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REVIEW

CHALLENGES IN PARTICLE PHYSICS AND 3-3-1 MODELS

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Abstract. *At present, particle physics faces problems related with disparity in diphoton decay of Higgs boson, discrepancy between theory and experiment of about* 3.6σ *in anomalous magnetic moment of the muon and neutrino physics. Moreover, the existence of Dark Matter and the Matter-Antimatter Asymmetry are challenges for any physical model. In this review I present some solutions in the framework of the 3-3-1 models. The simple (by Higgs sector) models contain the hybrid inflationary scenario and the first-order phase transitions, from which leptogenesis needed for BAU is followed. By these considerations, some bounds on model parameters are derived.*

Keywords: particle-theory and field-theory models of the early Universe, supersymmetric models, non-standard-model neutrinos, right-handed neutrinos.

I. INTRODUCTION

The discovery of the Higgs boson shows that the way we build model on theory of elementary particle interactions based in gauge group and Higgs mechanism is correct. Over the half of Century, the Standard Model (SM) of the electromagnetic, weak and strong interactions successfully possesses a great experimental examinations and is a standpoint for future development. However, the model still contains a number of problems such as the generation number of quarks and leptons, the neutrino mass and mixing, the electric charge quantization, the existence of about one quarter of DM, etc. To overcome the mentioned problems, the SM must be extended.

Among the extensions beyond the SM, the models based on $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ (3-3-1) gauge group [1, 2] have some interesting features including the ability to explain the generation problem [1, 2] and the electric charge quantization [3]. It is noted that in this scheme the gauge couplings can be unified at the scale of order TeV *without supersymmetry* [4]. The 3-3-1 models have two interesting properties needed for the mentioned aim, namely, the lepton-number violation due to the fact that lepton and anti-lepton lie in the same triplet [5].

It is well known that our Universe content is 68.3% of Dark Energy (DE), 26.8% of Dark Matter (DM) and of 4.9% of luminous matter [6]. With only one fact of accelerating Universe, the core origin of Dark Energy is still under question, while the existence of Dark Matter is unambiguous. According to the Standard Cosmology, in the moment at $10^{-36}s$ after the Big Bang (BB), there was an inflation, and our Universe has been expanded exponentially. The inflationary scenario solves a number of problems such as the Universe's flatness, horizon, primordial monopole, etc. It is well known that there is no anti-matter in our Universe, or other word speaking: at present there exists a Baryon Asymmetry of Universe (BAU). The baryon number vanishes ($n_B = 0$) at the BB, and this conflicts with the present BAU. Nowadays, the BAU is one of the greatest challenges in physics and any physical model has to give answer.

The 3-3-1 models are investigated in detail in wide area of particle physics. Hence, I only choose some aspects which I feel the most interesting. This review is organized as follows. In Sec. II, I give a brief review of the 3-3-1 models and their modified versions. In Sec. III, I discuss some aspect of collider physics concerned with diphoton Higgs decay, $(g-2)_\mu$ discrepancy. Sec. IV is devoted to neutrino mass and mixing and implication of the discrete symmetries. In Sec. V, I discuss about the dark matter. Sec. VI is devoted to the cosmological inflation in the supersymmetric economical 3-3-1 model and to inflationary scenario and leptogenesis in the 3-3-1-1 model. Phase transitions in two versions with minimal Higgs content are presented in Sec. VII. Here we find the parameter ranges where the EWPTs are the strongly first-order to provide B violation necessary for baryogenesis, and get the constraints on the mass of the charged Higgs boson. Sec. VIII is devoted for sphalerons in the reduced minimal 3-3-1 model. Finally, conclusion and outlook are given in the last section - section IX.

II. THE MODELS

In all versions of the 3-3-1 models, the strong interaction keeps the same as in the SM, while the electroweak part associated with $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ has two diagonal generators T_3 and T_8 from which the electric charge operator is based on

$$Q = T_3 + \beta T_8 + X. \tag{1}$$

The coefficient (=1) at the T_3 is defined to make the 3-3-1 models embed the SM. The lepton arrangement will define the parameter β which distinguishes two main versions: the minimal version with $\beta = \sqrt{3}$ and the version with neutral leptons/neutrinos $\beta = -1/\sqrt{3}$ at the bottom of the triplet.

II.1. The minimal 3-3-1 model

The minimal version [1] contains lepton triplet in the form

$$f_L = (\nu_l, l, l^c)^T_L \sim (1,3,0). \tag{2}$$

Name (minimal) of the version is followed from the fact that there is no new lepton in the model. Two first quark generations are in anti-triplet and the third one is in triplet:

$$Q_{iL} = (d_{iL}, -u_{iL}, D_{iL})^T \sim \left(3, \bar{3}, -\frac{1}{3}\right), \tag{3}$$

$$u_{iR} \sim (3,1,2/3), d_{iR} \sim (3,1,-1/3), D_{iR} \sim (3,1,-4/3),\ i = 1,2,$$
$$Q_{3L} = (u_{3L}, d_{3L}, T_L)^T \sim (3,3,2/3),$$
$$u_{3R} \sim (3,1,2/3), d_{3R} \sim (3,1,-1/3), T_R \sim (3,1,5/3).$$

To generate masses for all quarks and lepton, the Higgs sector needs three scalar triplets and one sextet:

$$\begin{aligned} \chi &= \left(\chi_1^-, \chi_2^{--}, \chi_3^0\right)^T \sim (1,3,-1), \\ \eta &= \left(\eta_1^0, \eta_2^-, \eta_3^+\right)^T \sim (1,3,0), \\ \rho &= \left(\rho_1^+, \rho_2^0, \rho_3^{++}\right)^T \sim (1,3,1), \\ S &\sim (1,6,0). \end{aligned} \tag{4}$$

with VEV: $\langle\rho_2^0\rangle = v/\sqrt{2}, \langle\eta_1^0\rangle = u/\sqrt{2}$, $\langle\chi_3^0\rangle = \omega/\sqrt{2}$ and $\langle S_{23}^0\rangle = v'/\sqrt{2}$.

The gauge sector of this model contains five new gauge bosons: one neutral Z' and two bileptons carrying lepton number 2: $Y^\pm$ and $X^{\pm\pm}$. In (2), lepton and antilepton lie in the same triplet, and this leads to lepton number violations in the model. Hence, it is better to deal with a new conserved charge $\mathscr{L}$ commuting with the gauge symmetry [5]

$$L = \frac{4}{\sqrt{3}} T_8 + \mathscr{L}. \tag{5}$$

The exotic quarks T and D_i have the electric charges, respectively, $5/3$ and $-4/3$ and carry both baryon and lepton numbers $L = \pm 2$ (we can call them *bilepton quarks*). Note that some Higgs bosons (including neutral ones) carry lepton number, but *only Higgs bosons without any charge can have* vacuum expectation value (VEV).

The singly charged bilepton $Y^\pm$ is responsible for the wrong muon decay

$$\mu \to e + \nu_e + \tilde{\nu_\mu},$$

while the doubly charged bilepton $X^{\pm\pm}$ connects with the decay

$$X^{--} \to l\, l$$

providing four leptons at the final states which are characteristic feature of the model. The model provides an interesting prediction for the Weinberg angle

$$\sin^2\theta_W(M_{Z'}) \leq \frac{1}{4}.$$

Besides the complication in the Higgs sector, the model also has one problem that it losses perturbative property at the scale above 5 TeV [7].

The above Higgs sector is complicated; and recently it is reduced to the minimal with only two Higgs triplets [8,9]. If the triplet ρ and χ are used then the obtained model is called reduced minimal 3-3-1 model (RM331) [8], while ρ is replaced by η then it is called the simple 3-3-1 model (S331) [9].

It has been recently shown that due to the ρ parameter and the Landau pole, the minimal and its reduced version should be ruled out [10]. It is noted that the RM331 has nonrenormalizable effective interactions. Thus, the situation has to be considered carefully.

II.2. The 3-3-1 model with right-handed neutrinos

In the SM, neutrinos are strictly massless which contradicts with the experimental data. To solve this problem, one needs right-handed neutrino. As a consequence, in the 3-3-1 model with right-handed neutrinos, leptons are in triplet [2]:

$$f^a_L = (\nu^a_L, e^a_L, (N_L)^a)^T \sim (1,3,-1/3), e^a_R \sim (1,1,-1), \tag{6}$$

where $a = 1,2,3$ is a generation index and N_L can be right-handed neutrino or neutral lepton. Two first generations of quarks are in antitriplets, and the third one is in triplet:

$$\begin{gathered}
Q_{iL} = (d_{iL}, -u_{iL}, D_{iL})^T \sim (3,\bar{3},0), \\
u_{iR} \sim (3,1,2/3), d_{iR} \sim (3,1,-1/3), D_{iR} \sim (3,1,-1/3),\ i = 1,2, \\
Q_{3L} = (u_{3L}, d_{3L}, T_L)^T \sim (3,3,1/3), \\
u_{3R} \sim (3,1,2/3), d_{3R} \sim (3,1,-1/3), T_R \sim (3,1,2/3).
\end{gathered} \tag{7}$$

The model with neutral lepton/neutrino ($\beta = -1/\sqrt{3}$) needs three scalar triplets to provide all fermions masses and the same for spontaneous symmetry breaking (SSB):

$$\begin{aligned}
\chi &= \left(\chi^0, \chi^-, \chi'^0\right)^T \sim (1,3,-1), \\
\rho &= \left(\rho^+, \rho^0, \rho'^+\right)^T \sim (1,3,2), \\
\eta &= \left(\eta^0, \eta^-, \eta'^0\right)^T \sim (1,3,-1).
\end{aligned} \tag{8}$$

The exotic quarks T and D_i have electric charges as usual one, i.e., $2/3$ and $-1/3$, respectively, and carry both baryon and lepton numbers $L = \pm 2$ (again, we can call them *bilepton quarks*). Since both usual and exotic quarks have the same electric charges, there are mixtures among them. This problem has been investigated in a series of works. The new gauge bosons are: the neutral Z' and two bileptons carrying lepton number 2: $Y^\pm$ and X^0. The neutral bilepton X^0 is non-Hermitian and is responsible for neutrino oscillation [11].

Note that two Higgs triplets η and χ have the same structure, so ones can reduce number of Higgs triplets from three to two, namely we can use only ρ and χ to produce masses for quarks and leptons; and the obtained model is called economical 3-3-1 model (E331) [12]. As in the RM331, the nonrenormalizable interactions, in this case, are needed for generation of quark masses [12].

III. COLLIDER PHYSICS AND 3-3-1 MODELS

Production of the bilepton gauge bosons at colliders such as e^+e^- and $p\tilde{p}$ was considered in [13], while production of the SM-like Higgs boson at the LHC was investigated in [14]. In [15], the authors have considered production of charged Higgs bosons in the 3-3-1 model at the LHC. In the 3-3-1 model, there are doubly charged Higgs bosons and their signal at future e^+e^- collider was considered in [16].

The last important element of the SM - the Higgs boson was discovered by the ATLAS and the CMS [17]. With mass of 126 GeV, the Higgs boson is quite heavy, therefore it can decay to all (except the top quark) particles of the SM. Due to the lepton number violation, there are some specific channels of neutral Higgs decays into muon and tauon in the 3-3-1 model [18]. It was showed that in the RM331 model, bileptons enhance the channel $\Gamma(h \to \gamma\gamma)$ and reduce the channel $\Gamma(h \to Z\gamma)$ [19]. At least, the RM331 model is able to explain the recent measurements of the LHC regarding the signal strength.

The muon anomalous magnetic moment $(g-2)_\mu$ is one of the most precisely measured quantities in particle physics. In a series of papers [20], it was shown that the 3-3-1 models have very tiny possibility to accommodate the reported $(g-2)_\mu$.

Due to lepton number violating interactions, the 3-3-1 models have rich phenomenology in Higgs and gauge sectors which have to be investigated in the future.

IV. Neutrino mass and mixing in 3-3-1 models

In the SM, neutrinos are strictly massless (even at the loop correction). Many experiments have recently showed that neutrinos are massive (with tiny value) and mixing with very special feature [21]. The neutrino experimental data are almost based on neutrino oscillation, from which one cannot determine the absolute values of neutrino masses which play an important role in our Universe evolution. Moreover, the neutrino oscillation is a source of family lepton number violation, hence, it causes baryogenesis [22].

Neutrino mass and mixing in the 3-3-1 models were widely considered in [23]. There are two famous methods to generate small neutrino masses, namely: the radiative and seesaw mechanisms. The seesaw mechanism relies in the violation of lepton number at a very high energy scale (M), giving a mass with the form $m_\nu = \frac{v_{SM}}{M}$. However, using inverse and double seesaw mechanisms, we can obtain small active neutrino masses at the TeV scale [24].

It is to be noted that neutrinos mix with very special form - the tribimaximal (TB) one [25]. There are a lot of attempts to explain this scheme, and they are mainly based on discrete symmetry such as $A_4, S_4, S_3, T', T_7, D_4$,... Note that to get the TB form, we should accept E. Ma's antzat: *only leptons of the SM can have lepton numbers [26].* This means that the right-handed neutrino does not carry lepton number. Hence the Dirac neutrino mass violates lepton numbers. This is a surprise!. This is a new feature of the models based in discrete symmetries.

F. Yin is the first person introducing the A_4 to the 3-3-1 model [27]. However, the quark sector was not treated in that paper. In [28], we applied the A_4 discrete symmetry to the 3-3-1 with right-handed neutrinos, where the TB was easily derived. Next step, we have applied the S_4 flavor symmetry to the model with neutral leptons. Both A_4 and S_4 can produce exactly TB, while the S_3 can provide approximately the TB [29]. The TB requires vanishing solar angle θ_{13}. However, recent data [21] shows that $\sin^2\theta_{13} = 0.0246$ that means $\theta_{13} \simeq 9^o$. In [30] we have introduced T_7 flavor symmetry to the model with neutral leptons, where the Dirac and Majorana CP phases are maximal without condition $\theta_{23} = \frac{\pi}{4}$. Besides the above discrete symmetries, the T' [31] and the D_4 [32] are also imposed to the 3-3-1 models. To conclude this section, we note that the 3-3-1 models can give good explanation to neutrino mass and mixing. The discrete symmetries give not only fair neutrino mixing but also the symmetry among generation which is absent in ordinary gauge model. However, the price we have to pay, is the large Higgs sector preventing predictivity of constructed models.

V. DARK MATTER IN 3-3-1 MODELS

Nowadays the existence of dark matter is unambiguous. However, in the SM there is no candidate for the DM. To be a candidate of DM, particle / object must be neutral by the electric charge, has a life-time larger than the Universe one ($\sim 13.7\times 10^9\, y$). Therefore, any physical model has to contain a candidate for the DM whose nature mainly is scalar field.

The self-interaction dark matter (SIDM) being scalar field was firstly observed by D. Fregolente and M. D. Tonasse in the minimal 3-3-1 model [33]. Note that SIDM has usually not an origin being from the gauge theories and it was firstly found in the gauge theory. The SIDM in the 3-3-1 model with right-handed neutrinos was found by H. N. Long and N. Q. Lan in [34]. The scalar bileptons being a candidate for DM were also studied in [35].

Recently the DM attracts physicists to search them at the LHC and they are a subject for experimental and theoretical physics. There are some criteria for direct and indirect searches for DM. Dong and collaborators have recently imposed a new symmetry called W-parity symmetry [36]. The obtained model is called by the 3-3-1-1 model, and its phenomenology including inflation and leptogenesis were considered in [37]. By adding inert scalar sextet $X=1$, the minimal 3-3-1 model with two Higgs triplets called *simple* one, contains a candidate for DM [9]. Phenomenology of inert dark matter is a subject of many recent studies.

VI. THE 3-3-1 MODELS AND COSMOLOGY

It is well known that our Universe is flat and without antimatter. These facts are key elements to which any physical model has to explain. In particle physics, we have the SM which very well describes high energy physics.; and the same situation in cosmology, where the Standard Cosmology (SC) is almost agree with astrophysical experimental data. To be consistent with observed data, the SC has to contain the cosmological inflation (CI), which was happened at $10^{-36} \div 10^{-34} s$ after the Big Bang (BB). It is noted that the inflationary scenario gives very good consistence with the WMAP data [38].

To be consistent with cosmological evolution, our strategy is the following: the model has to have an inflation or phase transition of the first-order. As a result, the leptogenesis or CP-violation will be existed. Then sphaleron completes to produce the BAU. Thus, the first step is to consider the CI in the model. To simplify our task, we choose to work with the model as compacted (by Higgs content) as possible. The first work in this direction is about inflation in supersymmetric economical 3-3-1 model [39]. With only two Higgs triplet, the economical 3-3-1 model [12] does not have a component playing a role of inflation - a key element of CI. The supersymmetric version of the economical 3-3-1 model has been constructed in [40], where χ has its supersymmetric *partner* χ'; and this version contains the needed for our aim element.

In [39], the authors have constructed a hybrid inflationary scheme based on a realistic supersymmetric $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model by adding a singlet superfield Φ which plays the role of the inflaton, namely the inflaton superfield. The inflaton superfield couples with a pair of Higgs superfields. Therefore, the additional global supersymmetric renorrmalizable superpotential for the inflation sector is chosen to be [41, 42]

$$W_{inf}(\Phi,\chi,\chi') = \alpha\Phi\chi\chi' - \mu^2\Phi. \tag{9}$$

The authors assumed that the initial value for the inflaton field is much greater than its critical value S_c. For $|S| > |S_c| \equiv \frac{\mu}{\sqrt{\alpha}}$ the potential is very flat in the $|S|$ direction, and the χ, χ' fields settle down to the local minimum of the potential, $\chi = \chi' = 0$, but it does not drive S to its minimum value. The Universe is dominated by a nonzero vacuum energy density, $V_0^{\frac{1}{4}} = \mu$, which can lead to an exponential expanding; inflation starts; and supersymmetry is broken.

As usual, the effective potential is needed for inflation; applying the Coleman-Weinberg formula in [43], at the one-loop level, one gets the effective potential along the inflaton direction

$$\Delta V = \frac{1}{64\pi^2}\sum_i (-1)^F m_i^4 \ln\left(\frac{m_i^2}{\Lambda^2}\right),$$

where $F=-1$ for the fermionic fields and $F=1$ for the bosonic fields. The coefficient $(-1)^F$ shows that bosons and fermions give opposite contributions.

The above model cannot resolve the horizon/flatness problems of the BB cosmology and violates the slow-roll conditions $\eta \ll 1$ (the η problem). To deal with these problems, we should consider the F-term inflation with the minimal Kähler potential.

The F-term inflation with Kähler potential is defined by

$$W_{stand}(\Phi,\chi,\chi') = \alpha\widehat{S}\left(\widehat{\chi}\widehat{\chi}' - M_X^2\right). \tag{10}$$

It is interesting to note that due the inflaton with mass in the GUT scale, the model can provide masses for neutrino differently from ones without inflationary scenario. With the help of the lepton-number-violating interactions among the inflaton and right-handed neutrinos, the non-thermal leptogenesis scenario is followed [44].

In recent work [45], the authors have considered the inflationary scenario and leptogenesis in newly proposed model based in $\mathrm{SU}(3)_C\otimes \mathrm{SU}(3)_L\otimes \mathrm{U}(1)_X\otimes \mathrm{U}(1)_N$ (3-3-1-1) gauge group. Here, the scalar field that spontaneously breaks the $U(1)_N$ symmetry plays the role of inflaton. The inflaton mass is in order of 10^{13} GeV, and it can dominantly decay into a pair of light Higgs bosons or a pair of heavy Majorana neutrinos which lead to a reheating temperature of 10^9 GeV order appropriate to a thermal leptogenesis scenario or to a reduced reheating temperature corresponding to a non-thermal leptogenesis scenario. Thus, the 3-3-1 models can provide the inflationary scenario or cosmological evolution of our Universe.

VII. ELECTROWEAK PHASE TRANSITION IN 3-3-1 MODELS

In theoretical particle physics, it is a well-known Higgs mechanism providing masses of gauge and matter fields. This is some kind of phase transitions in physical science, particularly in Cosmology. There are two kinds of phase transitions which are called by order of vanishing derivatives (according by L. Landau): first/second order phase transitions. The first order phase transition, to which the Higgs mechanism belongs, is very violent, while the second order transition is smooth. It is known that if baryon number is conserved and is equal to zero, it will equal to zero forever. If baryon number does not satisfy any conservation law, it vanishes in the state of thermal equilibrium. It is well known that the Sakharov's conditions is necessary for solving the BAU. For this aim, it is necessary the (strongly) first- order phase transition!

Why is the first order phase transition? For very large temperature, the effective potential has only one minimum at the zero. As temperature drops below the critical temperature (T_c), the second minimum appears. If the two minimums are separated by a potential barrier, the phase transition occurs with bubble nucleation. Inside the bubbles, the scalar field acquires a nonzero expectation value. If the bubble nucleation rate exceeds the universe's expansion rate, the bubbles collide and eventually fill all space. Such a transition is called the first order phase transition. It is very violent and one can expect large deviations from thermal equilibrium [46]. The other possible scenario takes place if the two minimums are never separated by a potential barrier. The phase transition is a smooth transition or the second order phase transition.

The electroweak phase transition (EWPT) is the transition between symmetric phase to asymmetric phase in order to generate mass for particles. Hence, *the phase transition is related to the mass of the Higgs boson* [46].

For the SM, although the EWPT strength is larger than unity at the electroweak scale, it is still too weak for the mass of the Higgs boson to be compatible with current experimental limits [46,47]; this suggests that electroweak baryogenesis (EWBG) requires new physics beyond the SM at the weak scale [48]. Our study shows that 3-3-1 schemes can give the EWPT consistent with experimental data!

VII.1. Phase transition in reduced minimal 3-3-1 model

Many extensions such as the two-Higgs-doublet model or Minimal Supersymmetric Standard Model have a more strongly first-order phase transition and the new sources of CP violation, which are necessary to account for the BAU; triggers for the first-order phase transition in the above mentioned models are heavy bosons or DM candidates [49–52].

To start, let us consider the hight-temperature effective potential

$$V_{eff} = D.(T^2 - T'^2_0)v^2 - E.Tv^3 + \frac{\lambda_T}{4}v^4,$$

where v is the VEV of Higgs boson. In order to have the strongly first-order phase transition, the strength of phase transition has to be larger than 1, i.e., $\frac{v_c}{T_c} \geq 1$.

The phase transition has been firstly investigated in the SM. But the difficulty of the SM is that the strength of the first-order electroweak phase transition, which must be larger than 1 at the electroweak scale, appears too weak for the experimentally allowed mass of the SM scalar Higgs boson [46, 47]. Therefore, it seems that EWBG requires a new physics beyond the SM at weak scale [48].

With the discovery of the Higgs boson, the study of phase transitions in the particle models is simplified: only to determine the order of phase transition. This opens a lot of hope for the extended models in examining the electroweak phase transition.

To give masses for all particles, the 3-3-1 models must have at least two Higgs triplets [8, 12]. Therefore, the number of bosons in the 3-3-1 models will many more than in the SM and symmetry breaking structure is different to the SM.

The physical scalar spectrum of the RM331 model is composed by a doubly charged scalar h^{++} and two neutral scalars h_1 and h_2 [8]. These new particles and exotic quarks can be triggers for the first order phase transition.

From the Higgs potential we can obtain V_0 that depends on VEVs as follows

$$V_0(v_\chi, v_\rho) = \mu_1^2 v_\chi^2 + \mu_2^2 v_\rho^2 + \lambda_1 v_\chi^4 + \lambda_2 v_\rho^4 + (\lambda_3 + \lambda_4) v_\chi^2 v_\rho^2.$$

The effective potential being a function of VEVs and temperature has the form

$$V = V_0(v_\chi, v_\rho) + \sum M^2_{boson}(v_\chi, v_\rho) W^\mu W_\mu + \sum m_{fermion}(v_\chi, v_\rho) \overline{f^c_L} f^c_L.$$

The effective potential can be rewritten as follows

$$V_{eff} = V_0 + V_{eff}^{hard} + V_{eff}^{light},$$

where

$$\begin{aligned}
V_{eff}^{hard} &= \frac{3}{64\pi^2}\left(m_{Z_2}^4 \ln\frac{m_{Z_2}^2}{Q^2} + m_{h_2}^4 \ln\frac{m_{h_2}^2}{Q^2} + 2m_{h^{++}}^4 \ln\frac{m_{h^{++}}^2}{Q^2}\right) \\
&+ \frac{3}{64\pi^2}\left(2m_U^4 \ln\frac{m_U^2}{Q^2} + 2m_V^4 \ln\frac{m_V^2}{Q^2} - 12m_Q^4 \ln\frac{m_Q^2}{Q^2}\right) \\
&+ \frac{T^4}{4\pi^2}\left[F_-\left(\frac{m_{h_2}}{T}\right) + 2F_-\left(\frac{m_{h^{++}}}{T}\right)\right] \\
&+ \frac{3T^4}{4\pi^2}\left[F_-\left(\frac{m_{Z_2}}{T}\right) + 2F_-\left(\frac{m_U}{T}\right) + 2F_-\left(\frac{m_V}{T}\right) + 12F_+\left(\frac{m_Q}{T}\right)\right],
\end{aligned}$$

and

$$\begin{aligned}
V_{eff}^{light} &= \frac{3}{64\pi^2}\left(m_{Z_1}^4 \ln\frac{m_{Z_1}^2}{Q^2} + 2m_W^4 \ln\frac{m_W^2}{Q^2} - 4m_t^4 \ln\frac{m_t^2}{Q^2}\right) \\
&+ \frac{3T^4}{4\pi^2}\left[F_-\left(\frac{m_{Z_1}}{T}\right) + 2F_-\left(\frac{m_W}{T}\right) + 4F_+\left(\frac{m_t}{T}\right)\right].
\end{aligned}$$

Here V_{eff}^{light} is like the effective potential of the SM, while V_{eff}^{hard} is contributions from heavy particles. We expect that V_{hard}^{eff} contributes heavily in the EWPT.

The symmetry breaking in the RM331 model happens in two steps: the first one $SU(3) \to SU(2)$ associated with v_{χ_0} and the second one $SU(2) \to U(1)$ associated with v_{ρ_0}. Because two scales of symmetry breaking are very different, $v_{\chi_0} \gg v_{\rho_0}$ ($v_{\chi_0} \sim 4-5$ TeV, $v_{\rho_0} = 246$ GeV) and because of the accelerating universe, the symmetry breaking $SU(3) \to SU(2)$ takes place before the symmetry breaking $SU(2) \to U(1)$. The symmetry breaking $SU(3) \to SU(2)$ through χ_0, generates the masses of the heavy gauge bosons such as $U^{\pm\pm}$, $V^{\pm}$, Z_2, and exotic quarks.

Through the boson mass formulations in the above sections, we see that boson $V^{\pm}$ only involves in the phase transition $SU(3) \to SU(2)$. $Z_1, W^{\pm}$ and h_1 only involve in the phase transition $SU(2) \to U(1)$. However, $U^{\pm\pm}$, Z_2 and h^{--} involve in both two phase transitions.

The critical temperature is determined as follows

$$T_c' = \frac{T_0'}{\sqrt{1 - E'^2/D'\lambda_{T_c'}'}}. \tag{11}$$

The second/last step is the phase transition $SU(2) \to U(1)$. This phase transition does not involve the exotic quarks and the bilepton boson $V^{\pm}$. Hence, in this case, v_χ is neglected, and the contribution of $U^{\mp\mp}$ is equal to $W^{\mp}$. Then [53]

$$\begin{aligned}
V^{eff}_{SU(2)\to U(1)} &= v_0(v_\rho)\frac{1}{64\pi^2}\left(m^4_{h_2}\ln\frac{m^2_{h_2}}{Q^2}+2m^4_{h^{++}}\ln\frac{m^2_{h^{++}}}{Q^2}\right)\\
&+ \frac{3}{64\pi^2}\left(2m^4_U\ln\frac{m^2_U}{Q^2}+m^4_{Z_1}\ln\frac{m^2_{Z_1}}{Q^2}\right.\\
&+ \left. m^4_{Z_2}\ln\frac{m^2_{Z_2}}{Q^2}+2m^4_W\ln\frac{m^2_W}{Q^2}-4m^4_t\ln\frac{m^2_t}{Q^2}\right)\\
&+ \frac{T^4}{4\pi^2}\left[F_-\left(\frac{m_{h_2}}{T}\right)+2F_-\left(\frac{m_{h^{++}}}{T}\right)\right]\\
&+ \frac{3T^4}{4\pi^2}\left[2F_-\left(\frac{m_U}{T}\right)+F_-\left(\frac{m_{Z_1}}{T}\right)\right.\\
&+ \left. F_-\left(\frac{m_{Z_2}}{T}\right)+2F_-\left(\frac{m_W}{T}\right)+4F_+\left(\frac{m_t}{T}\right)\right]
\end{aligned}$$

Denoting $V^{eff}_{SU(2)\to U(1)}\equiv V^{eff}_{SU(2)\to U(1)}(v_\rho,T)$, at high-temperature, one gets

$$V^{RM331}_{eff}=D(T^2-T_0^2).v_\rho^2-ET|v_\rho|^3+\frac{\lambda_T}{4}v_\rho^4,$$

where

$$\begin{aligned}
D &= \frac{1}{24{v_0}^2}\left\{6m^2_W+6m^2_U+3m^2_{Z_1}+3m^2_{Z_2}+6m^2_t+m^2_{h_2}+2m^2_{h^\pm}\right\},\\
T_0^2 &= \frac{1}{D}\left\{\frac{1}{4}m^2_{h_1}-\frac{1}{32\pi^2v_0^2}\left(6m^4_W+6m^4_U+3m^4_{Z_1}+3m^4_{Z_2}-12m^4_t\right.\right.\\
&\qquad \left.\left.+m^4_{h_2}+2m^4_{h^\pm}\right)\right\},\\
E &= \frac{1}{12\pi v_0^3}(6m^3_W+6m^3_U+3m^3_{Z_1}+3m^3_{Z_2}+m^3_{h_2}+2m^3_{h^\pm}), \qquad (12)\\
\lambda_T &= \frac{m^2_{h_1}}{2v_0^2}\left\{1-\frac{1}{8\pi^2v_0^2m_h^2}\left[6m^4_W\ln\frac{m^2_W}{bT^2}+3m^4_{Z_1}\ln\frac{m^2_{Z_1}}{bT^2}+3m^4_{Z_2}\ln\frac{m^2_{Z_2}}{bT^2}\right.\right.\\
&\qquad \left.\left.+6m^4_U\ln\frac{m^2_U}{bT^2}-12m^4_t\ln\frac{m^2_t}{b_FT^2}+m^4_{h_2}\ln\frac{m^2_{h_2}}{bT^2}+2m^4_{h^\pm}\ln\frac{m^2_{h^\pm}}{bT^2}\right]\right\},
\end{aligned}$$

here we have assumed $m_{H_2}=m_{h_{--}}=m_{Z_2}\equiv Y$ with boson Z_2 and used $Q\equiv v_{\rho_0}=v_0=246$ GeV.

With $m_{h_1}=125$ GeV and assuming $m_{Z_2}=m_{h_2}=m_{h^{--}}=Y$, we obtain $Y<344.718$ GeV [53]. When $\frac{v_{\rho c}}{T_c}=1$, i.e., $2E/\lambda_{T_c}=1$, we get $Y=203.825$ GeV, and the critical temperature is in range $0<T_c<111.473$ GeV.

We have got the following constraints on the mass of Higgs in RM331 [53]

$$285.56\,\text{GeV}<M_{h_2}<1.746\,\text{TeV},\qquad 3.32\,\text{TeV}<M_{h_{--}}<5.61\,\text{TeV}.$$

Thus we have used the effective potential at finite temperature to study the structure of the EWPT in the RM331 model. This phase transition is split into two phases, namely, the first transition is $SU(3) \to SU(2)$ or the symmetry breaking in the energy scale v_{χ_0} in order to generate masses for heavy particles and exotic quarks. The second phase transition is $SU(2) \to U(1)$ at v_{ρ_0}. The EWPT in this model may be the strongly first-order EWPT with $m_{h_1} = 125$ GeV if the heavy bosons masses are some few TeVs.

VII.2. Phase transition in economical 3-3-1 model

One follow the same approach for E331 model [12], whose lepton sector is more complicated than that of the RM331 model. The E331 model has the right-handed neutrino in the leptonic content, the bileptons (two singly charged gauge bosons $W^{\pm}$, $Y^{\pm}$, and a neutral gauge bosons X^0), the heavy neutral boson Z_2, and the exotic quarks. As in the RM331, here EWPT takes place with two transitions: i) $SU(3) \to SU(2)$ at the scale of ω_0 and the transition $SU(2) \to U(1)$ at the scale of v_0 [54].

The first phase transition $SU(3) \to SU(2)$ due to ω provides the bounds on parameters given in [54].

The new bosons and exotic quarks can be triggers for the EWPT $SU(3) \to SU(2)$ to be the first-order. It was shown that the EWPT $SU(2) \to U(1)$ is the first-order phase transition, but it seems quite weak [54].

VIII. ELECTROWEAK SPHALERONS IN THE REDUCED MINIMAL 3-3-1 MODEL

The 3-3-1 models have an inflation and EWPS to produce leptogenesis or CP-violation. Then sphaleron is next step to convert the leptogenesis into BAU. Sphaleron is a transition at high temperature where thermal fluctuations can bring the magnitude of the Higgs field from zero VEV over the barrier to nonzero VEV classically without tunneling. In the SM, the sphaleron rate is very small, about 10^{-60} [55] this rate is much smaller than the rate of BAU and smaller than the cosmological expansion rate.

To study the sphaleron processes, we consider the Lagrangian of the gauge- Higgs system

$$\mathscr{L}_{\text{gauge-Higgs}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \left(\mathscr{D}_\mu\chi\right)^\dagger (\mathscr{D}^\mu\chi) + \left(\mathscr{D}_\mu\rho\right)^\dagger (\mathscr{D}^\mu\rho) - V(\chi,\rho). \tag{13}$$

The sphaleron energies in the $SU(3) \to SU(2)$ and $SU(2) \to U(1)$ phase transitions, are given, respectively

$$\mathscr{E}_{sph.su(3)} = 4\pi\int\left[\frac{1}{2}\left(\frac{dv_\chi}{dr}\right)^2 + V_{eff}(v_\chi,T)\right]r^2dr, \tag{14}$$

and

$$\mathscr{E}_{sph.su(2)} = 4\pi\int\left[\frac{1}{2}\left(\frac{dv_\rho}{dr}\right)^2 + V_{eff}(v_\rho,T)\right]r^2dr. \tag{15}$$

The sphaleron rate per unit time per unit volume, Γ/V, is characterized by a Boltzmann factor, $\exp(-\mathscr{E}/T)$, as follows [56–58]:

$$\Gamma/V = \alpha^4 T^4 \exp(-\mathscr{E}/T), \tag{16}$$

where V is the volume of the EWPT's region, T is the temperature, $\mathscr{E}$ is the sphaleron energy, and $\alpha = 1/30$.

One will compare the sphaleron rate with the Hubble constant, which describes the cosmological expansion rate at the temperature T [59,60]

$$H^2 = \frac{\pi^2 g T^4}{90 M_{pl}^2}, \tag{17}$$

where $g = 106.75$, $M_{pl} = 2.43 \times 10^{18}$ GeV.

Assuming that the VEVs of the Higgs fields do not change from point to point in the universe, then one have $\frac{dv_\chi}{dr} = \frac{dv_\rho}{dr} = 0$, and

$$\frac{\partial V_{eff}(v_\chi)}{\partial v_\chi} = 0, \quad \frac{\partial V_{eff}(v_\rho)}{\partial v_\rho} = 0. \tag{18}$$

Eqs. (18) shows that v_χ and v_ρ are the extremes of the effective potentials. The sphaleron energies can be rewritten as

$$\mathscr{E}_{sph.su(3)} = 4\pi \int V_{eff}(v_\chi, T) r^2 dr = \frac{4\pi r^3}{3} V_{eff}(v_\chi, T)\bigg|_{v_{\chi m}}, \tag{19}$$

and

$$\mathscr{E}_{sph.su(2)} = 4\pi \int V_{eff}(v_\rho, T) r^2 dr = \frac{4\pi r^3}{3} V_{eff}(v_\rho, T)\bigg|_{v_{\rho m}}, \tag{20}$$

where v_{χ_m}, v_{ρ_m} are the VEVs at the maximum of the effective potentials. From (19) and (20), it follows that the sphaleron energies are equal to the maximum heights of the potential barriers.

The universe's volume at a temperature T is given by $V = \frac{4\pi r^3}{3} = \frac{1}{T^3}$. Because the whole universe is an identically thermal bath, the sphaleron energies are approximately

$$\mathscr{E}_{sph.su(3)} \sim \frac{E'^4 T}{4\lambda'^3_T}; \quad \mathscr{E}_{sph.su(2)} \sim \frac{E^4 T}{4\lambda^3_T}. \tag{21}$$

From the definitions (19) and (20), the sphaleron rates take the form, respectively

$$\Gamma_{su(3)} = \alpha_w^4 T \exp\left(-\frac{E'^4 T}{4\lambda'^3_T T}\right), \tag{22}$$

and

$$\Gamma_{su(2)} = \alpha_w^4 T \exp\left(-\frac{E^4 T}{4\lambda^3_T T}\right). \tag{23}$$

For the heavy particles, E, λ, E' and λ' are constant, and the sphaleron rates (for the the phase transition $SU(2) \to U(1)$) in this approximation are the linear functions of temperature [61].

Thus, the upper bounds of the sphaleron rates are much larger the Hubble constant [61]

$$\Gamma_{su(3)} \sim 10^{-3} \gg H; \quad \Gamma_{su(2)} \sim 10^{-4} \gg H \sim 10^{-13}. \tag{24}$$

In a thin-wall approximation, sphaleron rates are presented in Tables 1 and 2

Here $R_{b.su(3)}$ and $\Delta l'$ are respectively the radius and the wall thickness of a bubble which is nucleated in the phase transitions.

One can conclude that the sphaleron rates are larger than the cosmological expansion rate at temperatures above the critical temperature and are smaller than the cosmological expansion rate at temperatures below the critical temperature. For each transition, baryon violation rapidly takes

Table 1. The sphaleron rate in the EWPT $SU(3) \to SU(2)$ with $m_q(v_\chi) = m_{h2}(v_\chi) = 1500\,\text{GeV}$

T [GeV]	$\frac{R_{b.su(3)}}{\text{GeV}}$ $\times 10^{-6}$	$\frac{R_{b.su(3)}}{\Delta l'}$	$\mathscr{E}_{sph.SU(3)}$ [GeV]	$\Gamma_{SU(3)}$ $[10^{-11} \times \text{GeV}]$	$H \times 10^{-12}$ [GeV]	$\frac{\Gamma_{SU(3)}}{H}$
1479.48 (T_1')	10	10	6975.17	1.63719×10^6	3.08195	5.31×10^6
1450	12	12	12481.3	3.2702×10^4	2.96034	1.10×10^5
1400	13	13	17206.3	7.94481×10^2	2.7597	2.878×10^3
1390	15	15	23251.7	9.3264	2.72042	3.42
1388.4556 (T_c')	16.5	16.5	28135.1	0.2714	2.71438	1
1387	17	17	29854.0	0.07687	2.70869	0.28
1000	19	19	60590.8	5.98×10^{-19}	1.40801	4.25×10^{-18}
900	22	22	89250.8	9.50×10^{-36}	1.14049	8.33×10^{-35}
865.024 (T_0')	25	25	119110.36	1.69×10^{-52}	1.05357	1.60×10^{-51}

Table 2. The sphaleron rate in the EWPT $SU(2) \to U(1)$ with $m_{h2}(v_\rho) = 100\,\text{GeV}, m_{h^{\pm\pm}}(v_\rho) = 350\,\text{GeV}$

T [GeV]	$\frac{R_{s.su(2)}}{[\text{GeV}]}$ $\times 10^{-4}$	$R_{s.su(2)}/\Delta l$	$\mathscr{E}_{sph.SU(2)}$ [GeV]	$\Gamma_{SU(2)}$ $[10^{-12} \times \text{GeV}]$	H[GeV] $\times 10^{-14}$	$\frac{\Gamma_{SU(2)}}{H}$
141.574 (T_1)	6	10	742.838	919936.07	2.82211	3.25×10^7
141.5	8	10	1020.87	128525.28	2.81916	4.55×10^6
141	10	10	1442.75	6264.89	2.79927	2.23×10^5
140	12	12	2342.21	9.37289	2.7597	339.6
138.562 (T_c)	13.1	13	3135.75	0.02703	2.703	1
137	14	14	3922.29	0.0000622	2.6427	2.357×10^{-3}
130	16	16	6567.08	1.847×10^{-14}	2.379	7.76×10^{-13}
120	18	18	10068.2	5.403×10^{-29}	2.02754	2.66×10^{-27}
118.42 (T_0)	20	20	12656.7	5.595×10^{-39}	6.209	9.01×10^{-38}

place in the symmetric phase regions but it also quickly shuts off in the broken phase regions. This may provide B-violation necessary for baryogenesis, as required by the first of Sakharov's conditions, in the connection with non-equilibrium physics.

IX. CONCLUSIONS

In this review, I have showed that the 3-3-1 models are able to describe the cosmological evolution. The supersymmetric economical 3-3-1 model contains the hybrid inflationary scenario and the first-order phase transitions, while the modified 3-3-1-1 model consists an inflation connected with the Higgs scalar of the $U(1)_N$ symmetry breaking. The inflation happens in the GUT scale, while phase transition has two sequences corresponding two steps of symmetry breaking in the models. The sphaleron rates are much larger than the Hubble constant. They are larger than the cosmological expansion rate at temperatures above the critical temperature and are smaller than the cosmological expansion rate at temperatures below the critical temperature. From these considerations, some bound on model parameters are deduced. Note that the study on Cosmological aspects of the 3-3-1 models was done only for the simple, by the Higgs sector, versions, but it is easily to generalize for other versions.

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