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VACUUM STABILITY IN SUPERSYMMETRIC REDUCED MINIMAL 3-3-1 MODEL

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Abstract. *We investigate the vacuum stability conditions of the supersymmetric reduced minimal 331 model (SUSYRM331) that create important consequences on Higgs mass spectrum as well as soft-parameters of the model. We prove that if this condition is satisfied then all Higgses are massive. Furthermore, soft-parameters should be in order of $SU(3)_L$ scale. Based on this, we investigate in detail masses of CP-even neutral and doubly charged Higgses in the model. The neutral Higgs sector includes one light Higgs with mass at tree level $m_{H_1^0} \simeq m_Z|\cos 2\gamma| < 92.0$ GeV and three other heavy Higgses. For doubly charged Higgses, there may exist a light Higgs which can be observed by recent colliders such as LHC.*

I. INTRODUCTION

The discovery of a new scalar particle with mass around 126.5 GeV by the Large Hadron Collider (LHC) [1,2] is a milestone of particle physics at this time. Although this new particle inherits many properties of the Standard Model (SM) Higgs, both theoretical and experimental physics need more time and data to confirm whether it is really the SM Higgs or a Higgs of some model beyond SM. The fact now is that a new model beyond SM must contain at least one Higgs satisfying all properties of the new Higgs mentioned above. In this work, we show that the SUSYRM331 in the presence of B/μ-type term always contains a Higgs discovered by LHC and therefore it is a realistic model. We emphasize that Higgs sector was investigated in [3] without B/μ-type terms.

Another aim of this work is reinvestigating the Higgs spectrum of the SUSYRE331 [3] in the presence of b-type terms ($b_\rho \rho\rho' + b_\chi \chi\chi' + \mathrm{H.c}$). This will guarantee the vacuum stability of the model. But terms contributing to Higgs masses now become more complicating and the mass spectrums of different charged Higgs will directly depend on the scale of both b_ρ and b_χ. We only concentrate on the real neutral and double charged Higgs masses because they are not calculated exactly but we can find them with high accuracy.

II. HIGGS POTENTIAL

The SUSYRM331 model was built in [3]. In this work, we only investigate the minimum of the scalar potiential and the Higgs sector of the model. First, the scalar potential of the model is

$$V_{\mathrm{SUSYRM331}} = V_{\mathrm{D}} + V_{\mathrm{F}} + V_{\mathrm{soft}}, \tag{1}$$

$$V_{\mathrm{soft}} = -\mathcal{L}_{\mathrm{SMT}} = m^2_\rho \bar{\rho}\rho + m^2_\chi \bar{\chi}\chi + m^2_{\rho'} \bar{\rho}'\rho' + m^2_{\chi'} \bar{\chi}'\chi' - \left(b_\rho \rho\rho' + b_\chi \chi\chi' + \mathrm{h.c.}\right),$$

$$\begin{aligned}
V_{\mathrm{D}} &= \frac{g'^2}{12}(\bar{\rho}\rho - \bar{\rho}'\rho' - \bar{\chi}\chi + \bar{\chi}'\chi')^2 + \frac{g^2}{8}\sum_{i,j}\left(\bar{\rho}_i\lambda^a_{ij}\rho_j + \bar{\chi}_i\lambda^a_{ij}\chi_j - \bar{\rho}'_i\lambda^{*a}_{ij}\rho'_j - \bar{\chi}'_i\lambda^{*a}_{ij}\chi'_j\right)^2, \\
V_{\mathrm{F}} &= -\mathcal{L}_F = \sum_F \bar{F}_\mu F_\mu = \sum_i \left[\left|\frac{\mu_\rho}{2}\rho'_i\right|^2 + \left|\frac{\mu_\chi}{2}\chi'_i\right|^2 + \left|\frac{\mu_\rho}{2}\rho_i\right|^2 + \left|\frac{\mu_\chi}{2}\chi_i\right|^2\right],
\end{aligned}$$

where m_ρ, m_χ, $m_{\rho'}$ and $m_{\chi'}$ have the mass dimension; b_ρ as well as b_χ are assumed to be real and positive to make sure the non-zero and real values of VEVs. Note that V_{soft} in this work is revised by adding the B/μ-type terms.

In the Higgs sector, all of the four neutral scalar components ρ^0, χ^0, ρ'^0, χ'^0 gain non-zero vacuum expectation values (VEVs). The expansions of the neutral scalars around their VEVs are defined as

$$\begin{aligned}
<\rho> &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & v_\rho + H_\rho + iF_\rho & 0 \end{pmatrix}^T, <\rho'> = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & v_{\rho'} + H_{\rho'} + iF_{\rho'} & 0 \end{pmatrix}^T, \\
<\chi> &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & v_\chi + H_\chi + iF_\chi \end{pmatrix}^T, <\chi'> = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & v_{\chi'} + H_{\chi'} + iF_{\chi'} \end{pmatrix}^T.
\end{aligned}$$

The minimum of the Higgs potential (1) appears when all linear Higgs terms in this potential vanish, namely

$$\begin{aligned}
m^2_\rho + \frac{1}{4}\mu^2_\rho &= \frac{b_\rho}{t_\gamma} - \frac{2g^2+g_1^2}{12}v'^2(t^2_\gamma - 1) + \frac{g^2+g_1^2}{12}w'^2(t^2_\beta - 1), \\
m^2_\chi + \frac{1}{4}\mu^2_\chi &= \frac{b_\chi}{t_\beta} + \frac{g^2+g_1^2}{12}v'^2(t^2_\gamma - 1) - \frac{2g^2+g_1^2}{12}w'^2(t^2_\beta - 1), \\
m^2_\rho + m^2_{\rho'} + \frac{1}{2}\mu^2_\rho &= b_\rho\left(t_\gamma + \frac{1}{t_\gamma}\right), m^2_\chi + m^2_{\chi'} + \frac{1}{2}\mu^2_\chi = b_\chi\left(t_\beta + \frac{1}{t_\beta}\right),
\end{aligned} \tag{2}$$

where two new notations are used, $t_\gamma = \tan\gamma = \frac{v}{v'}, t_\beta = \tan\beta = \frac{w}{w'}$. Comparing with the gauge boson masses calculated in [3], we remind that

$$m^2_W = \frac{g^2}{4}(v^2+v'^2) = \frac{g^2}{4}v'^2(t^2_\gamma + 1); \quad m^2_V = \frac{g^2}{4}(w^2+w'^2) = \frac{g^2}{4}w'^2(t^2_\beta + 1). \tag{3}$$

Masses of these two gauge bosons will be used as independent parameters in our calculation. Four equations (2) now can be rewritten in forms of

$$m_\rho^2 + \frac{1}{4}\mu_\rho^2 \;=\; \frac{b_\rho}{t_\gamma} - \frac{1+t^2}{3} \times m_V^2 \cos 2\beta + \frac{t^2+2}{3} \times m_W^2 \cos 2\gamma, \tag{4}$$

$$m_\chi^2 + \frac{\mu_\chi^2}{4} \;=\; \frac{b_\chi}{t_\beta} - \frac{2+t^2}{3} \times m_V^2 \cos 2\beta + \frac{1+t^2}{3} \times m_W^2 \cos 2\gamma, \tag{5}$$

$$s_{2\gamma} \equiv \sin 2\gamma \;=\; \frac{2b_\rho}{m_\rho^2 + m_{\rho'}^2 + \frac{1}{2}\mu_\rho^2}, \quad s_{2\beta} \equiv \sin 2\beta = \frac{2b_\chi}{m_\chi^2 + m_{\chi'}^2 + \frac{1}{2}\mu_\chi^2}. \tag{6}$$

From two equations (4) and (5) we have

$$c_{2\gamma} \equiv \cos 2\gamma = \frac{-(m_\chi^2 + \frac{\mu_\chi^2}{4} - \frac{b_\chi}{t_\beta})(1+2s_W^2) + 2(m_\rho^2 + \frac{\mu_\rho^2}{4} - \frac{b_\rho}{t_\gamma})c_W^2}{m_W^2},$$

$$c_{2\beta} \equiv \cos 2\beta \;=\; \frac{(m_\rho^2 + \frac{\mu_\rho^2}{4} - \frac{b_\rho}{t_\gamma})(1+2s_W^2) - 2(m_\chi^2 + \frac{\mu_\chi^2}{4} - \frac{b_\chi}{t_\beta})c_W^2}{m_V^2}. \tag{7}$$

Because of the property of the general SUSY331 models, $m_W^2 \ll m_V^2$ and in addition $|c_{2\gamma}|,\ |c_{2\beta}| \leq 1$, it can be deduced that two quantities $(m_\chi^2 + \frac{\mu_\chi^2}{4} - \frac{b_\chi}{t_\beta})$ and $(m_\rho^2 + \frac{\mu_\rho^2}{4} - \frac{b_\rho}{t_\gamma})$ should be in the same order of $\mathcal{O}(m_W^2)$ or $\mathcal{O}(m_V^2)$. Similar to the case of the SUSYE331 model [6], to avoid the very light Higgses contained in the model, we only consider the case of two quantities having order of $\mathcal{O}(m_V^2)$, i.e the TeV scale.

III. HIGGS SPECTRUM

Before investigating the mass spectrum of real neutral and doubly charged Higgses, we note that the SUSYRM331 model also contain pseudo-scalar neutral and singly charged Higgses. It is easy to show the mass eigenvalues and eigenstates can be found exactly, for example see [7]. We will use masses of two pseudo-scalar Higgses as independent parameters characterized for soft and B/μ-terms. They are defined as follows

$$m_{A_1}^2 = \frac{2b_\rho}{s_{2\gamma}} = m_\rho^2 + m_{\rho'}^2 + \frac{1}{2}\mu_\rho^2, \quad m_{A_2}^2 = \frac{2b_\chi}{s_{2\beta}} = m_\chi^2 + m_{\chi'}^2 + \frac{1}{2}\mu_\chi^2. \tag{8}$$

III.1. Real scalar neutral Higgs

The squared mass matrix of these Higges can be written in the form of

$$\mathcal{L}_{\mathrm{H}}^{\mathrm{r}} \;=\; \frac{1}{2}(H_\rho,\ H_{\rho'},\ H_\chi,\ H_{\chi'}) \times\ \mathcal{M}_p^2(H_\rho,\ H_{\rho'},\ H_\chi,\ H_{\chi'})^T \tag{9}$$

with

$$M_{4S}^2 = \begin{pmatrix} m_{S11}^2 & m_{S12}^2 & m_{S13}^2 & m_{S14}^2 \\ & m_{S22}^2 & m_{S23}^2 & m_{S24}^2 \\ & & m_{S33}^2 & m_{S34}^2 \\ & & & m_{S44}^2 \end{pmatrix}, \tag{10}$$

where formulas of elements are

$$\begin{aligned}
m^2_{S11} &= \frac{b_\rho}{t_\gamma}+\frac{1}{6}g^2(2+t^2)t^2_\gamma v'^2, \quad m^2_{S12}=-b_\rho-\frac{1}{6}g^2(2+t^2)t_\gamma v'^2,\\
m^2_{S13} &= -\frac{1}{6}g^2(1+t^2)t_\beta t_\gamma v'w', \quad m^2_{S14}=\frac{1}{6}g^2(1+t^2)t_\gamma v'w',\\
m^2_{S22} &= b_\rho t_\gamma+\frac{1}{6}g^2(2+t^2)v'^2, \quad m^2_{S23}=\frac{1}{6}g^2(1+t^2)t_\beta v'w',\\
m^2_{S24} &= -\frac{1}{6}g^2(1+t^2)v'w', \quad m^2_{S33}=\frac{b_\chi}{t_\beta}+\frac{1}{6}g^2(2+t^2)t^2_\beta w'^2,\\
m^2_{S34} &= -b_\chi-\frac{1}{6}g^2(2+t^2)t_\beta w'^2, \quad m^2_{S44}=b_\chi t_\beta+\frac{1}{6}g^2(2+t^2)w'^2.
\end{aligned}$$

Each eigenvalues of this matrix $\lambda = m^2_{H^0}$ is one physical mass of CP-even neutral Higgs. It must be satisfied the equation $\det(M^2_{4S}-\lambda I_4)=0$, or equivalent $f(\lambda)=0$, where $f(\lambda)$ can be reduced to the more simple form by defining new variable and parameters as follows:

$$\lambda = Xm^2_V, m^2_W=\epsilon\times m^2_V,\ m^2_{A_1}=k_1m^2_V,\ m^2_{A_2}=k_2m^2_V \tag{11}$$

This equation now can be written in the form

$$f(X) = X^4+a_1X^3+a_2X^2+a_3X+a_4=0, \tag{12}$$

where

$$\begin{aligned}
a_1 &= -\frac{4}{3}-k_1-k_2+\frac{4s^2_W}{-1+4s^2_W}+\frac{4c^2_W}{-3+12s^2_W}\times\epsilon,\\
a_2 &= \frac{-4c^2_{2\beta}k_2c^2_W+k_1(-4-3k_2+4(1+3k_2)s^2_W)}{-3+12s^2_W}-\frac{4(1+c^2_{2\gamma}k_1c^2_W+k_2c^2_W)}{-3+12s^2_W}\times\epsilon,\\
a_3 &= -\frac{4c^2_{2\beta}k_1k_2c^2_W}{3-12s^2_W}-\frac{4(c^2_{2\beta}k_2+c^2_{2\gamma}k_1(1+k_2-k_2s^2_W))}{3-12s^2_W}\times\epsilon,\\
a_4 &= \frac{4c^2_{2\beta}c^2_{2\gamma}k_1k_2}{3-12s^2_W}\times\epsilon.
\end{aligned}\tag{13}$$

For light neutral Higgses, we denote its mass as $X_1=X'_1\times\epsilon$ where $X'_1\sim\mathcal{O}(1)$. Inserting this value into (12) then force the factor of lowest order of ϵ to be zero, we obtain

$$X'_1\simeq\frac{c^2_{2\gamma}}{c^2_W} \quad\Rightarrow\quad m^2_{H^0_1}\simeq M^2_Zc^2_{2\gamma}. \tag{14}$$

It is emphasized that although this value of light Higgs mass, $M_Z|c_{2\gamma}|<92.$ GeV, is much lighter than the value of 126.5 GeV discovered by LHC, the loop corrections will enhance this mass to the consistent value. This conclusion is the same as the case of the well-known Minimal Supersymmetric Standard Model (MSSM).

For heavy neutral Higgses, we denote their masses as $X_i=X'_i+X''_i\times\epsilon$ where both $X'_i,\ X''_i\sim\mathcal{O}(1)$ $(i=2,3,4)$. Then these masses can be written in the from

$$m^2_{H^0_i}=X'_im^2_V+X''_i\times m^2_W+\mathcal{O}(\epsilon)\times m^2_W. \tag{15}$$

where main contributions are

$$m^2_{H^0_2} \simeq X'_2 m^2_V = m^2_{A_1}, \tag{16}$$

$$m^2_{H^0_{3,4}} \simeq X'_{3,4} \times m^2_V = \frac{1}{6}\left(\frac{4c^2_W}{1-4s^2_W} + 3m^2_{A_2} \mp \sqrt{-\frac{48c^2_{2\beta}m^2_{A_2}m^2_V}{1-4s^2_W} + \left(3m^2_{A_2} + \frac{4c^2_W m^2_V}{1-4s^2_W}\right)^2}\right). \tag{17}$$

III.2. Doubly charged Higgs boson

The mass terms of the doubly charged boson are

$$\mathcal{L}_{H^{\pm\pm}} = \begin{pmatrix} \rho^{++} & \rho'^{++} & \chi^{++} & \chi'^{++} \end{pmatrix} \mathcal{M}^2_{H^{\pm\pm}} \begin{pmatrix} \rho^{--} & \rho'^{--} & \chi^{--} & \chi'^{--} \end{pmatrix}^T, \tag{18}$$

$$\mathcal{M}^2_{H^{\pm\pm}} = \frac{g^2}{4}\begin{pmatrix} m^2_{\rho^{++}\rho^{--}} & -\frac{4b_\rho}{g^2} - v'^2 t_\gamma & v'w't_\gamma t_\beta & -v'w't_\gamma \\ & m^2_{\rho'^{++}\rho'^{--}} & v'w't_\beta & v'w' \\ & & m^2_{\chi^{++}\chi^{--}} & -\frac{4b_\chi}{g^2} - w'^2 t^2_\beta \\ & & & m^2_{\chi'^{++}\chi'^{--}} \end{pmatrix} \tag{19}$$

$$\begin{aligned} m^2_{\rho^{++}\rho^{--}} &= \frac{4b_\rho}{g^2 t_\gamma} + v'^2 + w'^2(t^2_\beta - 1), \quad m^2_{\rho'^{++}\rho'^{--}} = v'^2 t_\gamma + \frac{4b_\rho t_\gamma}{g^2} + w'^2(1-t^2_\beta), \\ m^2_{\chi^{++}\chi^{--}} &= v'^2(t^2_\gamma - 1) + \frac{4b_\rho}{g^2 t_\beta} + w'^2, \quad m^2_{\chi'^{++}\chi'^{--}} = v'^2(1-t^2_\gamma) + w'^2 t^2_\beta + \frac{4b_\chi t_\beta}{g^2}. \end{aligned}$$

Finding the mass eigenvalues of doubly charged is similar to the case of the real scalar neutral Higgs shown above. The eigenvalues of this matrix $\lambda = m^2_{H^{\pm\pm}}$ must be satisfied the equation $\det(\mathcal{M}^2_{H^{\pm\pm}} - \lambda I_4) = 0$. Using parameters in (11), this equation can be written in the form of

$$g(X) = X^3 + AX^2 + BX + C = 0 \tag{20}$$

$$\begin{aligned} A &= -(1 + k_1 + k_2 + \epsilon), \\ B &= -c^2_{2\beta} - c^2_{2\gamma}\epsilon^2 + k_1 + (\epsilon + k_1)k_2 + c_{2\beta}c_{2\gamma}(k_1 + \epsilon(2 + k_2)), \\ C &= (1+\epsilon)(c_{2\beta} - c_{2\gamma}(\epsilon + k_1))(c_{2\beta} - c_{2\gamma}\epsilon + c_{2\beta}k_2). \end{aligned} \tag{21}$$

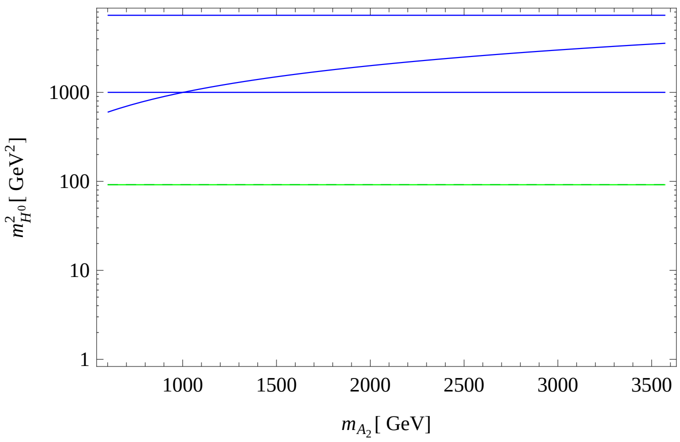
Masses of these Higgses are presented as

$$m^2_{H^{\pm\pm}} = X'm^2_V + X'' \times m^2_W + \mathcal{O}(\epsilon) \times m^2_W. \tag{22}$$

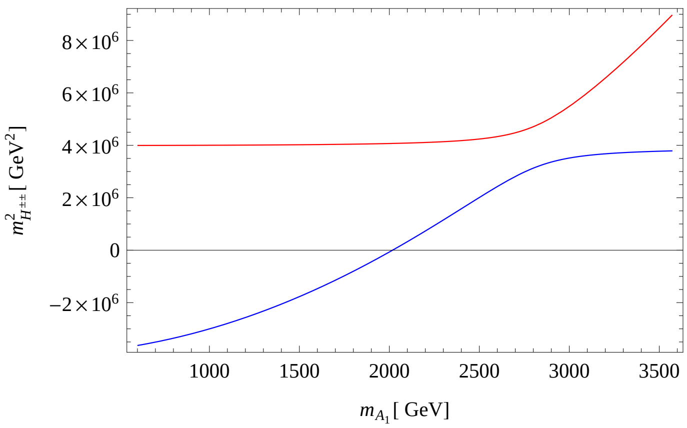
By analogy, it can be shown that main contributions to masses of doubly charged are

$$\begin{aligned} m^2_{H^{\pm\pm}_1} \simeq X'_1 \times m^2_V &= m^2_V + m^2_{A_2} \\ m^2_{H^{\pm\pm}_{2,3}} \simeq X'_{2,3} \times m^2_V &= m^2_{A_1} + \sqrt{4c^2_{2\beta}m^4_V - 4c_{2\beta}c_{2\gamma}m^2_V m^2_{A_1} + m^4_{A_1}} \\ X"_i = A/B, \end{aligned} \tag{23}$$

$$\begin{aligned} A &= c^2_{2\gamma}k_1 + c^2_{2\beta}(1+k_2) - c_{2\beta}c_{2\gamma}(k_1(1+k_2) - (2+k_2)(-1+X'_i)) + (k_2 - X'_i)X'_i, \\ B &= c^2_{2\beta} - c_{2\beta}c_{2\gamma}k_1 - k_1(1 + k_2 - 2X'_i) + (2 + 2k_2 - 3X'_i)X'_i. \end{aligned}$$



(a)



(b)

Fig. 1. Masses of Higgses as functions of m_{A_2} (a) or m_{A_1} (b), where $m_V = 2.0$ TeV, $m_W = 80.4$ GeV and $s_W^2 = 0.231$. The left panel presents masses of neutral Higgses with $m_{A_1} = 1.0$ TeV, $\tan\gamma = 50$, and $\tan\beta = 30$. The right panel presents squared masses of doubly charged Higgses with $m_V = 2.5$ TeV, $m_{A_2} = 1.0$ TeV, $\tan\gamma = 10$ and $\tan\beta = 30$.

For an illustration of ours result, masses of Higgses are shown in Fig. 1 with some fixed values of parameters. This choice of the parameters satisfies the scale of $SU(3)_L$ being TeV scale. The large values of $\tan\gamma$ and $\tan\beta$ make sure the loop corrections to the lightest neutral Higgs mass being consistent with present values of 126.5 GeV. In the left panel of the Fig.1, we draw four values of CP-even neutral Higgs masses, where the green line corresponds to the lightest mass and three blue curves present masses of three heavy masses of TeV scale. In the right panel, we see that in order to cancel doubly charged Higgs Tachyon, i.e $m^2_{H^{\pm\pm}}$ must be positive, m_{A_1} is bounded from below, 2.0 TeV for example in this case. In addition, if values of m_{A_1} is very close to 2.0 TeV, there exists a very light doubly charged Higgs. This is very important because this light doubly charged Higgs can be detected by recent colliders.

IV. CONCLUSIONS

To guarantee the stability of VEVs in the SUSYRM331, it is needed to add some b-type terms into V_{soft}. As a consequence, soft parameters are in order of $\mathrm{SU}(3)_L$ scale. Further more, all Higgses of the model are massive. There are only light neutral Higgs with mass at tree level is, the same mass of the light Higgs concerned in MSSM. The model contains three other heavy real neutral Higgses, two pseudo neutral, two singly charged, and three doubly charged Higgses. The model may contains one light doubly charged Higgs if m_{A_1} get a suitable values. This will be an important signal to distinguish the SUSYRM331 model with many other models without doubly charged Higgses.

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