$ is consistent [3].

SPK base B_1 is said to be more prioritized than to B_2 [3] if for all conflict sets $\mathcal{A} \subset B_1 \cup B_2$ then $Deg_{B_1}(\mathcal{A}) > Deg_{B_2}(\mathcal{A})$ here $Deg_B(\mathcal{A}) = \min\{a : (\phi,\ a) \in \mathcal{A} \cap B\}$, $Deg_B(\mathcal{A}) = 1$ if $\mathcal{A} \cap B$ is an empty set. Thus, $Deg_B(\mathcal{A})$ is a weight of the lowest certainty formula of $\mathcal{A}$. It can be seen that B_1 is more prioritized than B_2 if for $\forall \mathcal{A}$ in $B_1 \cup B_2$ the least certainty formula of $\mathcal{A}$ is in B_2. Two SPK bases B_1, B_2 are said to be equally prioritized if for every conflict set $\mathcal{A}$ of $B_1 \cup B_2$ then $Deg_{B_1}(\mathcal{A}) = Deg_{B_2}(\mathcal{A})$.

Example 2.8. Let $B_1 = \{(\phi \vee \psi \vee \xi,\ a_1),\ (\neg\psi,\ a_1),\ (\neg\sigma,\ a_1)\}$ and $B_2 = \{(\sigma \vee \xi,\ a_2), (\neg\xi,\ a_2), (\neg\phi,\ a_2),\ (\sigma \vee \psi,\ a_2)\}$ be two SPK bases, where $a_1,\ a_2$ are symbols. There are two inconsistent propositional knowledge bases $\mathcal{A}_1^*,\ \mathcal{A}_2^* \subset B_1^* \cup B_1^*$ so that after removing any proposition from each knowledge base, they will become consistent knowledge bases, namely $\mathcal{A}_1^* = \{\phi \vee \psi \vee \xi,\ \neg\phi,\ \neg\xi,\ \neg\psi\}$ and $\mathcal{A}_2^* = \{\neg\xi,\ \sigma \vee \xi,\ \neg\sigma\}$. So $\mathcal{A}_1 = \{(\phi \vee \psi \vee \xi,\ a_1),\ (\neg\phi,\ a_1),\ (\neg\xi,\ a_2),\ (\neg\psi,\ a_1)\}$ and $\mathcal{A}_2 = \{(\neg\xi,\ a_2),\ (\sigma \vee \xi,\ a_2),\ (\neg\sigma,\ a_1)\}$ are two inconsistent SPK bases and are also two conflict sets of $B = B_1 \cup B_2$. We have $Deg_{B_1}(\mathcal{A}_1) = a_1$, $Deg_{B_2}(\mathcal{A}_1) = a_2$ and $Deg_{B_1}(\mathcal{A}_2) = a_1$, $Deg_{B_2}(\mathcal{A}_2) = a_2$. Hence B_1 is more prioritized than to B_2 if $a_1 \geq a_2$ and B_2 is more prioritized than to B_1 if $a_1 < a_2$. In the case $a_1,\ a_2$ are not comparable, it is not possible to conclude which SPL base is more prioritized.

W_4 : If $B_1,\ B_2$ are inconsistent possibilistic knowledge bases and equally prioritized then $\mathcal{C}_{n_\pi}(\mathfrak{B}_\oplus) \not\models \mathcal{C}_{n_\pi}(B_1)$ and $\mathcal{C}_{n_\pi}(\mathfrak{B}_\oplus) \not\models \mathcal{C}_{n_\pi}(B_2)$.

For the sake of simplicity, if B and B' are SPK bases and E is a multi-set, instead of writing $E \bigsqcup \{B\}$ and $\{B\} \bigsqcup \{B'\}$, we can simply write $E \bigsqcup B$ and $B \bigsqcup B'$, respectively.

W_5 : $\mathcal{C}_{n_\pi}(\mathfrak{B}'_\oplus) \bigsqcup \mathcal{C}_{n_\pi}(\mathfrak{B}''_\oplus) \models \mathcal{C}_{n_\pi}(\mathfrak{B}_\oplus)$, here $\mathfrak{B} = \mathfrak{B}' \bigsqcup \mathfrak{B}''$, $\bigsqcup$ is a union of multi-sets.

W_6 : If $\mathcal{C}_{n_\pi}(\mathfrak{B}'_\oplus) \bigsqcup \mathcal{C}_{n_\pi}(\mathfrak{B}''_\oplus)$ is consistent then $\mathcal{C}_{n_\pi}(\mathfrak{B}_\oplus) \models \mathcal{C}_{n_\pi}(\mathfrak{B}'_\oplus) \bigsqcup \mathcal{C}_\pi(\mathfrak{B}''_\oplus)$.

In addition to these six postulates, there are two other postulates which can be satisfied by aggregation processes:

W_{arb} : $\forall B',\ \forall n,\ \mathcal{C}_{n_\pi}((\mathfrak{B} \bigsqcup B'^n)_\oplus) \equiv \mathcal{C}_{n_\pi}((\mathfrak{B} \bigsqcup B')_\oplus)$, here B'^n is a multi-set, $B'^n = \{\ B',\ B', \ldots,\ B'\}$ with size of n.

W_{maj} : $\forall\ B',\ \exists n,\ \mathcal{C}_{n_\pi}((\mathfrak{B} \bigsqcup B'^n)_\oplus) \models \mathcal{C}_{n_\pi}(B')$, here $\mathfrak{B} = \{B_1, B_2, \ldots, B_m\}$, B_i $(i = 1, 2, ..., m)$ and B' are SPK bases.

In a similar way as in the standard possibilistic logic [3], the meaning of the postulates aforementioned can be explained as follows: The postulate W_1 says that the result of merging of consistent SPK bases should be consistent; The postulate W_2 requires that when the sources are not conflicting, the result of merging should recover all the information provided by the sources; The postulate W_3 expresses the syntax independence of the merging process; The postulate W_4 says that when two SPK bases are equally prioritized then the result of merging should not give preference to any of the two bases; The postulates W_5 and W_6 express the decomposition of the merging process; The postulate W_{arb} means that the merging process should ignore redundancies; The postulate W_{maj} says that if a same symbolic possibilistic formula is believed to a weight α by two agents, it should be believed with a larger weight β in the result of merging.

3. AGGREGATION OF SPK BASES

Definition 3.1. SID τ_B is called a standard SID if there exists an interpretation ω so that $\tau_B(\omega) = 0$. SPK base B is consistent if and only if there does not exist ϕ in $\mathcal{L}$ so that $N_B(\phi) \geq a$ and $NB(\neg\phi) \geq b$ here $0 < a\ , b \in \wp$.

Proposition 3.2.
1) *SPK base* $B = \{(\phi_i, a_i),\ i = 1, \ldots,\ n\}$ *is consistent if and only if* $B^* = \{\phi_i,\ i = 1, \ldots,\ n\}$ *is consistent.*
2) *If* τ_B *is a standard SID, then* B *is consistent, and vice versa if* B *is consistent then* τ_B *is a standard SID.*

Proof.

1) We have, $B \models (\phi,\ a)$ if and only if $B^* \vdash \phi$ and $N_B(\phi) \geq a$. By definition, B is consistent iff $\nexists\phi \in\ \mathcal{L}:\ B \vdash (\phi,\ a)$ and $B\ \vdash (\neg\phi,\ b),\ 0 <\ a,\ b \in \wp$ iff $\nexists\phi \in\ \mathcal{L}:\ B^* \vdash \phi$ and $B^* \vdash \neg\phi$ iff B^* are consistent.

2) Suppose τ_B is a standard SID $\Rightarrow \exists\omega \in \Omega :\ \tau_B(\omega) = 0 \Rightarrow\ \exists\omega \in \Omega :\ \omega \vdash \bigwedge_{i=1-n}\phi_i$. (According to the formula (5)), so $\forall\phi \in \mathcal{L}$ obtained by applying the inference rules of the propositional logic on the formulas ϕ_i in B^* then $\omega \vdash \phi$ and $\omega \nvdash \neg\phi$ or $\forall\phi \in \mathcal{L}$, $B^* \vdash \phi$ and $B^* \nvdash \neg\phi$. So B^* is consistent. According to 1) we have B consistent.

Conversely, assume that B is consistent but τ_B is not a standard SID. Select (ϕ, α) so that $\phi \neq \perp$, $\alpha > 0$, $\exists C_1 \subset B^*$ and $C_1 \vdash \phi$. Denote $C^* = \{\cup_{i=1}^{k} C_i : C_i \vdash \phi,\ C_i \subset B^*\}$ and $\Omega^* = \{\omega \in \Omega :\ \exists i \text{ for } \omega \vdash C_i\}$ then $\forall\omega \in \Omega^*$, we have $\omega \vdash \phi$ which means $\omega \in [\phi]$.

According to the formula (2.6) we have $\beta = N_B(\neg\phi) = \min_{\omega\in[\phi]}\tau_B(\omega) = \min_{\omega\in\Omega^*}\tau_B(\omega)$. Because τ_B is not a standard SID so $\tau_B(\omega) > 0$ for every $\omega \in \Omega^*$ so $\beta > 0$.

On the other hand, $N_B(\phi) = \min_{\omega\notin[\phi]}\tau_B(\omega) = \min_{\omega\in\Omega/\Omega^*}\tau_B(\omega) = \alpha > 0$. Thus, $N_B(\perp) = \min(N_B(\phi),\ N_B(\neg\phi)) = \min(\alpha, \beta) > 0$, i.e. B is inconsistent. This is contradictory with the assumption that B is consistent. So τ_B must be a standard SID. ■

Back to Example 2.5 above, when information about the meeting comes from three agents with different confident degrees, to answer questions like: Should the meeting be held sooner or later? Who will attend? How will be the meeting, quiet or noisy?... it is neccesary to merge three SPK bases corresponding to the these agents into a new SPK base and basing on such an aggegated knowledge base to answer the arised questions. This paper will research the aggregation of SPK bases via most specificity SIDs of SPK bases.

Suppose that $B_1, \ldots, B_n$ are n SPK bases, where ${B_i}^*$ is the set of sentences in B_i, ${B_i}^* \subset \mathcal{L}$. The ${B_i}^*$ are generally different. Denote by τ_{B_i} $(i = 1, \ldots, n)$ a most specificity SID from SPK base B_i, $(i = 1, \ldots, n)$, the arised problem is that from the most specificity SIDs τ_{B_i} $(i = 1, \ldots, n)$ we need to generate an SID $\tau_{\mathfrak{B}_\oplus}$ of SPK base $\mathfrak{B}_\oplus$ aggregated from SPK bases B_i, $(i = 1, \ldots, n)$.

Definition 3.3. Merging operator of n SIDs τ_{B_i} $(i\ =\ 1,\ 2,\ ...,\ n)$ is a mapping $\oplus : \wp^n \to \wp$ satisfying two conditions:

$$\begin{aligned} &\bullet\ \oplus(1, \ldots, 1) = 1; \\ &\bullet \text{ If } a_i\ \geq b_i,\ \forall\ i = 1, \ldots, n \text{ then } \oplus(a_1,\ \ldots, a_n) \geq\ \oplus(b_1,\ \ldots, b_n). \end{aligned} \tag{3.1}$$

The second condition is that for every $i = 1, ..., n$, $a_i, b_i \in \wp$ and $\forall v : H \to (0,1]$, if $v(a_i) \geq v(b_i)$ then $v(\oplus(a_1, \ldots, a_n)) \geq v(\oplus(b_1, \ldots, b_n))$.

In fact, Definition 3.3 is similar to Definition 2.6 by adjusting the formula (2.2) to fit the context of defining of SIDs.

Example 3.4. Identifying most specificity SIDs of the 3 SPK bases given in Example 2.5 and of two aggregated SPK bases using the merging operators max and min.

The results are shown in Table 1 below.

As is known, calculations implemented on weights of formulas in SPL are only min and max, and a combination of these two calculations in a way, thus merging operators of SIDs can also only be min and max operators, and a combination of these two operators. The combination can be transformed into the forms as in the formula (2.4) above. From this, we have following remarks:

Remark 1. It is easy to see that $\oplus$ is commutative, associative, idempotent ($\oplus(a,\ a, ...,\ a) = a$) and monotonic but not strictly [3].

And from the Remark 1 we have following proposition.

Proposition 3.5. *Suppose* $\oplus$ *is operators* min, max *or a combination of the two operators, then* $\oplus$ *satisfies the postulates* $W_3,\ W_4,\ W_5$, *and* W_{arb}.

Proof. The way of proving that the merging operator $\oplus$ defined by the Definition 3.3 satisfies the postulates W_3, W_4, W_5, and W_{arb} is very similar to that the merging operator defined by Definition 2.6 satisfies the postulates P_3, P_4, P_5 and P_{arb} in [3] with some small adjustments to fit the context of SIDs, so it is ignored here. ∎

Remark 2. There exist some situations as follows: SIDs $\mathcal{T}_{B_i}$ $(i = 1,\ 2, ...,\ n)$ are standard SIDs but its aggregated SID may not be a standard SID. For example, with the operator $\oplus = \max$, consider the following example.

Example 3.6. Suppose $\phi \in \mathcal{L}$,

$$\mathcal{T}_{B_1}(\omega) = \begin{cases} 1 & \text{if } \omega \vdash \phi \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad \mathcal{T}_{B_2}(\omega) = \begin{cases} 0 & \text{if } \omega \vdash \phi \\ 1 & \text{otherwise} \end{cases}$$

are the two most specificity impossibility distributions of $B_1,\ B_2$.

Then $\mathcal{T}_{B_\oplus}(\omega) = \max(\mathcal{T}_{B_1}(\omega),\ \mathcal{T}_{B_2}(\omega)) = 1\ \forall \omega$, so $\mathcal{T}_{B_\oplus}$ is not standard distribution while $\mathcal{T}_{B_1}$ and $\mathcal{T}_{B_2}$ are standard SIDs. So $B_1,\ B_2$ are consistent SPK bases whereas $B_\bigoplus$ is an inconsistent SPK base. In other words, the operator $\bigoplus = \max$ does not satisfy the posttulate W_1 as in the standard possibilistic logic [3].

Example 3.6 also implies that when a merging operator is a combination in a way of the min and max operators, SID aggregated from standard SIDs may not be a standard SID. But for the operator min, that's not true. Specifically:

Proposition 3.7. $\oplus = \min$ *satisfies the postulates* $W_1,\ W_2$.

Proof.

1. For the postulate W_1: Suppose that $B_i,\ i = 1, ...,\ n$ are consistent SPK bases, to prove that $\mathcal{C}_{n_p}(\mathfrak{B}_\oplus)$ is also consistent we just need to prove that an aggregated SPK base $\mathfrak{B}_\oplus$

Table 1. Impossibility distribution of given and aggregated SPK bases

Ω	τ_{A_1}	τ_{A_2}	$\boldsymbol{\tau_{A_3}}$	$\boldsymbol{\tau}_{\max}$	$\boldsymbol{\tau}_{\min}$
$(\alpha, \beta, \gamma, \kappa, \lambda)$	a_1	a_2	a_3	$\max(a_1, a_2)$	a_3
$(\alpha, \beta, \gamma, \kappa, \neg\lambda)$	a_1	a_2	0	$\max(a_1, a_2)$	$\min(a_1, a_2)$
$(\alpha, \beta, \gamma, \neg\kappa, \lambda)$	0	0	a_3	a_3	0
$(\alpha, \beta, \gamma, \neg\kappa, \neg\lambda)$	0	0	0	0	0
$(\alpha, \beta, \neg\gamma, \kappa, \lambda)$	0	a_2	0	a_2	0
$(\alpha, \beta, \neg\gamma, \kappa, \neg\lambda)$	0	a_2	0	a_2	0
$(\alpha, \beta, \neg\gamma, \neg\kappa, \lambda)$	0	a_2	0	a_2	0
$(\alpha, \beta, \neg\gamma, \neg\kappa, \neg\lambda)$	0	a_2	0	a_2	0
$(\alpha, \neg\beta, \gamma, \kappa, \lambda)$	a_1	a_2	a_3	$\max(a_1, a_2)$	a_3
$(\alpha, \neg\beta, \gamma, \kappa, \neg\lambda)$	a_1	0	a_3	a_1	0
$(\alpha, \neg\beta, \gamma, \neg\kappa, \lambda)$	0	0	a_3	a_3	0
$(\alpha, \neg\beta, \gamma, \neg\kappa, \neg\lambda)$	0	0	a_3	a_3	0
$(\alpha, \neg\beta, \neg\gamma, \kappa, \lambda)$	0	a_2	0	a_2	0
$(\alpha, \neg\beta, \neg\gamma, \kappa, \neg\lambda)$	0	0	a_3	a_3	0
$(\alpha, \neg\beta, \neg\gamma, \neg\kappa, \lambda)$	0	0	0	0	0
$(\alpha, \neg\beta, \neg\gamma, \neg\kappa, \neg\lambda)$	0	0	a_3	a_3	0
$(\neg\alpha, \beta, \gamma, \kappa, \lambda)$	0	a_2	a_3	a_2	0
$(\neg\alpha, \beta, \gamma, \kappa, \neg\lambda)$	0	0	0	0	0
$(\neg\alpha, \beta, \gamma, \neg\kappa, \lambda)$	0	0	a_3	a_3	0
$(\neg\alpha, \beta, \gamma, \neg\kappa, \neg\lambda)$	0	0	0	0	0
$(\neg\alpha, \beta, \neg\gamma, \kappa, \lambda)$	0	a_2	0	a_2	0
$(\neg\alpha, \beta, \neg\gamma, \kappa, \neg\lambda)$	0	a_2	0	a_2	0
$(\neg\alpha, \beta, \neg\gamma, \neg\kappa, \lambda)$	0	a_2	0	a_2	0
$(\neg\alpha, \beta, \neg\gamma, \neg\kappa, \neg\lambda)$	0	a_2	0	a_2	0
$(\neg\alpha, \neg\beta, \gamma, \kappa, \lambda)$	0	a_2	a_3	a_2	0
$(\neg\alpha, \neg\beta, \gamma, \kappa, \neg\lambda)$	0	0	0	0	0
$(\neg\alpha, \neg\beta, \gamma, \neg\kappa, \lambda)$	0	0	a_3	a_3	0
$(\neg\alpha, \neg\beta, \gamma, \neg\kappa, \neg\lambda)$	0	0	0	0	0
$(\neg\alpha, \neg\beta, \neg\gamma, \kappa, \lambda)$	0	a_2	0	a_2	0
$(\neg\alpha, \neg\beta, \neg\gamma, \kappa, \neg\lambda)$	0	0	0	0	0
$(\neg\alpha, \neg\beta, \neg\gamma, \neg\kappa, \lambda)$	0	0	0	0	0
$(\neg\alpha, \neg\beta, \neg\gamma, \neg\kappa, \neg\lambda)$	0	0	0	0	0

is consistent. Indeed, because B_i is consistent so $\mathcal{T}_{B_i}$ is a standard SID, i.e. $\exists \omega_i \in \Omega$: $\mathcal{T}_{B_i}(\omega_i) = 0$. Since all $\mathcal{T}_{B_j}(\omega_i)$ $(j = 1, 2, ..., n)$ are comparable to 0, namely $\mathcal{T}_{B_j}(\omega_i) \geq 0$ and $\mathcal{T}_{B_i}(\omega_i)$, $= 0$, so $\mathcal{T}_{\mathfrak{B}_\oplus}(\omega_i) = \min$ $(\mathcal{T}_{B_1}(\omega_i)$, ..., $\mathcal{T}_{B_i}(\omega_i)$,..., $\mathcal{T}_{B_n}(\omega_i)) = 0$ $(i = 1, ..., n)$. Thus $\mathcal{T}_{\mathfrak{B}_\oplus}$ is a standard SID and arccording to the Proposition 3.2, $\mathfrak{B}_\oplus$ is consistent.

2. For the postulate W_2: First of all, it should be noted that, in the standard possibilistic logic, if $B_1 \cup B_2 \cup \cdots \cup B_n$ is consistent, then $\mathcal{C}_{n_p}(\mathfrak{B}_\oplus) \equiv \mathcal{C}_{n_p}(B_1 \cup B_2 \cup \cdots \cup B_n)$ if and

only if $\oplus$ is a conjunctive operator [3]. Essentially, the Definitions 2.6 and 3.3 about merging operators are related, and the condition that a merging operator of standard possibility distributions is a conjunctive operator is very similar to the condition that a merging operator of SIDs is a disjunctive operator. The min operator is disjunctive. So, the satisfaction of the postulate W_2 of min operator can be proven in the same way as in [3]. ∎

As we know, in the standard possibilistic logic, operator $\oplus$ satisfies W_6 if and only if $\oplus$ is a strictly monotonic operator; The operator $\oplus$ satisfies W_{maj} if and only if $\oplus$ is strict monotonic and reinforcement operator [3]. In the standard possibilistic logic, the min and max operators as well as a combination of the operators aren't strict by monotonic so they do not satisfy W_6 and W_{maj}.

In SPL, the operators min, max, and all combination of these two operators are also not strictly monotonic operators, and the same as in the standard possibilistic logic, we have following proposition.

Proposition 3.8. *Aggregation of SIDs does not satisfy* W_6, W_{maj}.

Proof. The idea is quite similar to the proof of the postulates P_6 and P_{maj} in [3], so it is ignored here. ∎

4. HIERARCHICAL AGGREGATION OF SPK BASES

Assume that $\mathcal{T}_{B_1}, \mathcal{T}_{B_2}, \ldots, \mathcal{T}_{B_n}$ are most specificity SIDs corresponding to the SPK bases $B_1, \ldots, B_n$. In these SIDs, there may be some situations that there are several groups of distributions having same characteristics, such as the order of interpretations (or possible words) sorted by values of SIDs or the reliability of the knowledge bases in each group are the same. In such situations, it would be more reasonable if the aggregation process of SIDs is implemented as follows: First of all, aggregating knowledge bases in each group and an aggregated knowledge base of each group is considered as a representative knowledge base of the group. This procedure can be performed by such some times and a final created knowledge base is an aggregated knowledge base of the merging process. The aggregation implemented by this way is called a hierarchical aggregation.

Assume that n SIDs $\mathcal{T}_{B_1}, \mathcal{T}_{B_2}, \ldots, \mathcal{T}_{B_n}$ are divided into m groups $(\mathcal{T}_{B_{i1_1}}, \mathcal{T}_{B_{i1_2}}, \ldots, \mathcal{T}_{B_{i1_{k1}}})$, $(\mathcal{T}_{B_{i2_1}}, \mathcal{T}_{B_{i2_2}}, \ldots, \mathcal{T}_{B_{i2_{k2}}})$,..., $(\mathcal{T}_{B_{im_1}}, \mathcal{T}_{B_{im_2}}, \ldots, \mathcal{T}_{B_{im_{km}}})$, so that SIDs in each group have common properties. Suppose that a merging operator $\oplus_2$ is used to aggregate SIDs in each group and other merging operator $\oplus_1$ is used to aggregate representative SIDs of the groups.

Definition 4.1. A hierarchical merging operator (2 hierarchies) denoted by $\oplus = \oplus_1 * \oplus_2$ is defined as follows

$\oplus(\mathcal{T}_{B_1}, \mathcal{T}_{B_2}, \ldots, \mathcal{T}_{B_n}) = \oplus_1(\oplus_2(\mathcal{T}_{B_{i1_1}}, \mathcal{T}_{B_{i1_2}}, \ldots, \mathcal{T}_{B_{i1_{k1}}}), \ldots, \oplus_2(\mathcal{T}_{B_{im_1}}, \mathcal{T}_{B_{im_2}}, \ldots, \mathcal{T}_{B_{im_{km}}}))$,
where $\oplus_1$, $\oplus_2$ are the merging operators of SIDs in low and high levels, respectively.

A merging operator of n - hierarchies is also defined in a similar way.

Example 4.2. Suppose that the process of hierarchical aggregation of 3 SPK bases in Example 2.5 is implemented as follows $\oplus(\mathcal{T}_{A1}, \mathcal{T}_{A2}, \mathcal{T}_{A3}) = \oplus_1(\oplus_2(\mathcal{T}_{A1}), \oplus_2(\mathcal{T}_{A2}, \mathcal{T}_{A3}))$, where $\oplus_1$, $\oplus_2$ are max or min operators. Then, there are 4 of aggregated SIDs corresponding to 4 combinations of min and max operators, as shown in Table 2 below. The table also shows that all 4 aggregated SPK bases are consistent.

Table 2. Aggregated impossibility distributions by min-min, min-max, max-min, max-max operators

Ω	$\mathcal{T}_{A1}$	$\mathcal{T}_{A2}$	$\mathcal{T}_{\boldsymbol{A}3}$	$\min-\min$	min-max	max-min	max-max
$(\alpha,\beta,\gamma,\kappa,\lambda)$	a_1	a_2	a_3	$\min(\min(a_1,a_2),a_3)$	a_3	$\max(\min(a_1,a_2),a_3)$	$\max(a_1,a_2)$
$(\alpha,\beta,\gamma,\kappa,\neg\lambda)$	a_1	a_2	0	0	0	0	$\max(a_1,a_2)$
$(\alpha,\beta,\gamma,\neg\kappa,\lambda)$	0	0	a_3	0	0	a_3	a_3
$(\alpha,\beta,\gamma,\neg\kappa,\neg\lambda)$	0	0	0	0	0	0	0
$(\alpha,\beta,\neg\gamma,\kappa,\lambda)$	0	a_2	0	0	0	0	a_2
$(\alpha,\beta,\neg\gamma,\kappa,\neg\lambda)$	0	a_2	0	0	0	0	a_2
$(\alpha,\beta,\neg\gamma,\neg\kappa,\lambda)$	0	a_2	0	0	0	0	a_2
$(\alpha,\beta,\neg\gamma,\neg\kappa,\neg\lambda)$	0	a_2	0	0	0	0	a_2
$(\alpha,\neg\beta,\gamma,\kappa,\lambda)$	a_1	a_2	a_3	$\min(\min(a_1,a_2),a_3)$	a_3	$\max(\min(a_1,a_2),a_3)$	$\max(a_1,a_2)$
$(\alpha,\neg\beta,\gamma,\kappa,\neg\lambda)$	a_1	0	a_3	0	a_3	a_3	a_1
$(\alpha,\neg\beta,\gamma,\neg\kappa,\lambda)$	0	0	a_3	0	0	a_3	a_3
$(\alpha,\neg\beta,\gamma,\neg\kappa,\neg\lambda)$	0	0	a_3	0	0	a_3	a_3
$(\alpha,\neg\beta,\neg\gamma,\kappa,\lambda)$	0	a_2	0	0	0	0	a_2
$(\alpha,\neg\beta,\neg\gamma,\kappa,\neg\lambda)$	0	0	a_3	0	0	a_3	a_3
$(\alpha,\neg\beta,\neg\gamma,\neg\kappa,\lambda)$	0	0	0	0	0	0	0
$(\alpha,\neg\beta,\neg\gamma,\neg\kappa,\neg\lambda)$	0	0	a_3	0	0	a_3	a_3
$(\neg\alpha,\beta,\gamma,\kappa,\lambda)$	0	a_2	a_3	0	a_3	a_3	a_2
$(\neg\alpha,\beta,\gamma,\kappa,\neg\lambda)$	0	0	0	0	0	0	0
$(\neg\alpha,\beta,\gamma,\neg\kappa,\lambda)$	0	0	a_3	0	0	a_3	a_3
$(\neg\alpha,\beta,\gamma,\neg\kappa,\neg\lambda)$	0	0	0	0	0	0	0
$(\neg\alpha,\beta,\neg\gamma,\kappa,\lambda)$	0	a_2	0	0	0	0	a_2
$(\neg\alpha,\beta,\neg\gamma,\kappa,\neg\lambda)$	0	a_2	0	0	0	0	a_2
$(\neg\alpha,\beta,\neg\gamma,\neg\kappa,\lambda)$	0	a_2	0	0	0	0	a_2
$(\neg\alpha,\beta,\neg\gamma,\neg\kappa,\neg\lambda)$	0	a_2	0	0	0	0	a_2
$(\neg\alpha,\neg\beta,\gamma,\kappa,\lambda)$	0	a_2	a_3	0	a_3	a_3	a_2
$(\neg\alpha,\neg\beta,\gamma,\kappa,\neg\lambda)$	0	0	0	0	0	0	0
$(\neg\alpha,\neg\beta,\gamma,\neg\kappa,\lambda)$	0	0	a_3	0	0	a_3	a_3
$(\neg\alpha,\neg\beta,\gamma,\neg\kappa,\neg\lambda)$	0	0	0	0	0	0	0
$(\neg\alpha,\neg\beta,\neg\gamma,\kappa,\lambda)$	0	a_2	0	0	0	0	a_2

Definition 4.3. In order to fit the context of hierarchical aggregation of SPK bases, the postulates W_4, W_5 in [3] are adjusted as follows:

W_4^*: Suppose that B_1, $B_2, .., B_n$ are consistent SPK bases, if $\{B_1, \ldots, B_k\}_{\oplus_2}$ and

$\{B_{k+1},\ldots,\ B_n\}_{\oplus_2}$ are equally prioritized, then $\mathcal{C}_{n_p}(\mathfrak{B}_\oplus)\not\models\{B_1,\ldots,\ B_k\}_{\oplus_2}$ and $\mathcal{C}_{n_p}(\mathfrak{B}_\oplus)\not\models\ \{B_{k+1},\ldots,\ B_n\}_{\oplus_2}$, here $\oplus\ =\ \oplus_1\times\oplus_2$.

W_5^*: $\mathcal{C}_{n_p}(\mathfrak{B}'_\oplus)\bigsqcup\mathcal{C}_{n_p}(\mathfrak{B}''_\oplus)\models\mathcal{C}_{n_p}(\mathfrak{B}_\oplus)$, here $\mathfrak{B}=\mathfrak{B}'\bigsqcup\mathfrak{B}''$, the symbol $\bigsqcup$ denotes the union on multi-sets, $\mathfrak{B}'$ and $\mathfrak{B}''$ are divided into the same number of groups.

Following propositions show properties of hierarchical merging operators from the postulate point of view.

Proposition 4.4. *If operators $\oplus_1,\ \oplus_2$ satisfy the postulates W_1, W_2, W_3, W_4, W_5, and W_{arb} then $\oplus$ also satisfies the postulates W_1, W_2, W_3, W_4^*, W_5^* and W_{arb}, respectively.*

Proof.

1) W_1: Because SPK bases $B_i,\ i=1,\ldots,n$ are consistent, so τ_{B_i} $(i=1,...,n)$ are standard SIDs. From the properties of $\oplus_2$, we can infer that the aggregated SID $\tau_{\oplus 2_{i_j}}$ of the group i_j $(j=1,\ldots,m)$ is a standard SID. Similarly, from the properties of $\oplus_1$ it can be seen that $\tau_\oplus$ is also a standard SID. Hence $\mathcal{C}_{n_p}(\mathfrak{B}_\oplus)$ is consistent.

2) W_2: Because $B_1\cup B_2\ \cup\cdots\cup B_n$ are consistent, so $B_{ij_1}\cup B_{ij_2}\cup\cdots\cup B_{ij_{kj}}$ are also consistent. Based on this and from Proposition 3.7, we infer that $\mathcal{C}_{n_p}(\mathfrak{B}_{ij\oplus_2})\equiv\mathcal{C}_{n_p}(B_{ij_1}\cup B_{ij_2}\cup\cdots\cup B_{ij_{kj}})$ and $\mathfrak{B}_{ij\oplus_2}$ is consistent for every $j=1,\ 2,...,\ m$.

Next we will prove that $\mathfrak{B}_{i1\oplus_2}\cup\mathfrak{B}_{i2\oplus_2}\ \cup\cdots\cup\mathfrak{B}_{im\oplus_2}$ is consistent. Suppose the opposite $\mathfrak{B}_{i1\oplus_2}\cup\mathfrak{B}_{i2\oplus_2}\cup\cdots\cup\mathfrak{B}_{im\oplus_2}$ is inconsistent iff $\exists\phi\in\mathcal{L}$ so that $\mathfrak{B}_{i1\oplus_2}\cup\mathfrak{B}_{i2\oplus_2}\cup\cdots\cup\mathfrak{B}_{im\oplus_2}\models(\phi,\ a)$ and $\mathfrak{B}_{i1\oplus_2}\cup\mathfrak{B}_{i2\oplus_2}\cup\cdots\cup\mathfrak{B}_{im\oplus_2}\models(\neg\phi,\ \beta)$, here a and $\beta>0$ iff $\exists p,\ q$ so that $\mathfrak{B}_{ip\oplus_2}\models(\phi,\ a)$ and $\mathfrak{B}_{iq\oplus_2}\models(\neg\phi,\ \beta)$ iff $(B_{ip_1}\cup B_{ip_2}\ \cup\cdots\cup B_{ip_{kp}})\models(\phi,\ a)$ and $(B_{iq_1}\cup B_{iq_2}\cup\cdots\cup B_{iq_{kq}})\models(\neg\phi,\ \beta)\Rightarrow(B_{ip_1}\cup B_{ip_2}\cup\cdots\cup B_{ip_{kp}}\cup\ B_{iq_1}\cup B_{iq_2}\cup\cdots\cup B_{iq_{kq}})$ is inconsistent $\Rightarrow B_1\cup B_2\cup\cdots\cup B_n$ is inconsistent. This is contrary to the assumption of SPK bases B_i $(i=1,...,n)$. And therefore we also have $\mathcal{C}_{n_p}(\mathfrak{B}_\oplus)\equiv\mathcal{C}_{n_p}(\mathfrak{B}_{i1\oplus_2}\cup\ \mathfrak{B}_{i2\oplus_2}\cup\cdots\cup\mathfrak{B}_{im\oplus_2})$.

With $(\phi,\ a)\in\mathcal{C}_{n_p}(\mathfrak{B}_\oplus)$ iff $(\phi,\ a)\in\mathcal{C}_{n_p}(\mathfrak{B}_{i1\oplus_2}\cup\ \mathfrak{B}_{i2\oplus_2}\cup\cdots\cup\mathfrak{B}_{im\oplus_2})$ iff $\mathfrak{B}_{i1\oplus_2}\cup\ \mathfrak{B}_{i2\oplus_2}\cup\cdots\cup\mathfrak{B}_{im\oplus_2}\models(\phi,\ a)$ iff $\exists p$ so that $\mathfrak{B}_{ip\oplus_2}\models(\phi,\ a)$ iff $\exists p:\ (B_{ip_1}\cup B_{ip_2}\cup\cdots\cup B_{ip_{kp}})\models(\phi,\ a)$ iff $B_1\cup B_2\cup\cdots\cup B_n\models(\phi,\ a)$ iff $(\phi,\ a)\in\mathcal{C}_{n_p}\ (B_1\cup B_2\ \cup\cdots\cup B_n)$.

3) W_3: If $\mathfrak{B}\Leftrightarrow\mathfrak{B}'$ then $\mathcal{C}_{n_p}(\mathfrak{B}_\oplus)\equiv\mathcal{C}_{n_p}(\mathfrak{B}'_\oplus)$.

By definition $\mathfrak{B}\Leftrightarrow\mathfrak{B}'\Rightarrow$ with each group $(B_{ij_1},\ B_{ij_2}\ ,\ldots,B_{ij_{kj}})$ in $\mathfrak{B},\ j=1,2,...,m$ there exists only the jth group $(B'_{hj_1},\ B'_{hj_2},\ldots,B'_{hj_{kj}})$ in $\mathfrak{B}'$ so that $\mathcal{C}_{n_p}(B_{ij_k})\equiv\mathcal{C}_{n_p}(B'_{hj_k})$ for $k=1,\ 2,...,\ k_j$. From $\mathcal{C}_{n_p}(B_{ij_k})\equiv\mathcal{C}_{n_p}(B'_{hj_k})\Rightarrow\tau_{B_{ij_k}}(\omega)=\tau_{B'_{hj_k}}(\omega)$ for $\forall\omega\in O$. Thus,

$$\tau_{\oplus_2}^{B_{ij}}(\omega)=\oplus_2(\tau_{B_{ij_1}}(\omega),\ \ldots,\ \tau_{B_{ij_j}}(\omega))=\oplus_2(\tau_{B'_{hj_1}}(\omega),\ \ldots,\ \tau_{B'_{hj_k}}(\omega))=\tau_{\oplus_2}^{B'_{hj}}(\omega)$$
$$\Rightarrow\ \tau_{\mathfrak{B}_\oplus}(\omega)=\oplus_1(\tau_{\oplus_2}^{B_{i1}}(\omega),\ \ldots,\ \tau_{\oplus_2}^{B_{im}}(\omega))=\oplus_1(\tau_{\oplus_2}^{B'_{h1}}(\omega),\ \ldots,\ \tau_{\oplus_2}^{B'_{hm}}(\omega))=\tau_{\mathfrak{B}'_\oplus}(\omega)$$

$$\Rightarrow\ \mathcal{C}_{n_p}(\mathfrak{B}_\oplus)\equiv\mathcal{C}_{n_p}(\mathfrak{B}'_\oplus).$$

4) W_4^* : Because $\oplus_1$ satisfies the postulate W_4, so the proof of this postulate is directly inferred.

5) W_5^* : Suppore that $\mathfrak{B}'$ and $\mathfrak{B}''$ are divided into m groups $\Rightarrow\mathfrak{B}=\mathfrak{B}'\bigsqcup\mathfrak{B}''$ is also divided into m groups, $\mathfrak{B}=\{\ \mathfrak{B}_1,\ \ldots,\ \mathfrak{B}_m\}$,

$$\begin{aligned}\mathfrak{B}_j = \{B_{j_1},\ B_{j_2}, \ldots, B_{j_k}\} &= \{\{B'_{j_1},\ B'_{j_2}, \ldots, B'_{j_h}\} \bigsqcup \{B''_{j_1},\ B''_{j_2}, \ldots, B''_{j_q}\} \\ &= \mathfrak{B}'_j \bigsqcup \mathfrak{B}''_j,\end{aligned}$$

here, $B'_{j_h} \in \mathfrak{B}'$ and $B''_{j_q} \in \mathfrak{B}''$.

From the properties of $\oplus_2$, we have

$$\mathcal{C}_{n_p}(\mathfrak{B}'_{j_{\oplus_2}}) \bigsqcup \mathcal{C}_{n_p}(\mathfrak{B}''_{j_{\oplus_2}}) \models \mathcal{C}_{n_p}(\mathfrak{B}_{j_{\oplus_2}}). \tag{4.1}$$

For each $(\phi, a) \in \mathcal{C}_{n_p}(\mathfrak{B}_\oplus) \Rightarrow \mathfrak{B}_\oplus \models (\phi, a) \Rightarrow \exists h$ so that $\mathfrak{B}_{h\oplus_1} \models (\phi, a) \Rightarrow (\phi, a) \in \mathcal{C}_{n_p}(\mathfrak{B}_{h_{\oplus_2}}) \Rightarrow$ according to (4.1), we obtain $\mathcal{C}_{n_p}(\mathfrak{B}'_{h\oplus_2}) \bigsqcup \mathcal{C}_{n_p}(\mathfrak{B}''_{h\oplus_2}) \models (\phi, a)$ and from the hypothesis of $\oplus_1 \Rightarrow \mathcal{C}_{n_p}(\mathfrak{B}'_\oplus) \bigsqcup \mathcal{C}_{n_p}(\mathfrak{B}''_\oplus) \models (\phi, a)$.

6) $W_{arb} : \forall B',\ \forall n,\ \mathcal{C}_{n_p}((\mathfrak{B} \bigsqcup B'^n)_\oplus) \equiv \mathcal{C}_{n_p}((\mathfrak{B} \bigsqcup B')_\oplus)$.

Suppose that $\mathfrak{B} = \bigsqcup_{j=1,\ \ldots,m}\{B_{ij_1},\ B_{ij_2}\ , \ldots, B_{ij_{kj}}\}$,

$$\begin{aligned}(\mathfrak{B} \bigsqcup B'^n)_\oplus &= \{\{B_{i1_1},\ B_{i1_2}, \ldots, B_{i1_{k1}}\}_{\oplus_2}, ..., \{B_{im_1},\ B_{i1_2}, \ldots, B_{im_{km}} \bigsqcup B'^n\}_{\oplus_2}\}_{\oplus_1} \\ &= \{\{B_{i1_1},\ B_{i1_2}, \ldots, B_{i1_{k1}}\}_{\oplus_2}, ..., \{B_{im_1},\ B_{i1_2}\ , \ldots, B_{im_{k1}} \bigsqcup B'\}_{\oplus_2}\}_{\oplus_1} \\ &= (\mathfrak{B} \bigsqcup B')_\oplus \text{ (by the properties of } \oplus_2).\end{aligned}$$

So $\mathcal{C}_{n_p}((\mathfrak{B} \bigsqcup B'^n)_\oplus\ ) \equiv \mathcal{C}_{n_p}((\mathfrak{B} \bigsqcup B')_\oplus)$. ■

5. CONCLUSIONS

The soundness and completeness of the inference system in SPL [4] has enabled to implement the aggregation of SPK bases via their most specificity impossibility distributions.

This paper focused on researching and clarifying the nature of merging operators of the SIDs from the postulate point of view. The postulates of aggregation processes are adjusted from the accepted postulates widely in aggregation processes of knowledge bases in classical propositional language. The properties of merging operators as well as hierarchically merging operators of SIDs from the postulate point of view are also clarified.

Since only the min and max calculations and a combination of these two calculations are performed on the weights (which are symbols), so merging operators as well as hierarchically merging operators of SIDs are only min, max, and a combination of these two operators in a way. The poverty of merging operators in SPL shows a limitation of this logic. This suggests that it is necessary to continue developing a SPL by using weights (symbols) having an algebraic structure with more calculations.

However, it can be said that with the proposed SPL, we had a complete logic language for expressing and reasoning as well as building smart systems on symbolic knowledges.

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