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# EFFECT OF HIGHER LANDAU LEVELS ON THE TRANSVERSE THERMOELECTRIC CONDUCTIVITY IN HIGH-$T_c$ SUPERCONDUCTORS

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**Abstract.** *We investigate effect of higher Landau levels on the transverse thermoelectric conductivity $\alpha_{xy}$, describing the Nernst effect in high-$T_c$ superconductors, by using the time-dependent Ginzburg-Landau theory in two-dimensional model with thermal noise. The transverse thermoelectric conductivity is calculated in the self-consistent Gaussian approximation. Our results indicate that the contribution of higher Landau levels is less than that of lower Landau levels to the transverse thermoelectric conductivity. Our results are in good agreement with experimental data on high-$T_c$ superconductor.*

*Keywords: transverse thermoelectric conductivity, time-dependent Ginzburg-Landau theory, high-$T_c$ superconductor.*

*Classification numbers: 74.40.+k, 74.25.Ha, 74.25.Dw.*

## I. INTRODUCTION

Recently the Nernst effect in high-$T_c$ superconductors has attracted attention both theoretically [1–5] and experimentally [6–11]. The electric field is induced in a metal under magnetic field by the temperature gradient $\nabla T$ perpendicular to the magnetic field $\mathbf{H}$, phenomenon known as the Nernst effect [1]. In the mixed state the Nernst effect is large due to vortex motion, while in the normal state and in the vortex lattice or glass states it is typically smaller. The appearance of a fluctuation tail above the critical temperature in the Nernst signal was observed in strongly type-II superconductors, both low-$T_c$ like $NbSe_2$ and $Nb_{0.15}Si_{0.85}$ films [12] and several different high-temperature materials [6, 7, 10, 11]. The Nernst effect therefore is a probe of thermal fluctuations phenomena in the vortex matter.

The time-dependent Ginzburg-Landau equation has been remarkably successful in describing various transport properties (including the Nernst effect) [1, 3, 5, 13, 14]. However, this description becomes very complicated when fluctuations are of importance. Some progress can be achieved when certain additional assumptions are made. One of the often made additional assumption is that only the lowest Landau level (LLL) significantly contributes to physical quantities of interest [3, 13, 15, 17]. There is a debate however on how restrictive the LLL approximation actually is. When fluctuations are included one can argue using Hartree approximation [18] that the LLL range of validity is even smaller. Therefore, one should consider higher Landau levels (HLL) contributions to physical quantities.

In this paper we explicitly calculate the effects of HLL on the transverse thermoelectric conductivity $\alpha_{xy}$ by using the time-dependent Ginzburg-Landau theory in the two-dimensional (2D) model with thermal noise. We obtain explicit expressions for the transverse thermoelectric conductivity $\alpha_{xy}$ depending on Landau levels. The HLL contribution to the transverse thermoelectric conductivity $\alpha_{xy}$ is analyzed. We also compare the result including HLL with experimental data on high-Tc superconductor.

The paper is organized as follows. The model is defined in Sec. II. The comparison with experiment is presented in Sec. III. We conclude in Sec. IV.

## II. THEORY

### II.1. The Ginzburg - Landau Model in 2D

The Ginzburg-Landau free energy in 2D is

$$F = s\int d^2r\left\{\frac{\hbar^2}{2m^*}|\mathbf{D}\psi|^2 + a|\psi|^2 + \frac{b'}{2}|\psi|^4\right\}, \tag{1}$$

where $s$ is the order parameter effective "thickness", the covariant derivatives are defined by $\mathbf{D} \equiv \nabla - i(2\pi/\Phi_0)\mathbf{A}$ with $\mathbf{A} = (-By, 0)$ describing a constant and practically homogeneous magnetic field and $\Phi_0 = hc/e^*$, $e^* = -2e > 0$. For simplicity we assume linear dependence $a(T) = \alpha T_c^{mf}(t-1), t = T/T_c^{mf}$, although the temperature dependence can be easily modified to better describe the experimental coherence length. It is higher than measured critical temperature due to strong thermal fluctuations on the mesoscopic scale.

In order to study transport phenomena in superconductors, one uses the time-dependent Ginzburg-Landau (TDGL) equation

$$\Gamma_0^{-1}\left(\frac{\partial}{\partial t} + i\frac{e^*}{\hbar}\phi\right)\psi = -\frac{\delta F}{\delta\psi^*} + \zeta. \tag{2}$$

Explicitly the TDGL equation for the superconducting order parameter is

$$\Gamma_0^{-1}\left(\frac{\partial}{\partial t} + i\frac{e^*}{\hbar}\phi\right)\psi = \frac{\hbar^2}{2m^*}\mathbf{D}^2\psi - a\psi - b'|\psi|^2\psi + \zeta, \tag{3}$$

where $\phi(\mathbf{r})$ is the scalar potential describing electric field. To incorporate the thermal fluctuations via Langeven method, the noise term $\zeta(\mathbf{r}, t)$, having Gaussian correlations

$$s\langle\zeta^*(\mathbf{r}, t)\zeta(\mathbf{r}', t')\rangle = 2T\Gamma_0^{-1}\delta(\mathbf{r} - \mathbf{r}')\delta(t - t'), \tag{4}$$

is introduced. Here $\delta(\mathbf{r}-\mathbf{r}')$ is the two dimensional $\delta$ function of the in-plane coordinates, and $\Gamma_0^{-1}$ is the relaxation time rate.

The heat current density in GL model reads

$$\mathbf{j}^h = -\frac{\hbar^2}{2m^*}\left\langle\left(\frac{\partial}{\partial t} - i\frac{e^*}{\hbar}\phi\right)\Psi^*\left(\nabla - i\frac{2\pi}{\Phi_0}\mathbf{A}\right)\Psi\right\rangle + c.c.\cdot \tag{5}$$

## II.2. Solution of TDGL in the self-consistent Gaussian approximation

A simple approximation which captures the most interesting fluctuations effects in the self-consistent Gaussian approximation (see [16] for details), in which the cubic term in the GL equation Eq. (3) $b'|\psi|^2\psi$ is replaced by a linear one $2b'\left\langle|\psi|^2\right\rangle\psi$

$$\Gamma_0^{-1}\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \left(\frac{\hbar^2}{2m^*}\mathbf{D}^2 - \widetilde{a}\right)\psi(\mathbf{r},t) + \zeta(\mathbf{r},t), \tag{6}$$

lead to the "renormalized" value of the coefficient:

$$\widetilde{a} = a + 2b'\langle|\psi|^2\rangle. \tag{7}$$

The formal solution of this equation is

$$\psi(\mathbf{r},t) = \int d\mathbf{r}'\int dt' R_0(\mathbf{r},t;\mathbf{r}',t')\zeta(\mathbf{r}',t'), \tag{8}$$

where $R_0$ is the equilibrium Green function.

In the Landau gauge, one has

$$R_0(\mathbf{r},t;\mathbf{r}',t') = \frac{1}{4\pi^2}\left(\frac{m^*\omega_B}{\hbar}\right)^{1/2}\int_{\omega,\widetilde{y}_0} R_0(\widetilde{y},\widetilde{y}',\omega,\widetilde{y}_0)e^{-i(m^*\omega_B/\hbar)^{1/2}\widetilde{y}_0(x-x')}e^{i\omega(t-t')}, \tag{9}$$

where $\widetilde{y} = (m^*\omega_B/\hbar)^{1/2}y$ with $\omega_B = e^*B/m^*c$, and $\widetilde{y}_0 = -(\hbar/m^*\omega_B)^{1/2}k_x$, $k_x$ is the $x$ component of the vector momentum and

$$R_0(\widetilde{y},\widetilde{y}',\omega,\widetilde{y}_0) = \left(\frac{m^*\omega_B}{\hbar\pi}\right)^{1/2}\exp\left[-(\widetilde{y}-\widetilde{y}_0)^2/2 - (\widetilde{y}'-\widetilde{y}_0)^2/2\right]\sum_n \frac{1}{2^n n!}\frac{H_n(\widetilde{y}-\widetilde{y}_0)H_n(\widetilde{y}'-\widetilde{y}_0)}{(i\Gamma_0^{-1}\omega + E_n)}, \tag{10}$$

with the energy eigenvalues

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega_B + \widetilde{a}, \tag{11}$$

while $H_n$ are the Hermite polynomials.

In equilibrium, $\langle|\psi(\mathbf{r},t)|^2\rangle$ is

$$\langle|\psi(\mathbf{r},t)|^2\rangle = \frac{T}{2\pi s}\frac{m^*\omega_B}{\hbar}\sum_{n=0}^{N}\frac{1}{E_n}. \tag{12}$$

Thus equation (7) becomes

$$\varepsilon_b = \widetilde{\varepsilon}_b - 2\omega b\frac{T}{T_c^{mf}}\sum_{n=0}^{N}\frac{1}{\widetilde{\varepsilon}_b + 2nb}, \tag{13}$$

where the reduced temperature is defined as $\varepsilon = a/\alpha T_c^{mf}$, $\varepsilon_b = \varepsilon + b$, (with similar expression for $\widetilde{\varepsilon}$ and $\widetilde{\varepsilon}_b$), $\omega = \sqrt{2Gi}\pi$ with $Gi \equiv \frac{1}{2}\left(\frac{8e^2\kappa^2\xi^2 T_c^{mf}}{c^2\hbar^2 s}\right)^2$ being the Ginzburg parameter characterizing the strength of thermal fluctuations on the mesoscopic scale. The ultraviolet cutoff $\Lambda$ was introduced. It effectively limits the number of Landau levels to $N = \frac{\Lambda}{b} - 1$.

The equation (13) can be written as follows

$$\varepsilon_b^r = \widetilde{\varepsilon}_b - \frac{\omega t_c}{2\pi}\left[f(\widetilde{\varepsilon}_b/2b) - \log(2b)\right], \tag{14}$$

where the critical temperature $T_c$ is significantly renormalized: $\varepsilon_b^r = \varepsilon_b + \omega t_c \log\Lambda$, $\varepsilon_b^r = a/\alpha T_c + b, t_c = T/T_c$, $\omega = \sqrt{2Gi}\pi$ with $Gi \equiv \frac{1}{2}\left(\frac{8e^2\kappa^2\xi^2 T_c}{c^2\hbar^2 s}\right)^2$ ($T_c^{mf}$ is now replaced by $T_c$) and $f(x)$ is the polygamma function.

### II.3. The transverse thermoelectric conductivity

We assume that the weak electric field $\mathbf{E}$ is along the $y$ axis, generated by the scalar potential $\phi = -E_y y$. The heat and the electric current in the vortex liquid phase can be written

$$\mathbf{j}^h = -\frac{\hbar^2}{2m^*}\left[\mathbf{D}(\mathbf{r})\left(\frac{\partial}{\partial t'} - i\frac{e^*}{\hbar}\phi(\mathbf{r}')\right) + \mathbf{D}^*(\mathbf{r}')\left(\frac{\partial}{\partial t} + i\frac{e^*}{\hbar}\phi(\mathbf{r})\right)\right] C(\mathbf{r},t;\mathbf{r}',t')|_{\mathbf{r}=\mathbf{r}';t=t'}, \tag{15}$$

where

$$C(\mathbf{r},t;\mathbf{r}',t') = \frac{2\Gamma_0^{-1}T}{s}\int_{\mathbf{r}_1,t_1} R(\mathbf{r},t;\mathbf{r}_1,t_1)R^*(\mathbf{r}',t';\mathbf{r}_1,t_1), \tag{16}$$

with $R$ is the Green function of the linearized TDGL equation in the presence of the scalar potential. One finds correction to the Green function to linear order in the electric field

$$R(\mathbf{r},t;\mathbf{r}',t') = R_0(\mathbf{r},t;\mathbf{r}',t') - i\frac{e^*\Gamma_0^{-1}}{\hbar}\int_{\mathbf{r}_1,t_1}\phi(\mathbf{r}_1)R_0(\mathbf{r},t;\mathbf{r}_1,t_1)R_0(\mathbf{r}_1,t_1;\mathbf{r}',t'). \tag{17}$$

The transverse thermoelectric conductivity is obtained by expanding the correlation function to linear order in the electric field. The correlation function $C$ in terms of the Green function $R_0$ using Eqs. (19), (16) and (17) takes a form

$$C(\mathbf{r},t;\mathbf{r}',t') = C_0(\mathbf{r},t;\mathbf{r}',t') + C_1\left(\mathbf{r},t;\mathbf{r}',t'\right), \tag{18}$$

where

$$C_0(\mathbf{r},t;\mathbf{r}',t') = \frac{2\Gamma_0^{-1}T}{s}\int_{\mathbf{r}_1,t_1} R_0(\mathbf{r},t;\mathbf{r}_1,t_1)R_0^*(\mathbf{r}',t';\mathbf{r}_1,t_1), \tag{19}$$

$$C_1(\mathbf{r},t;\mathbf{r}',t') = i\frac{e^*\Gamma_0^{-1}}{\hbar}\int_{\mathbf{r}_1,t_1}\phi(\mathbf{r}_1)\left[R_0^*(\mathbf{r}',t';\mathbf{r}_1,t_1)C_0(\mathbf{r},t;\mathbf{r}_1,t_1) - R_0(\mathbf{r},t;\mathbf{r}_1,t_1)C_0^*(\mathbf{r}',t';\mathbf{r}_1,t_1)\right]. \tag{20}$$

In order to determine the transverse thermoelectric conductivity, we need to compute the $x$ component of the heat current to the first order in the electric field. In the chosen gauge, the heat current along the $x$ direction also contains two terms. The term coming from $C_0$ vanishes: $j_0^{(h)x} = 0$. It is possible to interpret easily that $C_0$ is the equilibrium correlation function which does not contribute to the current. Considering the $C_1$

$$j_1^{(h)x} = \frac{(e^*)^2}{\pi mcs} E_y B k_B T \sum_{n=0} \left[ \frac{n+1/2}{2E_n} - \frac{n+1}{E_n + E_{n+1}} \right]. \tag{21}$$

By an Onsager relation [1, 3], $\alpha_{xy}$ can be obtained from the heat current response to an electric field

$$\begin{aligned} \alpha_{xy} &= \frac{e^* k_B b}{2\hbar\pi s} \sum_{n=0}^{N} \left[ \frac{n+1/2}{2nb+\widetilde{\varepsilon}_b} - \frac{n+1}{2(n+1/2)b+\widetilde{\varepsilon}_b} \right] \\ &= \frac{e^* k_B (b-\widetilde{\varepsilon}_b)}{4\hbar\pi bs} \left[ f\left(\frac{\widetilde{\varepsilon}_b}{2b}\right) - f\left(\frac{\widetilde{\varepsilon}_b}{2b} + \frac{1}{2}\right) \right]. \end{aligned} \tag{22}$$

## III. COMPARISON WITH EXPERIMENT

We compare the transverse thermoelectric conductivity equation (22) including all Landau levels with the experimental data of Wang et al. [8] on an overdoped $La_{2-x}Sr_xCuO_4$ sample with $T_c = 28$ K. The comparison is presented in Fig. 1. The parameters we obtained from the fit are: $H_{c2}(0) = 45$ T (corresponding to $\xi = 27Å$), $\kappa = 69, s = 7.6Å$ (corresponding to $\omega \simeq 0.1$), which is roughly in agreement in magnitude with the value of $s = 6.6Å$ in [19]. With these values, our calculation gives good agreement with the experimental data.

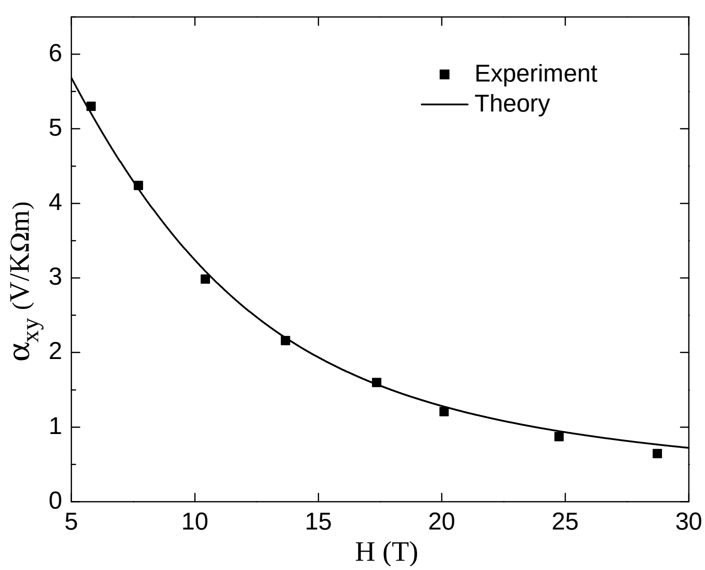


**Fig. 1.** Points are the transverse thermoelectric conductivity at temperatures T=20 K in reference [8]. The solid line is our result including all Landau levels.

Using the parameters specified above we plot several theoretical curves. In Fig. 2, dependence of the transverse thermoelectric conductivity on magnetic field is shown for different temperatures. At given magnetic field, as the temperature increases, the transverse thermoelectric conductivity decreases. In Fig. 3, we estimate contribution of HLL to the transverse thermoelectric conductivity. When the number of Landau levels $N$ increases, the curves get closer together which means the contribution of HLL with $N \geq 4$ to the transverse thermoelectric conductivity is not significant.

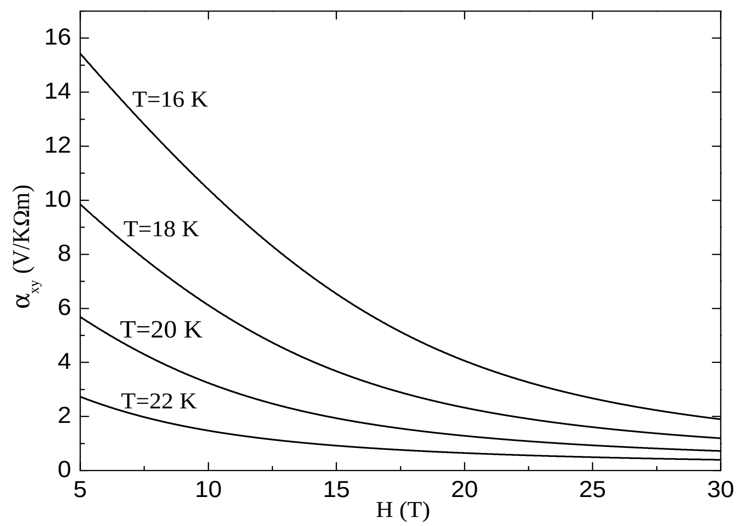


**Fig. 2.** The thermoelectric conductivity including all Landau levels as a function of magnetic field for different temperatures.

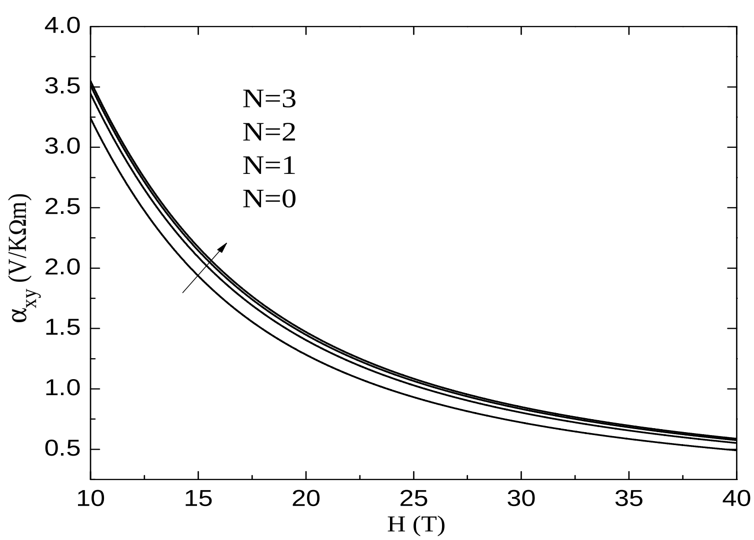


**Fig. 3.** The thermoelectric conductivity as a function of magnetic field at temperature T=20 K. The arrow indicates the increasing number of Landau levels.

## IV. CONCLUSIONS

We have investigated the contribution of higher Landau levels to the transverse thermoelectric conductivity of high-Tc superconductor using the self-consistent Gaussian approximation within the time-dependent Ginzburg- Landau theory with thermal noise.

Our results include higher Landau levels and are presented using both the strength of the thermal fluctuation $\omega$ and more often used Ginzburg number $Gi$. The results are compared to the experimental data on an overdoped $La_{2-x}Sr_xCuO_4$ materials for temperatures close to $T_c$. This comparison is also in good quantitative agreement. Our results also show that the contribution of higher Landau levels with $N \geq 4$ to the transverse thermoelectric conductivity is not significant.

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