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THE UNSTEADY FLOW AFTER DAM BREAKING*

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Abstract. The following problems are presented: the unsteady flow on a river system and reservoirs, the discontinuous wave and unsteady flow after the dam breaking, numerical experiments for some test cases and for natural Da river.

Tóm tắt. Bài báo này trình bày mô hình toán học và thuật toán tính dòng chảy không dừng trên hệ thống sông và hồ chứa, sóng gián đoạn và dòng chảy không dừng sau vỡ đập bằng phương pháp đặc trưng, áp dụng tính thử nghiệm số cho bốn bài toán kiểm tra có nghiệm giải tích và các trường hợp vỡ hoàn toàn cũng như không hoàn toàn của đập Sơn La trên hệ thống sông Đà.

INTRODUCTION

There are several algorithms and softwares for calculating the unsteady flow on a river after dam breaking. Some of them allow calculating the unsteady flow after gradual dam breaking, but cannot exactly determine the position of discontinuous front ξ and the height Δh of the front ξ (see [8-11]). Other ones determine the accuracy of the front ξ and the height Δh, but the unsteady flow is calculated only on one river branch after the instant dam breaking (see [1, 5, 6, 7]).

In this paper the algorithm basing on the [5] permits to calculate the unsteady flow on the river system, connecting with reservoirs after instant or gradual dam breaking.

1. MATHEMATICAL MODELLING

The equation system describing the unsteady flows is established from the laws of conservation (see [1]) and has the following form:

$$\oint_{\partial S} Qdt - \omega dx = \iint_S q dx dt,$$

$$\oint_{\partial S} \Big[P + \frac{Q^2}{\omega}\Big] dt - Qdx = \iint_S \Big[g\omega\Big(i - \frac{Q|Q|}{K^2} + R_x\Big)\Big] dx dt, \tag{1.1}$$

where

$$P = g\int_0^h (h-\zeta) b(x,\zeta) d\zeta, \qquad R_x = g\int_0^h (h-\zeta)\frac{\partial b(x,\zeta)}{\partial x} d\zeta,$$

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x - the coordinate along channel, t - time,
q - lateral flow, ω - cross-section area,
K - conveyance factor, h - the depth,
i - bottom slope, $b(x,\zeta)$ - width on the distance ζ from the bottom,
g - acceleration due to gravity, S - consideration region,
Q - discharge, ∂S - boundary of S.

1.1. One dimensional Saint–Venant equation system

If the flow is continuous, from (1.1) we get the Saint–Venant equation system

$$B\frac{\partial Z}{\partial t} + \frac{\partial Q}{\partial x} = q,$$

$$\frac{\partial Q}{\partial t} + 2v\frac{\partial Q}{\partial x} + B(c^2 - v^2)\frac{\partial Z}{\partial x} = \Phi, \tag{1.2}$$

where

$$\Phi = \left[iB + \left(\frac{\partial \omega}{\partial x}\right)_{h=const}\right]v^2 - \frac{g\omega Q|Q|}{K^2} = \left(\frac{\partial \omega}{\partial x} - B\frac{\partial Z}{\partial x}\right)v^2 - \frac{g\omega Q|Q|}{K^2},$$

Z - level of free surface,
v - velocity,
B - width of the water surface,
c - celerity of small wave propagation.

Equation system (1.2) is quasilinear and of hyperbolic type, which can be rewritten in the characteristic form:

$$\frac{\partial Q}{\partial t} + (v-c)\frac{\partial Q}{\partial x} + B(-v-c)\Big[\frac{\partial Z}{\partial t} + (v-c)\frac{\partial Z}{\partial x}\Big] = \Phi + (-v-c)q, \tag{1.3}$$

$$\frac{\partial Q}{\partial t} + (v+c)\frac{\partial Q}{\partial x} + B(-v+c)\Big[\frac{\partial Z}{\partial t} + (v+c)\frac{\partial Z}{\partial x}\Big] = \Phi + (-v+c)q. \tag{1.4}$$

For solving the equation system (1.2) or (1.3)–(1.4), it is necessary to give initial conditions at $t = 0 : Z(x,0) = Z^0(x), Q(x,0) = Q^0(x)$ and the boundary conditions, adjoint conditions.

a. Boundary conditions

For subcritical flow, one boundary condition is needed:
- At the upstream boundary:

$$Q(x_b,t) = Q_b(t). \tag{1.5}$$

- At the downstream boundary:

$$Z(x_b,t) = Z_b(t) \;\text{ or }\; Q(x_b,t) = f(Z_b(t)). \tag{1.6}$$

For supercritical flow:
- At the upstream boundary, two boundary conditions are needed:

$$Q(x_b,t) = Q_b(t) \;\text{ and }\; Z(x_b,t) = Z_b(t). \tag{1.7}$$

- At the downstream boundary:
No boundary condition is needed.

b. Adjoint conditions at the internal node of river systems for the continuous flow (for example, nodes D, E, F in Fig. 1.1).

At every internal node it is necessary to give the following adjoint condition (for example, adjoint conditions at D):

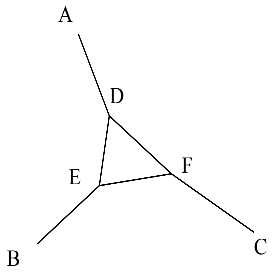

Fig. 1.1

$$\sum_{j\in J_D} \alpha_{D_j} Q_{D_j},$$

$$Z_{D_j} = Z_D,\ j \in J_D, \tag{1.8}$$

where J_D is the set of the river branches having common node D.

$$\alpha_{D_j} = \begin{cases} -1 \text{ if } D \text{ is left boundary of the river branch } j, \\ +1 \text{ if } D \text{ is right boundary of the river branch } j. \end{cases}$$

c. Adjoint conditions at the common node A of a river and a reservoir for the continuous flow

Suppose that the reservoir has volume V depending on the elevation Z_H :

$$V = V(Z_H).$$

The adjoint conditions are (see Fig. 1.2)

$$\sum_{j=1}^{2} \alpha_j Q_j + Q_3 = 0,\ Z_{Aj} = Z_H,\ j = 1, 2... \tag{1.9}$$

where $Q_3 = -\dfrac{dV(Z_H)}{dt}$.

1.2. Adjoint condition at the discontinuous front

One adjoint condition at the discontinuous front is needed: (see [1, 5, 7])

$$[P].\left[\frac{1}{\omega}\right] + [v]^2 = 0, \tag{1.10}$$

where, $[f] = f^+ - f^-$, f^- is the value f at the left side of ξ, f^+ is the value f at the right side of ξ.

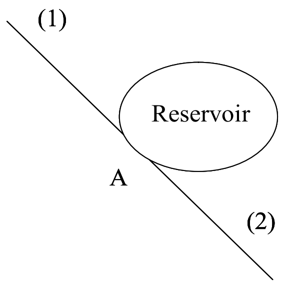

Fig. 1.2

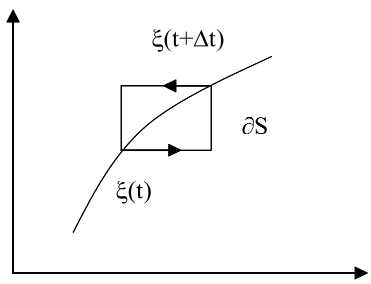

Fig. 1.3

The velocity of the discontinuous front ξ is (see Fig. 1.3)

$$C_* = v^+ + \sqrt{\frac{\omega^-}{\omega^+}\frac{P^+ - P^-}{\omega^+ - \omega^-}} = v^- + \sqrt{\frac{\omega^+}{\omega^-}\frac{P^+ - P^-}{\omega^+ - \omega^-}} = \frac{Q^+ - Q^-}{\omega^+ - \omega^-},$$

$$v^+ + c^+ < C_*,\ v^- - c^- < C_* < v^- + c^-. \tag{1.11}$$

In the case when the height of discontinuous front is very small ($\Delta h \ll 1$), the adjoint condition and velocity C_* are:

$$Q^+ - Q^- - B^+(v^- + c^+)(Z^+ - Z^-) = 0, \quad (1.12)$$

$$C_* = v^- + c^+ \approx v^+ + c^-. \quad (1.13)$$

2. THE ALGORITHMS

2.1. Calculation of the one dimensional unsteady flows (see[2, 4, 5, 6])

Equations (1.3) and (1.4) may be rewritten as follows:

$$\frac{dQ}{dt} + a_1\frac{dZ}{dt} = b_1, \ \frac{dx}{dt} = c_1, \quad (2.1)$$

$$\frac{dQ}{dt} + a_2\frac{dZ}{dt} = b_2, \ \frac{dx}{dt} = c_2, \quad (2.2)$$

where

$$a_1 = B(-v - c), \ b_1 = \Phi + (-v - c)q, \ c_1 = v - c,$$

$$a_2 = B(-v + c), \ b_2 = \Phi + (-v + c)q, \ c_2 = v + c,$$

a. Calculation of the values Z_0^{k+1} and Q_0^{k+1} at left boundary L_0

• Determine the coordination of point $A^{(i)}$, (i.e. the intersection of a characteristics line $dx/dt = v - c$ and the line $t = t_k$) at the iterative step (i) (see Fig. 2.1)

$$x_{A^{(i)}} = x_0 + \frac{\tau}{2}\left[(c_1)^{i-1}_{L_0^*} + (c_1)_{A^{(i-1)}}\right], \ (c_1)^{(0)}_{L_0^*} = (c_1)_{A^{(0)}} = (c_1)^k_{L_0}.$$

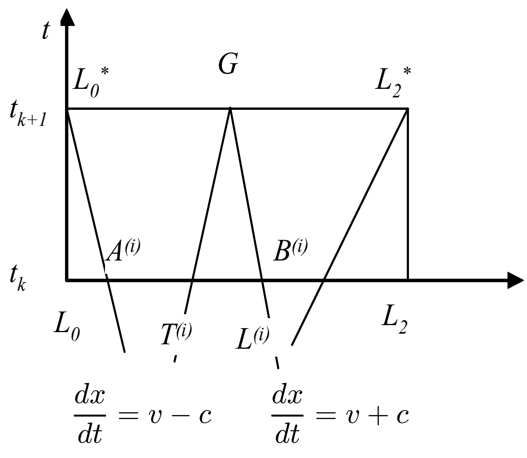

Fig. 2.1

• Determine the values $Z_{A^{(i)}}$ and $Q_{A^{(i)}}$ by the linear interpolation.
• Substituting these values into equation (2.1) we get

$$Q_0^{(i)} + a_0^{(i)} Z_0^{(i)} = d_0^{(i)}. \quad (2.3)$$

• From the equation (2.3) and boundary conditions (1.5) one deduces $Z_0^{(i)}$.

The iterative process is stopped if $|Z_0^{(i)} - Z_0^{(i-1)}| < \varepsilon |Z_0^{(i-1)}|, \varepsilon \leqslant 0.01$.

b. Calculation of the values Z_N^{k+1} and Q_N^{k+1} at right boundary L_2

By the analogous argument from the equation (2.2), we have the following equation at the iterative step (i) (see Fig. 2.1)

$$Q_N^{(i)} + a_N^{(i)} Z_N^{(i)} = d_N^{(i)}. \tag{2.4}$$

Solving this equation (2.4) and the boundary condition (1.6) $Z_N^{k+1} = Z_b(t_{k+1})$ or linearized boundary condition $Q_N^{(i)} + \alpha_N^{(i)} Z_N^{(i)} = \beta_N^{(i)}$, where

$$\alpha_N^{(i)} = -\frac{\partial f}{\partial Z}\bigg|_N^{(i-1)}; \ \beta_N^{(i)} = Q_N^{(i-1)} - \frac{\partial f}{\partial Z}\bigg|_N^{(i-1)} .Z_N^{(i-1)}, \text{ we get } Q_N^{(i)},\ Z_N^{(i)}.$$

The iterative process is stopped if

$$|Z_N^{(i)} - Z_N^{(i-1)}| < \varepsilon |Z_N^{(i-1)}|, |Q_N^{(i)} - Q_N^{(i-1)}| < \varepsilon |Q_N^{(i-1)}|.$$

c. Calculation of the values Z^{k+1} and Q^{k+1} at the internal node of river system (for example, D on Fig. 11)

For each river branch j ($j = 1, 2, ..., J_D$) having common internal node D, we have one linear equation

$$Q_{D_j}^{(i)} = a_{D_j}^{(i)} Z_{D_j}^{(i)} + d_{D_j}^{(i)}, \tag{2.5}$$

where
$a_{D_j}^{(i)} = -(a_0^{(i)})_j$ and $d_{D_j}^{(i)} = (d_0^{(i)})_j$ if D is left boundary of the branch j,
$a_{D_j}^{(i)} = -(a_N^{(i)})_j$ and $d_{D_j}^{(i)} = (d_N^{(i)})_j$ if D is right boundary of the branch j.
From adjoint conditions (1.8) and (2.5), we obtain

$$Z_{D_j}^{(i)} = Z_D^{(i)} = \frac{-\sum\limits_{j=1}^{J_D} \alpha_{D_j} d_{D_j}^{(i)}}{\sum\limits_{j=1}^{J_D} \alpha_{D_j} d_{D_j}^{(i)}}. \tag{2.6}$$

The interative process is stopped if $|Z_D^{(i)} - Z_D^{(i-1)}| < \varepsilon |Z_D^{(i-1)}|$.

d. Calculation of the values Z^{k+1} and Q^{k+1} at the common node of a river and a reservoir (for example, node A on the Fig. 1.2)

Linearizing the equation $Q_3 = -\dfrac{dV(Z)}{dt}$, we have

$$Q_3^{k+1} \approx -\frac{V(Z^{k+1}) - V(Z^k)}{\tau} \approx -\frac{1}{\tau}\Big[V(Z^k) + \frac{dV}{dZ}(Z^{k+1} - Z^k) - V(Z^k)\Big], \tag{2.7}$$

$$Q_3^{(i)} \approx \beta^{(i)}(Z^{(i)} - Z^k),$$

where $\beta^{(i)} = -\dfrac{1}{2\tau}\left[\left(\dfrac{dV}{dZ}\right)^{(i-1)} + \left(\dfrac{dV}{dZ}\right)^{(k)}\right].$

From the equation (2.5) for each river branch, adjoint conditions (1.9) and equation (2.7) we get

$$Z_H^{(i)} = \frac{\beta^{(i)} Z_H^k - \sum\limits_{j=1}^{2} \alpha_j d_{A_j}^{(i)}}{\beta^{(i)} + \sum\limits_{j=1}^{2} \alpha_j a_{A_j}^{(i)}}. \tag{2.8}$$

Iterative process is stopped if $|Z_H^{(i)} - Z_H^{(i-1)}| < \varepsilon |Z_H^{(i-1)}|$.

The discharge is calculated from (2.5) and (2.7).

e. Calculation of Z and Q at interior nodes of river branch (For example, node G on Fig. 2.1)

From the equations (2.1) and (2.2), by method of characteristic we get the following equations for determining the values Z and Q at the iterative step (i) .

$$Q_G^{(i)} + a_L^{(i)} Z_G^{(i)} = d_L^{(i)},$$

$$Q_G^{(i)} + a_T^{(i)} Z_G^{(i)} = d_T^{(i)}.$$

Solving this equation system we obtain $Z_G^{(i)}$ and $Q_G^{(i)}$.

Iterative process is stopped if

$$\left|Q_G^{(i)} - Q_G^{(i-1)}\right| < \varepsilon \left|Q_G^{(i-1)}\right| \text{ and } \left|Z_G^{(i)} - Z_G^{(i-1)}\right| < \varepsilon \left|Z_G^{(i-1)}\right|.$$

2.2. Discontinuous wave on a river

Suppose that the dam sitting at the point L_1 is totally and instantaneously broken. The computational process includes (see [1, 6, 7, 8]).

a. Calculation of Z^-, Q^- at the moment of dam breaking

According to references [3, 5, 6, 7] these values can be calculated by an iterative method using formulas

$$V_i = v^+ + \sqrt{\frac{g}{2}[(h^{(i)})^2 - (h^+)^2].\left(\frac{1}{h^+} - \frac{1}{h^{(i)}}\right)} \text{ and } h_s = \frac{1}{4g}\left(v_1 + 2\sqrt{gh_1} - V_i\right)^2,$$

where

$v_1 = v(L_1 - 0.0), \quad h_1 = h(L_1 - 0.0), \quad h^{(i)} = h^{(i-1)} + 0.01h^+, \quad h^{(0)} = h^+ = h(L_1 + 0.0).$

Iterative process is stopped if $h_s \leqslant h^{(i)}$.

b. Determine the position of the discontinuous front ξ

$$\xi^{k+1} = \xi(t_{k+1}) = \xi^k + C_\bullet^k \tau,$$

where

$$C_* = v^+ + \sqrt{\frac{\omega^-}{\omega^+} . \frac{P^+ - P^-}{\omega^+ - \omega^-}}.$$

c. Determine the values $(Z^+)^{k+1}, (Q^+)^{k+1}$ *at the right side of the discontinuous front* ξ

From the Saint–Venant equation system in the characteristic form (2.1), (2.2) one deduces the equations at the iterative step (i)

$$(Q^+)^{(i)} + a_L^{(i)}(Z^+)^{(i)} = d_L^{(i)},$$

$$(Q^+)^{(i)} + a_T^{(i)}(Z^+)^{(i)} = d_T^{(i)}.$$

Solving this equation system we get $(Z^+)^{(i)}$ and $(Q^+)^{(i)}$.

We take $(Z^+)^{k+1} = (Z^+)^{(i)}$, $(Q^+)^{k+1} = (Q^+)^{(i)}$ if $|(Z^+)^{(i)} - (Z^+)^{(i-1)}| < \varepsilon|(Z^+)^{(i-1)}|$ and $|(Q^+)^{(i)} - (Q^+)^{(i-1)}| < \varepsilon|(Q^+)^{(i-1)}|$.

d. Determine the values $(Z^-)^{k+1}, (Q^-)^{k+1}$ *at the left side of the discontinuous front* ξ

From the equation (2.2) it yields

$$(Q^-)^{(i)} + a_T^{(i)}(Z^-)^{(i)} = d_T^{(i)}.$$

Linearizing adjoint condition (1.10) one deduces

$$\gamma^{(i)}(Q^-)^{(i)} + \mu^{(i)}(Z^-)^{(i)} = \theta^{(i)},$$

where $\gamma^{(i)}, \mu^{(i)}, \theta^{(i)}$, are known coefficients.

Solving this equation system we obtain $(Z^-)^{(i)}$ and $(Q^-)^{(i)}$.

If

$$|(Z^-)^{(i)} - (Z^-)^{(i-1)}| < \varepsilon|(Z^-)^{(i-1)}|, \quad |(Q^-)^{(i)} - (Q^-)^{(i-1)}| < \varepsilon|(Q^-)^{(i-1)}|,$$

we take

$$(Z^-)^{K+1} = (Z^-)^{(i)}, \; (Q^-)^{K+1} = (Q^-)^{(i)}.$$

e. The values Z^{k+1} and Q^{k+1} at the boundary nodes, internal nodes of river system, common nodes of a river and a reservoir or interior nodes of each river branch are calculated by the method of characteristic as in the point 1.

2.3. Unsteady flow after the dam breaking on river

Suppose that the dam breaking is gradual. The condition at dam is the function:

$$Q = f(Z_T, Z_D), \tag{2.9}$$

and

$$Q_T = Q_D = Q. \tag{2.10}$$

Linearizing the equation (2.9) we get

$$Q_T^{(i)} = \alpha^{(i)} Z_T^{(i)} + \beta^{(i)} Z_D^{(i)} + \gamma^i. \tag{2.11}$$

a. For the supercritical flow

Analogously, from the equation (2.1), (2.2) one deduces two following equations at the left side of the dam

$$Q_T^{(i)} + a_T^{(i)} Z_T^{(i)} = d_T^{(i)}, \tag{2.12}$$

$$Q_T^{(i)} + a_L^{(i)} Z_T^{(i)} = d_L^{(i)}. \tag{2.13}$$

Solving the equations (2.10) - (2.13) we obtain the values $Z_T^{(i)}, Z_D^{(i)}, Q_T^{(i)}, Q_D^{(i)}$.
The iterative process is stopped if the error is small enough and we take

$$Z_T^{k+1} = Z_T^{(i)},\ \ Z_D^{k+1} = Z_D^{(i)},\ \ Q_T^{k+1} = Q_T^{(i)}.$$

b. For the subcritical flow

From the equation (2.2) at the right side and (2.1) at the left side of the dam we have

$$Q_T^{(i)} + a_T^{(i)} Z_T^{(i)} = d_T^{(i)}, \tag{2.14}$$

$$Q_D^{(i)} + a_L^{(i)} Z_D^{(i)} = d_L^{(i)}. \tag{2.15}$$

Solving the equations (2.10), (2.11), (2.14), (2.15) we obtain the values $Z_T^{(i)}, Z_D^{(i)}, Q_T^{(i)}, Q_D^{(i)}$.
The iterative is stopped if the error is small enough.

c. The values Z^{k+1} and Q^{k+1} at the boundary, internal, common nodes or interior nodes of each river branch are calculated by the same method as in the point 1.

3. NUMERICAL EXPERIMENTS

The method of characteristic is applied to solve some test problems and natural Da river system problem (see [8]).

3.1. Test case 1

Channel of 1.5 km long in which every section is rectangular. Its geometry is described in Fig. 3.1 and Fig. 3.2. The bed slop is about 10% with reverse gradients. One can notice the important contracting section at $x = 800\,\text{m}$ which creates an acceleration of the flow.

This test enables to check that these source terms are correctly evaluated, in the case of flat water at rest.

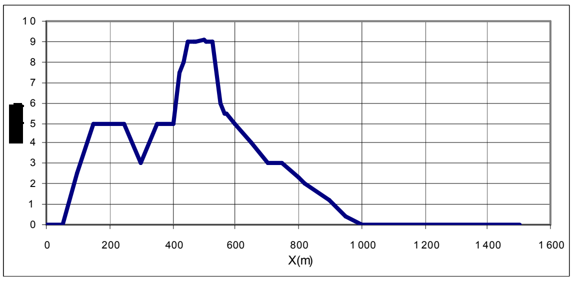

Fig. 3.1. Channel geometry - Profile view

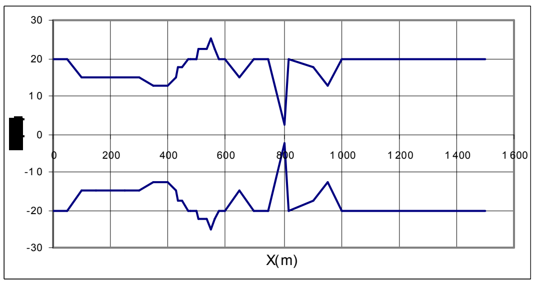

Fig. 3.2. Channel geometry - Top view

The complete description of the geometry is given in the Table 1.

* In each configuration the boundary and initial conditions are as follows:

- Downstream boundary and initial condition: level imposed equal to 12 m.
- Upstream boundary condition: no discharge.
- Initial condition: water at rest at the level 12 m.

Table 1

Cross-sec	$X(m)$	$Z_b(m)$	$B(m)$	Cross-sec	$X(m)$	$Z_b(m)$	$B(m)$
1	0	0	40	16	530	9	45
2	50	0	40	17	550	6	50
3	100	2.5	30	18	565	5.5	45
4	150	5	30	19	575	5.5	40
5	250	5	30	20	600	5	40
6	300	3	30	21	650	4	30
7	350	5	25	22	700	3	40
8	400	5	25	23	750	3	40
9	425	7.5	30	24	800	2.3	5
10	435	8	35	25	820	2	40
11	450	9	35	26	900	1.2	35
12	470	9	40	27	950	0.4	25
13	475	9	40	28	1000	0	40
14	500	9.1	40	29	1500	0	40
15	505	9	45				

* The analytical solution is very simple in this test case.
  - Water at rest: discharge and flow velocity must be equal to zero.
  - Flat free surface water level stays at the initial level of 12 m.
* The numerical solution (see Fig. 3.3):
  - Discharge flow is $0\,\mathrm{m}^3/\mathrm{s}$.
  - Water surface level is 12 m.

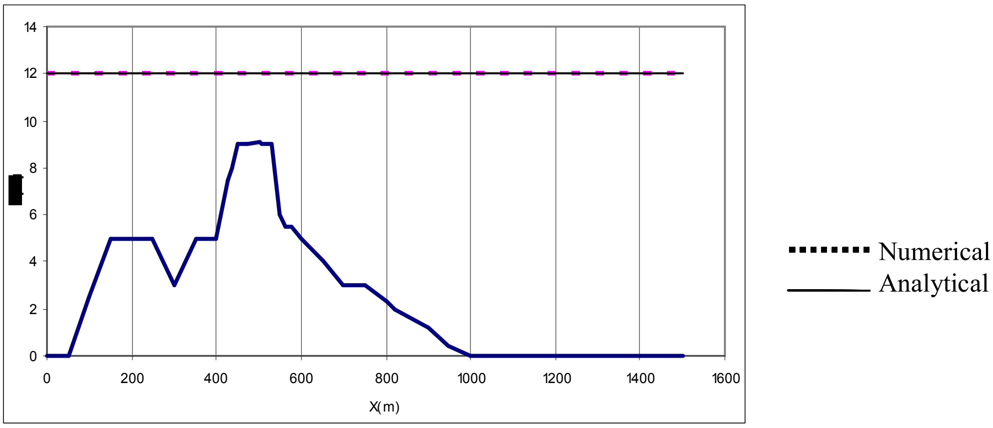

Fig. 3.3. The numerical solution and the analytical solution

3.2. Test case 2

The steady flow over a bump in a rectangular channel with a constant width. According to the boundary and initial condition, the flow may be subcritical, transcritical with a steady shock, supercritical or at rest.

* Geometry data:
  - The channel width $B = 1\,\mathrm{m}$.

- The channel length $L = 25\,\text{m}$.
- Bottom Z_b equation $x < 8\,\text{m}$ and $x > 12\,\text{m}$: $Z_f = 0$,
  $8\,\text{m} < x < 12\,\text{m}$: $Z_f = 0.2 - 0.05(x - 10)^2$.

* Transcritical flow without shock:

- Downstream: level imposed equal to 0.66 m, no level imposed when the flow becomes supercritical.
- Upstream: discharge imposed equal to $1.53\,\text{m}^3/\text{s}$.
- Analytic and numerical solution (see Fig. 3.4).

* Transcritical flow with shock:

- Downstream: level imposed equal to 0.33 m.
- Upstream: discharge imposed equal to $0.18\,\text{m}^3/\text{s}$.
- Analytic and numerical solution (see Fig. 3.5).

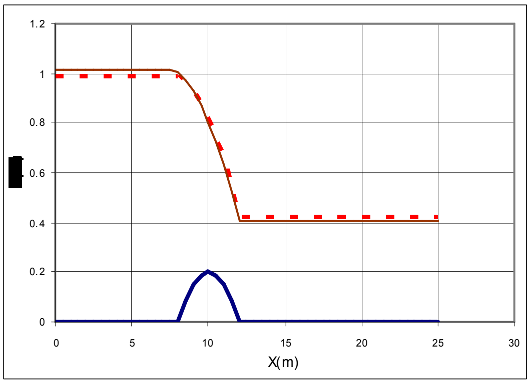

Fig. 3.4

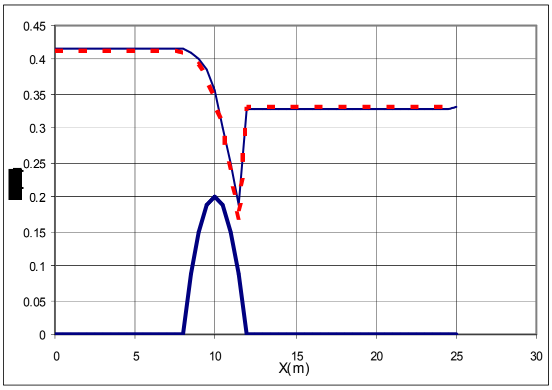

Fig. 3.5

* Subcritical flow

- Downstream: level imposed equal to 2 m.
- Upstream: discharge imposed equal to $4.42\,\text{m}^3/\text{s}$.
- Analytic and numerical solution (see Fig. 3.6).

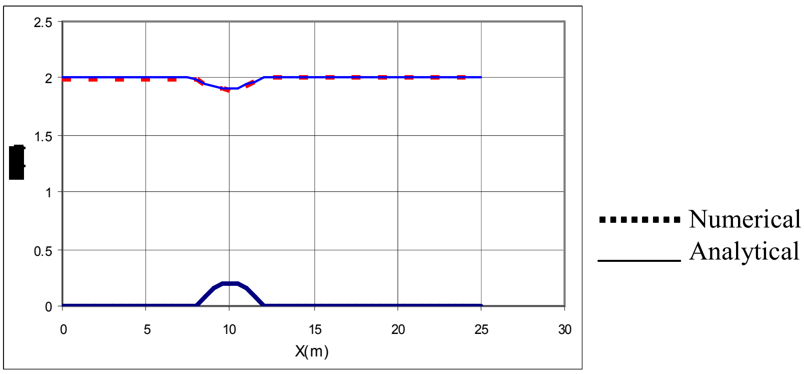

Fig.3.6. The numerical solution and the analytical solution

* Initial conditions

- Constant level equal to the level imposed downstream.
- Discharge equal to zero.

- Friction term equal to zero.

3.3. Test case 3

Our purpose is to calculate the unsteady flow resulting from an instantaneous dam breaking in a rectangular channel with constant width.

* Geometrical data (see Fig. 3.7):

- Channel length 2000 m.
- Dam position $x = 0$ m.
- Channel width L=1 m.

* Physical parameters

- No friction.
- Boundary conditions.

Downstream: level imposed equal to y_2.

Upstream: no discharge.

* Initial conditions

$y = y_1 = 6$ if $x < 0$.

$y = y_2 = 0$m if $x > 0$.

* Analytic and numerical solution (see Fig. 3.8).

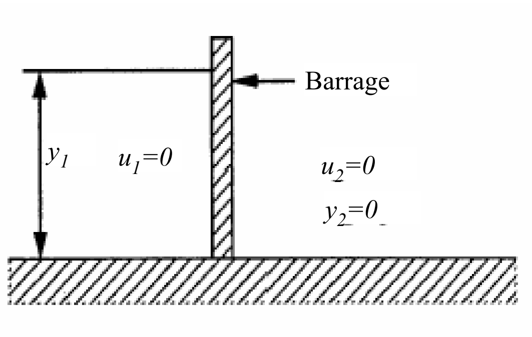

Fig. 3.7
Dam break on dry bed, initial state

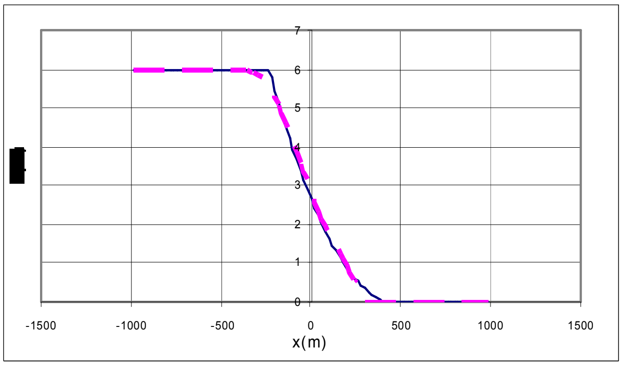

Fig. 3.8
The numerical solution and the analytical solution at $t = 30$ s

3.4. Test case 4

Our purpose is to calculate the unsteady flow of an instantaneous dam break on an already wet bed.

* Geometrical data (see Fig. 3.9):

- Channel length 2000 m.
- Dam position $x = 0$ m.
- Channel width $L = 1$ m.

* Physical parameters

- No friction.
- Boundary conditions.

Downstream: level imposed equal to y_2.

Upstream: no discharge.

* Initial conditions

$$y = y_1 = 6 \text{ if } x < 0.$$

$$y = y_2 = 2\text{m if } x > 0.$$

* Analytic and numerical solution (see Fig. 3.10).

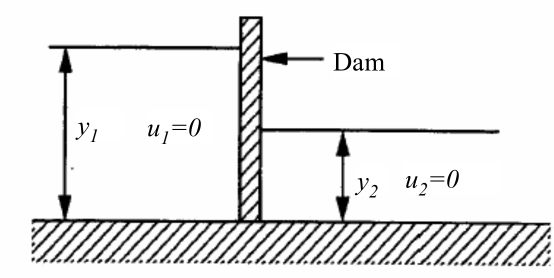

Fig. 3.9
Dam break on wet bed, initial state

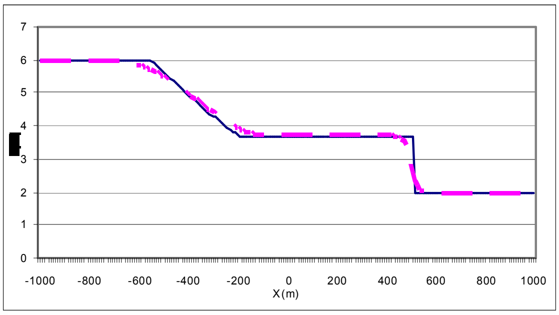

Fig. 3.10
The numerical and the analytical solution at t =72 s

3.5. Instant dam break and discontinuous wave on the Da river (see Fig. 3.11)

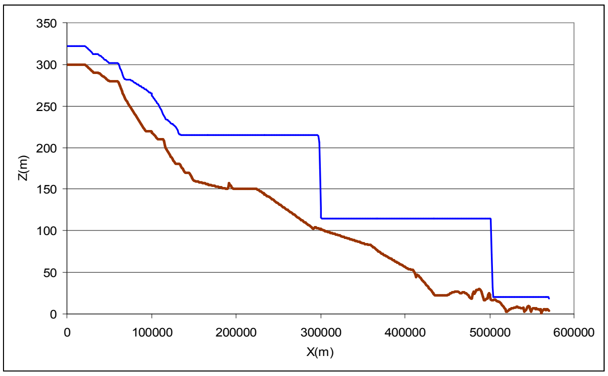

Fig. 3.11a
SonLa dam on the Da river, initial state

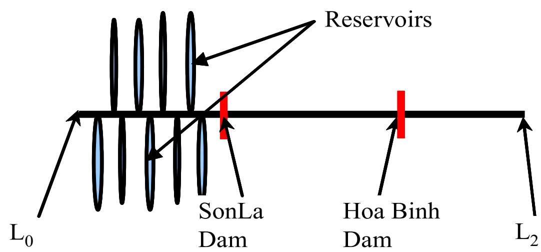

Fig. 3.11b
Da river and the reservoirs

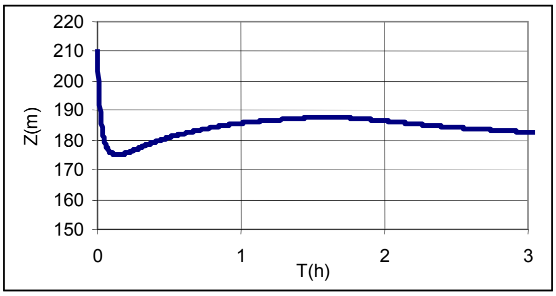

Fig.3.12
$Z(t)$ at Son La dam

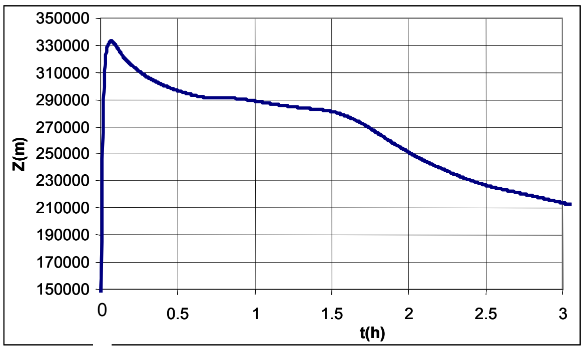

Fig.3.13
$Q(t)$ at Son La dam

The Son La dam is situated at the distance of 300 km from the upstream boundary L_0. The water level on upstream side of dam is 215 m and on the other one is 116 m. The cross-section areas are constructed according to the information of field measurements. The water volume of Main River and some reservoirs at upstream side of SonLa dam is 9.10^9 m^3. Suppose that the dam is totally and instantly broken. The numerical solution is presented in the Fig. 3.12, Fig. 3.13 and Fig. 3.14a, Fig. 3.14b.

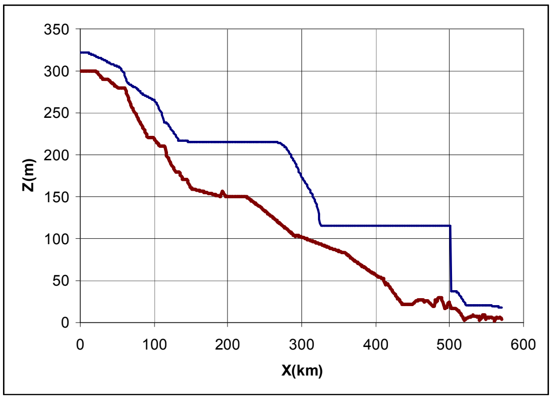

Fig. 3.14a
Water surface elevation at $t = 0.25$ h

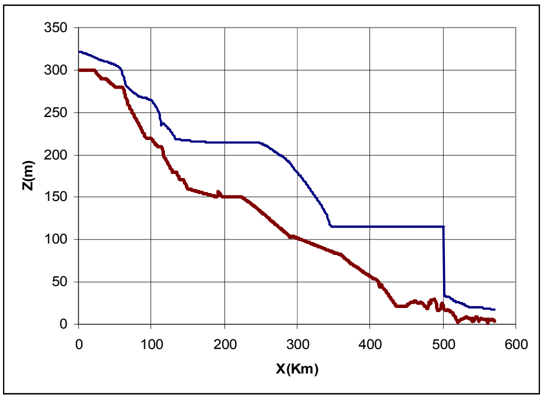

Fig. 3.14b
Water surface elevation at $t = 0.5$ h

3.6. Gradual dam break and the unsteady flow on the Da river (see Fig. 3.11b)

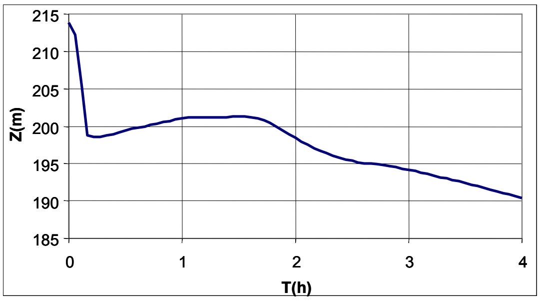

Fig. 3.15a. $Z(t)$ at Son La dam

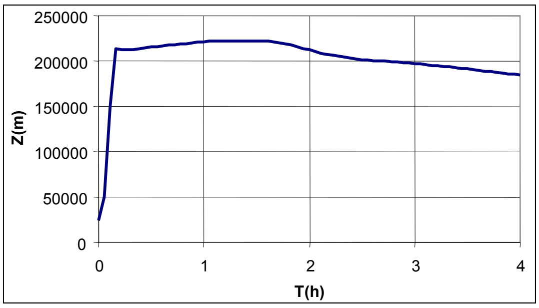

Fig. 3.15b. $Q(t)$ at Son La dam

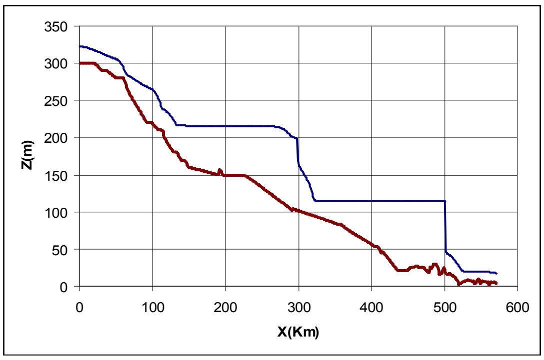

Fig. 3.16a
Water surface elevation at $t = 0.25$ h

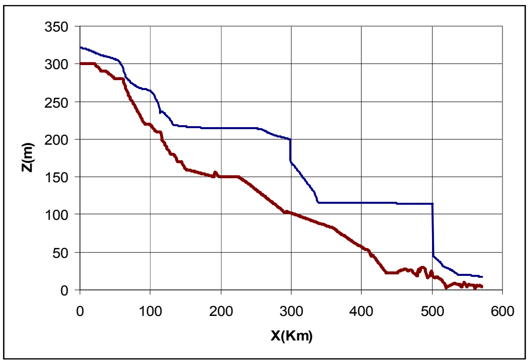

Fig. 3.16b
Water surface elevation at $t = 0.5$ h

The data are given as in the problem 5, we suppose that the dam is gradual failure and rectangular breach $135\,\mathrm{m} \times 105\,\mathrm{m}$ (width $\times$ depth). Maximum of breach size at $t = 0.25\,\mathrm{h}$. The numerical solution is presented in the Fig. 3.15. and Fig. 3.16.

CONCLUSIONS

The algorithm is applied for calculating:

- The unsteady flows of some test cases having the analytical solution.
- The unsteady flow on the Da river system after instant or gradual dam breaking.

The computational results show that:

- The algorithm is easy executed and numerical solutions for the test cases have height accuracy in comparison with the analytical solution.
- The unsteady flows on Da river system, connecting with reservoir after gradual dam breaking is corresponded with the ones of other algorithms in [9].

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