Journal of Computer Science and Cybernetics, V.35, N.2 (2019), 167–184
DOI 10.15625/1813-9663/35/2/13095

NASH EQUILIBRIUM MODEL FOR CONFLICTS IN PROJECT MANAGEMENT

BAO NGOC TRINH[1], QUYET THANG HUYNH[1,*], XUAN THANG NGUYEN[2]
[1]*School of Information and Communication Technology, HUST*
[2]*Faculty of Information Technology, Hanoi University, Vietnam*
[*]*thanghq@soict.hust.edu.vn*

Abstract. There are many types of concern to project management, and we used to deal with them by using Risk management activities. But in complicated situations, there are many factors and preconditions of the issues that make the problems unrecognizable with risk consultants, and the consequences of these issues getting more and more enormous. The cause of the problems can be a conflict between the project teams and their partners or maybe inside their team, or between groups. The solution for these unseen conflicts can result in cost, schedule, and quality of the project. In recent study trends, the application of Game theory to conflict resolution in project management began to be applied. In this research, we propose a Unified model of Game theory to solve the conflict problems in project management. We present solution of using the Unified model for three typical conflict problems: project payments, multi-round procurement, and management of risk responses. Experimental results show that the proposed model has been applied effectively for these three problems.

Keywords. Project Conflicts; Game Theory; Nash Equilibrium.

1. INTRODUCTION

Project management is the core process of most current business activities, which include the projects that have the goal of creating products, services or original results. Project management tasks include project scope management, quality, project schedule, budget, resources, and risks. In which Risk management examines the functions which would happen including plan scheduling processes, risk identification, analyzing, resource planning and project control monitoring. Risk management aims for increasing the probability and influences of positive events, also decreasing the likelihood and consequences of adverse events of the project, in which risks determined as uncertain events or conditions which when occurs, would affect the project [1]. Still, there are more internal problems affecting the project which is out of the management of Risk management parts such as the conflicts between the project partners or the conflicts in Risk management itself. Therefore, detecting and analyzing these problems brings a necessary supplement for the Project management tasks, thus ensuring all matters arising to be controlled and also enhancing the quality and chances of success of the project.

Of all the problem-solving methods involving the conflict of interests type, Game theory emerges as a highly potential approach, whereas many of the Game theory applications have achieved remarkable scientific achievements such as the Nobel Prize. Game theory is a branch of applied mathematics that studies tactical situations in which players choose different actions to try to maximize the results received. Therefore, the outcome will depend on the decision of all players, in which each player will try to predict the choice of the remaining players to be able to make the best choice for themselves [2]. Nash Equilibrium is one of the most commonly used concepts in the applications of Game theory in economics when it comes to studying interdependent behaviors of agents with mutually contradictory goals. It is the central concept of non-cooperative Game theory with perfect information, proposed by J.F. Nash (1951). Thus, the objectives of this study include:

(i) Analysis of existing conflicts and conflicts within the project that are beyond the scope of the risk and not adequately managed;

(ii) The research will investigate some of the existing models regarding some problems that have resulted in the publication;

(iii) And finally, to synthesize and propose a Unified model for conflict problems in the project according to Game theory and also suggest a Nash Equilibrium model for this problem.

The results of the current research on the type of conflicts in the project under the direction of Game theory are not much. It may include the previous studies of Brent Lagesse [3] on the model of Game theory's delivery of the project in which to consider the conflicts when assigning personnel but does not have the appropriate skills for the job. It also includes the problem of conflict in schedule payments [4] in addition to some author's claims on the subject matter such as improved articles and algorithmic modifications and the model of multi-round procurement [5], or issues when implementing payment scheduling [6]. It is clear that the problems of the project are numerous and complex, with many variations, and moreover the results are needed for the management of the project so much that further similar researches on other issue are required, particularly a joint study on the overview of all existing conflicts, as well as build a general unified model. Based on conventional models of Game theory, we can apply a variety of solutions, namely multi-objective optimization strategies or evolutionary algorithms to find a Nash Equilibrium. In this solution, players composed of members and organizations inside and outside the project are satisfied or have a balanced benefit, no one is worse, and no one is more profitable than the many criteria of the problem. Given the different layers of the project, the solution to Nash Equilibrium is very relevant and necessary. The rest of this paper is organizing as follows. Section 2 presents a classification of conflicts in project management; In Section 3 we show the game-theoretical model of conflicts and present status of applying Game theory to three typical conflict problems: Project Payments, Multi-round Procurement, and Management of Risk responses. Section 4 presents applying the unified model for the three mentioned problems. We conduct the experimental evaluation in Section 5. Section 6 concludes with a summary and shows our future work.

2. CLASSIFICATION OF CONFLICTS IN PROJECT MANAGEMENT

Based on various factors such as the source, cause, role or function of conflicts in the project, conflicts can be classified into several categories as follows [7, 8]:

- By source of conflict, conflicts are classified as (i) conflict from plan scheduling, (ii) conflict from determining the priority in performing project task, (iii) conflict from the power sources, (iv) conflict from technical problems, (v) conflict from administrative procedure, (vi) conflict from private issue and (vii) conflict from expenditure.
- By cause of conflict, conflicts are classified as: (i) conflict from different goals, (ii) conflict from resource disparity, (iii) conflict by other people's obstruction, (iv) conflict due to stress and psychological pressure from many people, (v) conflict due to ambiguity of jurisdiction and (vi) conflict due to misleading communication.
- By role, conflicts are classified as (i) positive conflict and (ii) negative conflict.
- By function, conflicts are classified as (i) functional conflict and (ii) dysfunctional conflict.

In the conflict classification system, research would choose to determine and list the conflict by origin [8] such as the Table 1.

3. GAME-THEORETICAL MODEL OF CONFLICTS

Game theory is a methodology using mathematical tools to model and analyze situations involving several decision makers (DMs), called players [2]. Game theoretic models arise in numerous application domains including Board and field games, Marketing and commercial operations, Politics, Defense, Robotics and multi-agent systems, Social networks [2, 9].

3.1. Game representation

The strategic form of a game

Strategic form of a game with finite n players is a tuple of three parts as follows [9]

$$G = \Big\langle N, \big(A_i\big)_{i\in N}, \big(u_i\big)_{i\in N}\Big\rangle \tag{1}$$

where,

G is the game in strategic form;

$N = \big\{1, 2, .., n\big\}$ is the set of players;

A_i is the set of action of player i ($i \in N$);

$A := \big\{a|a = (a_i)_{i\in N}, a_i \in A_i, \forall i \in N\big\}$ is the set of action profiles;

$u_i : A \to \mathbb{R}$ is the payoff function of player i: $(a_1, a_2, ..., a_n) \to u_i(a_1, a_2, ..., a_n)$.

The payoff function u_i is a profit (to maximize) but can also be a cost (to minimize).
An equivalent way of writing the action profiles is $(a_j)_{j\in N} = (a_1, a_2, ..., a_n) = (a_i, a_{-i})$, where $a_{-i} = (a_j)_{j\in N, j\neq i}$ is the action profile of all players except i.

Nash Equilibrium

The basic concept of the Nash Equilibrium is "unilateral deviations": Only one player changes its own decision while the others stick to their current choices. The action profile/outcome $a^* = (a_1^*, .., a_i^*, .., a_n^*)$ is Nash Equilibrium (NE) when

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*),\ \forall a_i \in A_i,\ \forall i \in N, \tag{2}$$

Table 1. Classification by source of conflicts

Source	Conflict description
Plan scheduling	Conflict between the project scheduling conditions Conflict in project payment schedule Conflict between the plan and other project management activities such as: expense, time, human resources and power sources Conflict between the changes of plans
Determining priority in performing project task	Conflict in assigning roles for performing the task Conflict between the binding factors of the project task Not defining the priority level of each task Conflict between the priority levels when implementing risk management methods
Power sources	Conflict in assigning roles for project implementation and capability Conflict in attributing project power sources Conflict in multi-round procurement Conflict in procurement
Technical problems	Conflict between communicating channels Conflict between technology complexity and project completion deadline Conflict between staff capability and project technology Conflict between technology and educating process
Administrative procedure	Conflict between project management methods Conflict between the tasks of the project implementation units Conflict between the project implementation procedures and project implementing speed Conflict between applying standards of the project
Private issue	Conflict between the goal of the investors Conflict between the board of managers and the project implementation members Conflict between the project members Conflict between project experience and member payment expense
Expenditure	Conflict in attributing project expenses Conflict in finance management and project changes Conflict between financial management and project quality Conflict between finance management and educating human resources of the project

NE is the point where no player can get a higher profit by his unilateral tactics. That is, a player who wants to maximize their benefits will not deviate from the NE. In fact, a non-NE strategy means that there is a player who can get higher returns by choosing a strategy $a^{''}$ when others choose the strategy $a^{'}$ that $u_i(a_i^{''}, a_{-i}^{'}) \geq u_i(a_i^{'}, a_{-i}^{'})$.

However, the current model with the class of conflicting problems in project management has the following issues: In a game of conflict which affect to owner benefit, the interests of the owner and the partners also have the opposite; the investor is also the person who keeps the game fair to players who are counterparts. For games without the participation of the owner, all players are equal in the game, there should be one more person play an essential role in balancing the interests of the players. On the other hand, to evaluate a particular players specific tactics is good or not, it must be based on the factors affecting the gain considered in the problem. For example, these factors in the risk problem are: severity, priority, cost of risk, duration of implementation, difficulty of the method; and these in the Multi-objective procurement problem are: the confidence of the contractor with the project owner, the prestige of the contractor, the time of implementation, the price of the tender package. Therefore, each strategy characterized by a set represented by the influencing factors.

3.2. Applying Game theory to some typical conflict problems in project management - present status

According to the classification of conflict problems in project management as discussed above in Section 2, objects in conflict can be model as players in a game; however, there are two main issues as follows:

- The way to explain the model, describe the game, describe the strategy and determine the Nash Equilibrium are different in each research.
- In some classes of games in which no player is the owner, it is unreasonable to consider the balance of interests among players without regard to the general interest of the project (project owner).

Typical representatives of these problems may be listed: conflict in project payments, conflict in multi-round procurement, conflict in the management of risk processing solutions [2, 8]:

Conflict in project payments

Scheduling project payments are one of the problems that have caught much attention in project management [4]. In managing the project, it is always necessary to unify the owner's payment and the contractor's schedule so that both parties benefit. While the investor wants to delay paying the project, the contractor expects the amount to take place as soon as possible. The involvement of the investor and the contractor is a game of two players in which each party wants to achieve their interests. The payment plan is divided into payment periods with different amounts of money by the investor. The contractor runs the project through several stages with various activities, which have certain constraints. Each activity has several ways of proceeding, each of which consumes a specified amount of resources over a specified period. Conflicts occur in the payment of project funds when both contractor and investor want to maximize their financial returns. The optimal payment schedule with the contractor is to obtain a one-time payment at the start of the project.

After receiving the total initial amount, the contractor will try to minimize their costs by scheduling work such that higher cash flow. The optimal payment schedule for both contractor and investor is called the ideal solution for both, having a payment schedule that corresponds to the completed work of the project [4]. The main problem is solving the schedule of the project, ensuring both the interests of the investor and the contractor during the project implementation which helps to resolve the conflict between the contractor and the project owner during the project payment process based on information from both of them.

In [4] there assumed that the terms of project payment in the contract are executed in stages, namely, the project payment is making at a node of an activity network. The ideal protocol for the owner must be that the money is paid once for all only when the project is completing. It is evident that in most cases, any single profitable scheme is hard to be accepted by both sides and compromise has to be considered [4]. A more practical situation is that during the process of this compromise (or gaming), the owner puts forward a schedule of payment first, and then the client proposes a corresponding activity schedule according to that payment schedule. The strategic game is presented as follows [4]:

$$G = \{S_0, S_0 \to S_c, u_0(S_0, S_c), u_c(S_0, S_c)\}, \tag{3}$$

where,

S_0: the strategy of the project owner;

S_c: the strategy of the project team;

$u_0(S_0, S_c)$: payoff function of the owner;

$u_c(S_0, S_c)$: payoff function of the contractor.

The difference in formula (3) and formula (1) is the role of S_0 - project owner.

Conflict in multi-round procurement

Multi-round procurement with many bidders is the process of the project owner, and other bidders join negotiation, persuasion to bring benefits to themselves [5]. Bidding is a process whereby an investor chooses a contractor who meets his or her requirements. The buyer organizes the tender so that the seller (the contractor) can compete for each other. Indeed, for large projects, time stretching is usually divided into smaller categories. The project owner (investor) will not find the contractor for the whole project at a single time but will hold the bid for each item at different times. The purpose is to maximize the benefits to the contractor while minimizing risks during project implementation. The contractor will select the time of bidding and choose the significant level of the tender package. Since the raw material prices fluctuate over time, the contractor must ensure that they have sufficient capacity to implement the tender package. The investor will choose to distribute the parts of the package to the appropriate contractor [2, 5].

Conflict occurs when both builders and contractors participating in the bidding will try to get the most significant benefit for themselves from the tender package. Specifically, for the contractors, the benefit that they wish to receive from the tender package is to find a reliable investor with the most reasonable price to reduce the cost of the project, minimize the cost of the project but it does not offend their partners. For the investors, the critical goal is to be selected. To achieve that they should provide the most suitable conditions and prices for the offered goods; their last benefit is the profit from the project after winning the bid. The problem is to solve the Multi-round procurement problem, ensure the benefits for both the contractor and the investor, that is, help to resolve conflicts between the contractor and the project owner during the Multi-round procurement engagement based on the available information of the project, the project owner and the contractor. In practical situations, the process of this compromise is significantly difficult and depends mainly on project investor [5]. Solving the problem of multi-round procurement brings the win-win result for all parties. The strategic game is presented as following [5]:

$$G = \{S_0, S_c, F_0, F_c\}, \tag{4}$$

where,

S_0: the strategy of contractor;

S_c: the strategy of the bidder;

F_0: payoff function of the contractor;

F_c: payoff function of the bidder.

The game presentations of both problems, Project payments, and Multi-round procurement are similar (see (3) and (4)).

Conflict in the management of risk processing solutions

Risk factors always exist in all activities of project implementation. However, identifying and control the risks in the project are not simple. Many projects have overlooked or controlled a risky, prone to failure, customer complaints about quality or loss of capital due to rising costs. Risk management in projects will increase the positive impact and mitigate negative consequences from the risks. Risks will be identified, analyzed, monitored and to minimize the risks arising before, during and after project implementation [8, 10].

When planning to address risk, this may occur in conflict with another risk handling plan. Therefore, the choice of how to deal with risks will directly affect the progress and benefits of the project [10]. The strategic game for conflicts in the management of risk processing solutions is as following [10]

$$G = \{R_0 \to R_n; S_{ij}(t_{ij}, p_{ij}, c_{ij}, d_{ij});\ C_0 \to C_m;\ p_i(C_x \to C_y)\}, \tag{5}$$

where,

$R_0 \to R_n$: denotes all the risks that a conflict may have while being resolved;

n: number of risks;

$S_{ij}(t_{ij}, p_{ij}, c_{ij}, d_{ij})$: denotes the method of handling j for risk R_i;

t_{ij} : denotes the time to implement method j for risk R_i;

p_{ij}: denotes the priority of implementing method j for risk R_i;

c_{ij}: denotes the cost when implementing method j for risk R_i;

d_{ij}: denotes the difficulty of method j for the risk R_i;

$C_0 \to C_m$: denotes conflicting groups of handling between risks;

m: denotes the number of conflicts in the risk management plan;

$p_i(C_x \to C_y)$: is payoff matrix for groups of risk $C_x \to C_y$ in conflict.

3.3. Proposed solution for a Unified model of Game theory for conflict problems

The idea of the Unified model is to create a special player with different interests than the rest of the players. For games involving the investor, this particular player is called the investor. For non-investor games, this special player is acquired by adding a virtual player called a referee in the game. At the same time, each player's specific tactics will also have a characteristic vector that indicates the extent to which elements of the strategy are affected. In the Unified model, this player is represented with $\{P_0,\ S_0,\ u_0\}$, where P_0 is special player (investor in games with the participation of investors, virtual player in the game without the involvement of investors); S_0 is the set of pure strategies of a special player, and u_0 is a payoff function of the special player P_0.

Other N normal players will be modeled as $\{P,\ S_i,\ u_i\}$, where P is a set of N normal players (partners) joining the game; S_i is a set of pure strategies of the players number i. u_i is a payoff function of the normal player number i.

We propose adding component R^c, which presents set of players conflicts.

Thus, by combining three components: (i) special player, (ii) regular/normal players and (iii) set of players conflicts, we propose unified model as follows

$$G = \{P_0, S_0, u_0, P, S_i, u_i, R^c\}, \tag{6}$$

where,

G : represents the game;

P_0 is a special player who is a stakeholder of project or a virtual player who represent the benefit of entire project when we analyze the characteristics of project elements;

$S_0 = \{s_{01}, .., s_{0j}, .., s_{0M_0}\}$ is the set of strategies of the special player; M_0 is the number of strategies of the special player;

$u_0 : S_0 \to \mathbb{R}$ is a payoff function that maps special player's strategy to payoff value in a real number;

$P = \{P_1, .., P_i, .., P_N\}$ is a set of normal players; N: number of normal players;

$S_i = \{s_{i1}, ..s_{ij}, ..s_{iM_i}\}$ is a set of strategies of normal player $i(1 \le i \le N)$ and M_i is the number of strategy of normal player i;

$u_i : S_i \to \mathbb{R}$ is a payoff function that maps the strategy of player i to payoff value in a real number $(1 \le i \le N)$;

R^c: is vector space denotes a set of conflicts in the problem, and the non-empty vector $\vec{v} \in R^c$ denotes a conflict between strategies of M players $(1 \le M \le N)$, in a strategic form Game, $\vec{v} \in R^c$ can be explained in detail as follows $(s_{0k}, s_{pq}, .., s_{xy})$, in which $s_{0k} \in S_0$ and $s_{pq}, s_{xy} \in S_i,\ (1 \le p, x \le N)$ and $(1 \le q \le M_p)(1 \le y \le M_x)$;

Nash Equilibrium is determined as follows:

When a player $i, (1 \le i \le N)$ chooses a pure strategy $s_i \in S_i,\ S_i = \{s_{i1}, .., s_{ij}, ..., s_{iM_i}\}$ and $s_{-i} \in S_i$ is the pure strategy of other players except i $(1 \le i \le N)$ and $s_{-i} \supset S_0 = \{s_{01}, .., s_{0i}, .., s_{0N}\}$. The payoff function of player i can also be described in the form $u_i(s_i, s_{-i})$. The set of strategies $s^* = (s_1^*, .., s_i^*, .., s_n^*)$ is Nash Equilibrium when $\forall (s_i^*, s_j^*) \notin R^c, (1 \le i, j \le N)$ and

$$u_i(s_i^*, s_{-i}^*) \ge u_i(s_i, s_{-i}^*),\ \forall s_i \in S_i,\ \forall i \in N. \tag{7}$$

NE is the point where no player can get a higher profit by his unilateral tactics. That is a player who wants to maximize their benefits will not deviate from the NE. A non-NE strategy means that there is a player who can get higher returns by choosing a strategy s'' when others want the strategy s' that $u_i(s_i'', s_{-i}') \ge u_i(s_i', s_{-i}')$.

4. USING UNIFIED MODEL FOR SOLVING CONFLICT PROBLEMS IN PROJECT MANAGEMENT

4.1. Project payments

The problem of optimizing the conflict in all billing project can be modeled into a game consisting of two persons (the investor and the contractor) with the adequate information represented by the Unified model from the (6)

$$G = \{P_0, S_0, u_0, P_1, S_1, u_1, R^c\}, \tag{8}$$

where,

P_0 is a project owner;

$S_0 = \{s_{01}, .., s_{0j}, .., s_{0N}\}$ is a set of the project owners strategies; N is number of activity in a project;

$u_0 : S_0 \to \mathbb{R}$ is a payoff function that maps project owner's strategy to payoff value in a real number;

P_1 is a contractor;

$S_1 = \{s_{11}, .., s_{1j}, .., s_{1N}\}$ is a set of the contractors strategies;

$u_1 : S_1 \to \mathbb{R}$ is a payoff function that maps contractor's strategy to payoff value in a real number;

R^c is vector space denotes a set of conflicts in payment order, and the non-empty vector $\vec{v} \in R^c$ denotes a conflict between desired payment order of 2 players (project owner and contractor), in a strategic form, it represents a conflict between 2 strategy (s_{0i}, s_{1j}), in which $s_{0i} \in S_0$ and $s_{1j} \in S_1$ and $(1 \leq i, j \leq N)$;

To set up a project schedule, the investor's strategy $S_0 = \{s_{01}, .., s_{0j}, .., s_{0N}\}$ is the list of the payments, each s_{0j} of which is represented as follows $s_{0j} = (pay_1, .., pay_j, .., pay_M)$. In the above expression, $pay_j (1 \leq j \leq M)$ is the percentage of the budget that the investor intended to pay to the contractor at the event i, and M is the total period of payment for the project. The list of payments needs to meet the following constraint:

$$\sum_{i=1}^{M} pay_i = 1. \tag{9}$$

In the set of the contractors strategies $S_1 = \{s_{11}, .., s_{1j}, .., s_{1N}\}$, the strategy named s_{1j} represented by the vector $\vec{w}$ which has the H dimension $\vec{w} = \{s_{1j1}, .., s_{1jk}, .., s_{1jH}\}$ denotes H kinds of resources (human resource, facilities, environment, etc.) which the project activity j needs to complete the task.

4.2. Multi-round procurement

The problem of optimizing the conflict in the multi-round procurement problem can be modeled by a game having $1 + N$ players, which consists of one investor and N remaining players are contractors with full information represented by the strategic set as follows [5]

$$G = \{P_0, S_0, u_0, P, S_i, u_i, R^c\}, \tag{10}$$

where,

P_0 is a project owner;

$S_0 = \{s_{01},.., s_{0j},.., s_{0K}\}$ is a set of the project owners strategies; K is a number of project owner's strategies;

$u_0 : S_0 \rightarrow \mathbb{R}$ is a payoff function that maps project owner's strategy to payoff value in a real number;

$P = \{P_1, ..P_i, ..P_N\}$ is a set of contractors;

$S_i = \{S_{i1}, ..S_{ij}, ..S_{iM_i}\}$ is a set of strategies of contractor number $i(1 \leq i \leq N)$ and M_i is the number of strategy of player i and can be also known as number of category player i want to submit;

$u_i : S_i \rightarrow \mathbb{R}$ is a payoff function that maps the strategy of player i to payoff value in a real number $(1 \leq i \leq N)$;

R^c: is vector space denotes a set of conflicts in the problem, and the non-empty vector $\vec{v} \in R^c$ denotes a conflict between strategies of M contractors who submit in a same procurement packaging $\{P_{i1}, ..P_{ij}, ..P_{iM}\}$, $(1 \leq M \leq N)$. In a strategic form, $\vec{v} \in R^c$ can be explained in detail as follows $(s_{0k}, s_{i1}, .., s_{ij}, .., s_{iB})$, in which $s_{0k} \in S_0$ and $(s_{i1}, .., s_{ij}, .., s_{iB}) \in S_i$, $(1 \leq i, j \leq N)$ and $(1 \leq B \leq M_B)$;

To solve the bidding for the entire project which has Q types of material to be purchased in K categories, the project owner must define a procurement requirements for each category, the strategic set of the investor $S_0 = \{s_{01}, .., s_{0j}, .., s_{0K}\}$ is the scheduling of procurement requirement and the time-line for each key milestone, which each s_{0j} corresponds to procurement requirement of category j, on which includes the name, quantity and cost of items of the category j. And s_{0j} can be described in detail as follows: $(mate_{j1}, .., mate_{ji}, .., mate_{jK})$ and K is the number of items in category j. In the above expression, $mate_{ji}$ contains a tuple of three elements $(nameofitem, quantity, cost)$ of items j which the project owner is going to buys. In the set of the contractor's strategies $S_i = \{s_{i1}, .., s_{ij}, .., s_{iM_i}\}$, the element s_{ij} denotes a decision and procurement bidding plan for category j over M_i categories of contractor i, it can be explained in detail by: $s_{ij} = \{s_{ij1}, .., s_{sijk}, .., s_{ijH}\}$, H is number of items in the category j, and s_{ijk} is a bidding information for the item k of category j.

4.3. Risk responses

We add virtual player P_0, which plays the role of referee in the game to keep the balance between normal players (the risk). The game includes other N risks (players) which conflict with each other. Both two kind of player has a single goal to find the best solution for the game which can protect the players from conflicts with another one. Based on the unified model (formula (6)), the problem of conflict optimization in the risk conflicting problem can be modeled into a game, consisting of N conflicting risk and one virtual player as follows

$$G = \{P_0, S_0, u_0, P, S_i, u_i, R^c\}, \tag{11}$$

where,

P_0 is a virtual player who represent the benefit of entire project;

$S_0 = \{s_{01}, .., s_{0j}, .., s_{0M_0}\}$ is the set of strategies of the special player; M_0 is the number of strategies of the special player;

$u_0 : S_0 \rightarrow \mathbb{R}$ is a payoff function that maps special player's strategy to payoff value in a real number;

$P = \{P_1, .., P_i, .., P_N\}$ is a set of all risks which have the conflicts in response methods;

N: number of risks which have a conflict to each other;

$S_i = \{s_{i1}, ..s_{ij}, ..s_{iM_i}\}$ is a set of response of risk $i (1 \leq i \leq N)$ and M_i is the number of response for risk i;

$u_i : S_i \to \mathbb{R}$ is a payoff function that maps the effectiveness in selecting S_i for risk i to payoff value in a real number $(1 \leq i \leq N)$;

R^c: is vector space denotes a set of conflicts in the problem, and the non-empty vector $\vec{v} \in R^c$ denotes a conflict between response methods of M risks. This group of risks can be defined as $\{P_{i1}, ..P_{ij}, .., P_{iB}\}$, $(1 \leq B \leq N)$. In a strategic form, $\vec{v} \in R^c$ can be explain in detail as follows $(s_{0k}, s_{i1}, .., s_{ij}, .., s_{iB})$, in which $s_{0k} \in S_0$ and $(s_{i1}, .., s_{ij}, .., s_{iB}) \in S_i$, $(1 \leq i,\ j \leq N)$ and $(1 \leq B \leq M_B)$;

The strategy for risk j of the virtual player is the tuple of four elements: minimal cost, minimal time that project owner plan for risk j, maximal cost, maximal time that project owner can afford

$$S_{0j} = \{cost_{jmin}, cost_{jmax}, time_{jmin}, time_{jmax}\}. \tag{12}$$

Set of risks $P = \{P_1, .., P_i, .., P_N\}$ with each P_i is characterized by the performance vector $\vec{w}$ with the elements: (i) the amount of money that must be incurred in the event of the impact (impact) and (ii) the level of risk (level). In detail $\vec{w} = (w_{impact}, w_{level})$.

The payoff value of the project owner will be

$$u_0 = a_1 w_{impact} + a_2 w_{level} = a_1 \sum_{i=1}^{N} w_{impact_i}.e^{-rt_i} + a_2 \sum_{i=1}^{N} w_{level_i} \tag{13}$$

where a_1, a_2 are the expert setting value to balance the importance of two factors impact, level. e^{-rt_i} often to be use in defining the real value of money with interest rate r in t time.

Set of strategies to solve all the risk $S_i = \{S_{i1}, .., S_{ij}, .., S_{iM_i}\}$ with each S_{ij} is characterized by the performance vector $\vec{u} = (u_{cost}, u_{priority}, u_{diff}, u_{time})$ indicates the influence of four factors in selecting right response method for risk i, $(1 \leq i \leq N)$. These four factors are: (i) cost, (ii) priority, (iii) difficulty (diff), and (iv) time.

The payoff value of risk i in case of selecting the response method k is the combination of these four risk response's characteristics: (i) cost, (ii) priority, (iii) difficult, and (iv) time. In detail, the payoff function of the risk i will be

$$\begin{aligned} u_i &= b_1 u_{cost} + b_2 u_{priority} + b_3 u_{diff} + b_4 u_{time} \\ &= b_1 \frac{w_{impact_i} - u_{cost_i}}{(1+r)^{t_i}} + b_2 u_{priority_i} + b_3 u_{diff_i} + b_4 u_{time_i}. \end{aligned} \tag{14}$$

The smaller the u_i function, the greater the benefit for the risks (normal players).

5. EXPERIMENTAL RESULTS

5.1. Project payments

A software project includes 20 tasks, each task needs a specific amount of time to finish, and it requires one or two type of resource. The quantity of each resource and working time will define the cost of performing this task. The cost per unit for Resource 1 is 1 million, and the cost per unit for Resource 2 is 2 million dollars. The monthly discount rate for both the owner and the client is estimated to be 0.7%. By computer-based calculation, we obtain optimal results in a Nash Equilibrium pattern for both investors and developers and results are given in Table 2.

Table 2. Project tasks and efforts

Activity	Time	Resource 1	Resource 2	Activity	Time	Resource 1	Resource 2
1	10	6	2	11	80	2	2
2	30	6	1	12	30	3	0
3	50	3	0	13	30	3	0
4	50	3	0	14	50	3	2
5	10	3	1	15	150	3	2
6	60	6	2	16	50	3	2
7	60	6	2	17	40	0	1
8	50	3	1	18	110	4	1
9	40	6	0	19	90	6	2
10	90	3	2	20	50	5	1

With both the payment schedule and activities schedule are optimal, both parties of the game meet at the equilibrium. The amount of money to pay by the investors is 82.27 million dollars. It is saving 1.26 million dollars compared to 83.52 million dollars of the first plan and 3.07 million dollars compared to 85.33 of the second plan, whereas with the developers, their net profit is 43.12 million dollars, more than 12.01 million dollars compared to 31.11 million dollars of the first plan and 7.81 million dollars compared to 35.31 million dollars of the second plan. We want to emphasize that the area of Game theory and project management has a successful established practical model that can cover more issues than those treated in this introduction. The Nash Equilibrium has resolved the conflict of the payment schedule found in the application.

5.2. Multi-round procurement

Using the above model and put it into the input of the genetic algorithm in the problem Multi-round procurement, the data of the problem include information about the bidding packages, the strategies of 3 contractors [5]. The Multi-round procurement problem belongs to the participant class, so there is no need to simulate a virtual player. After running in program ten times, a table of result has been produced to compare fitness function values, investors and contractor benefits. Through experiments on simulation data set, it was found out some points:

- The results after the runs are not identical, but are fairly similar and adaptive functions have approximate values. The average standard deviation of investor is only 10.259%. With a better computer and longer running time, we will have a smaller deviation.

- The wide range of fitness value (investor: 657,093 - 971,988 and bidder: 106,279 -

Table 3. Schedule 1

Node	Activity	Payment (%)
1	1, 2	15
2	3	15
3	4	15
4	5, 6	5
5	7, 8	3
6	9, 10, 11, 12	5
7	13, 14	5
8	15, 16, 17, 18, 19	10
9	20	27

Table 4. Schedule 2

Node	Activity	Payment (%)
1	1, 2	25
2	3, 4, 5	5
3	6, 7	5
4	8, 9	15
5	10, 11, 12	10
6	13, 14, 15	10
7	16, 17	15
8	18	5
9	19, 20	10

Table 5. Information about the bidding package

Round	Stage	Product ID	Product name	Quantity
1	ST01	H01	PC	100
		H02	Laptop	25
2	ST02	H03	Projector	50
3	ST03	H04	Printer	25
		H05	Color Printer	2
		H06	Scanner	10
4	ST04	H01	PC	12
		H02	Laptop	2
5	ST05	H04	Printer	26
		H05	Color Printer	1
		H06	Scanner	22
6	ST06	H01	PC	21

Table 6. Strategies of contractor 1

Product ID	Starting point	Ending point	Original price	Sale price	Discount
H01	1/6/2016	31/5/2017	10,720	12,800	10
H02	1/6/2016	31/5/2017	14,800	17,400	10
H03	1/6/2016	31/5/2017	3,500	4,150	10
H04	1/6/2016	31/5/2017	11,400	13,350	10
H05	1/6/2016	31/5/2017	2,880	3,400	10
H06	1/6/2016	31/5/2017	31,050	36,900	10
H01	1/6/2017	31/12/2018	11,680	13,760	8
H02	1/6/2017	31/12/2018	13,600	16,200	8
H03	1/6/2017	31/12/2018	3,750	4,400	8
H04	1/6/2017	31/12/2018	10,650	12,600	8
H05	1/6/2017	31/12/2018	3,080	3,600	8
H06	1/6/2017	31/12/2018	31,500	37,350	8

106,279) shows us that all generations have a good candidate with a variety of characteristic.

- Looking at the value of profitability of each contractor, contractors benefits are balanced, to show us that an optimal solution meets Nash Equilibrium criterion.

Table 7. Testing result

Product ID	Starting point	Investor (1000VND)			Benefit of contractor (1000VND)		
		Estimated cost	Payment	Benefit	Bidder 1	Bidder 2	Bidder 3
10:01.5	3,570,381	5,386,000	3,896,428	767,244	133,715	132,481	135,468
11:47.4	3,879,695	5,386,000	3,792,720	870,952	108,000	106,766	106,279
08:18.8	2,363,866	5,386,000	4,006,579	657,093	137,443	136,209	138,599
09:41.4	4,299,435	5,386,000	3,874,651	789,020	135,191	133,957	133,093
08:11.6	2,920,003	5,386,000	3,881,353	782,318	134,814	133,580	133,517
09:45.6	2,133,330	5,386,000	3,883,464	780,208	134,641	133,407	134,290
25:14.1	2,120,500	5,386,000	3,898,911	764,761	135,080	133,846	134,642
24:14.9	4,183,533	5,386,000	3,890,816	772,855	135,565	134,331	133,499
08:18.8	2,363,866	5,386,000	3,691,684	971,988	125,952	124,718	125,753
09:41.4	4,299,435	5,386,000	3,874,651	789,020	135,191	133,957	133,093

At this time, we can say that we successfully find a balanced point Nash Equilibrium for a game of bidders and investor. The paper has proposed a useful method in solving the problem of multi-round procurement which needs the right decision-making in each round of bidding. It also proved the potentiality when applying Game-theoretical model in real-world procurement projects.

5.3. Risk responses

In solving risk management problem, we must find a set of risk management characteristics in the project. These options will help us to define the value of the response of risks based on the matrix of Game theory. In 25 response methods o 7 risks; there are response methods of one risk does not get along with another, these clash of risk responses are described in the following tables [10]

Table 8. Risks - players of the game

Risk ID	Financial impact	Risk level
R01	20,000	Extreme
R02	10,000	High
R03	400,000	Extreme
R04	30,000	High
R05	6,000	Medium
R06	12,000	Medium
R07	34000	Low

Table 9. Conflict between risk responses

Conflict ID	Risk responses 1	Risk responses 2
Con01	S11	S24
Con02	S15	S33
Con03	S41	S53
Con04	S61	S72

After five times of experiments running, a table of result has been produced to compare fitness function values, investors and contractor benefits (See Table 11). Through experiments on simulation data set, it was found out some points:

- The fitness functions after 100 generations are almost the same; The average value of Fitness function is 13,732. The average of the absolute deviations is only 3.33%. An applying Genetic Algorithm to this problem showed positive results.
- The running time is a little bit low because of this paper apply traditional Genetic Algorithm. The running time can be decreased if we implement a better approach in Genetic Algorithms such as NSGA II or NSGA III.
- In the sample data, the quantity of risk, and risk responses are small, so that the Nash Equilibrium found look similar, but it will be more diversified in more significant data set.

This experiment presents that we successfully explored a new optimization method by modeling the problem with Game theory and finding a Nash Equilibrium of the game by a

Table 10. Risk responses - strategies of players

Risk response	Cost($)	Difficulty	Time	Priority
S11	30,000	7	23	9
S12	356,000	4	38	1
S13	11,000	8	42	7
S14	184,000	2	34	5
S15	4,000	8	29	8
S21	2,000	9	9	2
S22	49,000	6	30	3
S23	7,000	3	51	6
S24	5,000	3	33	3
S25	1,000	9	58	4
S31	4,000	6	26	4
S32	8,000	2	23	9
S33	23,000	4	20	7
S34	37,000	5	20	8
S35	19,000	5	21	8
S41	18,000	2	48	2
S42	25,000	1	60	0
S43	2,000	6	25	5
S51	22,000	2	17	7
S52	5,000	3	33	4
S53	1,000	6	13	8
S61	108,000	0	40	8
S62	6,000	6	44	8
S71	123,000	3	37	8
S72	8,000	9	30	9

Table 11. Nash Equilibrium of the Game

No	Time	Fitness value	Nash Equilibria - Recommended risk responses
1	11:21.5	13,375	5, 4, 1, 2, 3, 2, 2
2	10:47.4	14,101	4, 4, 2, 2, 3, 1, 2
3	09:58.9	13,986	2, 1, 5, 3, 1, 1, 2
4	09:41.4	12,945	2, 4, 2, 1, 2, 1, 1
5	10:11.6	14,253	4, 1, 5, 3, 3, 2, 1

Multi-objectives optimization algorithm. We believe that the general model for all conflicts introduced in formula (6) will help in finding a solution for the complex issue of risk responses.

6. CONCLUSIONS

In this paper, we proposed an efficient approach for solving three typical conflict problems in project management: project payments, multi-round procurement, and conflicts in risk responses. Firstly, we develop a unified model of Game theory for conflict problems, which contains three components: an individual player, N regular players and a set of conflicts. Then, we propose the approach to apply the Nash Equilibrium for the model based on calculation the payoff function $u_i(s_i, s_{-i})$ and the set of risk conflicts R^c.

Finally, we apply a Unified model using Nash Equilibrium method to solve three problems. We detail the application of the Unified model of Game theory and construct the Nash Equilibrium for each of the above problems. Experimental results show that our approach works effectively.

In the future, we will focus on how to apply multi-objective optimization algorithms in finding the Nash Equilibrium for multi-round procurement and risk responses conflicts problems..

ACKNOWLEDGMENTS

This research was supported by Vietnams National Foundation for Science and Technology Development (NAFOSTED), funded by the Ministry of Science and Technology (Project Code 102.03-2019.10).

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Received on September 13, 2018
Revised on March 10, 2019